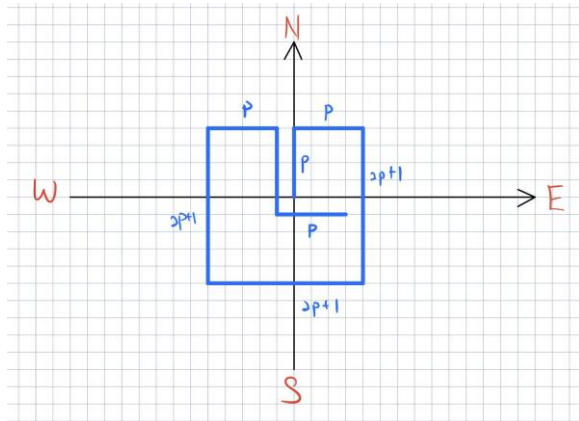


## Problem 8

To disprove that SAW is context-free, we can first assume that it is a context-free language.

And the string  $w$  be a string with  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$  where  $p = 2^k$



Also, we need to assume that the string  $w$  is satisfied with these conditions:

1. where for  $i \geq 0$ ,  $u(v^i)x(y^i)z$
2.  $|vwx| \leq 2^k$
3. And  $v$  and  $y$  are not empty string.

Case 1 Consider either  $v$  or  $y$  in the portion of each section.

1. For string with  $v$  or  $y$  in range  $N$ . For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump  $N$  with  $i=2$  where  $N^{p+1} E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$  it will not be a self-avoiding walk which contradicts the context-free language of SAW as the  $N$  string will have the same length as the  $S$  string portion, then  $E$  string portion will collide with  $N$  string portion.

2. For string with  $v$  or  $y$  as  $E$ . For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump the  $E$  portion down to  $i=0$ , we will get the last  $E$  portion to collide with the  $S$  portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

3. For string with  $v$  or  $y$  as  $S$ . For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump the  $S$  portion down to  $i=0$ , we will get the last  $E$  portion to collide with the  $N$  portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

4. For string with  $v$  or  $y$  as  $W$ . For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump the  $W$  portion down to  $i=0$ , we will get the last  $E$  portion to collide with the  $S$  portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

5. For string with v or y as N. For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump N with  $i=2$  where  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+2} E^p S^{p+1} E^p$  it will not be a self-avoiding walk which contradicts the context-free language of SAW. After pumping the N the E portion will collide with the first N portion.

6. For string with v or y as E. For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+2} E^p S^{p+1} E^p$

When we pump E with  $i=2$  where  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+2} E^p S^{p+1} E^p$  it will not be a self-avoiding walk which contradicts the context-free language of SAW. After pumping the N the S portion will collide with the first N portion.

7. For string with v or y as S. For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump the S portion down to  $i=0$ , we will get the last E portion to collide with the N portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

8. For string with v or y as E. For example, when a string of  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

When we pump the E portion to  $i=2$ , the E portion will collide with the S portion and it will not be a self-avoiding walk which contradicts the SAW being a CFL.

Case 2 either v or y contains two consequences of direction.

1. For string with v or y contains NE, ES,
  - a.  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$ ,
  - b.  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$
  - c.  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$
  - d.  $N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$ .

While pumping it  $i=2$  we will get

$N^{p+1} E^{p+1} S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

$N^p E^{p+1} S^{2p+2} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

$N^p E^p S^{2p+1} W^{2p+1} N^{2p+2} E^{p+1} S^{p+1} E^p$

$N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^{p+1} S^{p+2} E^p$

which will not be a self-avoiding walk since the S portion will collide with the N portion.

2. For string with v or y contains SW and WN,
 

$N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

$N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$ .

While pumping it down to  $i=0$ , in these two situations the collision will occur on the S portion it will collide with the N portion.

3. For string with v or y contains SE,
 

$N^p E^p S^{2p+1} W^{2p+1} N^{2p+1} E^p S^{p+1} E^p$

While pumping it down to  $i=0$ , the E portion will collide with the N portion which will not be a self-avoiding walk.

For all the cases, we can conclude that it contradicted the assumption that we made earlier where SAW is a context-free language but by proving by using pumping lemma.

In conclusion, SAW is not a context free language.