Question 01:

Sol:

Goods	Services
.2	.7
.8	.3

For First:

income

$P_g = .2P_g + .7P_s$

For Second:

income expenses

$P_s = .8P_g + .3P_s$

Move all variables to left side

$.8P_g - .7P_s = 0$

$-.8P_g + .7P_s = 0$

Row reduced argumented matrix:

$$\begin{bmatrix} .8 & -.7 & 0 \\ -.8 & .7 & 0 \end{bmatrix} \sim \begin{bmatrix} .8 & -.7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -.875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

General solution is $P_g = .875P_s$.

$P_s = 1000$

$\therefore P_g = .875P_s$

Questions:-

(a) Fill in the exchange Table one column at a time.

Distribution of output from.

Fuels and Power	Manufacturing	Services	Purchased by
output 0.10	0.10	0.20 input	Fuels and power
0.80	0.10	0.40	Manufacturing
0.10	0.80	0.40	Services

(b): Denote the total annual output amount of sectors P_F, P_M and P_S.

income expenses

$P_F = 0.1P_F + 0.1P_M + 0.2P_S$

From the second and third rows of table, the income/expense requirement for the:

$P_M = 0.8P_S + 0.1P_M + 0.4P_S$

$P_S = 0.1P_S + 0.8P_M + 0.4P_S$

Move all variables to left side.

$0.9P_F - 0.1P_M - 0.2P_S = 0$

$-0.8P_F + 0.9P_M - 0.4P_S = 0$

$-0.1P_F - 0.8P_M + 0.6P_S = 0$

$$\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix}$$

(c) You can obtain reduced echelon form.

$$\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -.301 & 0 \\ 0 & 1 & -.712 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is $P_F = .301P_S$, $P_M = .712P_S$ with P_S free. If P_S is 100, then $P_F = 30.1$ and $P_M = 71.2$.

Question 4 :-

(a): Fill in the exchange table.

Distribution.

Minings	Lumber	Energy	Transportation	input	Purchased By
.30	.15	.20	.20	→	Mining
.10	.15	.15	.10	→	Lumber
.60	.50	.45	.50	→	Energy
0	.20	.20	.20	→	Transportation

output

(b): Donate the total annual output:

income   expenses

$$P_M = .30P_M + .15P_L + .20P_E + .20P_T$$

$$P_L = .10P_M + .15P_L + .15P_E + .10P_T$$

$$P_E = .60P_M + .50P_L + .45P_E + .50P_T$$

$$P_T = .20P_L + .20P_E + .20P_T$$

Move all variables:

$$.70P_M - .15P_L - .20P_T = 0$$

$$-.10P_M + .85P_L - .15P_E - .10P_T = 0$$

$$-.60P_M - .50P_L + .55P_E - .50P_T = 0$$

$$-.20P_L - .20P_E + .80P_T = 0$$

$$\begin{bmatrix} .70 & -.15 & -.20 & -.20 & 0 \\ -.10 & .85 & -1.5 & -.10 & 0 \\ -.60 & -.50 & .5.5 & -.50 & 0 \\ 0 & -.20 & -2.0 & .80 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.37 & 0 \\ 0 & 1 & 0 & -.84 & 0 \\ 0 & 0 & 1 & -3.16 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solve for basic variables in Terms of free variable P_T and obtain $P_M = 1.37P_T$, $P_L = .84P_T$ and

$P_E = 3.16 P_T \cdot 80$, take $P_T = 100$, round off other prices to $P_M = 137$, $P_L = 84$ and $P_E = 316$

Question 12:

Write equation for each intersection.

Intersection	Flow in		Flow out
A :	$x_1 + x_4$	=	x_2
B :	x_2	=	$x_3 + 100$
C	$x_3 + 80$	=	x_4

Rearrange the equation

$$x_1 - x_2 + x_4 = 0$$
$$x_2 - x_3 = 100$$
$$x_3 - x_4 = -80$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

The general equation

$$\begin{cases} x_1 = 20 \\ x_2 = 20 + x_4 \\ x_3 = -80 + x_4 \\ x_4 \text{ is free} \end{cases}$$

Since x_3 cannot be negative, the minimum value of x_4 is 80.

Q. NO. 2

solution

let

$P_s = 200$ million dollars

$P_c = 188$ million

$P_E = 170$ million

Any constant nonegative multiple of these prices is a set of prices, because solution set is multiples of one vector. Changing the unit of measurement to another currency such as chinese yen has the same effect as multplying all equilibrium prices by a constant. Ratios will remain same.

Q. NO. 6

Solution

The following vectors list the numbers of atoms of (Al), (O) (C).

$$Al_2O_3: \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad C: \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; Al: \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad CO_2: \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

coefficient in the equation

$$x_1 \cdot Al_2O_3 + x_2 \cdot C \longrightarrow x_3 \cdot Al + x_4 \cdot CO$$

$$x_1\begin{bmatrix}2\\3\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\1\end{bmatrix}=x_3\begin{bmatrix}1\\0\\0\end{bmatrix}+x_4\begin{bmatrix}0\\2\\1\end{bmatrix}$$

moving right terms to left side

$$\begin{bmatrix}2&0&-1&0&0\\3&0&0&-2&0\\0&1&0&-1&0\end{bmatrix}\sim\begin{bmatrix}1&0&-1/2&0&0\\3&0&0&-2&0\\0&1&0&-1&0\end{bmatrix}\sim$$

$$\begin{bmatrix}1&0&-1/2&0&0\\0&0&3/2&-2&0\\0&1&0&-1&0\end{bmatrix}\sim\begin{bmatrix}1&0&-1/2&0&0\\0&1&0&-1&0\\0&0&3/2&-2&0\end{bmatrix}\sim$$

$$\begin{bmatrix}1&0&-1/2&0&0\\0&1&0&-1&0\\0&0&1&-4/3&0\end{bmatrix}\sim\begin{bmatrix}1&0&0&-2/3&0\\0&1&0&-1&0\\0&0&1&-4/3&0\end{bmatrix}$$

$x_1=(2/3)x_4,\ x_2=x_4,\ x_3=(4/3)x_4$ with x_4 free

Take

$x_4=3$, Then $x_1=2$, $x_2=3$ & $x_3=4$.

Balanced equation

$2Al_2O_3+3C\longrightarrow 4Al+3CO_2$

Q. No. 7

Solution

Atoms of (Na), (H), (C) (O),

$NaHCO_3:\begin{bmatrix}1\\1\\1\\3\end{bmatrix},\ H_3C_6H_5O_7:\begin{bmatrix}0\\8\\6\\7\end{bmatrix},\ Na_3C_6H_5O_7:\begin{bmatrix}3\\5\\6\\7\end{bmatrix},\ H_2$

$$\begin{bmatrix}0\\2\\0\\1\end{bmatrix}, \quad CO_2: \begin{array}{c}\begin{bmatrix}0\\0\\1\\2\end{bmatrix}\end{array}\begin{array}{l}Na\\H\\C\\O\end{array}$$

reading left to right.

$$x_1 \cdot NaHCO_3 + x_2 \cdot H_3C_6H_5O_7 \rightarrow x_3 \cdot Na_3C_6H_5O_7 + x_4 \cdot H_2O + x_5 \cdot CO_2$$

$$x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3\begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4\begin{bmatrix}0\\2\\0\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

Reducing Augmented matrix:

$$\begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\1 & 8 & -5 & -2 & 0 & 0\\1 & 6 & -6 & 0 & -1 & 0\\3 & 7 & -7 & -1 & -2 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & -1 & 0\\0 & 1 & 0 & 0 & -1/3 & 0\\0 & 0 & 1 & 0 & -1/3 & 0\\0 & 0 & 0 & 1 & -1 & 0\end{bmatrix}$$

The general solution:

$x_1 = x_5$, $x_2 = (1/3)x_5$, $x_3 = (1/3)x_5$, $x_4 = x_5$, and x_5 is free. Take $x_5 = 3$. Then $x_1 = x_4 = 3$, $x_2 = x_3 = 1$. Now Balance equation

$$3NaHCO_3 + H_3C_6H_5O_7 \longrightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

Q-No-8

Solution:-

Vector List

$$H_3O: \begin{bmatrix}3\\1\\0\\0\end{bmatrix}, \; CaCO_3: \begin{bmatrix}0\\3\\1\\1\end{bmatrix}, \; H_2O \begin{bmatrix}2\\1\\0\\0\end{bmatrix}, \; Ca \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \; CO_2: \begin{bmatrix}0\\2\\0\\1\end{bmatrix}\begin{array}{l}H\\O\\Ca\\C\end{array}$$

Coefficients in chemical equation

$x_1 . H_3O + x_2 . CaCO_3 \longrightarrow x_3 . H_2O + x_4 . Ca + x_5 . CO_2$

$$x_1\begin{bmatrix}3\\1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\3\\1\\1\end{bmatrix} = x_3\begin{bmatrix}2\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\0\\1\\0\end{bmatrix} + x_5\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$$

Moving the terms to left side
reducing Augmented matrix:

$$\begin{bmatrix}3 & 0 & -2 & 0 & 0 & 0\\1 & 3 & -1 & 0 & -2 & 0\\0 & 1 & 0 & -1 & 0 & 0\\0 & 1 & 0 & 0 & -1 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & -2 & 0\\0 & 1 & 0 & 0 & -1 & 0\\0 & 0 & 1 & 0 & -3 & 0\\0 & 0 & 0 & 1 & -1 & 0\end{bmatrix}$$

$x_1 = 2x_5$, $x_2 = x_5$, $x_3 = 3x_5$, $x_4 = x_5$ and x_5 is free
Take
$x_5 = 1$. Then $x_1 = 2$, and $x_2 = x_4 = 1$ & $x_3 = 3$

$2H_3O + CaCO_3 \longrightarrow 3H_2O + Ca + CO_2$

Q. NO. 9

Solution
Vector list of atoms of numbers

$$_2S_3: \begin{bmatrix}2\\3\\0\\0\end{bmatrix}, H_2O\begin{bmatrix}0\\0\\2\\1\end{bmatrix}, H_3BO_3: \begin{bmatrix}1\\0\\3\\3\end{bmatrix}, H_2S: \begin{bmatrix}0\\1\\2\\0\end{bmatrix} \begin{matrix}B\\S\\H\\O\end{matrix}$$

$x_1 . B_2S_3 + x_2 . H_2O \rightarrow x_3 . H_3BO_3 + x_4 . H_2S$

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Now Reducing Augmented matrix

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now general solution

$x_1 = 1/3\, x_4$, $x_2 = 2x_4$, $x_3 = 2/3\, x_4$. x_4 = free

Now Take

$x_4 = 3$, $x_1 = 1$

$x_2 = 6$, $x_3 = 2$

Balanced equation

$B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$

Q. No. 10

Solution

Using the order lead (Pb), (N) (Cr), (Mn) (O) Now vector equation

$$x_1 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 8 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 3 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} + x_6 \begin{array}{c} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} Pb \\ N \\ Cr \\ Mn \\ O \end{array}$$

General solution

$x_1 = (1/6)x_6$, $x_2 = (22/45)x_6$, $x_3 = (1/18)x_6$,

$x_4 = (11/45)x_6$

$x_5 = (44/45)x_6$ $\quad x_6 = \text{free}$

Take

$x_6 = 90$ Then $x_1 = 15$, $x_2 = 44$, $x_3 = 5$, $x_4 = 22$

$x_5 = 88$.

Balanced equation

$15PbN_6 + 44CrMn_2O_8 \longrightarrow 5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO$

Q. NO. 11

Solution

Vector list of atoms per molecule of (Mn), (S), (As), (Cr) (O) & (H). Vector equation

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} = x_4\begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix} + x_6\begin{bmatrix}0\\3\\6\\1\\12\\0\end{bmatrix}$$

$$+ x_7\begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}\begin{array}{l}Mn\\S\\As\\Cr\\O\\H\end{array}$$

General solution is

$x_1 = (16/327)x_7$, $x_2 = (13/327)x_7$, $x_3 = (374/327)x_7$,

$x_4 = (16/327)x_7$, $x_5 = (26/327)x_7$

$x_6 = (130/327)x_7$, $x_7 =$ free

Take $x_7 = 327$ to make other variables whole Numbers

Balanced equation

$16MnS + 13As_2Cr_{10}O_{35} + 374H_2SO_4 \rightarrow 16HMnO_4 + 26As$

$+ 130CrSO_{12} + 327H_2O$

Question 13::

Write the equation for each intersection.

Intersection	Flow in		Flow out
A	$x_2 + 30$	=	$x_1 + 80$
B	$x_3 + x_2$	=	$x_2 + x_4$
C	$x_6 + 100$	=	$x_5 + 40$
D	$x_4 + 40$	=	$x_6 + 90$
E	$x_1 + 60$	=	$x_3 + 20$

Rearranged the equation:

$$x_1 - x_2 = -50$$

$$x_2 - x_3 + x_4 - x_5 = 0$$

$$x_5 - x_6 = 60$$

$$x_4 - x_6 = 50$$

$$x_1 - x_3 = -40$$

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The general equation is:

$$\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$$

(b): To find minimum flows, note that since x_1 cannot be negative, $x_3 \geq 40$. This implies that $x_2 \geq 50$. Also since x_6 cannot be negative, $x_4 \geq 50$ and $x_5 \geq 60$. The minimum flows are $x_2 = 50, x_3 = 40, x_4 = 50, x_5 = 60$. (when $x_1 = 0$ and $x_6 = 0$).

Question: 14.

Intersection	Flow in		Flow out
A	80	=	$x_1 + x_2$
B	$x_1 + x_2 + 100$	=	x_4
C	x_3	=	$x_2 + 90$
D	$x_4 + x_5$	=	$x_3 + 90$

Rearranged the equation.

$$\begin{aligned}
x_1 \qquad\qquad\qquad\qquad + x_5 &= 80 \\
x_1 + x_2 \qquad\quad - x_4 \qquad &= -100 \\
x_2 - x_3 \qquad\qquad &= 90 \\
x_3 - x_4 - x_5 &= -90
\end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 80 \\ 1 & 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & 0 & -90 \\ 0 & 0 & 1 & -1 & -1 & -90 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 80 \\ 0 & 1 & 0 & -1 & -1 & -180 \\ 0 & 0 & 1 & -1 & -1 & -90 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. The general equation is
$$\begin{cases} x_1 = 80 - x_5 \\ x_2 = x_4 + x_5 - 180 \\ x_3 = x_4 + x_5 - 90 \\ x_4 \text{ is free} \\ x_5 \text{ is free} \end{cases}$$

b.

If $x_5 = 0$ then eq. is
$$\begin{cases} x_1 = 80 \\ x_2 = x_4 - 80 \\ x_3 = x_4 - 90 \\ x_4 \text{ is free} \end{cases}$$

c.

Since x_2 cannot be negative, The minimum value of x_4 when $x_5 = 0$ is 180.