The equation to be solved is

in (1)

(2)

Solver with separated angles

Solve equation for different angles:

(3)

let , multiply equation (3) with test function and integral it on

(4)

Focus on the first term, perform integral by part:

Then

Then (4) becomes:

(5)

**Modal Representation**

Considering the space span by ,

Let

Then (5) becomes:

Assume:

Choose with , assume test function is

As a summary: denote , where

Let

Where ,

Then

Where

Thus let ,

Thus

Then let’s consider boundary terms. We need to introduce global index settings first.

Then about the numerical flux

Let us define

Then

Then we can define:

And

For boundary surfaces, substitute with Dirichlet boundary condition.

Implement Design:

let , and , multiply equation（1 with test function and integral it on

(3)

Focus on the first term，perform integral by part we got：

Such that

(4)

Project into the space span by ： ，substitute (4) into（3）we have, for , Let assume it be

Let , , ,

First about three local terms:

If we project into the same space, we have:

where

Non-local term:

First we need to define the geometry:

Assume we are considering , then we define .

Then consider the interface of .:

About numerical flux, we use:

Thus

With the expansion of and assume

Where

Since we are using uniform Cartesian grid,

Define

Then

(local)

(adjacent)

The complete formulation is:

Then , where

Let come with the details about calculating those matrices.

For simplicity, we use modal based DG first. Using , with normalization such that

1. Index of basis
2. Calculate

Where , by orthogonal property of ,

1. Calculate

We need to separate calculate of into three parts.

And

1. Calculate

First we need to define numerical flux and

We need to separate calculate of into six parts