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Problem 5

Base case:

When n =1, A slithy Boolean expression in CNF with at most 1 variable has at most 1 clause and is satisfiable. For example, if we have a variable 'A' the expression can have at most one clause, which is either 'A' or '~A' Both of these clauses satisfy the slithy property, as each clause contains only one variable.

Induction Hypothesis:

Assume that for k vertices a slithy Boolean expression in CNF will have at most k clause is true.

Inductive step:

We want to prove that k+1 variables have at most k+1 clauses. Let G be the slithy Boolean expression in CNF with k+1 variables.

Now we have two cases to consider:

When G has only one clause which is related to the $(k+1)^{th}$ variable, we can remove the clause and we will get an expression with k variables. By the inductive hypothesis, a slithy expression with k variables will have at most k clauses. Since we removed a clause from G, we now add back the clause back which will end up getting a k+1 clause and it is satisfiable.

In the other case, we can have multiple clauses which contain the $(k+1)^{th}$ variable. And we remove all the clauses which are related to $(k+1)^{th}$ variables and we will get k variables with less than k clauses is already assumed in the induction hypothesis. By adding the clauses back, we can still ensure that the variable still satisfies the slithy condition. Thus, the expression is still satisfiable.

By considering these cases, expression G satisfies the slithy condition with at most k+1 variables will have at most (k+1) clauses and is satisfiable.

By the principle of mathematical induction, the statement for all integers n >=1 is true for a slithy Boolean expression in CNF with at most n variable has at most n clauses and is statisfiable.