KangHongBo 5/5/2023
32684673

Assignment 2
Question 3
Elias Omega code (see Week 5 lecture slides) is a decodable code over all possible positive integers Z = {1, 2, 3, . . .∞}. The goal of this question is for you to mathematically prove that no Elias codeword of any n ∈ Z can be a prefix of any other codeword for m =/= n ∈ Z

Let's assume that there exist two positive integers n and m where n is not equal to m such that the codeword of n is a prefix of a codeword of m in Elias Omega code.

As we know Elias encodes the integer to a codeword with a kind of format as shown below

Elias(n)= length component(L) + component(C)

The length component determines how long the component is and the component is the binary form of n (eg Elias(4)=0 00 100)

Let both n and m write in this method.

Elias(n) = S1 = L1+C1
Elias(m) = S2 = L2+C2

Since we assumed that the codeword of n is a prefix of the codeword of m we can also know that C1<C2.
From this information, there are multiple possible:

1) When the length(S1) = length(S2)
   That means that L1=L2 because L denotes the length of the components and C1 and C2 will be the same components to achieve the target of the codeword of n is a prefix of the codeword of m but this violated the assumption that n is not equal to m so there is no way that the length(S1) == length(S2)

2) When length (S1)< length(S2) there is some possible condition:
   a) Length(L1)=Length(L2) and Length(C1)< Length(C2)
      This condition generally means L1=/=L2 which will directly violate the codeword of n is a prefix of the codeword of m

   b) Length(L1)+Length(C1)>Length(L2) where L1<L2
      In this condition, we can note that each L can be written to D1 + D2 + D3 + ,,, Dn where n term is the binary of the length of C but having the left most bit be 0 and D1 have the same relation of Dn to c with D2 and so on. Having this thought we can assume that in this L1 will not have the same Dn-1 term to get Length(L1) + Length(C1) > Length(L2) and since it will not be the same for the Dn-1 term we can also conclude that the codeword of n cannot be a prefix of the codeword of m

c) Length(L1)+Length(C1)<=Length(L2)
For this case, we know that every C starts with a 1bit and based on what we know in 2b) we can know that there can be the same till the Dn term for n but the next term of codeword n will be a 1bit where it is the component of C1 and we know that for m the comparing part is still a Dm or maybe Dm-k part and its leftmost bit will be 0bit so there is no possibility that the codeword of n can be a prefix of the codeword of m in this case.

By proving with contradiction, we can see that all the above possible having a result of codeword n cannot be a prefix of codeword m or n is equal to m but our assumption is the opposite meaning of the result. Therefore, we can have a solution in which no Elias codeword of any n ∈ Z can be a prefix of any other codeword for m =/= n ∈ Z.