Untitled-1

March 12, 2022

1 Midterm - Hong Nguyen - USC ID 3335891045

2 Question 1

Libraries

[5]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy.spatial.distance import cdist as cdist
```

Functions

[13]:
```
# Function load data set
# input: # of dataset
def read_midterm_dataset(number):
        if(number == 1):
                train = pd.read_csv("Pr1_dataset1/train.csv")
                val = pd.read_csv("Pr1_dataset1/val.csv")
                test = pd.read_csv("Pr1_dataset1/test.csv")
        else:
                train = pd.read_csv("Pr1_dataset2/train_2.csv")
                val = pd.read_csv("Pr1_dataset2/val_2.csv")
                test = pd.read_csv("Pr1_dataset2/test_2.csv")
        X_train, y_train = train.iloc[:,:2].to_numpy(), train.iloc[:,2].
 ↪to_numpy()
        X_val, y_val = val.iloc[:,:2].to_numpy(), val.iloc[:,2].to_numpy()
        X_test, y_test = test.iloc[:,:2].to_numpy(), test.iloc[:,2].to_numpy()
        return X_train, y_train, X_val, y_val, X_test, y_test

# Define a RBF kernel
# input:
#       x1, x2: Data point
#       gamma : Coefficient of RBF kernel
def RBF_kernel(x1, x2, gamma):
    return np.exp(-gamma * cdist(x1, x2, 'euclidean'))

# Define a linear kernel
# input:
#       x1, x2: Data point
#       gamma : No meaning (just for sync to pass function as parametter)
def liner_kernel(x1, x2, gamma):
    return np.dot(x1,x2.T)

# Define a nearest mean classifier object
# input:
#       X_train : train data
#       y_train : label
#       kernel  : kernel function which define above
class nearest_mean_2class():
    def __init__(self, X_train, y_train, kernel) -> None:
        self.X_train = X_train
        self.y_train = y_train
        self.kernel = kernel
        self.class1_idx = np.where(y_train==1)[0]
        self.class2_idx = np.where(y_train==2)[0]
        self.N1 = self.class1_idx.shape[0]
        self.N2 = self.class2_idx.shape[0]

    def g(self,x,gamma):
        ans = 2/self.N1 * np.sum(self.kernel(x.reshape((1,2)),self.X_train[self.
↪class1_idx],gamma)) \
        - 2/self.N2 * np.sum(self.kernel(x.reshape((1,2)),self.X_train[self.
↪class2_idx],gamma)) \
        - 1/self.N1**2 * np.sum(np.sum(self.kernel(self.X_train[self.
↪class1_idx],self.X_train[self.class1_idx],gamma))) \
        + 1/self.N2**2 * np.sum(np.sum(self.kernel(self.X_train[self.
↪class2_idx],self.X_train[self.class2_idx],gamma)))
        return ans

# Function that plot the data plot the training
# data and decision regions in the original feature space
# Input :
#       training: train data
#       label_train: train label
#       mode: Nearest mean classifier model object only
def plotDecBoundaries(training, label_train, model, gamma):
        nclass =  max(np.unique(label_train))

        # Set the feature range for ploting
        max_x = np.ceil(max(training[:, 0])) + 1
        min_x = np.floor(min(training[:, 0])) - 1
        max_y = np.ceil(max(training[:, 1])) + 1
        min_y = np.floor(min(training[:, 1])) - 1
        xrange = (min_x, max_x)
        yrange = (min_y, max_y)

        # step size for how finely you want to visualize the decision boundary.
        inc = 0.05

        # generate grid coordinates. this will be the basis of the decision
        # boundary visualization.
        (x, y) = np.meshgrid(np.arange(xrange[0], xrange[1]+inc/100, inc), np.
↪arange(yrange[0], yrange[1]+inc/100, inc))

        # size of the (x, y) image, which will also be the size of the
        # decision boundary image that is used as the plot background.
        image_size = x.shape
        xy = np.hstack( (x.reshape(x.shape[0]*x.shape[1], 1, order='F'), y.
↪reshape(y.shape[0]*y.shape[1], 1, order='F')) ) # make (x,y) pairs as a␣
↪bunch of row vectors.
        print(xy.shape)

        # distance measure evaluations for each (x,y) pair.
        pred_label = []
        for i in range(len(xy)):
                pred_label.append(np.where(model.g(xy[i],gamma) > 0,0,1).
↪tolist())
        pred_label = np.array(pred_label)
        pred_label.shape

        # reshape the idx (which contains the class label) into an image.
        decisionmap = pred_label.reshape(image_size, order='F')

        #show the image, give each coordinate a color according to its class␣
↪label
        plt.imshow(decisionmap, extent=[xrange[0], xrange[1], yrange[0],␣
↪yrange[1]], origin='lower')

        # plot the class training data.
        plt.plot(training[label_train == 1, 0],training[label_train == 1, 1],␣
↪'rx')
        plt.plot(training[label_train == 2, 0],training[label_train == 2, 1],␣
↪'go')
        if nclass == 3:
                plt.plot(training[label_train == 3, 0],training[label_train ==␣
↪3, 1], 'b*')

        # include legend for training data
        if nclass == 3:
                l = plt.legend(('Class 1', 'Class 2', 'Class 3'), loc=2)
        else:
                l = plt.legend(('Class 1', 'Class 2'), loc=2)
        plt.gca().add_artist(l)
        plt.show()
```

Dataset 1

Import dataset 1

[3]:
```
X_train1, y_train1, X_val1, y_val1, X_test1, y_test1 = read_midterm_dataset(1)
```

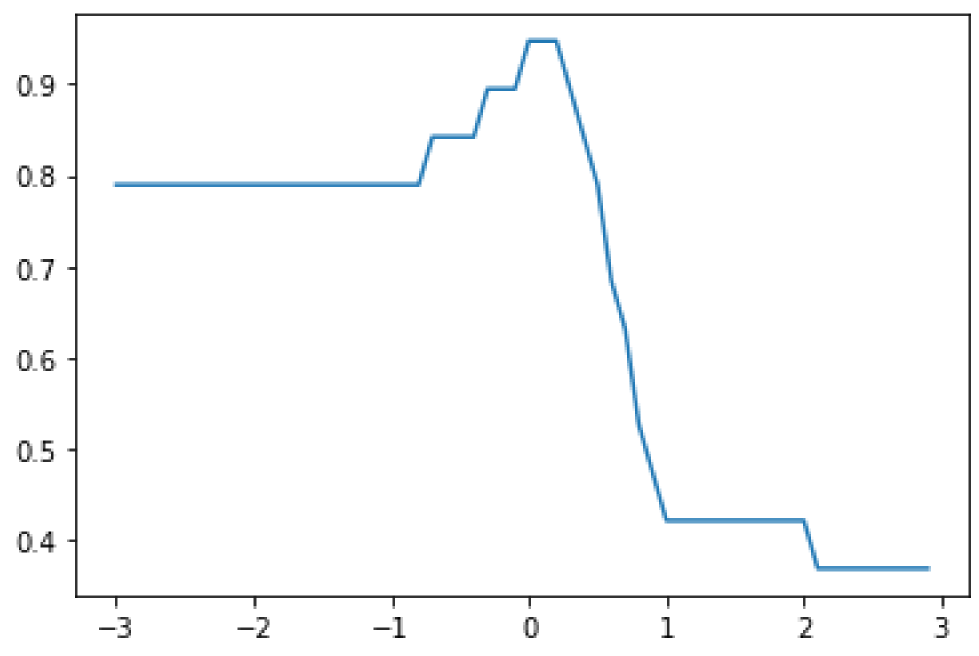
d) Code a 2-class kernel nearest means classifier

[6]:
```
model1 = nearest_mean_2class(X_train1, y_train1, RBF_kernel)
k = np.arange(-3,3, 0.1)
gamma = np.power(10,k)
acc = np.zeros(len(gamma))
for j in range(len(gamma)):
        predicted = []
        for i in range(len(X_val1)):
                predicted.append(np.where(model1.g(X_val1[i],gamma[j]) > 0,1,2).
 ↪tolist())
        acc[j] = np.mean(predicted == y_val1)
```

e) Plot the validation-set classification error as a function of gamma for dataset 1, for RBF kernel

[7]:
```
plt.plot(k,acc)
```

[7]:
```
[<matplotlib.lines.Line2D at 0x7f33cd9c42b0>]
```



Pick the optimal value of gamma

```
[12]: gamma_opt = gamma[np.argmax(acc)]
      gamma_opt
```

```
[12]: 1.0000000000000062
```

f) Compare test-set error using the linear kernel with test-set error using the RBF kernel, for each dataset. Comment on the results.

```
[27]: predicted = []
      for i in range(len(X_test1)):
              predicted.append(np.where(model1.g(X_test1[i],1) > 0,1,2).tolist())
      acc_test = np.mean(predicted == y_test1)
      acc_test
```

```
[27]: 0.7575757575757576
```

Comment: the accuracy is not good in compare with validation set

g) For the linear kernel, plot the training data, decision regions and boundary, in the feature space, for dataset 1.

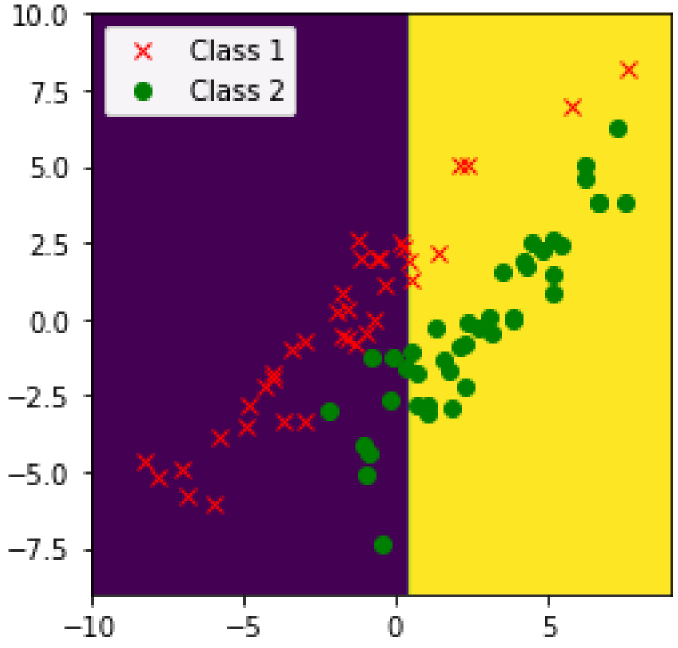
```
[8]: model1_linear = nearest_mean_2class(X_train1, y_train1, liner_kernel)
     predicted = []
     for i in range(len(X_val1)):
             predicted.append(np.where(model1_linear.g(X_val1[i],1) > 0,1,2).
      ↪tolist())
     acc_linear = np.mean(predicted == y_val1)
```

[9]:
```
# Linear model accuracy
acc_linear
```

[9]:
```
0.6842105263157895
```

[14]:
```
plotDecBoundaries(X_train1, y_train1, model1_linear, 1)
```

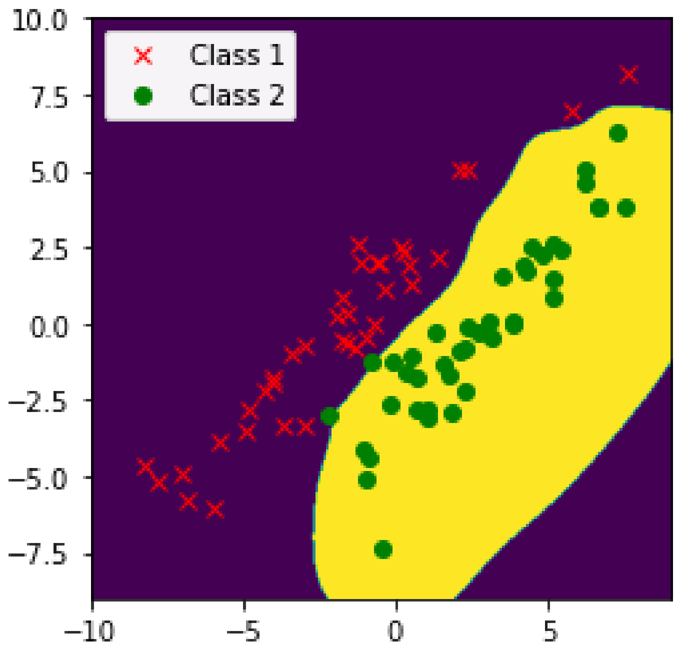
```
(145161, 2)
```



h) For the RBF kernel with optimal , plot the training data and decision regions in the original feature space, for each dataset

[15]:
```
plotDecBoundaries(X_train1, y_train1, model1, gamma_opt)
```

```
(145161, 2)
```

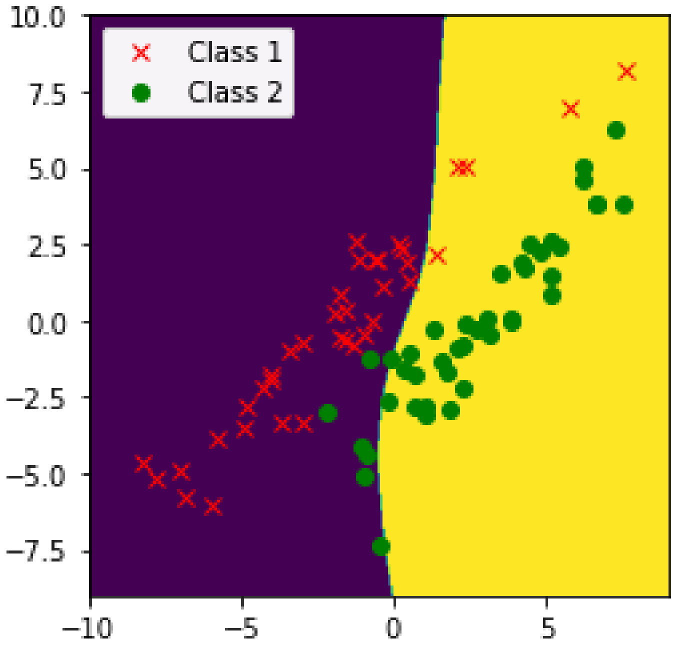


i) For the RBF kernel, repeat part (h) except for different gamma

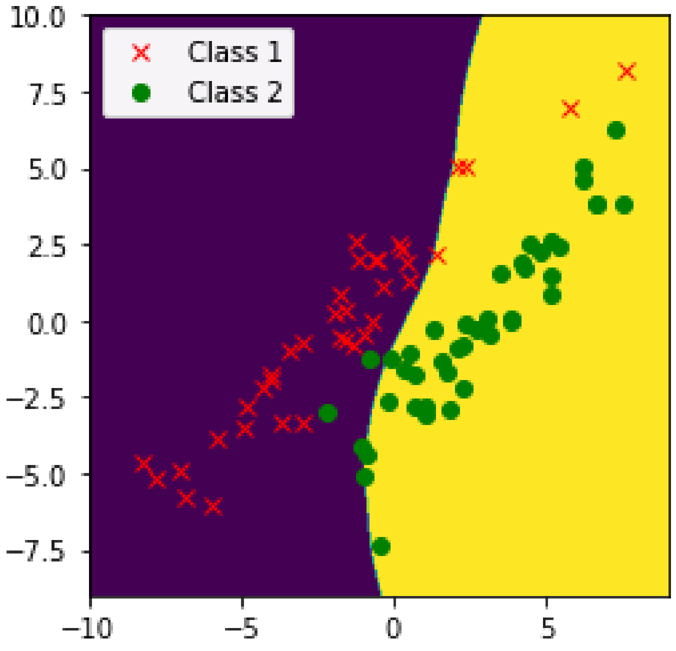
[17]:
```
# Because we see that, gamma_opt approximately = 1 and 1 also make highest␣
↪accuracy.
# Lets pick gamma_opt = 1
gamma_opt = 1
```

[18]:
```
gamma = gamma_opt*[0.01, 0.1, 0.3, 3, 10,100]
for j in gamma:
        plotDecBoundaries(X_train1, y_train1, model1, j)
```

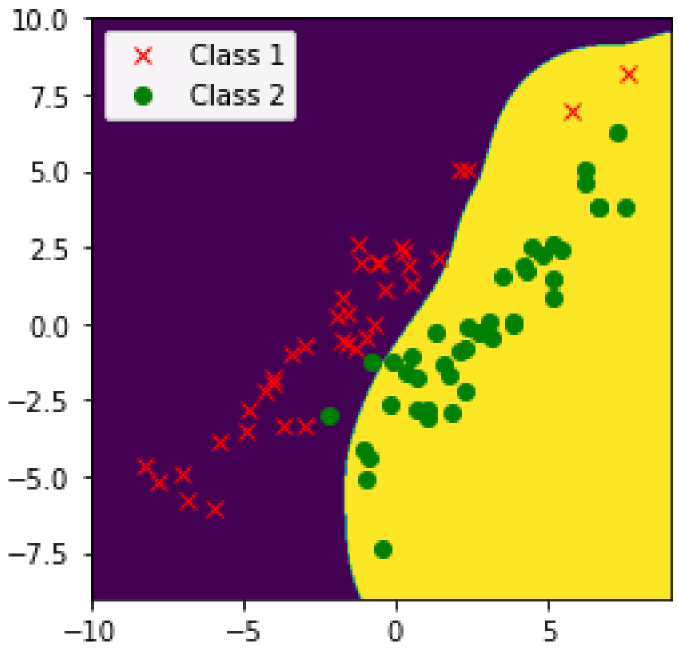
```
(145161, 2)
```



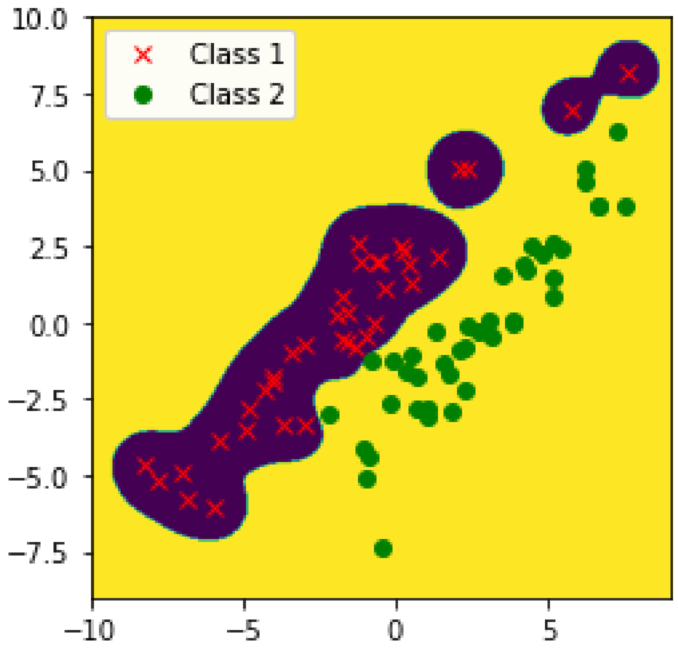
```
(145161, 2)
```



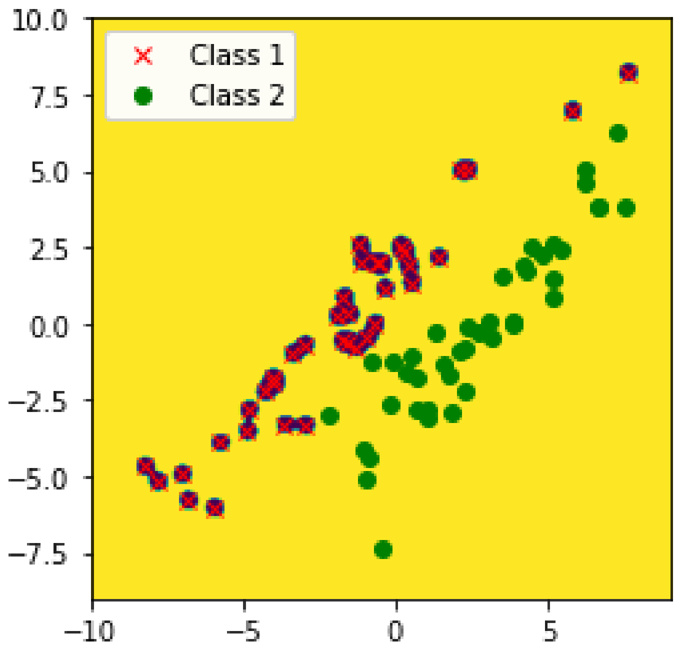
```
(145161, 2)
```



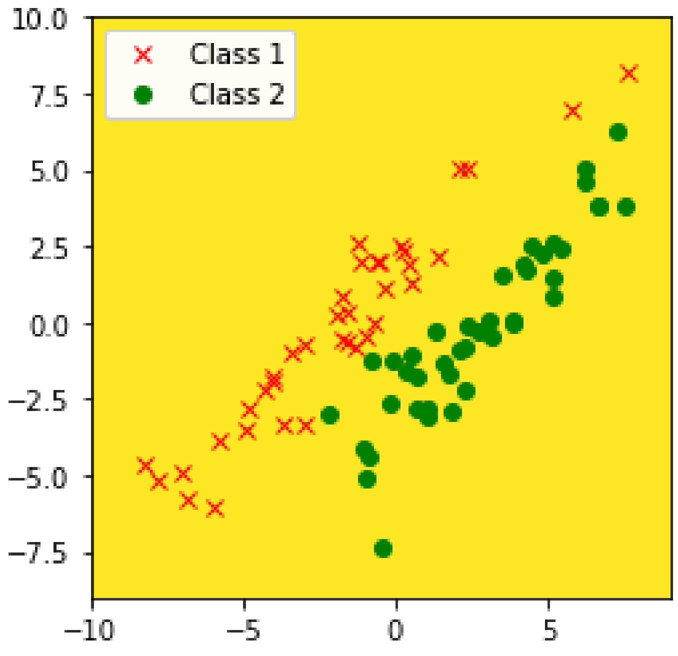
```
(145161, 2)
```



```
(145161, 2)
```



```
(145161, 2)
```



dataset 2

Import dataset 2

[19]:
```
X_train2, y_train2, X_val2, y_val2, X_test2, y_test2 = read_midterm_dataset(2)
# Change label from (0,1) to (1,2) to match with dataset 1
y_train2 = y_train2 + 1
y_val2 = y_val2 + 1
y_test2 = y_test2 + 1
```

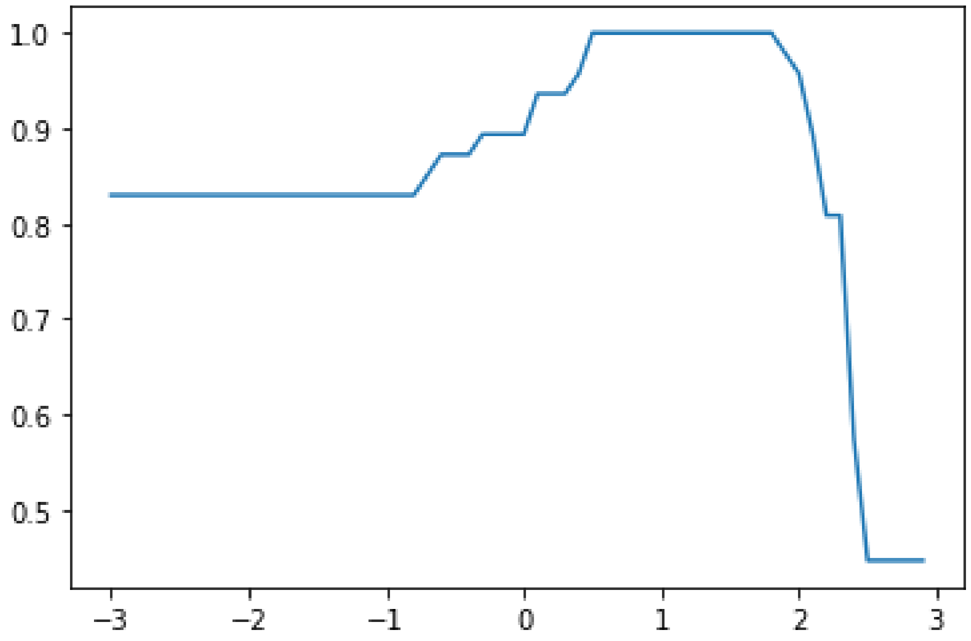
d) Train a 2-class kernel nearest means classifier with RBF

[20]:
```
model2 = nearest_mean_2class(X_train2, y_train2, RBF_kernel)
k = np.arange(-3,3, 0.1)
gamma = np.power(10,k)
acc = np.zeros(len(gamma))
for j in range(len(gamma)):
        predicted = []
        for i in range(len(X_val2)):
                predicted.append(np.where(model2.g(X_val2[i],gamma[j]) > 0,1,2).
 ↪tolist())
        acc[j] = np.mean(predicted == y_val2)
```

e) Plot the validation-set classification error as a function of , for each dataset, for RBF kernel

[107]:
```
plt.plot(k,acc)
```

[107]:
```
[<matplotlib.lines.Line2D at 0x7f20770c4550>]
```



Pick the optimal value of gamma

```
[110]: np.max(acc)
```

```
[110]: 1.0
```

```
[25]: gamma2_opt = gamma[np.argmax(acc)]
```

f) Compare test-set error using the linear kernel with test-set error using the RBF kernel, for each dataset.

```
[111]: predicted = []
       for i in range(len(X_test2)):
               predicted.append(np.where(model2.g(X_test2[i],1) > 0,1,2).tolist())
       acc_test = np.mean(predicted == y_test2)
       acc_test
```

```
[111]: 0.8930817610062893
```

Comment: The test set error is good with respect to dataset 1

g) For the linear kernel, train and plot the training data, decision regions and boundary, in the feature space, for each dataset

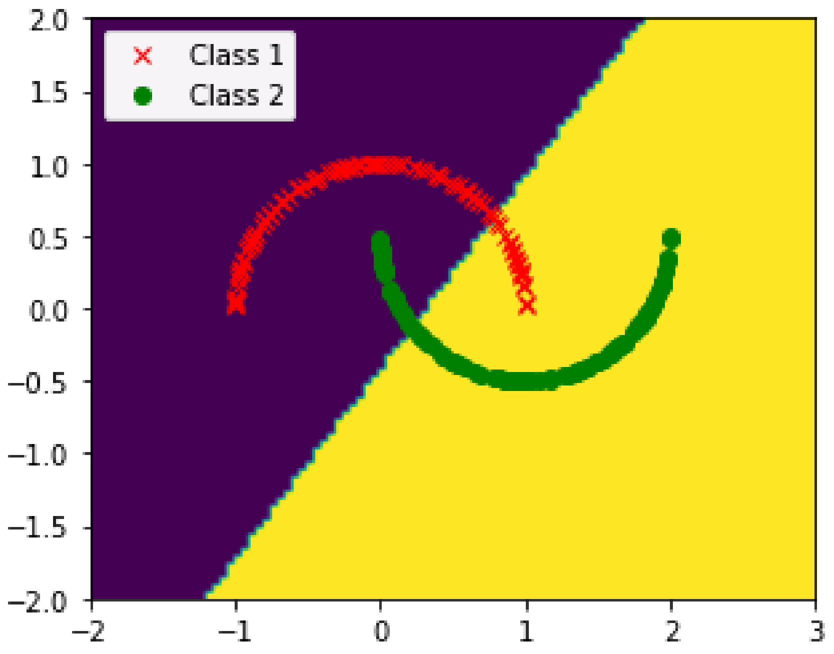
```
[22]: model2_linear = nearest_mean_2class(X_train2, y_train2, liner_kernel)
      predicted = []
      for i in range(len(X_val2)):
              predicted.append(np.where(model2_linear.g(X_val2[i],1) > 0,1,2).
       ↪tolist())
      acc_linear = np.mean(predicted == y_val2)
```

```
[23]: acc_linear
```

```
[23]: 0.8085106382978723
```

```
[24]: plotDecBoundaries(X_train2, y_train2, model2_linear, 1)
```

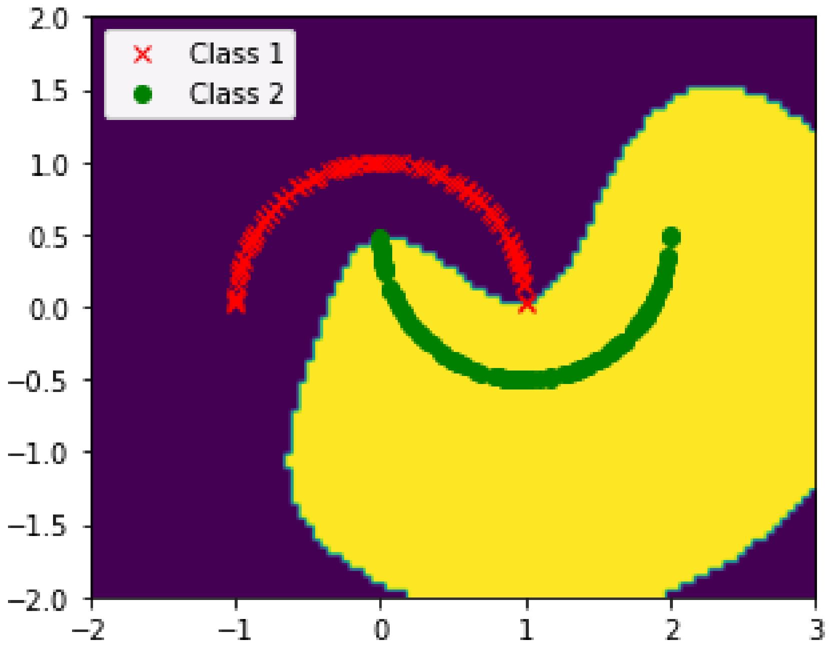
```
(8181, 2)
```



h) For the RBF kernel with optimal , plot the training data and decision regions in the original feature space, for dataset 2.

[26]:
```
plotDecBoundaries(X_train2, y_train2, model2, gamma2_opt)
```

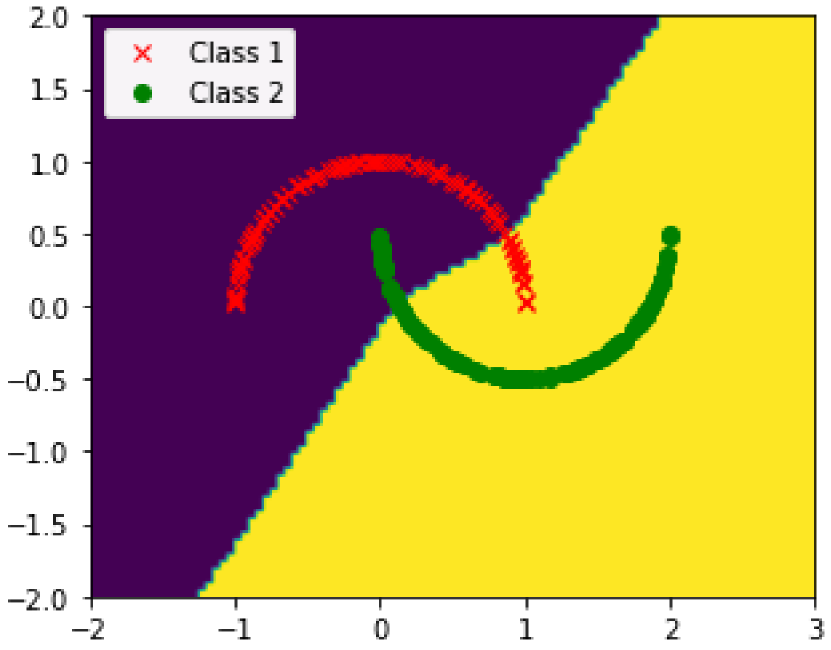
```
(8181, 2)
```



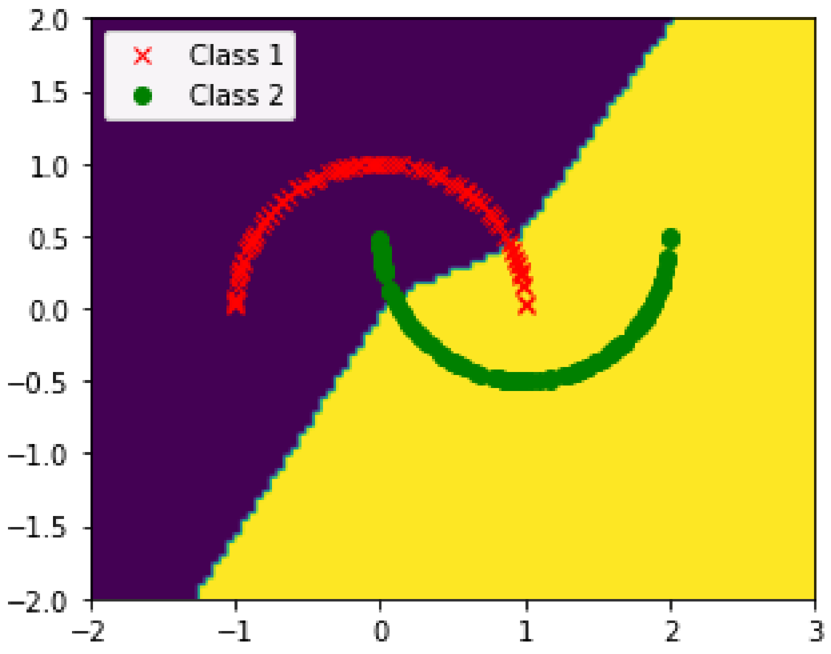
i) For the RBF kernel, repeat part (h) except for different gamma

```
[28]: gamma2 = gamma2_opt*np.array([0.01, 0.1, 0.3, 3, 10,100])
      for j in gamma2:
              plotDecBoundaries(X_train2, y_train2, model2, j)
```

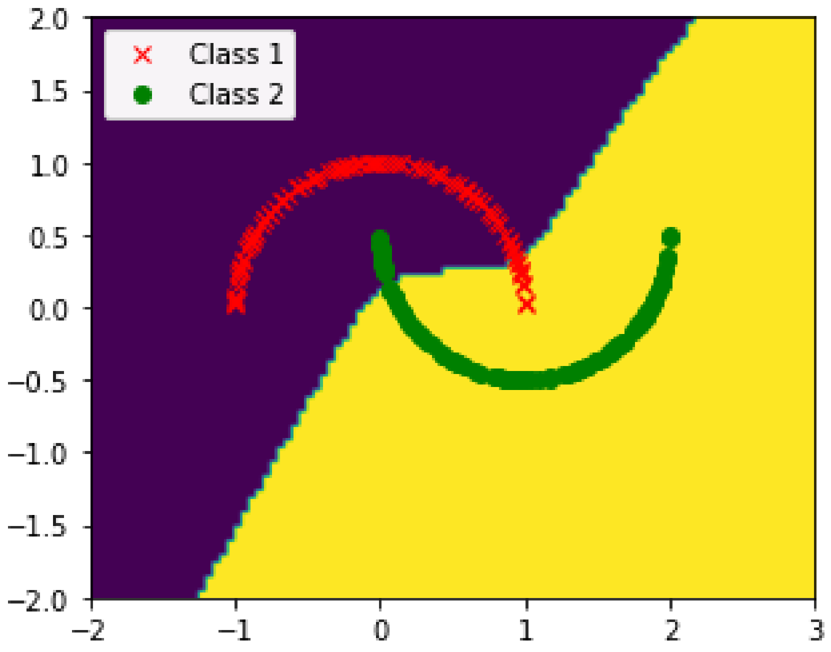
```
(8181, 2)
```



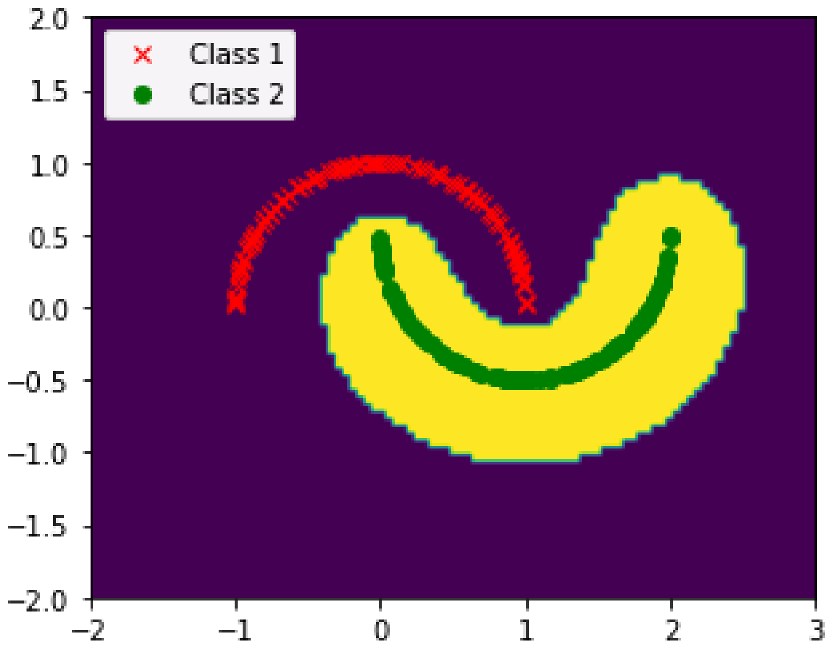
```
(8181, 2)
```



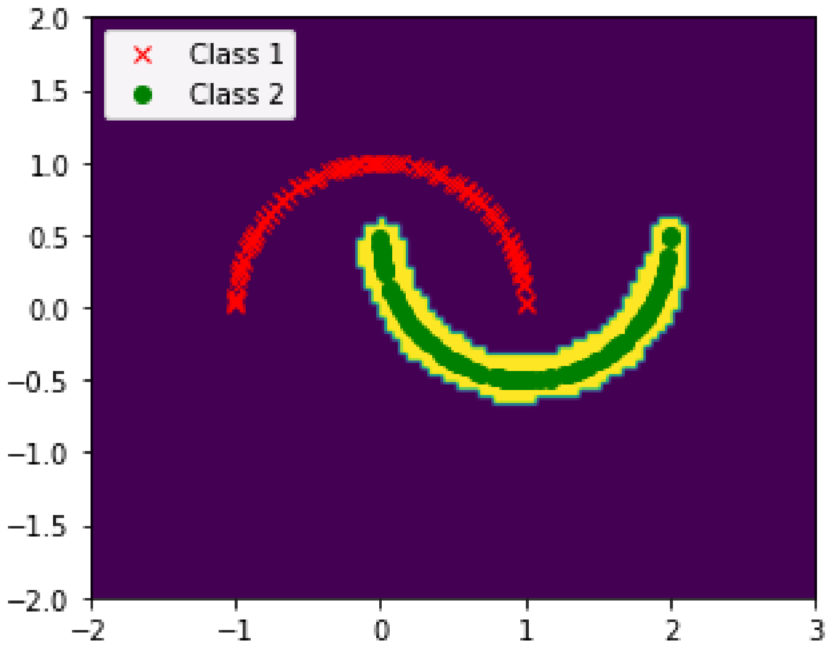
```
(8181, 2)
```



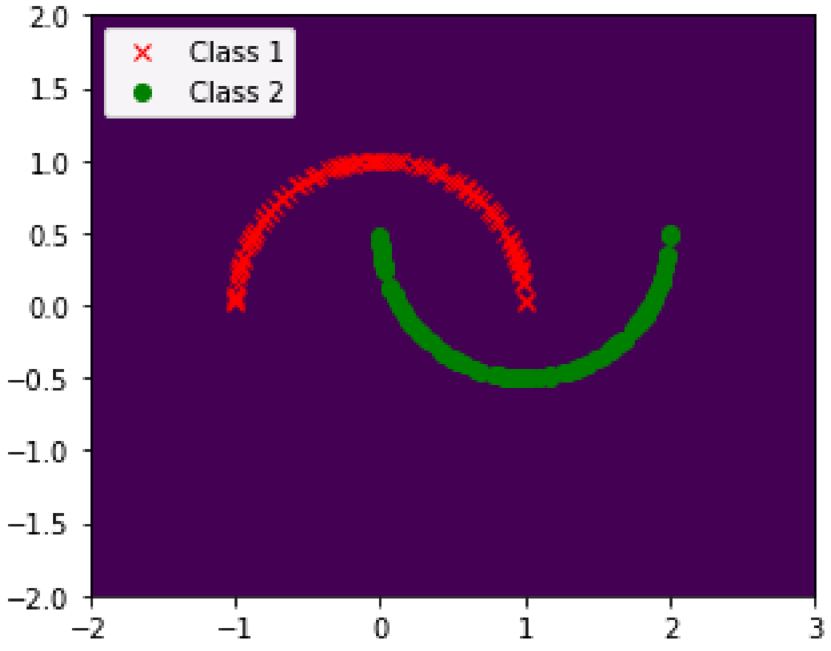
```
(8181, 2)
```



```
(8181, 2)
```



```
(8181, 2)
```



Class 1
Class 2
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2
−1
0
1
2
3