Name:

M. Hanzla Javed

Reg NO:

FA20-BCS-050

Subject:

Linear Algebra

Submitted To:

Umair Umer

Assignment # 2

What is Determinant? Write its property with examples..?

Define:-

The determinant is a scaler value that is a function of entries of a square matrix. In particular the determinant is non-zero if and only if the matrix is invertible. The determinant of a matrix A is denoted by. det(A), det A or |A|.

Example:-

In case of 2x2 matrix the determinant can be defined as.

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Rightarrow ad - cb$$

Properties Of Determinant:

The properties of determinant are given below:

Date:

M T W T F S

1.

If any two rows (or columns) of a determinant are interchanged then sign of determinant changes.

Example:

If a matrix of 3x3, then.

$$|A| = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= 1(2-8) - 2(3-20) + 4(6-10)$$

$$|A| \Rightarrow 12$$

$$|A| = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

Interchange (Row 2) with (Row 3),

$$|A'| = \begin{vmatrix} 1 & 2 & 4 \\ 5 & 2 & 1 \\ 3 & 2 & 4 \end{vmatrix}$$

$$= 1(8-2) - 2(20-3) + 4(10-6)$$

$$\Rightarrow 6 - 34 + 16$$

$$\Rightarrow -12$$

So, it proved that

$$|A| = -|A'|$$

Date:

M T W T F S

2.

If all elements of a row (or column) are zero, then determinant is 0.

Example:

$$|A| = \begin{vmatrix} 0 & 2 & 5 \\ 0 & 8 & 8 \\ 0 & 5 & 2 \end{vmatrix}$$

$$\Rightarrow 0 - 2(0) + 5(0)$$

$$\Rightarrow 0$$

So, it is proved by a above matrix, the determinant of any zero row or column is zero.

3.

The value of the determinant remains unchanged if its rows and columns are interchanged.

Example:

$$|A^T| = |A|$$

$$A = \begin{bmatrix} 2 & 5 \\ 8 & 0 \end{bmatrix}, \quad A^T = \begin{bmatrix} 2 & 8 \\ 5 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 8 & 0 \end{vmatrix} \qquad |A^T| = \begin{vmatrix} 2 & 8 \\ 5 & 0 \end{vmatrix}$$

$\|A\| = (2\times 0) - (8\times 5)$	$\|A^T\| = (2\times 0) - (5\times 8)$
$\Rightarrow 0 - 40$	$= 0 - 40$
$\|A\| \Rightarrow -40$	$\|A^T\| \Rightarrow -40$

Hence, it is proved in above example that

$$|A| = |A^T|$$

4.

If elements of a row or column of a determinant are expressed as sum of Two (or more) terms, then The determinant can be expressed a sum of two (or more) determinants.

Example:

$$|A| = \begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix}$$

___/___/202   Day:______

As we see in above example, if two rows and columns are same then determinant will be zero.
So,

$$|A| = 0 + 2(0)$$
$$|A| = 0$$

5.

If in a determinant all the elements above or below, the diagonal is zero, then value of determinant is equal to product of diagonal elements.

Example :

$$\begin{vmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ d & e & 0 \\ g & h & i \end{vmatrix}$$

$$= a \times e \times i$$

6.

If any two rows (or columns) of a determinant are identical. the value of determinant is zero.

Date: ______ M T W T F S

Example -

$$|A| = \begin{vmatrix} 5 & 2 & 8 \\ 1 & 0 & 1 \\ 5 & 2 & 8 \end{vmatrix}$$

As R_1 and R_3 has same values so,

$|A| = 5(0-2) - 2(8-5) + 8(2-0)$

$\Rightarrow 5(-2) - 2(3) + 8(2)$

$\Rightarrow -10 - 6 + 16$

$|A| \Rightarrow 0$

So, it is proved that the determent of a matrix is zero which has same rows or colums.

7. If each element of a row (or a column) of a determinant is multiplied by a constant k, then determinant's value get multiplied by k.

Scanned with CamScanner

Date:

M T W T F S

Example :

$$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

L.H.S

$$|A| = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

$\Rightarrow 2(0-6) - 4(5-4) + 6(3-0)$

$\Rightarrow -12 \quad -4 \quad +18$

$\Rightarrow 2$

R.H.S :

$$|A| = 2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

$\Rightarrow 2[1(0-6) - 2(5-4) + 3(3-0)]$

$\Rightarrow 2[-6 - 2 + 9]$

$\Rightarrow 2[-8 + 9]$

$\Rightarrow 2[1]$

$|A| \Rightarrow 2$

Hence, in above example, it is proved.