Assignment 2 **Non-uniform flow**
Apply non-uniform flow calculation method to actual river
Version based on Assignment 1
With more concise comments and self-improvement

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Improvements

• Add references web where the data was collected.
↬ **GIS maps and tables for Manning's n values.**
• Wrong in input calculation.
↬ **Correctly, the value of data Table 1 in the previous version was entered incorrectly, leading to a confusing result. The value had been positioned in the correct way.**
• Analysis is not focused and some spelling and grammar mistakes.
↬ **Some superfluous comments have been omitted, and some analysis has been added instead.**

1 What to do

Longitudinal profile table (river bed height and river width)

Measure river bed height and channel width (distance between levees) along the target river using topographic maps. Result should be given in a table.

Table 1.1: Longitudinal profile table

No.	Distance [km]	Elevation [m]	Width [m]	Notes
1	0.0	3	110	Bridge
2	0.1	3	127	
3	0.2	3	123	
4	0.3	3	118	
5	0.4	4	110	
6	0.5	4	109	
7	0.6	4	109	
8	0.7	3	107	
9	0.8	3	107	
10	0.9	4	112	
11	1.0	4	111	
12	1.1	4	111	
13	1.2	4	106	
14	1.3	4	114	
15	1.4	4	132	
16	1.5	3	145	
17	1.6	3	134	
18	1.7	3	129	
19	1.8	3	125	
20	1.9	2	138	

Figure 1.1: Sakura river section and selected area shown by standard map

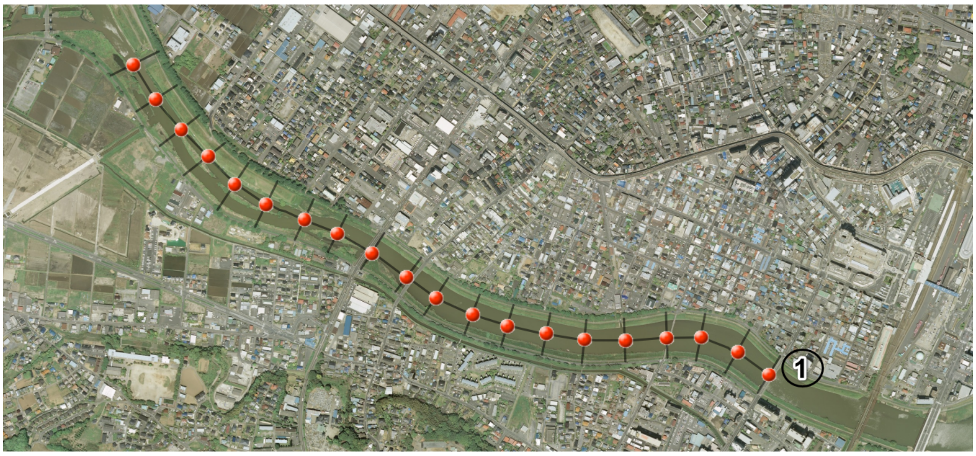


Figure 1.2: Sakura river section and selected area shown by image

Figure 1.3: The starting point of the survey is presented via Google Earth as a street view (point 1 is marked on the standard map and image)

2 One-dimensional calculation as non-uniform flow

Set Manning's n, discharge (m^3/s) and boundary condition (water level at downstream) as you like. You can use Manning's equation to set boundary condition. Weir would give a control section (Froude number = 1) where you get critical depth.

Starting from the lowest boundary and control sections, calculate water level at next cross section (toward upstream in subcritical flow, downstream in supercritical flow) using Bernoulli equation with friction loss. Hydraulic jump occurs at certain point downstream to supercritical flow.

Bernoulli equation for cross-section:

$$\frac{\alpha}{2g}\frac{Q^2}{A_1^2} + h_1 + h_f = \frac{\alpha}{2g}\frac{Q^2}{A_2^2} + h_2 + iL \tag{2.1}$$

Manning's equation:

$$h_f = \frac{Q^2}{2}\left(\frac{n_1^2}{R_1^{\frac{4}{3}}A_1^2} + \frac{n_2^2}{R_2^{\frac{4}{3}}A_2^2}\right)L \tag{2.2}$$

From 2.1 and 2.2, the non-uniform flow calculation:

$$h_2 + iL = h_1 + \frac{\alpha Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) + \frac{Q^2}{2}\left(\frac{n_1^2}{R_1^{\frac{4}{3}}A_1^2} + \frac{n_2^2}{R_2^{\frac{4}{3}}A_2^2}\right)L \tag{2.3}$$

Assumption $\alpha = 1$, a channel slope of $i_1 = \frac{1}{400}$, Q = 1000 m^3/s, L = 100 m.

$$h_2 = h_1 + \frac{\alpha Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) + \frac{Q^2}{2}\left(\frac{n_1^2}{R_1^{\frac{4}{3}}A_1^2} + \frac{n_2^2}{R_2^{\frac{4}{3}}A_2^2}\right)L - iL$$

$$\Longleftrightarrow \boxed{h_2 = h_1 + \frac{1\times 1000^2}{2\times 9.81}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) + \frac{1000^2}{2}\left(\frac{n_1^2}{R_1^{\frac{4}{3}}A_1^2} + \frac{n_2^2}{R_2^{\frac{4}{3}}A_2^2}\right)100 - \frac{100}{400}}$$

According to Manning's n for Channels (Chow, 1959) assuming that at base level, clean, straight, full stage, no cracks or chasms (as figure), so $n_1 = 0.03$.

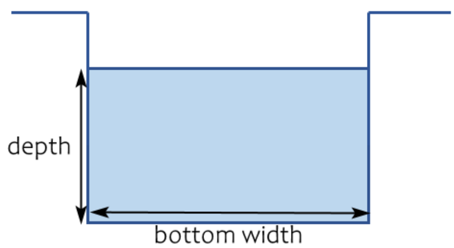


Figure 2.1: Assume that the cross-sectional area of rectangular open channel with $h_1 = 1.39m, w_1 = 110m$. Then,

- The flow area $A_1 = w_1 h_1 [m^2]$
- The wetted perimeter $P_1 = 2h_1 + w_1 [m]$
- The hydraulic radius $R_1 = \frac{A_1}{P_1} = \frac{w_1 h_1}{2h_1 + w_1} [m]$
- The hydraulic depth = $h_1 [m]$

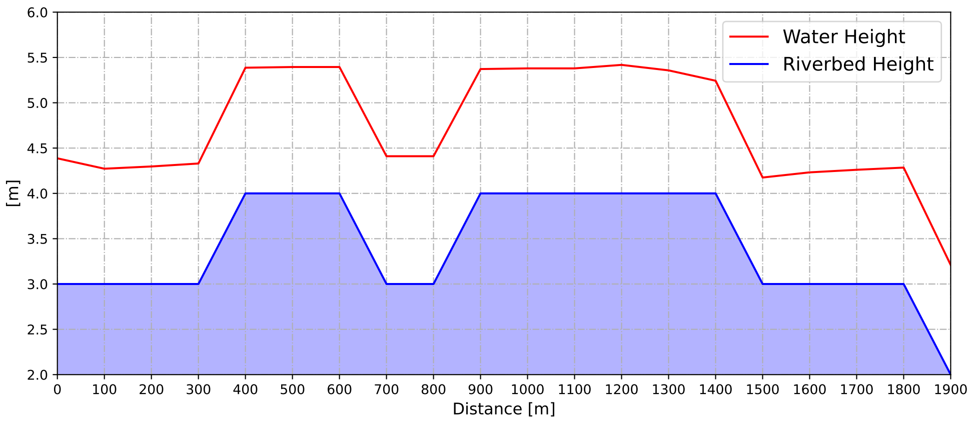
Assume that $Q_1 = 1000 m^3/s$
The Froude number is used to categorize open-channel flow in the following way:
(a) Fr < 1: subcritical (tranquil) flow
(b) Fr = 1: critical flow
(c) Fr > 1: supercritical (rapid) flow

3 Water level profile

Water level profile

Take longitudinal distance in horizontal axis, show water level and river bed height in vertical axis. Show all the conditions you used in calculation.

Parameters	Cross section	Q $[m^3/s]$	h_1 [m]	w_1 [m]	n
Assumed value	The same with initial h_1, w_1 value	1000	1.39	110	0.03
Explanation	Rectangular	a guess	a guess	as width of Table 1.1	clean, straight, full stage, no cracks or chasms



When measuring the elevation, the elevation may not be accurate above the water. The data is given based on taking points on the GIS-map, Google Earth view map by considering the surrounding land and cross-section to determine the height. In addition, the data recording for the next height (i.e h_2) is also rounded during the calculation to the nearest hundredth.

One should note that on this stretch, $F_R < 1$ and y_c for each section has been calculated at each step for determining the hydraulic jump. When $F_R < 1 \iff \sqrt{gh} > V$. As a result, the wave's velocity exceeds the velocity of the river, allowing it to proceed upstream. The downstream conditions are controlled. As a result, the wave's velocity exceeds that of the river, allowing it to proceed upstream. The control section is located downstream-end for this subcritical flow. Hence downstream conditions are affected by condition at upstream.

$$y_c = \left(\frac{Q^2}{gB^2}\right)^{\frac{1}{3}} = \left(\frac{1000^2}{9.8 \times 110^2}\right)^{\frac{1}{3}} = 2.0355[m]$$

4 Change some conditions

Change some conditions

Assume some kinds of river works are done.
Ex) Channel improvement (change channel width, cross section area, wetted perimeter), remove vegetation (change Manning's n), remove a weir, dam construction (change discharge), etc.
Run another calculation for a new condition, and compare the results. How high should the level be?

4.1 Using the trapezoidal shape as cross section

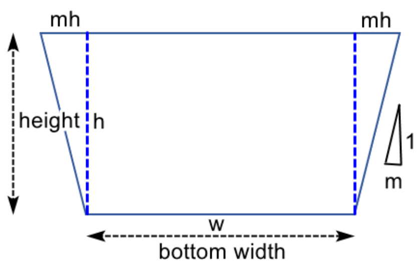
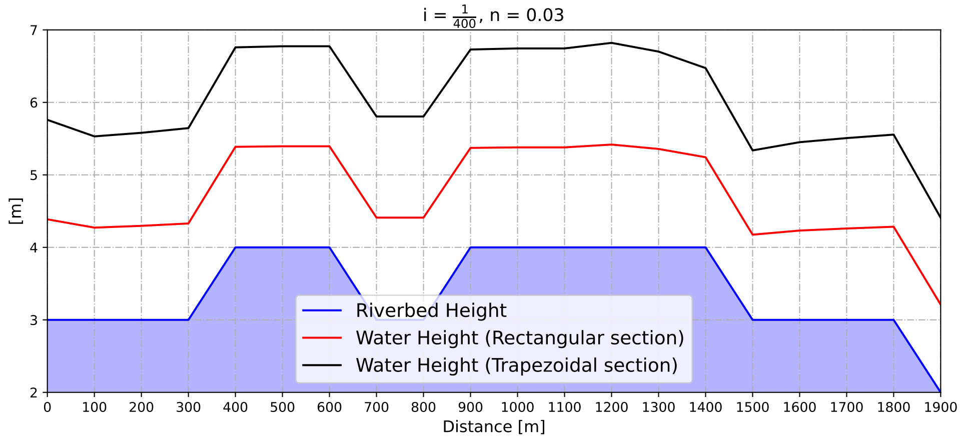


Figure 4.1: Assume that the cross-sectional area of trapezoidal open channel with $h_1 = 1.39m, w_1 = 110m$. Then,

- The flow area $A_1 = w_1 h_1 + m h_1^2$
- The wetted perimeter $P_1 = w_1 + 2h\sqrt{1+m^2}$
- The hydraulic radius $R_1 = \dfrac{A_1}{P_1} = \dfrac{w_1 h_1 + m h_1^2}{w_1 + 2h\sqrt{1+m^2}}$
- The hydraulic depth $\dfrac{wh + mh^2}{w + 2mh}$

Through the map, it can be observed that the trapezoidal cross section shape is the closest to reality. The value of m is supposed to 5 [m] in calculation.

Parameters	Cross section	Q [m^3/s]	h_1 [m]	w_1 [m]	n
Assumed value	The same with initial h_1, w_1 value	1000	1.39	110	0.03
Explanation	Trapezoid	a guess	a guess	as width of Table 1.1	clean, straight, full stage, no cracks or chasms



In the initial expectations, the trapezoidal channel section is the closest to the visual images through the map system and Google Earth. That is, it should give a result closer to rectangular, but simulation results have not shown that. The water level value is almost double that of rectangular and four times more than the actual one. This can also be inferred from the fact that the trapezoidal soil channel has additional factors such as the lateral slope ratio. So it is possible that these factors have a certain influence.
The ideal hydraulic channel section produces the highest flow rate for a given flow cross-area as well as the smallest cross-area and wetted perimeter for a given discharge. At the same time, the cost of a building may be decreased almost entirely. And, a detailed parameter to improve the calculation for the trapezoidal section is as follows: R = y/2; $\theta = 60^o$; the most efficient trapezoidal section is half hexagon.

4.2 Replacing Manning's n factor

The Manning's n values for Channels, Closed Conduits Flowing Partially Full, and Corrugated Metal Pipes as reference tables value for n . It is supposed that the channels is not maintained well, that is clean, winding, some pools, shoals, some weeds and more stones. In this case, n = 0.05. Besides, a case where the constant n is larger is also calculated (n = 0.07). This value means the main channels including jagged precipices, weeds, abysses.

Parameters	Crossed section	Q [m^3/s]	h_1 [m]	w_1 [m]	n	n
Assumed value	The same with initial h_1, w_1 value	1000	1.39	110	0.05	0.07
Explanation	Rectangular	a guess	a guess	as width of Table 1.1	clean, winding, some pools, shoals, some weeds and more stones	sluggish reaches, weedy, deep pools

Roughness, which is Manning's value of n, determines flow resistance in the physical sense. The roughness of the surface varies from section to section and over time (e.g. brushes grow in channels, drains age and deteriorate). One should note that each river section has some different properties, like reeds, splits, actual dunes. So that, the way to choose the Manning number must be considered carefully. To be more precise, and to be able to integrate many properties of the river section, the Cowan's equation is often used.

On the Figure 4.2, it can be seen that from the change of Mann's constant there is also a sensitive effect. Mann constant difference is 0.02 but causes elevation difference to about 0.05m according to rectangular section. All three options have diminishing results based on the length of the river considered for validation. The larger the indicator n, the greater the depth to starting with (and it decreases). It can be spotted the same trend as in the trapezoidal cross section. However, there is a big difference between the magnitudes. First, when comparing different values of n, they differ by about 1 m. Besides, compared to the same value of n, in each section case, this difference is about 4 m.

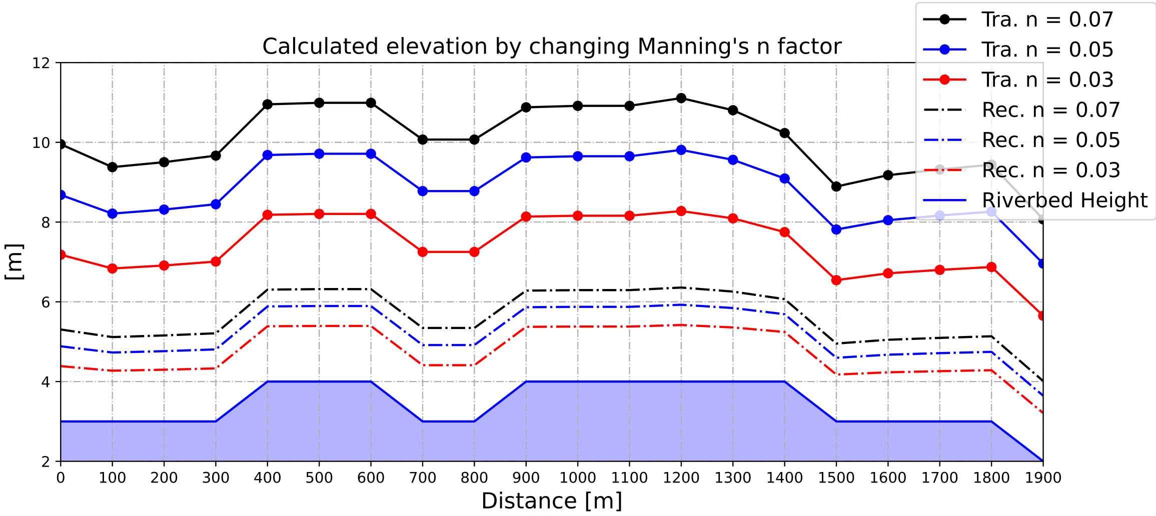


Figure 4.2: Changing n factor for both rectangular and trapezoidal section. The black, green and blue represent n values of 0.07, 0.05 and 0.03, respectively.

4.3 Replacing slope

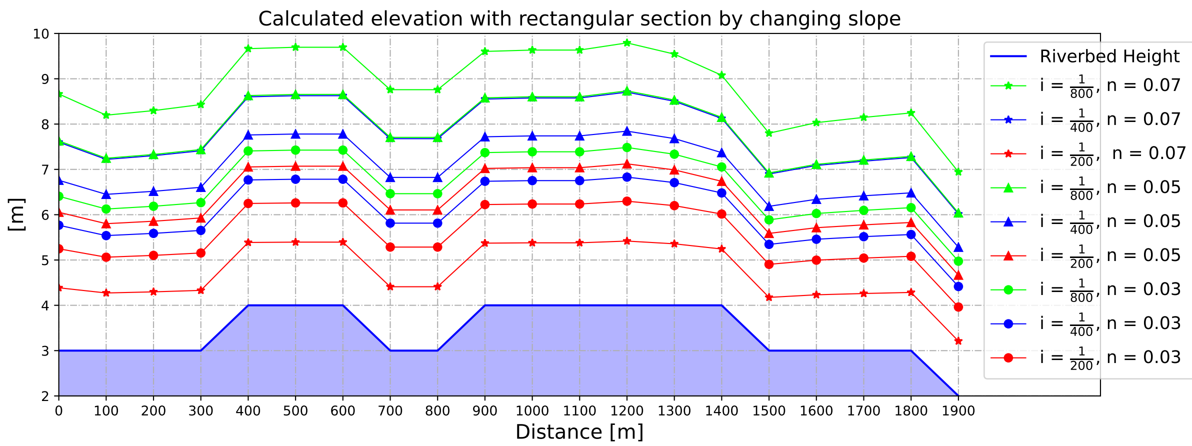


Figure 4.3: Changing i factor in the case of the rectangular section

Thus, it can be seen that the slope change also leads to a change in the perception of the river section being evaluated. That is, they can suddenly change the varying flow profiles of the river bed to mild slope, steep slope or adverse slope. In this calculation, a value of i is used for the entire section but in practice the river may have more complex topography and bends.

An interesting result in Figure 4.3 is that i = 1/800, n = 0.05 and i = 1/400, n = 0.07 give almost the same results, when checking their absolute values, they do not differ much (thousandths). However, the relationship between them has not been carefully examined, so there are still no concrete conclusions.

4.4 Changing discharge Q

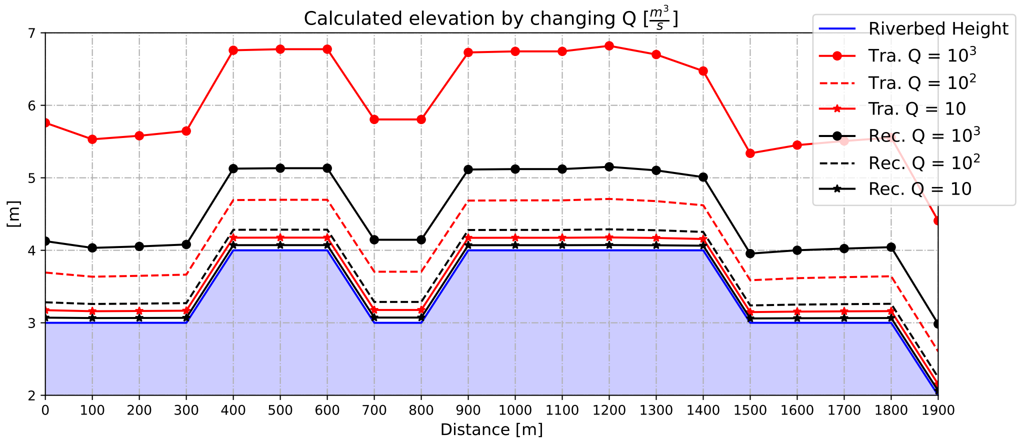


Figure 4.4: Changing Q factor for two kind of section. The red line belongs to the trapezoidal section and the black line belongs to the rectangular section. And the sign of the circle, dash and star is Q = 1000, 100 and 10 m^3/s respectively.

An other notable result shown in Figure 4.4. At the same value of Q, the trapezoidal section format is always larger than the rectangular one. In particular, the initial guess of 1000 m^3/s does not seem to be the correct number, however Q = 10 m^3/s is more reliable. It can be seen that with three trials and errors that give an approximate value to reality is also a good sign for the case n = 0.03. However, when Q is changed, one should note that the y critical value also changes and needs to be calculated in turn to consider the flow. In reality, the discharge Q should not be constant all the time. At least, in the simplest way, the mathematical description for the stage-discharge relation is given as the equation below, where h is the gauge corresponding to a discharge Q and h_0 is the corresponding to zero discharge k and m are constants.

$$Q = k(h - h_0)^m$$

5 Conclusions

In an open channel with a subcritical flow, the downstream end is the control section for progressively varied flow profiles. And, the desired survey river is relatively short for validating the subcritical. For improvement in the future, one should have chosen a longer segment and each selection step was also longer (i.e 300-500m) .

A pure rectangular cross section is extremely difficult to achieve. Rectangles should only be used to study calculations and become familiar with open channel flow. The trapezoidal cross-section, on the other hand, is frequently seen to be the most similar to the river section, yet it also produces poor outcomes. It's possible that the mistake was hesitant to pick the first w_1, h_1 value. For improvement, a combination of sectional shapes will give better results suggested as Figure 5.1.

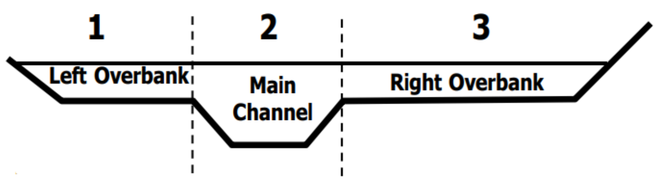


Figure 5.1: An example of combining and dividing sections into small parts to simulate closer to reality. And, this kind of section is widely calculated by Divided channel method.

The n value has a significant influence on the height calculation. The results show that the larger the value of n, the larger the calculated value of height and they decrease with the survey length. In the calculation, the dependence between the roughness coefficient n and Q has not been tested. But it can be seen in Mann's equation that they are inversely proportional to each other. Besides n, factors i and Q also play an important role in the re-simulation and some of their influence has been mentioned in the analysis sections.

Thus, it can be seen that the dyke bank must be higher or at least equal to the highest water level for this river section. Depending on each case and calculation can only conclude. In more detail, by my calculation in rectangular, the dyke should be at least about 5m, 6m and 9m height for n = 0.03, n = 0.05 and 0.07 respectively. And, these numbers are higher in trapezoidal section as 9, 10 ans 11m height for n = 0.03, n = 0.05 and 0.07 respectively. Regarding future work for this calculation, it is necessary to improve the cross section and clarify the details for the n value to get the closest simulation to reality.

6 References

GIS maps. https://maps.gsi.go.jp/

Reference tables for Manning's n values for Channels, Closed Conduits Flowing Partially Full, and Corrugated Metal Pipes. http://www.fsl.orst.edu/geowater/FX3/help/