International Astronomy and Astrophysics Competition

INTERNATIONAL ASTRONOMY AND ASTROPHYSICS COMPETITION

SOLUTION

INTAKE YEAR: 2021

Qualification Round 2021

Author:
Nguyen Ngoc Kim Hong

Email:
hongnguyen.ngock2@gmail.com

Acknowledgement

I would like to sincerely thank the organizers and those who have supported to create this useful contest for young generation.

Contents

1 INTRODUCTION

Inspire and Challenge Students
The International Astronomy and Astrophysics Competition is a great way to sharpen the mind of your students, to give them motivation and to challenge them with creative astronomy and astrophysics problems. The problems can be used as homework or project work as well: The competition includes various topics and difficulties. Furthermore, teachers and schools may use IAAC competition for after-school classes or workshops. The problems will inspire students to get engaged with science and will provide a unique opportunity for talented students to challenge their skills.

2 Problem A : Observing the Night Sky

2.1 QUESTION

Fill in the blank spaces with the correct answers:
Approximately how many stars are visible with the naked eye in the night sky?
(1) ____________________
Where in the night sky can you observe the famous double star system Mizar and Alcor?
(2) ____________________
What kind of celestial object is Neowise C/2020 F3 and what makes it special?
(3) ____________________
Which very intense meteor shower is taking place annually in December?
(4) ____________________
What are the names of the following three well-known constellations?

(5) ____________ (6) ____________ (7) ____________

2.2 SOLUTION

(1) About 5000 visible stars
(2)it can be seen in constellation of Ursa Minor.
(3) It is actually a comet with its period 4400 years.
(4) It is known as Geminids!
(5) It's Cassiopeia
(6) It's Cygnus
(7) It's Andromeda

3 Problem B : Shock Wave Escape

3.1 QUESTION

The star of a distant solar system explodes as a supernova. At the moment of the explosion, an resting exploration spaceship is 15 AU away from the shock wave. The shock wave of the explosion travels with 25000 km/s towards the spaceship. To save the crew, the spacecraft makes use of a special booster that uniformly accelerates at 150 m/s^2 in the opposite direction.
Determine if the crew manages to escape from the shock wave. (Neglect relativistic effect.)

3.2 SOLUTION

- 15 AU = $2.244 \times 10^{1}2$m.

- $d_{wave} = 25000 \times 10^3 \delta t$.
- $d_{spaceship} = \frac{1}{2} \times 150 \times \delta t^2$

When the shock wave rach the spaceship:

$$75t^2 + 2.244 \times 10^{12} = 25000 \times 10^3 t$$

$$\leftrightarrow t^2 - 333333.333t + 2.992 \times 10^{10} = 0$$

Now, this is the 2- order equation with $\Delta = b^2 - 4ac = (-333333.333)^2 - 4 \times 2.992 \times 10^{10} < 0$.
The shock wave never reach the space ship

4 Problem C : Mysterious Planet

4.1 QUESTION

A research team has discovered that a moon is circling a planet of our solar system: The moon orbits the planet once every 7 hours on a nearly circular orbit in a distance R of 48000 km from the centre of the planet. Unfortunately, the mass m of the moon is not known. Use Newton's law of gravitation with G = 6.67 $\times 10^{-11}$ m^3 /(kg·s^2) to approach the following questions:

$$F = G\frac{mM}{R^2} \tag{1}$$

Based on the observations, determine the total mass M of the planet?
Which moon and planet of our solar system is the team observing? (Use literature.)

4.2 SOLUTION

Since,

$$G\frac{mM}{R^2} = m\frac{v^2}{R}$$

Hence,

$$M = \frac{v^2 G}{R}$$

For a circular motion,
• Period: T = $\frac{2\pi R}{v} \longrightarrow v = \frac{2\pi R}{T}$
Finally,

$$M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 (48000)^3}{(6.67 \times 10^{-11})(7 \times 3600)^2} = 1.03 \times 10^{26} kg$$

Neptune planet and Naiad moon

5 Problem D : Gravitational Constant

5.1 QUESTION

An astronaut working on the Moon tries to determine the gravitational constant G by throwing a Moon rock of mass m with a velocity of v vertically into the sky. The astronaut knows that the Moon has a density ρ of 3340 $\frac{kg}{m^3}$ and a radius R of 1740 km.
(a) Show with (1) that the potential energy of the rock at height h above the surface is given by:

$$E = \frac{-4\pi G}{3} m\rho \frac{R^3}{R+h}$$

(b) Next, show that the gravitational constant can be determined by:

$$G = \frac{3}{8\pi} \frac{v^2}{\rho R^2} \frac{R+h}{h}$$

(c) What is the resulting G if the rock is thrown with 30 km/h and reaches 21.5 m?

5.2 SOLUTION

(a) Gravitational potential energy is the potential energy a massive object has in relation to another massive object due to gravity.

$$E(h) - E(\infty) = \int_{\infty}^{h} F\,dr = \int_{\infty}^{h} \frac{GMm}{(R+r)^2}\,dr = GMm\int_{\infty}^{h} \frac{1}{(R+r)^2}\,dr = -GMm\frac{1}{R+h} = \frac{-GMm}{R+h}$$

where E(∞) = 0 & M =$\frac{4}{3}\pi R^3\rho$. Finally,

$$E(h) = -\frac{4\pi G}{3} m\rho \frac{R^3}{R+h}$$

(b) The gravitational constant can be determined by:

$$E(h=0) + \frac{1}{2}mv^2 = E(h) \Longleftrightarrow -\frac{4\pi G}{3} m\rho \frac{R^3}{R+0} + \frac{1}{2}mv^2 = -\frac{4\pi G}{3} m\rho \frac{R^3}{R+h}$$

Then,

$$\frac{1}{2}mv^2 = \frac{4\pi G}{3} m\rho R^2 (1 - \frac{R}{R+h}) \Longleftrightarrow G = \frac{3}{8\pi}\frac{v^2}{\rho R^2}\frac{R+h}{h}$$

(c) Replaces v = 30 km/h = 8.33 m/s and h = 21.5 m to the equation of G:

$$G = \frac{3}{8\pi}\frac{v^2}{\rho R^2}\frac{R+h}{h} = \frac{3}{8\pi}\frac{8.33^2}{3340\times(1.74\times10^6)^2}\frac{1.74\times10^6+21.5}{21.5}$$

6 Problem E : Pulsars

6.1 QUESTION

Radio telescopes are an essential tool for modern astrophysics. They played a crucial role in discovering a fascinating astronomical object: Pulsars - highly compact objects that periodically emit radiation. Pulsars are still an active part of astrophysical research.

Explain how pulsars are formed and the causes for their pulsating behaviour.

6.2 SOLUTION

Pulsars are simply named for the "pulses" of radio waves that we see here on Earth. In fact, when pulsars were first detected, in 1967, nobody knew what was causing the pulses. Pulsars are easy to detect if you are doing a radio survey, since they are strong radio sources, and emit their bursts of radio emission at very regular intervals.

6.2.1 The forming of Pulsar

Pulsars belong to a family of objects called neutron stars that form when a star more massive than the sun runs out of fuel in its core and collapses in on itself. This stellar death typically creates a massive explosion called a supernova.

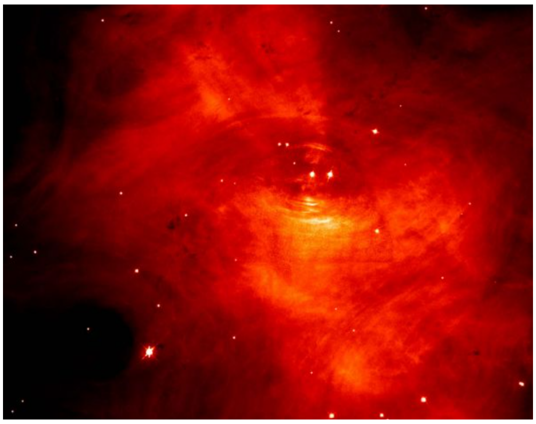

Figure 6.1: The Pulsar Powered Crab Credit J. Hester and P. Scowen (ASU), NASA
In the Summer of 1054 A.D. Chinese astronomers reported that a star in the constellation of Taurus suddenly became as bright as the full Moon. Fading slowly, it remained visible for over a year. It is now understood that a spectacular supernova explosion - the detonation of a massive star whose remains are now visible as the Crab Nebula- was responsible for the apparition. The core of the star collapsed to form a rotating neutron star or pulsar, one of the most exotic objects known to modern astronomers.

6.2.2 The Pulsar behavior [1]

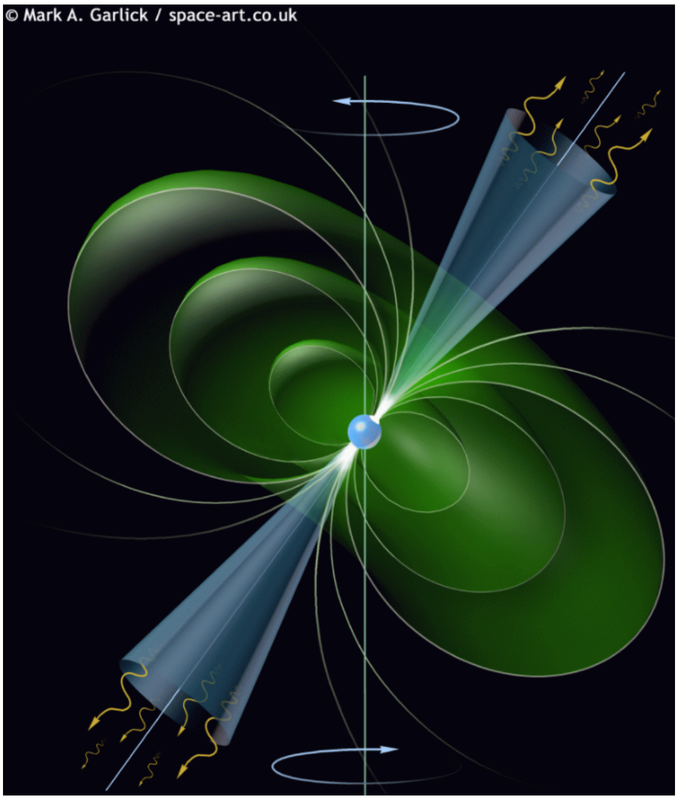

Figure 6.2: This is how people model a pulsar.

■ Pulsars are rotating neutron stars whose beams of radiation sweep across us.
Neutron stars have very strong magnetic fields, and are generally rotating quite rapidly. The electric field that is generated by the rotating, magnetic neutron star rips away charged particles from the surface of the neutron star, accelerates the particles along the magnetic field lines, and produces two beams of electromagnetic radiation
■ Pulsars rotate more slowly as they radiate away energy.
Most spinning objects tend to slow down with time. For example, the Earth spins more slowly, thanks to tidal braking. And, pulsars also tend to spin more slowly as they age.
■ Pulsars in close binary systems accrete gas and emit X-rays.
Pulsating X-ray sources, also called X-ray pulsars are close binary systems in which an ordinary star pours gas onto a neutron star. Following the magnetic field lines, the gas is funneled to the magnetic poles, producing beams of X-rays. Thus, we can get the signal of X-ray light if the beam of an X-ray pulsar sweeps across the signal- receiver

References

[1] Professor Barbara Ryden. Pulsars. http://www.astronomy.ohio-state.edu/~ryden/ast162_5/notes22.html.