MEMORANDUM

TO: Dr. Bedillion
FROM: Dajun Tao, Peize Hong, Jiatong Zhang
DATE: 4/22/2022
RE: MLC Laboratory

This report showed the results of four different controller synthesis methods (Inverse loop shaping, H2 synthesis, H_∞ synthesis, and μ synthesis) on a DINO Quanser Aero system. A nominal plant Simulink model tested the four controller performance and also robust stability and performance. Then the controllers were tested on the Quanser Aero hardware to test both 0 uncertainty and uncertainty response.

This report has been proofread by all members of the group:

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Introduction

The goal of this lab was to test four different controllers on the DIDO Quanser Aero system to make the system track the reference pitch and yaw angles and maintain stability. The four controllers were synthesized from four different methods: inverse loop shaping, H2 synthesis, H infinite synthesis, and μ synthesis. The four controllers were run on both nominal and uncertain conditions. After completing running these different controllers, the results would be compared with each other to determine the performance and also control usage of these different methods.

Equipment Specification

The hardware used was the DIDO Quanser Aero system, which has two fans, perpendicular to each other, to control the yaw and pitch angles of the whole device. The two fans were mounted on the two ends of the horizontal shaft on the top of the center vertical shaft. One fan faces horizontally to control the yaw angle; one faces vertically to control the pitch angle. Both the main center shaft and the top horizontal shaft would spin freely.

Procedure and Results

Preliminaries

1.) The Nominal Simulink Model

A nominal Simulink model was built based on the nominal plant G and the provided uncertain plant Simulink model as shown in Figure 1. A saturation nonlinearity was added before the plant to limit the control voltages to ±25 V. This nominal Simulink model would be used for all the initial response and usage testing of four different methods.

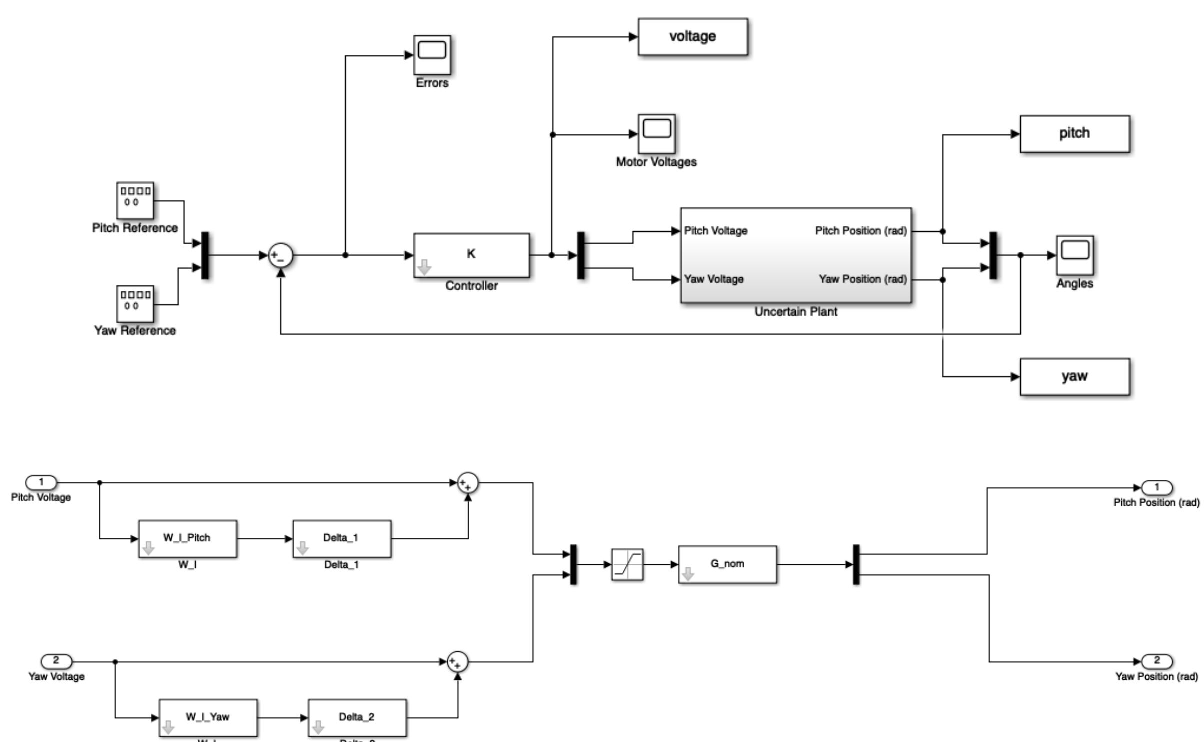

Figure 1. The Nominal Simulink Model

2.) Uncertain Plant

The uncertain plant was provided in the LabStartup, parameter, and state space files. Multiplicative input uncertainty weights were fitted for the uncertain model using ucover.

__Loop Shaping__

1.) Angles Response and Usage Plots

Inverse loop shaping was used to derive the desired controller. As the inverse of the nominal plant was improper, a second order high frequency pole was added to the inverse plant to make it proper. A first order integrator transfer function was chosen as the desired loop transfer function (L). The inverse controller was calculated by multiplying the proper inverse plant and the desired loop transfer function. Δ_1 and Δ_2 were set to 0 for the simulation.

Figure 2 shows the output of the pitch angle in radians in relation to time in seconds. There were noticeable oscillations at the beginning of each steady state of the pitch angle response. Figure 3 shows the output of the yaw angle in radians in relation to time in seconds. The yaw angle response had smaller oscillations than the pitch angle response. Figure 4 shows the control usage in voltage in relationship with time in seconds. The control usages were extremely high (the yaw usage had a peak of about 2300V). As the inverse loop shaping had no consideration for system usage and only aimed to match the desired loop transfer function, high usage was expected.

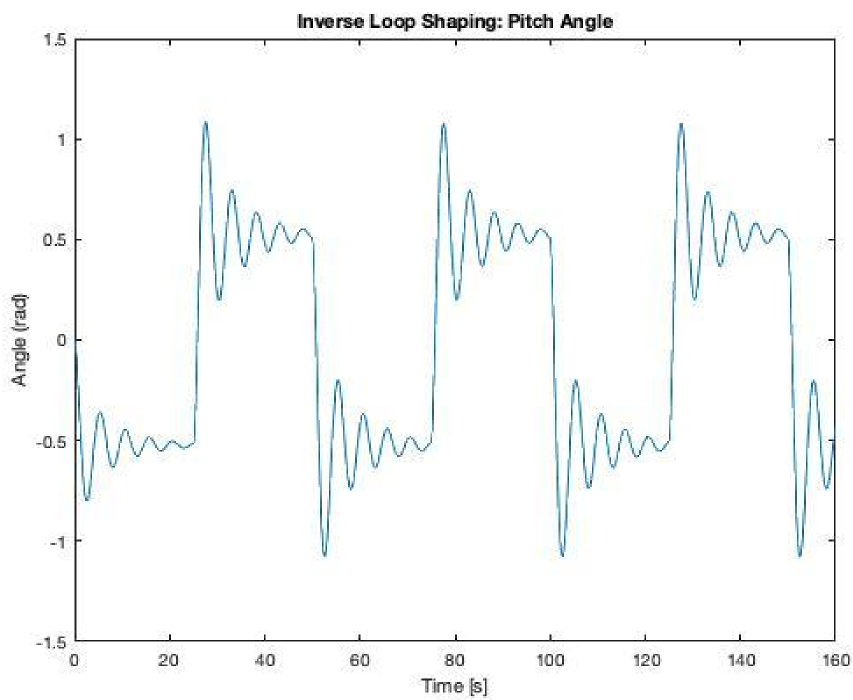

Figure 2. Inverse Loop Shaping Nominal Model Pitch Angle Response

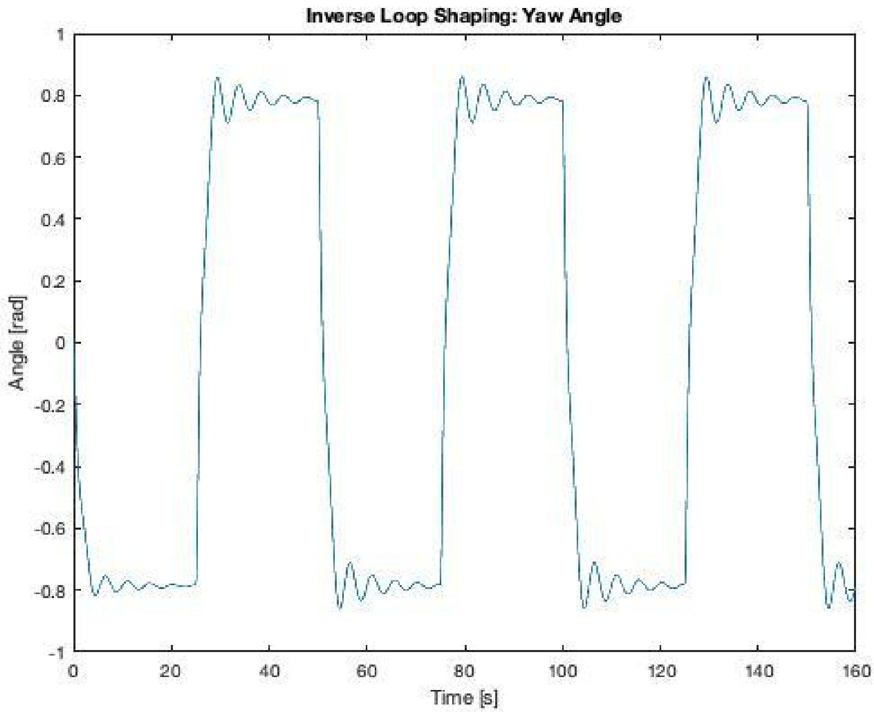

Figure 3. Inverse Loop Shaping Nominal Model Yaw Angle Response

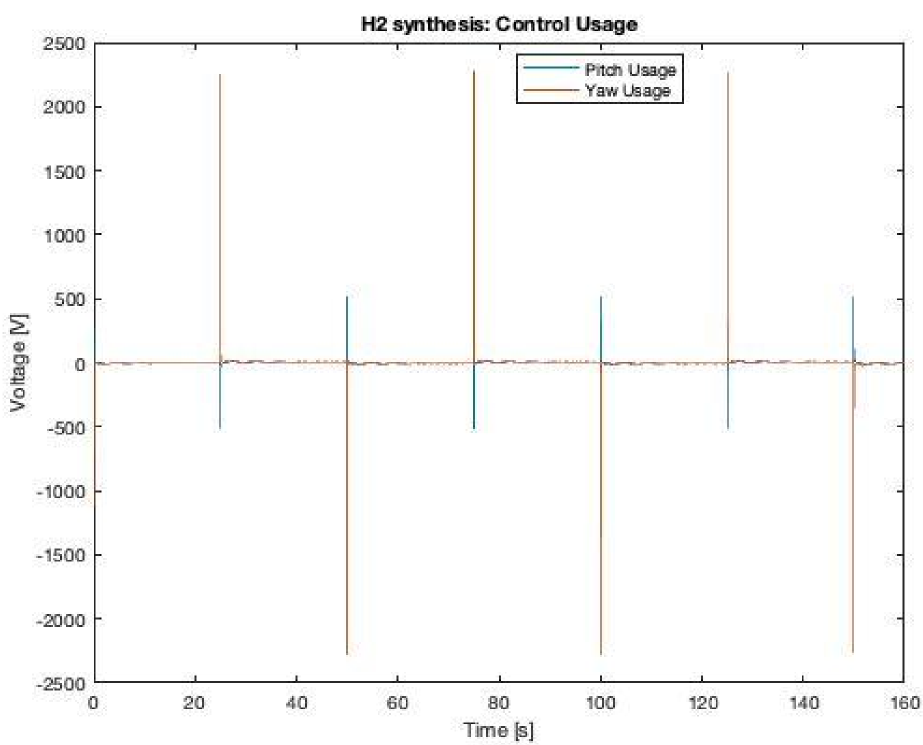

Figure 4. Inverse Loop Shaping Nominal Model Control Usage

2.) Robust Stability and Performance Test with 10 Uncertainty Samples

10 values of Δ_1 and Δ_2 were taken from samples of an uncertain LTI system for the simulation. Figure 5 shows the relationship between angles in radians and time in seconds for 10 uncertainty samples. The pitch angle is in blue, and the yaw angle is in red. The angles didn't go unstable for any of the inputs. The controller was robustly stable based on the response from simulation. The robust performance cannot be determined based on the plot.

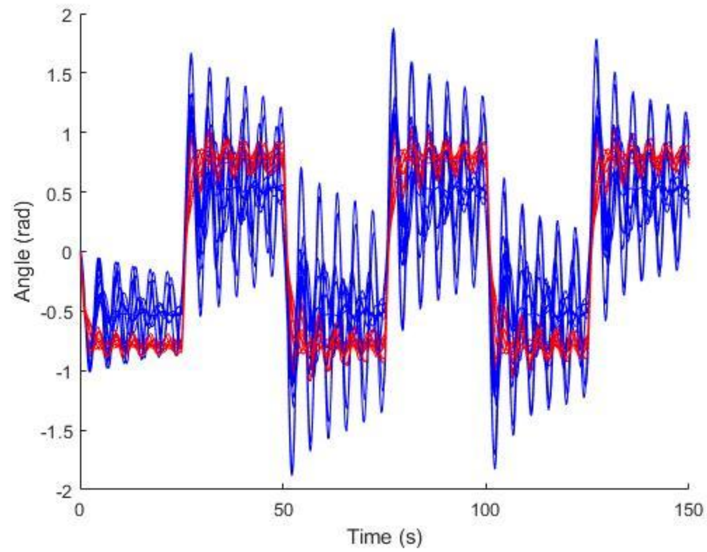

Figure 5. Inverse Loop Shaping with 10 Uncertainty Samples

3.) Hardware Simulations with 0 Uncertainty

The provided "MLC_Aero_Simulink" file was used with 0 uncertainty values (Delta_1 and Delta_2). The Simulink model was run on the Quanser Aero system.

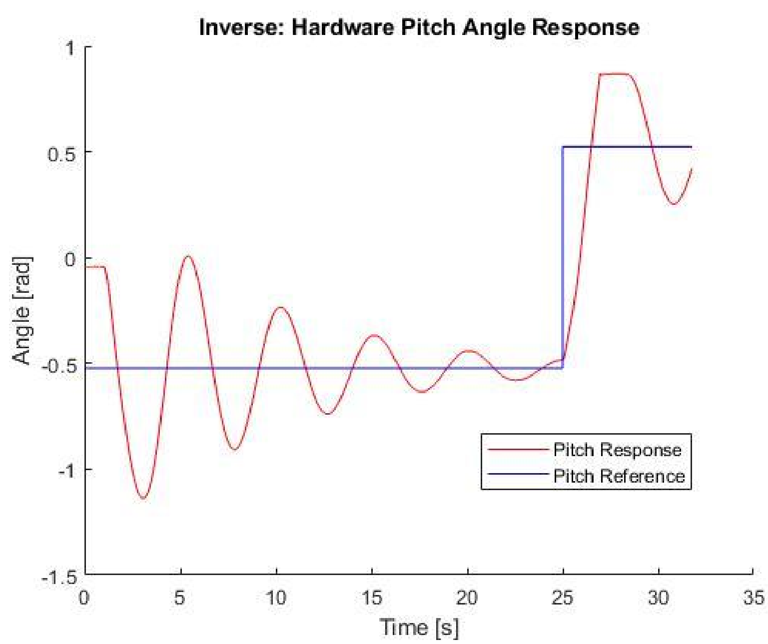

Figure 6. Hardware Inverse Loop Shaping Pitch Angle Response

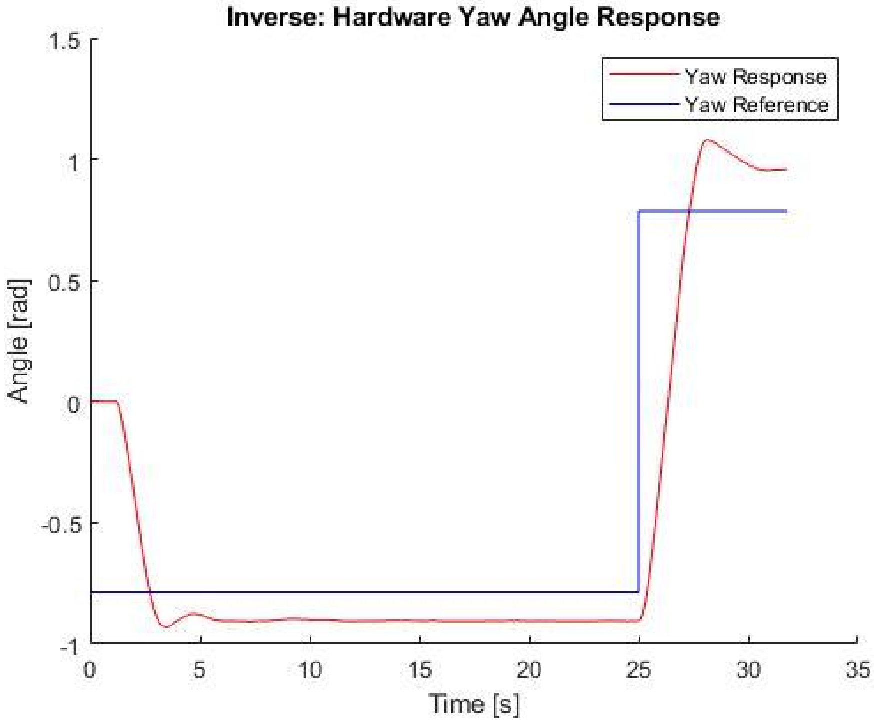

Figure 7. Hardware Inverse Loop Shaping Yaw Angle Response

Figure 6 and figure 7 show the pitch and yaw angle response with the reference signal. The pitch angle response slowly converged to the reference signal with huge initial oscillations. The yaw angle response had less overshoot than the pitch angle response, but there were steady errors throughout the initial steady state. The hardware result was similar to the simulation result in part 1).

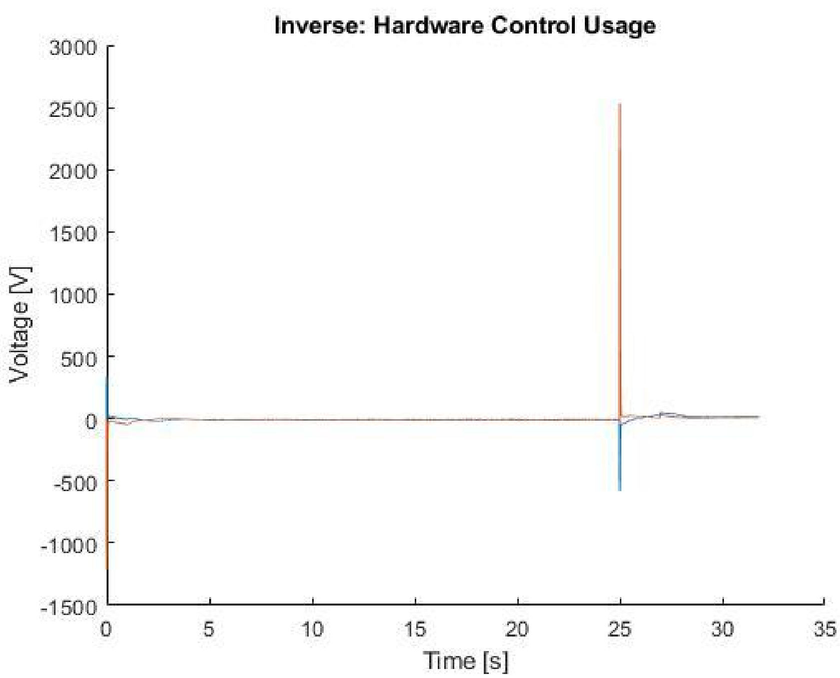

Figure 8. Hardware Inverse Loop Shaping Control Usage

Figure 8 shows the voltage response from the hardware. This is similar to the simulation result in part 1). The yaw voltage had a peak value of about 2500V, and the pitch voltage had a peak value of -500V.

4.) Hardware Simulation with 10 Uncertain Samples

The “MLC_Aero_Simulink” file was used with 5 uncertain samples to test the performance of the controller. New samples were generated when there were unstable samples or the system stopped in less than 10 seconds. Figure 9 shows the angle response in radians in relation to the time in seconds with 5 uncertainty samples for the inverse loop shaping controller. The yaw angle is in blue, and the pitch angle is in red. As shown in the plot, the controller performance remained relatively the same after uncertainty gets involved. The yaw response had a huge initial overshoot which was not present in the nominal response.

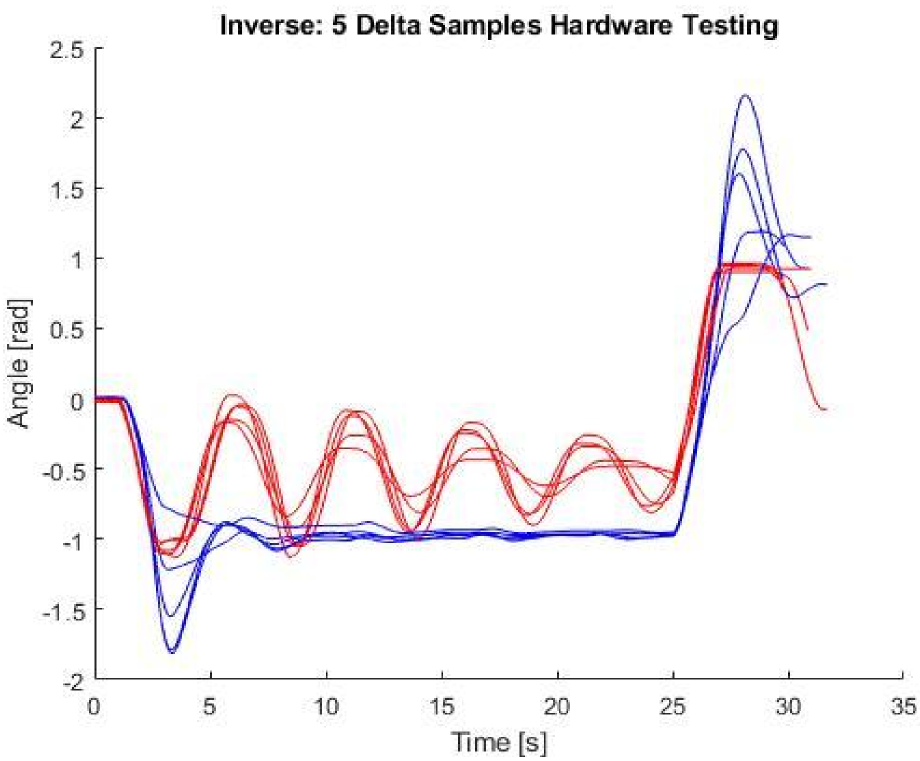

Figure 9. Hardware Inverse Loop Shaping with 5 Uncertainty Samples

<u>H_2 Optimal Control</u>

1.) Angles Response and Usage Plots

The nominal plant Simulink model was used to obtain the angles response and control usage plots for H2 synthesis controller. To synthesize the performance weight, the sensitivity peak was limited to below 6db, steady state disturbance rejection was set to a factor of 100, and the crossover frequency was set to 5 rad/s. Moreover, as the voltage was set to below 25V, controller weight was determined to be a 2-by-2 matrix with 1/25 at the diagonal elements. Finally, a generalized plant was built with sysic, and the controller was synthesized by h2syn. Then this controller was implemented in the nominal plant Simulink model to get response and usage plots.

Figure 10 shows the output of the pitch angle in radians in relation to time in seconds. The steady state oscillations of the pitch angle response were much smaller than the steady state pitch angle response oscillations of the inverse loop shaping. Figure 11 shows the output of the yaw angle in radians in relationship with time in seconds. There were some more prominent overshoots at the square input. Figure 12 shows the control usage in voltage in relation to time in seconds. The control usage had a peak of ±8 V, which was also much better than the inverse loop shaping response. Interestingly the voltage usage of the H2 synthesis had a similar shape to the pitch angle response.

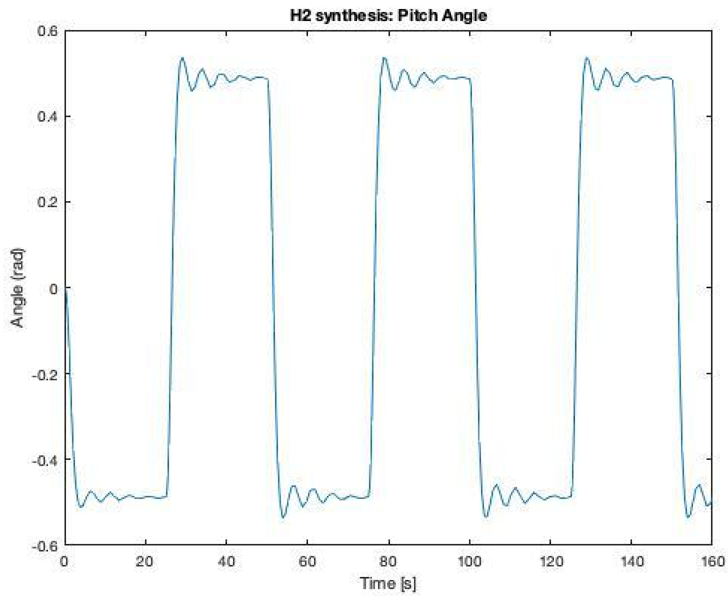

Figure 10. H2 Synthesis Nominal Model Pitch Angle Response

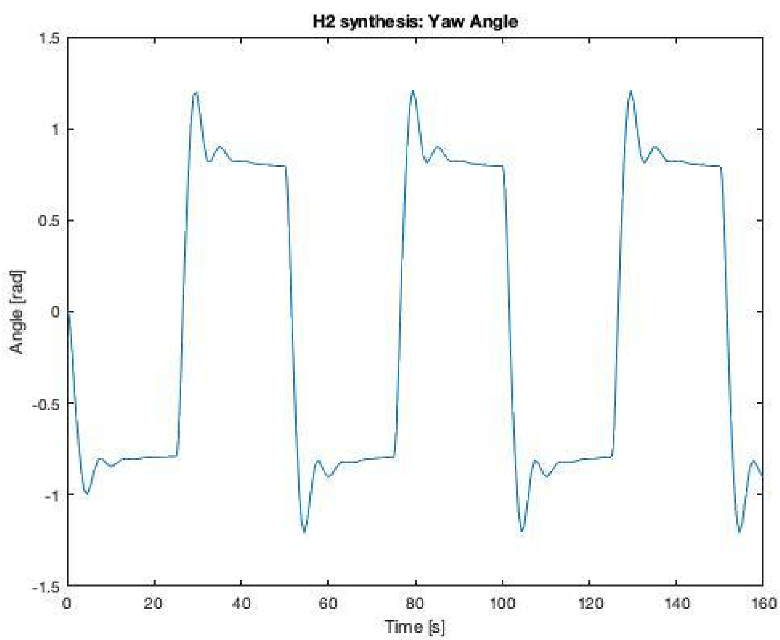

Figure 11. H2 Synthesis Nominal Model Yaw Angle Response

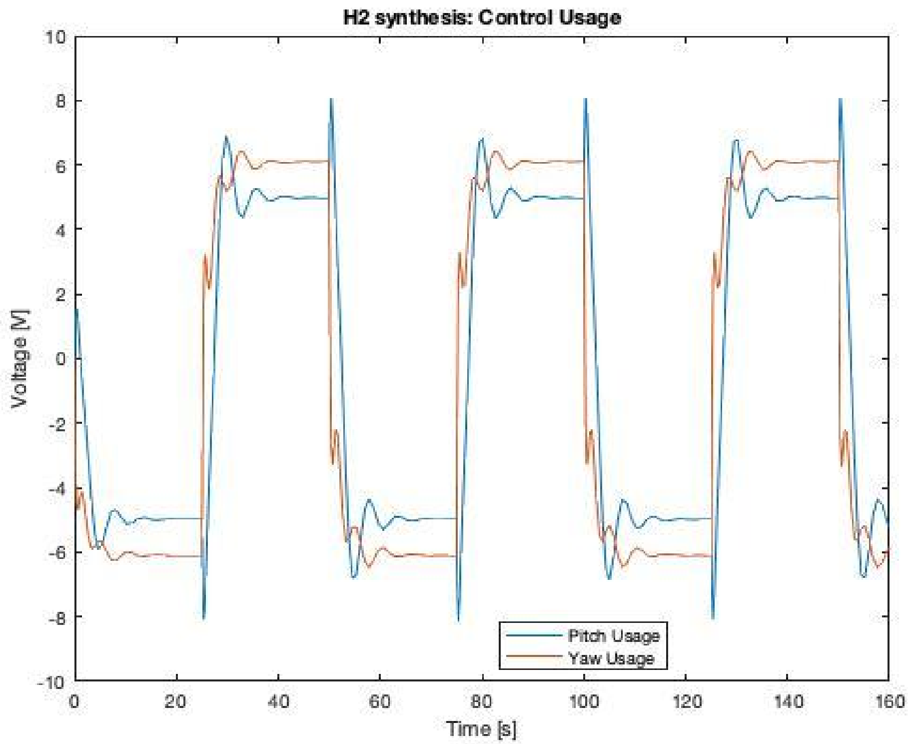

Figure 12. H2 Synthesis Nominal Model Pitch Control Usage

2.) Robust Stability and Performance Test

An uncertainty plant model was built with sysic and an uncertain lft was built with this uncertainty plant model and H2 controller. Both robust stability and performance were tested with this controller. As Figure 13 shows, the system had robust stability with a smaller than one result from robstab. However, the system didn't have robust performance as a result from robustperf was larger than one.

```
RS_H2 = 1/robstab(N_unc).LowerBound
```

```
RS_H2 = 0.3674
```

```
RP_H2 = 1/robustperf(N_unc).LowerBound
```

```
RP_H2 = 8.2438
```

Figure 13. H2 Synthesis Robust Stability and Performance Tests

3.) Hardware Simulations with 0 Uncertainty

Part 3 of the H2 synthesis followed the same procedure as the one in the inverse loop shaping.

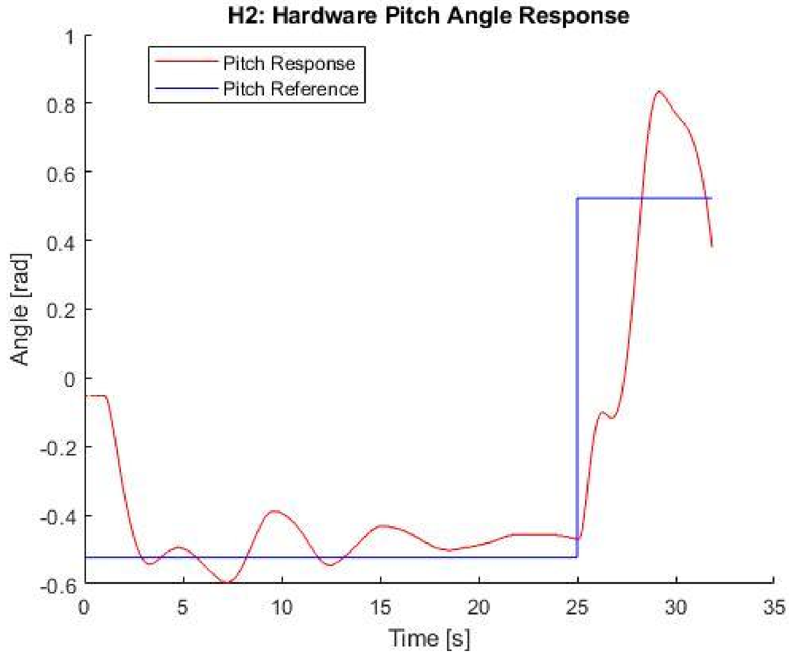

Figure 14. Hardware H2 Pitch Angle Response

From the above graphs we can see that the H2 synthesis controller made the angles slowly converge to follow the reference signal. For the pitch angle, it had much smaller oscillations than the pitch angle response of the inverse loop shaping. Although the steady state error between reference pitch angle and actual pitch was large at the beginning, the controller still tried to converge to the reference. However, the convergence was not responsive enough. The overshoot seems didn't change very much compared to the inverse loop shaping.

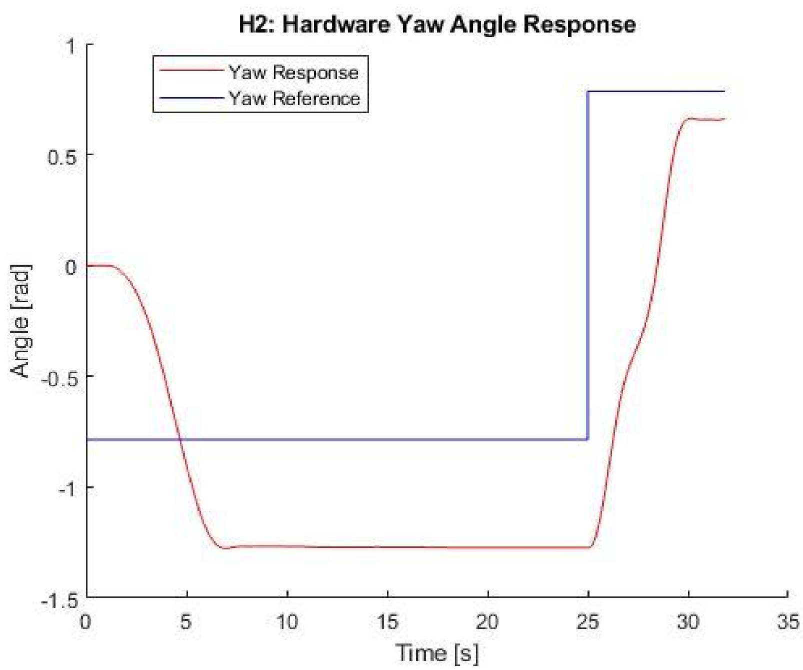

Figure 15. Hardware H2 Yaw Angle Response

The yaw reference tracking was worse than the yaw reference tracking of inverse loop shaping. There was a huge steady state error at the initial steady state; the controller couldn't convert to the reference. Also as Figure 15 shows, the rise time after the square input was also worse than the inverse loop shaping, but there was no overshooting. As it was later discovered the yaw motor had some potential issues, there might not be enough voltage imputed to the yaw motor, causing the worse performance of the yaw angle response. Also, the yaw motor issue caused the system to be shut down if the input voltage exceeds 6V.

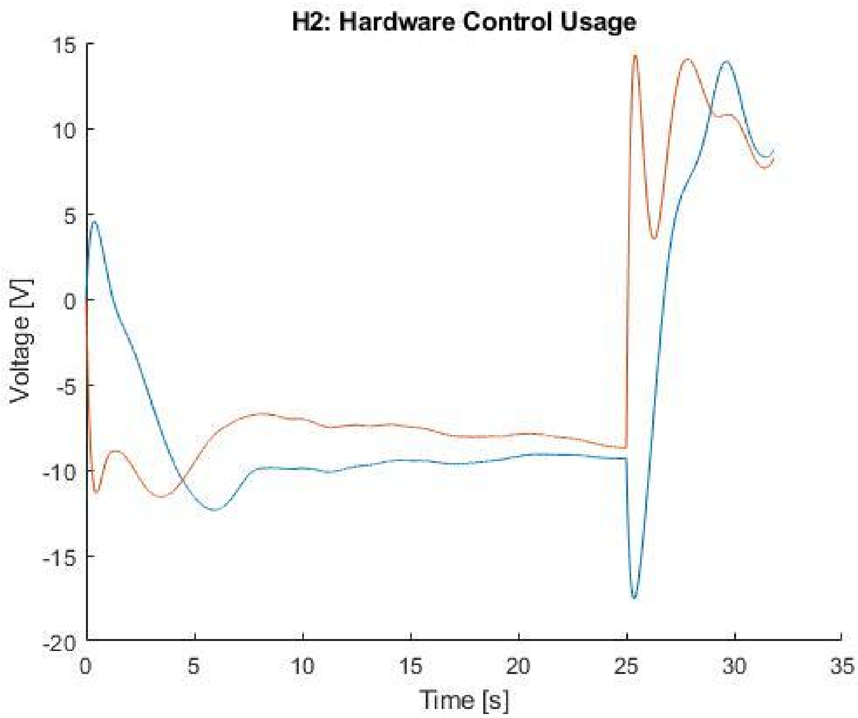

Figure 16. Hardware H2 Control Usage

The voltage usage of the H2 synthesis seems well below the voltage limit of +/-25V. This is much better than the voltage usage of the inverse loop shaping.

4.) Hardware Simulation with 10 Uncertain Samples

For this part of the problem, it followed the same procedure as the one in part 4 of the inverse loop shaping. Figure 17 shows the angle response in radians in relation to the time in second with 5 uncertainty samples for the H2 Synthesis controller. The yaw angle is in blue, and the pitch angle is in red. The pitch response of the uncertain samples was more oscillated than the previous nominal response. The yaw response had a huge initial overshoot which was not present in the nominal response.

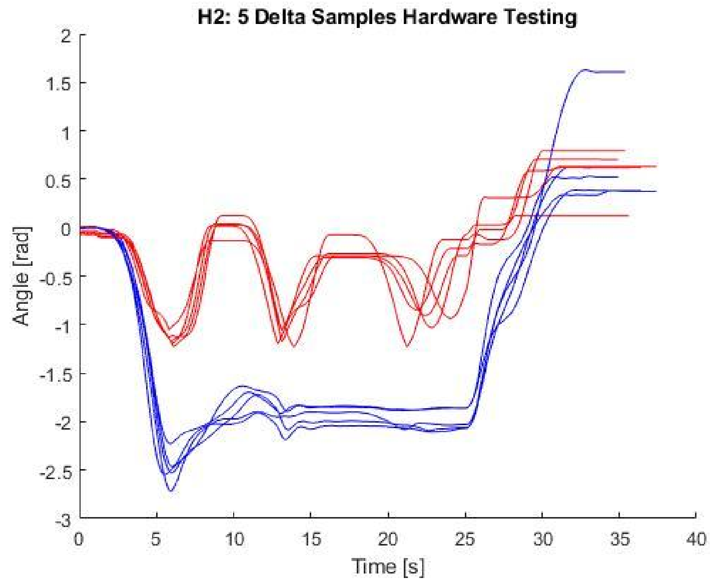

Figure 17. Hardware H2 Synthesis with 5 Uncertainty Samples

Due to the hardware issues, the consecutive 5 runs of uncertainty sample caused the hardware to constantly shut down and create more oscillated results. These consecutive runs were more oscillated than the previous nominal response. As time was limited for each group, each uncertainty run could not be spaced out to achieve better results.

<u>H_∞ Optimal Control</u>

1.) Angles Response and Usage Plots

The H_∞ synthesis followed the same steps as the H2 synthesis, with the same weights and generalized plant model. The only difference was this synthesis used hinfsyn to obtain the final H_∞ controller. As shown in figures 18 and 17, both the pitch and the yaw angle response had less overshoot and oscillations than the result from H2. The control usage for pitch angle was ±30 V, which was larger than the H2 control usage.

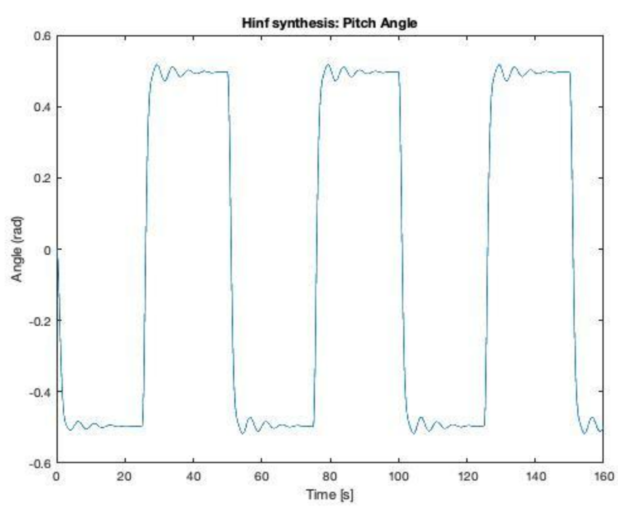

Figure 18. H_∞ Synthesis Nominal Model Pitch Angle Response

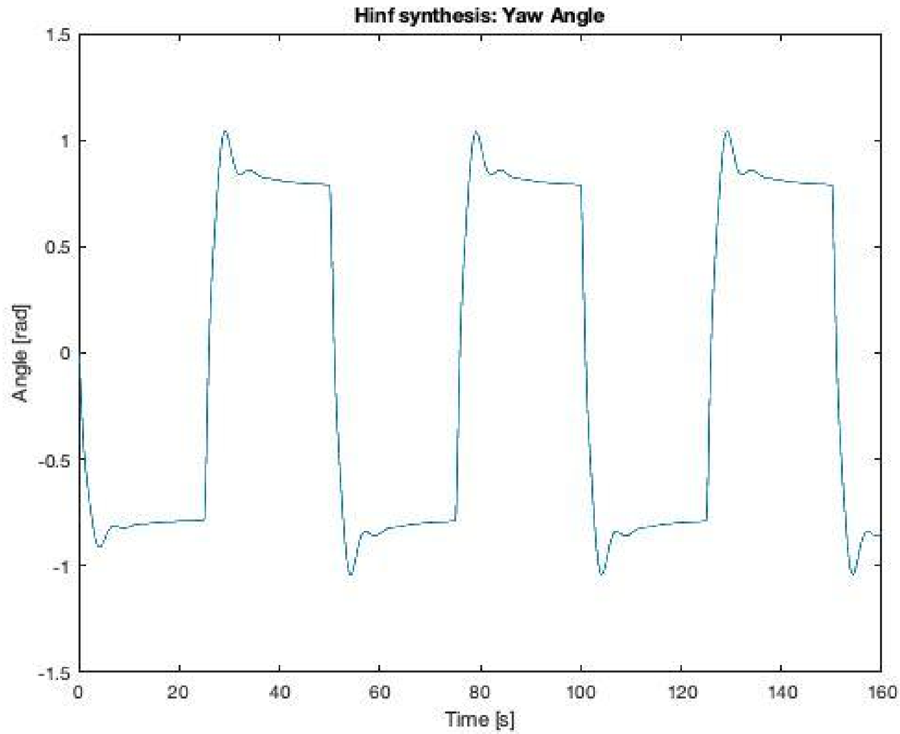

Figure 19. H_∞ Synthesis Nominal Model Yaw Angle Response

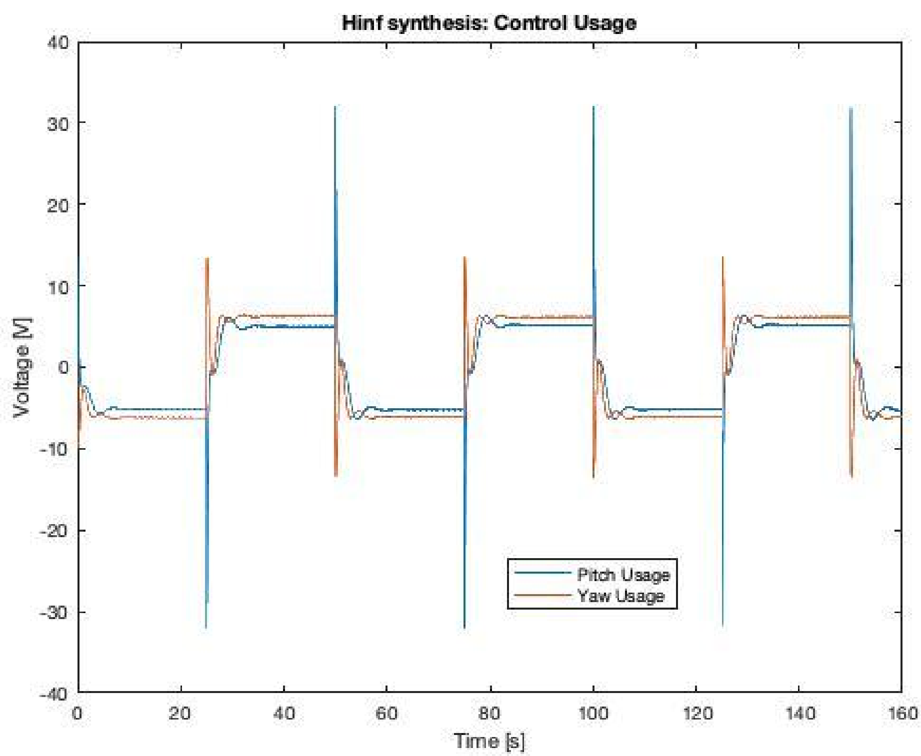

Figure 20. H_∞ Synthesis Nominal Model Control Usage

2.) Robust Stability and Performance Test

The robust stability and performance tests of H_∞ synthesis also followed the same steps as the H2 synthesis. As Figure 21 shows, the system had robust stability with a smaller than one result from robstab. However, the system didn't have robust performance as the result from robustperf was bigger than one. H_∞ synthesis had better robust performance since the result was smaller than H2.

```
RS_Hinf = 1/robstab(N_unc).LowerBound
```

```
RS_Hinf = 0.4222
```

```
RP_Hinf = 1/robustperf(N_unc).LowerBound
```

```
RP_Hinf = 5.4885
```

Figure 21. H_∞ Synthesis Robust Stability and Performance Tests

3.) Hardware Simulations with 0 Uncertainty

Part 3 of the H_∞ synthesis followed the same procedure as the one in the inverse loop shaping.

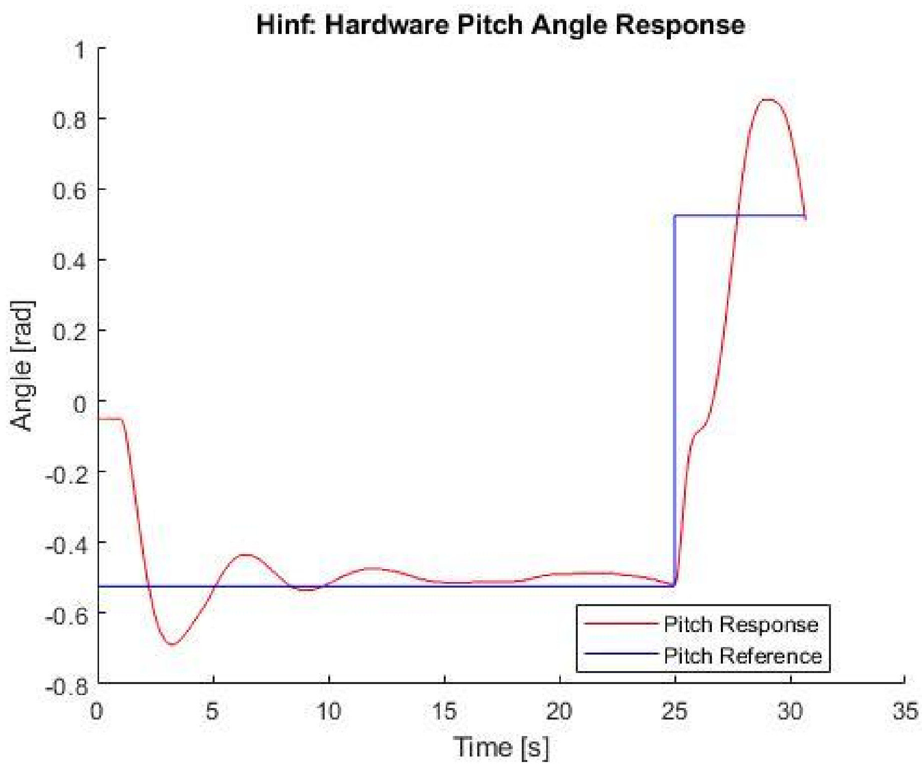

Figure 22. Hardware H_∞ Pitch Angle Response

The H_∞ synthesis results of the pitch angle show that H_∞ had a better ability to track the reference input before the square input (Figure 22). It had smaller oscillations and more steady results towards the end of the initial steady state than the pitch angle response of H2 synthesis.

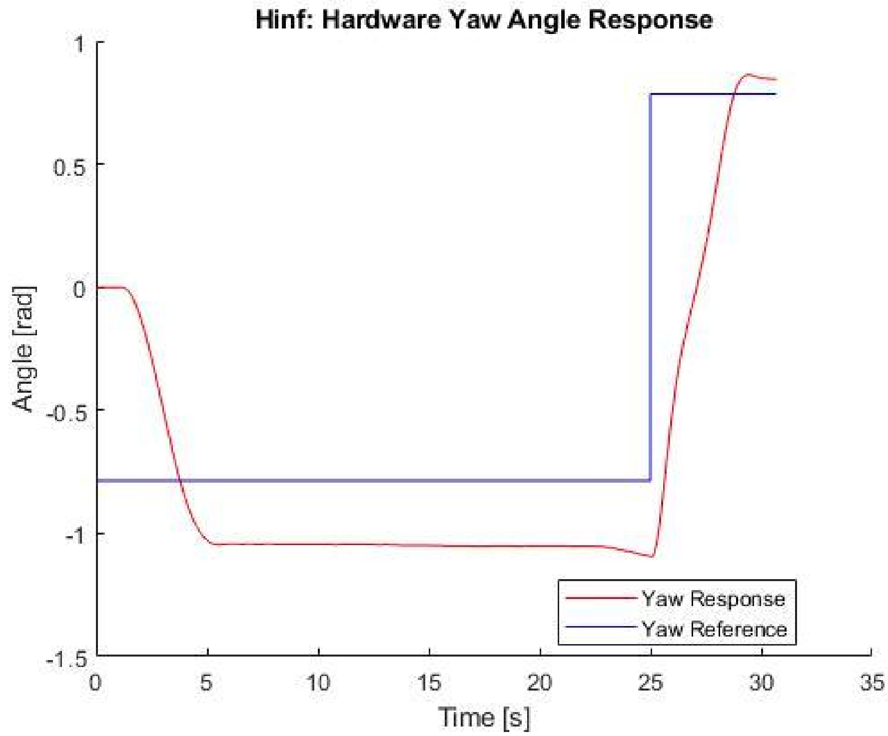

Figure 23. Hardware H$_\infty$ Yaw Angle Response

Due to the hardware issues of the yaw motor, the yaw angle response would not track the reference yaw angle. The initial steady-state error seems smaller than the steady error of the H2 synthesis. It also had a much smaller steady-state error at the square input than the steady error of the H2 synthesis.

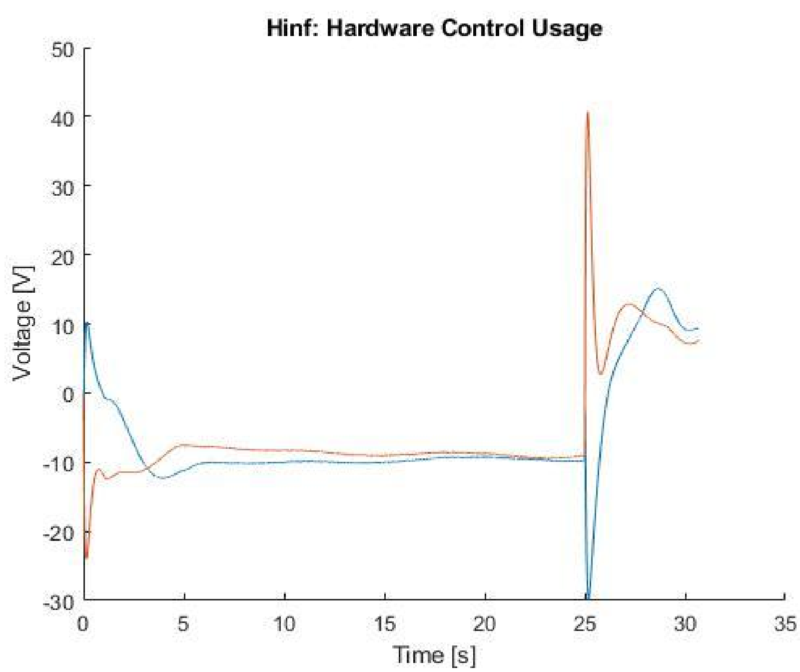

Figure 24. Hardware H$_\infty$ Control Usage

The voltage usage of the yaw motor of the H_∞ synthesis was larger than the voltage usage of the yaw motor of the H2 synthesis. The peak voltage was around 40 V which well exceeded the voltage limit of ±25 V. The nominal voltage usage was very similar to the voltage usage of the actual hardware response.

4.) Hardware Simulation with 10 Uncertain Samples

For this part of the problem, it followed the same procedure as one in the part 4 of the inverse loop shaping. Figure 25 shows the angle response in radians in relation to the time in seconds with 5 uncertainty samples for the H_∞ Synthesis controller. The yaw angle is in blue, and the pitch angle is in red. The pitch response of the uncertain samples was more oscillated than the previous nominal response. The yaw response had a huge initial overshoot which was not present in the nominal response.

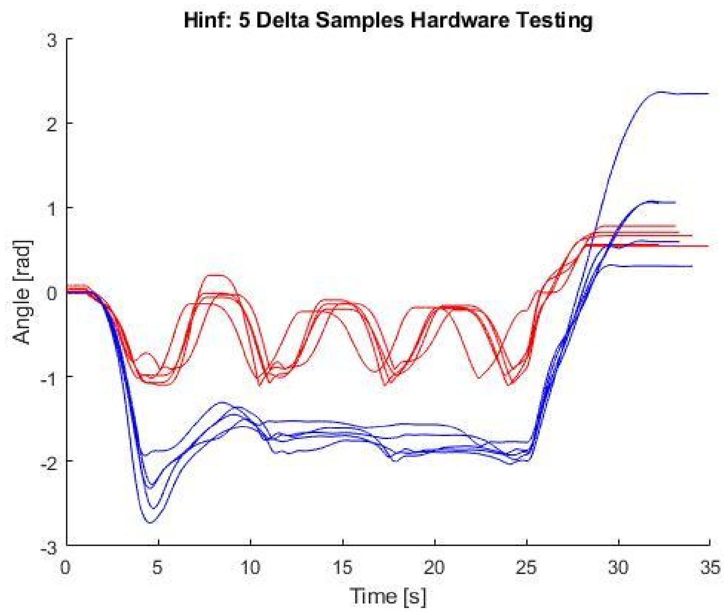

Figure 25. Hardware H_∞ with 5 Uncertainty Samples

The same hardware issues also plagued the H_∞ uncertainty test. Still, the time limit prevents us from doing more runs.

<u>μ-Synthesis</u>

1.) Angles Response and Usage Plots

μ-Synthesis followed a similar procedure as the previous H2 and H_∞. The difference was the μ-Synthesis used the uncertain generalized plant and also used musyn to get the final μ-Synthesis controller.

Figures 26 and 27 show the relationship between angle responses in radians and time in seconds. The pitch angle response had more oscillations than H2 and Hinf, and it takes peak ±30 V control usage. The yaw angle response was the most stable in all four methods, but it takes ±60 V control usage.

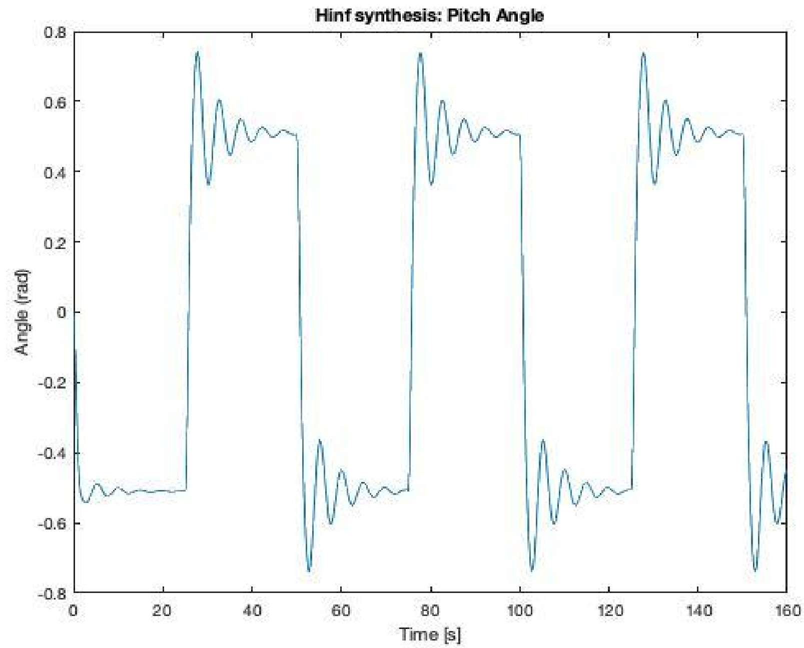

Figure 26. μ-Synthesis Nominal Model Pitch Angle Response

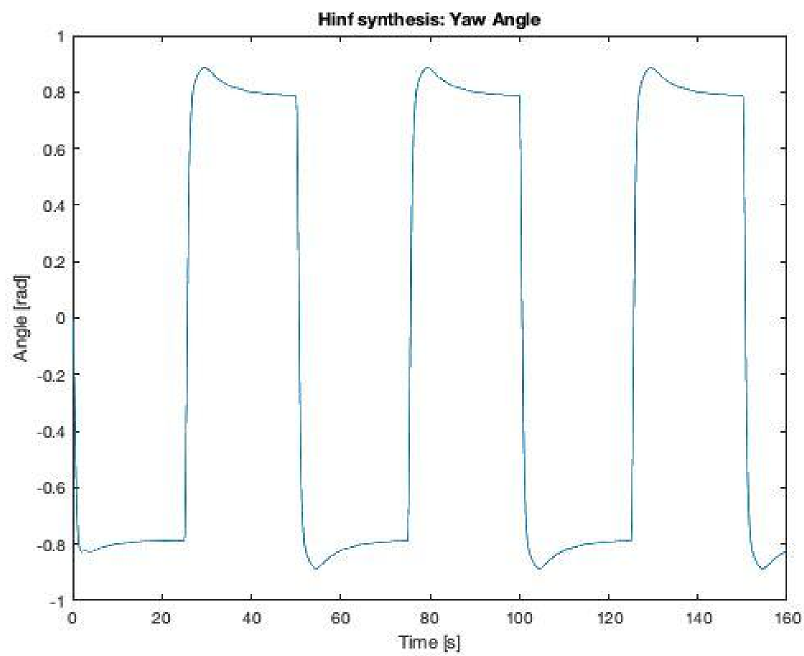

Figure 27. μ-Synthesis Nominal Model Yaw Angle Response

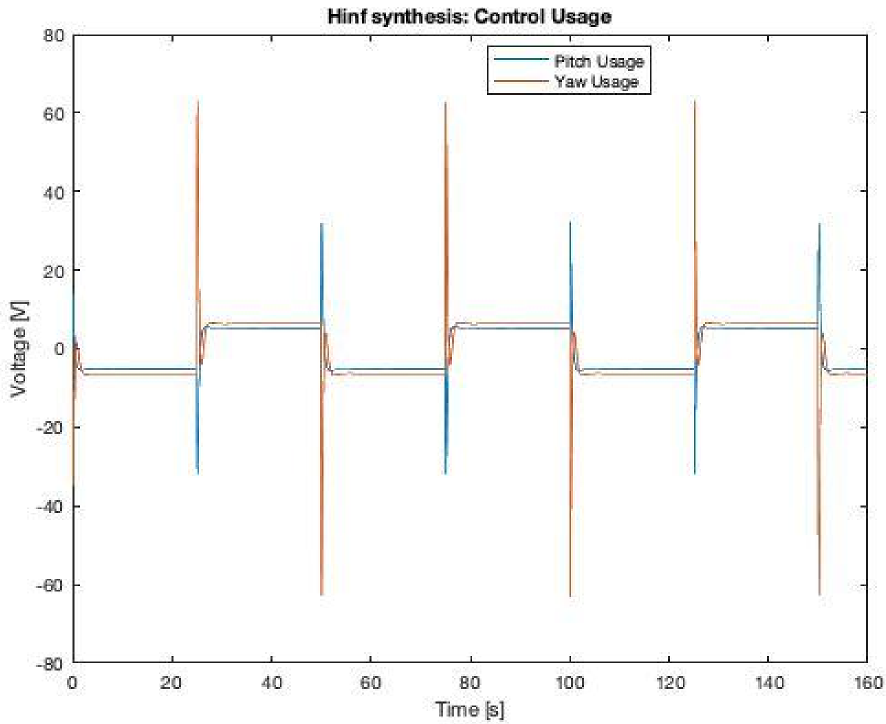

Figure 28. μ-Synthesis Nominal Model Control Usage

2.) Robust Stability and Performance Test

The robust stability and performance tests of μ-Synthesis also followed the same steps as the previous synthesis. As Figure 29 shows, the system had robust stability with a smaller than one result from robstab. However, the system didn't have robust performance as the result from robustperf was bigger than one. In this case, μ-Synthesis had the best robust stability and robust performance.

```
N_unc = minreal(lft(P_unc, Kmu));

2 states removed.

RS_mu = 1/robstab(N_unc).LowerBound

RS_mu = 0.3026

RP_mu = 1/robustperf(N_unc).LowerBound

RP_mu = 2.6789
```

Figure 29. μ-Synthesis Robust Stability and Performance Tests

10 values of Δ_1 and Δ_2 were taken from samples of an uncertain LTI system for the simulation. Figure 30 shows the relationship between angles in radians and time in seconds for 10 uncertainty samples. The pitch angle is in blue and the yaw angle is in red. The angles didn't go unstable for any of the inputs. The controller had robustly stable based on the response from simulation.

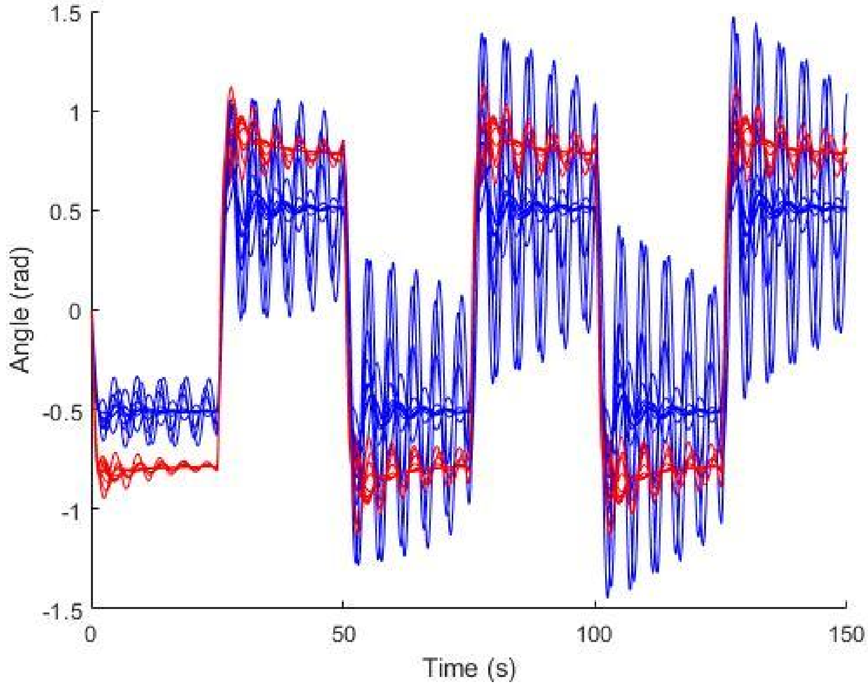

Figure 30. μ-Synthesis with 10 Uncertainty Samples

3.) Hardware Simulations with 0 Uncertainty

Part 3 of the μ-Synthesis followed the same procedure as the one in the inverse loop shaping.

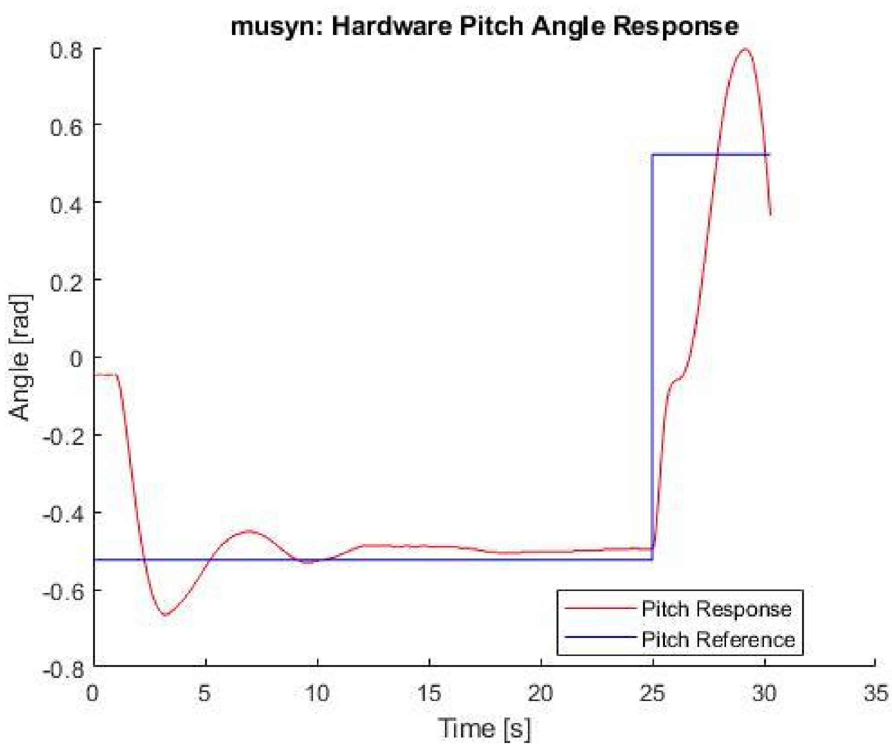

Figure 31. Hardware μ-Synthesis Pitch Angle Response

The μ-Synthesis shares similar results as the H_∞; At the initial steady state, the pitch angle response of the μ-Synthesis seems less oscillated than the pitch angle response of the H_∞. It seems the initial response almost converged to the reference. Also, the overshoot at square input seems smaller than the overshoot of the H_∞.

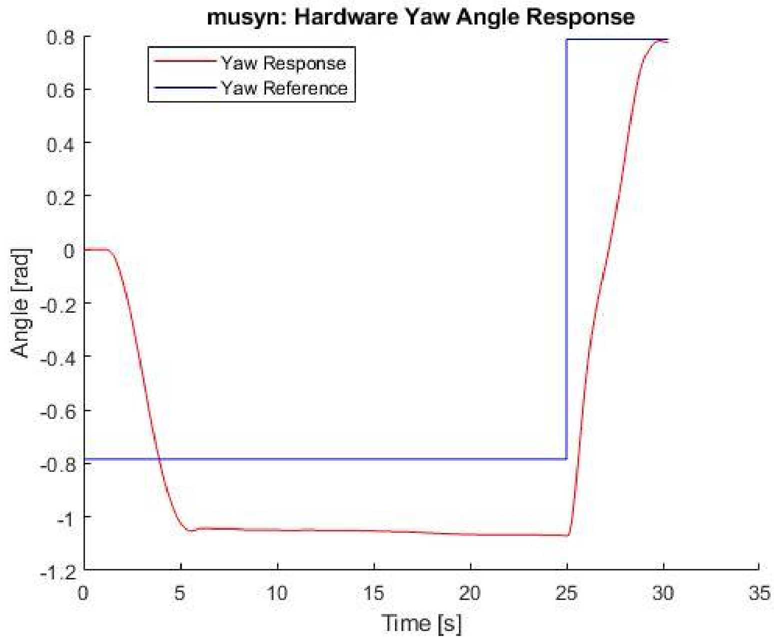

Figure 32. Hardware μ-Synthesis Yaw Angle Response

The initial steady state error seems similar to the steady error of the H_∞ synthesis and was larger than the steady error of the H2 synthesis. Moreover, μ-Synthesis had a better convergence after the square input than both H_∞ and H2 synthesis.

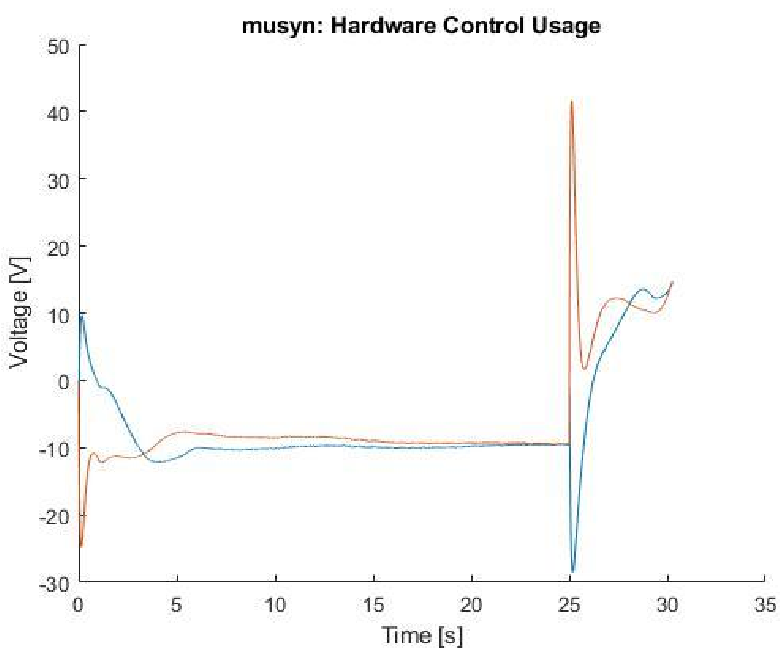

Figure 33. Hardware μ-Synthesis Control Usage

The voltage usage of the yaw motor of the μ-synthesis seems similar to the yaw motor voltage usage of the H_∞ synthesis. The peak voltage was also around 40 V which well exceeded the ±25 V voltage limit. The nominal voltage usage was slightly larger than the voltage usage of the actual hardware response.

4.) Hardware Simulation with 10 Uncertain Samples

For this part of the problem, it followed the same procedure as one in the part 4 of the inverse loop shaping. Figure 34 shows the angle response in radians in relation to the time in second with 5 uncertainty samples for the μ-Synthesis controller. The yaw angle is in blue and the pitch angle is in red. The pitch response of the uncertain samples was more oscillated than the previous nominal response. The yaw response had an even more pronounced huge initial overshoot which was not present in the nominal response.

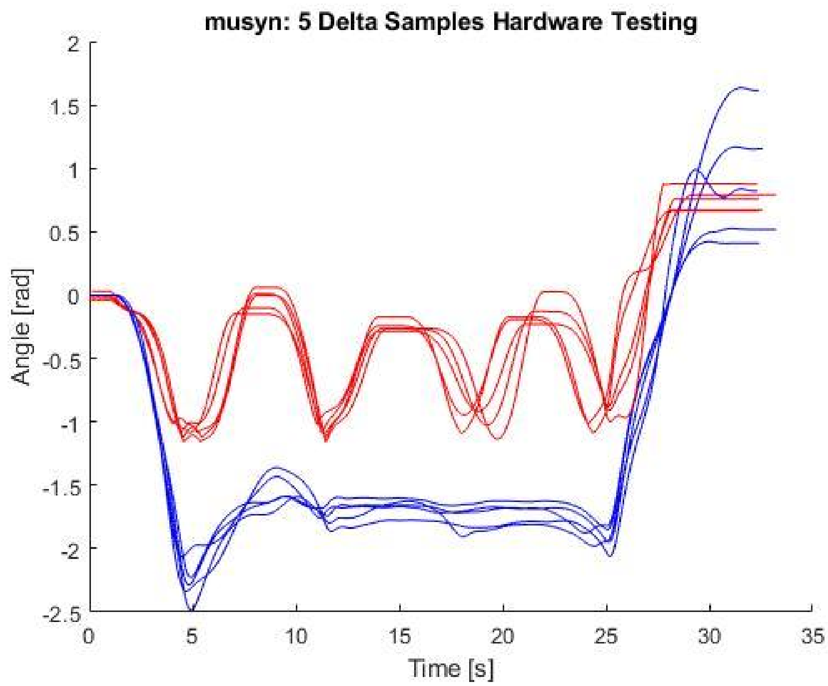

Figure 34. Hardware μ-Synthesis with 5 Uncertainty Samples

Just like the previous tests, hardware issues might contribute to the oscillated results.

<u>Conclusions</u>

The initial nominal plant simulations showed that all synthesis methods have great potential to track the reference with robust stability. The H_∞ synthesis seems to have the least oscillation, and the inverse loop shaping seems the most oscillated. The real hardware simulation showed some similarities. Due to some potential hardware issues, only the inverse loop shaping could maintain a relatively long run time; all the other synthesis methods would not run past the first square input. The inverse loop shaping was still the most oscillated one, and H2 seems to be more oscillated than both H_∞ and μ-Synthesis. However, the hardware issues prevent the yaw motor from converging the yaw angle to the reference.

Work Distribution

Dajun Tao: Controllers Design, Angles Response Plots, Robust Performance and Stability Test and Hardware Simulations

Jiatong Zhang: Nominal Plant Building, Hardware Simulations

Peize Hong: Controller Design, Hardware Simulations