

Question 01:

• 16. Salient features of market mechanism with its merits and demerits

• 2 7 Feb. 1961 34 39 hrs. old

schwärzende Befunde ~~8~~ Reaktionen 54mehr als 34 Tumoren mit Befunden

~~Confidential~~ For First: - 01-01-2023 00:00

~~income~~ income 83.0 81.0

$$P_g = 2P_a + 7P_s$$

... a banda de motoqueiros. Infelizmente os bandidos fizeram

For Second: ~~negative~~ ~~positive~~

~~income - expenses~~

$$P_c = .8 P_{eq} + .3 P_S$$

Move all variables to left side

$$8P_4 - 7P_5 = 0$$

$$-8P_4 + 7P_3 = 0$$

Row reduced augmented matrix:

$$\begin{bmatrix} .8 & -7 & 0 \\ -8 & 7 & 0 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} .8 & -7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & -875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

General solution is $Pg = .875 Ps$.

$$P_s = 1000 \text{ kPa}, \quad T_{ref} = 293 \text{ K}$$

$$m_{\text{ref}} = \rho_{\text{air}} \cdot 878 \rho_{\text{sub}} \cdot \text{quido} \text{ von } 100 \text{ Vol. } (2)$$

Question :-

(a) Fill in the exchange Table one column at a time.

Distribution of output from:

Fuels and Power	Manufacturing	Services	Purchased by
output	0.10	0.20 input	Fuels and power
income	0.80	0.40	Manufacturing
expenses	0.10	0.40	Services

(b): Donote the total annual output of sectors P_F , P_M and P_S .

$$P_F = 0.1P_F + 0.1P_M + 0.2P_S$$

From the second and third rows of table, the income/expense requirement for the

$$P_M = 0.8P_F + 0.1P_M + 0.4P_S$$

$$P_S = 0.1P_F + 0.8P_M + 0.4P_S$$

Move all variables to left side.

$$0.9P_F - 0.1P_M - 0.2P_S = 0$$

$$-0.8P_F + 0.9P_M - 0.4P_S = 0$$

$$-0.1P_F - 0.8P_M + 0.6P_S = 0$$

$$\begin{bmatrix} 0.9 & -0.1 & -0.2 & 0 \\ -0.8 & 0.9 & -0.4 & 0 \\ -0.1 & -0.8 & 0.6 & 0 \end{bmatrix}$$

(c) You can obtain reduced echelon form.

$$\begin{bmatrix} 0.9 & -0.1 & -0.2 & 0 \\ -0.8 & 0.9 & -0.4 & 0 \\ -0.1 & -0.8 & 0.6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -0.301 & 0 \\ 0 & 1 & -0.712 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is $P_F = 30.1P_S$, $P_M = 71.2P_S$ with P_S free. If P_S is 100, then $P_F = 30.1$ and $P_M = 71.2$.

Question 4:

(a): Fill in the exchange table.

Distribution:

	Minings	Lumber	Energy	Transportation	input	Purchased By
output	.30	.15	.20	.20	→	Mining
	.10	.15	.15	.10	→	Lumber
	.60	.50	.45	.50	→	Energy
	0	.20	.20	.20	→	Transportation

(b): Calculate the total annual output:

income expenses

$$P_M = .30P_M + .15P_L + .20P_E + .20P_T$$

$$P_L = .10P_M + .15P_L + .15P_E + .10P_T$$

$$P_E = .60P_M + .50P_L + .45P_E + .50P_T$$

$$P_T = .20P_L + .20P_E + .20P_T$$

Move all variables:

$$.70P_M - .15P_L - .20P_T = 0$$

$$-.10P_M + .85P_L - .15P_E - .10P_T = 0$$

$$-.60P_M + .50P_L + .55P_E - .50P_T = 0$$

$$-.20P_L + .20P_E + .80P_T = 0$$

$$\left[\begin{array}{cccc|c} .70 & -.15 & -.20 & -.20 & 0 \\ -.10 & .85 & -.15 & -.10 & 0 \\ -.60 & .50 & .55 & -.50 & 0 \\ 0 & -.20 & .20 & .80 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1.37 & 0 \\ 0 & 1 & 0 & -1.84 & 0 \\ 0 & 0 & 1 & -3.16 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Solve for basic variables in terms of free variable P_T and obtain $P_M = 1.37P_T$, $P_L = .844P_T$ and

$$P_E = 3 \cdot 16 D_T \cdot 80, \text{ take } P_T = 100, \text{ round off other prices}$$

$$\text{So } P_M = 137, P_L = 84 \text{ and } P_E = 316$$

Question 12: A network diagram is given below.

Write equation for each intersection.

Intersection	Flow in	Flow out
A	$u_1 + u_4$	u_2
B	u_2	$u_3 + 100$
C	$u_3 + 80$	u_4

Rearrange the equations

$$u_1 - u_2 + u_4 = 0$$

$$u_2 - u_3 - 100 = 0$$

$$u_3 + u_4 = -80$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -100 \\ 0 & 0 & 1 & 1 & -80 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

The general equation

$$\begin{cases} u_1 = 20 \\ u_2 = 20 + u_4 \\ u_3 = -80 + u_4 \\ u_4 \text{ is free} \end{cases}$$

Since u_3 cannot be negative, the minimum value of u_4 is 80.

Q. No. 2

Solution

Let

$$P_S = 200 \text{ million } \text{ dollars}$$

$$P_C = 188 \text{ million }$$

$$P_E = 170 \text{ million }$$

Any constant nonnegative multiple of these prices is a set of prices, because solution set is multiples of one vector.

Changing the unit of measurement to another currency such as Chinese yen has the same effect as multiplying all equilibrium prices by a constant. Ratios will remain same.

Q. No. 6

Solution

The following vectors list the numbers of atoms of (Al_2O_3) , (O) , (C) .

$$Al_2O_3: \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, C: \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; Al: \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, CO_2: \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

CO efficient in the equation



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - x_3 \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

moving right terms to left side

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 3/2 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 3/2 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -4/3 & 0 \end{bmatrix}$$

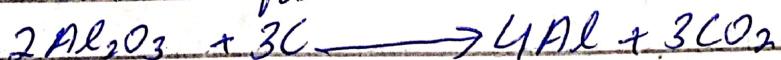
$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & -4/3 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$$

$x_1 = (2/3)x_4$, $x_2 = x_4$, $x_3 = -4(1/3)x_4$ with x_4 free
Take

$x_4 = 3$, Then $x_1 = 2 \rightarrow x_1 = 3$ & $x_3 = 4$.

Balanced equation



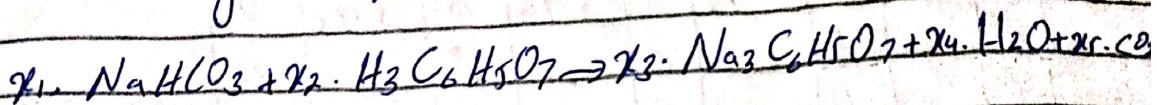
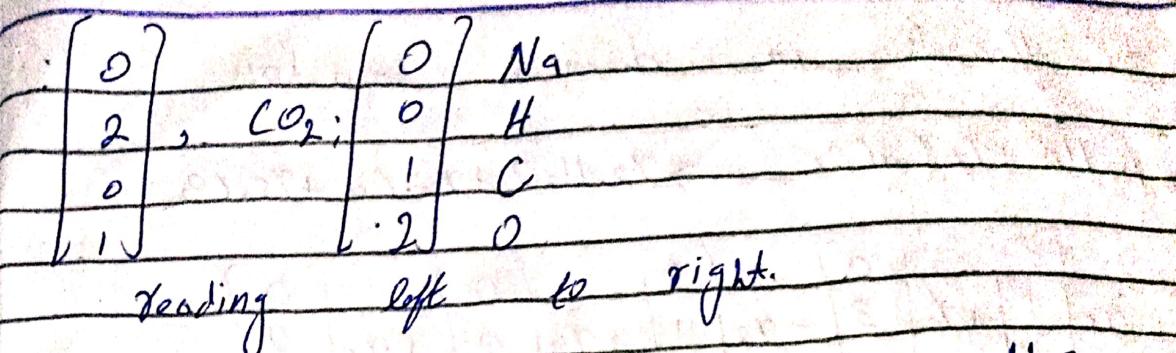
Q. No. 7

Solution

Atoms of (Na) , (H) , (C)
(O),

$$NaHCO_3: \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, H_3C_6H_5O_7: \begin{bmatrix} 0 \\ 8 \\ 6 \end{bmatrix}, Na_3C_6H_5O_7: \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}, H_2:$$

$$\begin{bmatrix} 7 \end{bmatrix}$$



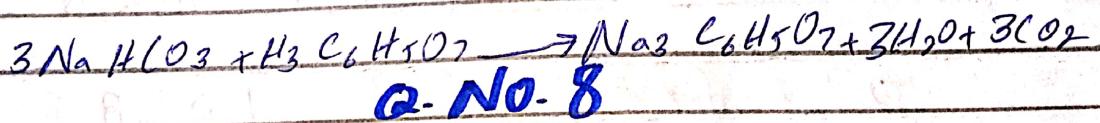
$$x_1 \left[\begin{array}{c} 1 \\ 1 \\ 1 \\ 3 \end{array} \right] + x_2 \left[\begin{array}{c} 0 \\ 8 \\ 6 \\ 7 \end{array} \right] = x_3 \left[\begin{array}{c} 3 \\ 5 \\ 6 \\ 7 \end{array} \right] + x_4 \left[\begin{array}{c} 0 \\ 2 \\ 0 \\ 1 \end{array} \right] + x_5 \left[\begin{array}{c} 0 \\ 0 \\ 1 \\ 2 \end{array} \right]$$

Reducing Augmented matrix:

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & -3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 3 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & -1 & 3 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

The general solution

$x_1 = x_5, x_2 = (1/3)x_5, x_3 = (1/3)x_5, x_4 = x_5$, and
 x_5 is free. Take $x_5 = 3$. Then $x_1 = x_4 = 3$
 $, x_2 = x_3 = 1$. Now Balanced equation

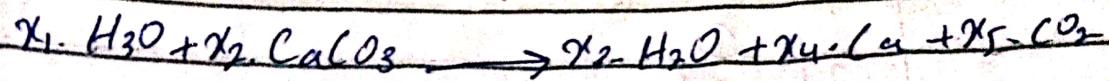


Solution:-

Vector List

$$\text{H}_2\text{O}: \left[\begin{array}{c} 3 \\ 1 \\ 0 \\ 0 \end{array} \right], \text{CaCO}_3: \left[\begin{array}{c} 0 \\ 3 \\ 1 \\ 0 \end{array} \right], \text{H}_2\text{O}: \left[\begin{array}{c} 2 \\ 1 \\ 0 \\ 0 \end{array} \right], \text{Ca}: \left[\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} \right], \text{CO}_2: \left[\begin{array}{c} 0 \\ 0 \\ 1 \\ 6 \end{array} \right] \text{H}_2\text{O}: \left[\begin{array}{c} 2 \\ 0 \\ 0 \\ 0 \end{array} \right] \text{CaCO}_3: \left[\begin{array}{c} 0 \\ 2 \\ 0 \\ 0 \end{array} \right] \text{C}$$

Coefficients in chemical equation



$$\begin{matrix} x_1 & \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} & + x_2 & \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} & = x_3 & \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} & + x_4 & \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & + x_5 & \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \end{matrix}$$

Moving the terms to left side
using Augmented matrix:

$$\left[\begin{array}{cccccc|cccccc} 3 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$x_1 = 2x_5$, $x_2 = x_5$, $x_3 = 3x_5$, $x_4 = x_5$ and x_5 is free
Take

$x_5 = 1$. Then $x_1 = 2$, and $x_2 = x_4 = 1$ & $x_3 = 3$

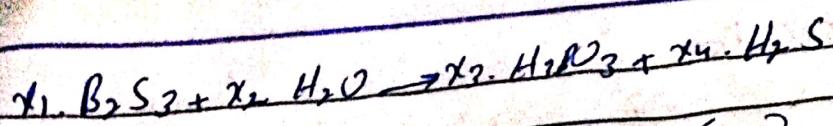


Q. NO. 9

Solution

Vector list of atoms of numbers

$$S_3: \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, H_2O: \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, H_3BO_3: \begin{bmatrix} -1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, H_2S: \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} B \\ S \\ H \\ O$$



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Now Reducing Augmented matrix

$$\left[\begin{array}{cccc|cccc} 2 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & -1/3 & 0 \\ 3 & 0 & 0 & -1 & 0 & 0 & 1 & 6 & -2 & 0 \\ 0 & 2 & -3 & -2 & 0 & 0 & 0 & 1 & -1/3 & 0 \\ 6 & 1 & -3 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Now general solution

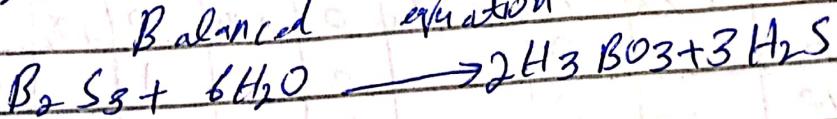
$$x_1 = 1/3 x_4, \quad x_2 = 2x_4, \quad x_3 = 2/3 x_4, \quad x_4 = \text{free}$$

Now Take

$$x_4 = 3, \quad x_1 = 1$$

$$x_2 = 6, \quad x_3 = 2$$

Balanced equation



Q. No. 10

Solution

Using the order lead (Pb), (N)

(C_r), (M_n) (O)

Now Vector equation

$$x_1 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} \quad \begin{matrix} \text{Pb} \\ \text{N} \\ \text{C} \\ \text{M}_n \end{matrix}$$

General Solution

$$x_1 = (1/6)x_6, x_2 = (22/45)x_6, x_3 = (1/18)x_6,$$

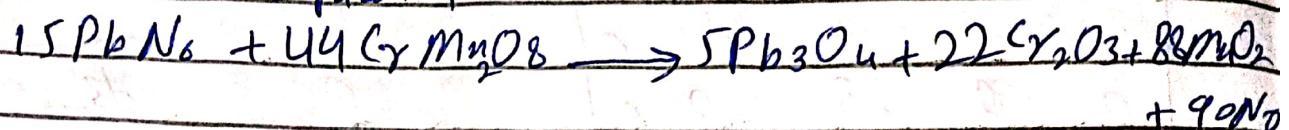
$$x_4 = (11/45)x_6$$

$$x_5 = (44/45)x_6 \quad x_6 = \text{free}$$

Take

$$x_6 = 90 \quad \text{Then } x_1 = 15, x_2 = 44, x_3 = 5, x_4 = 22 \\ x_5 = 88.$$

Balanced equation



Q. No. 11

Solution

Vector list of atoms per molecule
 ab (Mn), (S), (As), (Cr) (O)
 e (H). Vector equation

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 10 \\ 35 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 4 \end{bmatrix} = x_4 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 4 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_7 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 12 \\ 0 \end{bmatrix}$$

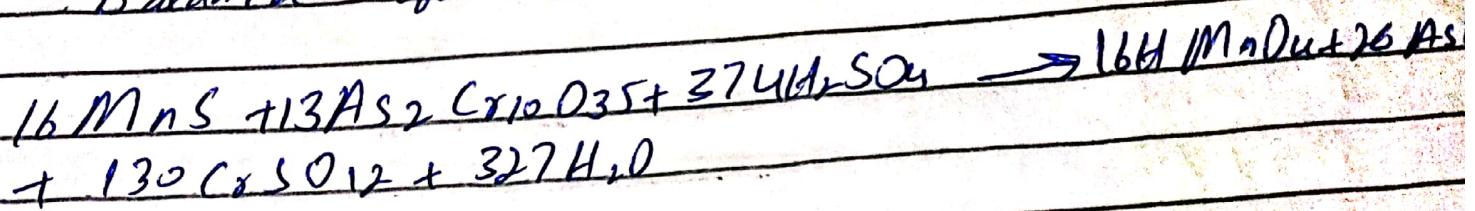
$$+ x_7 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{array}{l} \text{Mn} \\ \text{S} \\ \text{As} \\ \text{Cr} \\ \text{O} \\ \text{H} \end{array}$$

General solution

$$x_1 = \frac{16}{327}x_7, x_2 = \frac{13}{327}x_7, x_3 = \frac{374}{327}x_7,$$
$$x_4 = \frac{16}{327}x_7, x_5 = \frac{26}{327}x_7.$$
$$x_6 = \frac{130}{327}x_7, x_7 = \text{free}$$

Take $x_7 = 327$ to make
other variables whole numbers

Balanced equation



Question 13: Write the equation for each intersection.

Intersection	Flow in	Flow out
A	$u_2 + 30$	$u_1 - 100$
B	$u_2 + u_3$	$u_2 + u_4$
C	$u_6 + 100$	$u_5 + 40$
D	$u_4 + 40$	$u_6 + 90$
E	$u_1 + 60$	$u_3 + 20$

Rearranged the equation:

$$u_1 - u_2 = -50$$

$$u_2 + u_3 - u_2 + u_4 - u_5 = 0$$

$$u_2 + u_3 - u_5 - u_6 = 60$$

$$u_4 - u_2 - u_4 - u_6 = 50$$

$$u_1 + 60 - u_3 - u_5 = -40$$

$$\left[\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccccc} 1 & 0 & -1 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

The general equation is:

$$\left\{ \begin{array}{l} u_1 - u_3 = 40 \\ u_2 = u_3 + 10 \end{array} \right.$$

$$\left\{ \begin{array}{l} u_3 \text{ is free} \\ u_4 = u_3 + 50 \end{array} \right.$$

$$u_5 = u_6 + 60$$

$$u_6 \text{ is free}$$

(b): To find minimum flows, note that since u_i cannot be negative, $u_3 \geq 40$. This implies that $u_2 \geq 50$. Also since u_6 cannot be negative, $u_4 \geq 50$ and $u_5 \geq 60$. The minimum flows are $u_2 = 50$, $u_3 = 40$, $u_4 = 50$, $u_5 = 60$. (when $u_1 = 0$ and $u_6 = 0$)

Question: 14.

Intersection	Flow in	Flow out
A	80	$= u_1 + u_2$
B	$u_1 + u_2 + 100$	$= u_4$
C	u_3	$= u_2 + 90$
D	$u_4 + u_5$	$= u_7 + 90$

Rearranged the equation.

$$\begin{aligned} u_1 + u_5 &= 80 \\ u_1 + u_2 - u_4 &= -90 \\ u_2 - u_3 &= 90 \\ u_3 - u_4 - u_5 &= -90 \end{aligned}$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 80 \\ 1 & 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & 0 & -90 \\ 0 & 0 & 1 & -1 & -1 & -90 \end{array} \right]$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 80 \\ 0 & 1 & 0 & -1 & -1 & -180 \\ 0 & 0 & 1 & -1 & -1 & -90 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

a. The general equation is

$$\left\{ \begin{array}{l} u_1 = 80 - u_5 \\ u_2 = u_4 + u_5 - 180 \\ u_3 = u_4 + u_5 - 90 \\ u_4 \text{ is free} \\ u_5 \text{ is free} \end{array} \right.$$

b.

If $u_5 = 0$ then eq. is

$$\left\{ \begin{array}{l} u_1 = 80 \\ u_2 = u_4 - 80 \\ u_3 = u_4 - 90 \\ u_6 \text{ is free} \end{array} \right.$$

c.

Since u_2 cannot be negative, the minimum value of u_4 when $u_5 = 0$ is 18° .