Homework 7: Massless Dynamics

24-760 Robot Dynamics & Analysis
Fall 2021

Name: Solution

For this homework, please compose everything in a single Matlab script, except helper functions. In the main script, you need to include all reasoning (either type it or insert a picture of your hand written result), calculation, and required output (with the same output variable names given in problem statements). Please fill in all the `TODO` sections and clearly label sections based on which part they are for. Please use the precise variable names that we define in the template and do not overwrite them in later sections. If you used any helper functions, please put them together with the main script in a zip file named as andrewID_24760_HW7.zip, where andrewID is your Andrew ID.

Please make sure to use the predefined symbolic variables in the code template, especially the differential state, for example, we defined `q1` and its first and second derivative `dq1` and `ddq1` there. You should be able to complete the homework without defining any new symbolic variables.

Problem 1) Massless Fingers

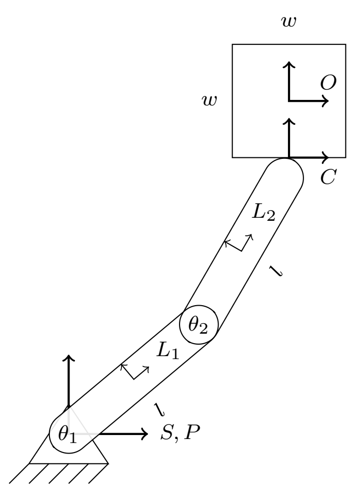

Figure 1: Two joint manipulation robot. For each frame, the label indicates the x-axis.

Consider a planar two link manipulation robot, as shown in Figure 1. Each link has length l, mass m_l, and inertia I_l, while the object is a square with width w, mass m_o, and inertia I_o. The combined state in local coordinates is $q = [\theta_1, \theta_2, x_o, y_o, \phi_o]^T$ (where the object position o is in local coordinates relative to the P frame). Each joint i of the robot is actuated with some torque τ_i. The object is balanced centered on the finger tip with frictional contact. The gravity vector points in the $-y$ direction in the world frame. The S frame is coincident with the P frame. You may use Matlab and the results of prior homework or book/lecture examples to help with the calculations.

1.1) What are the $\bar{M}, \bar{C}, \bar{N}, A, \Upsilon$, and $\dot{A}$ matrices in the equations of motion,

$$
\begin{aligned}
&\bar{M}(q)\ddot{q} + \bar{C}(q,\dot{q})\dot{q} + \bar{N}(q,\dot{q}) + A(q)^T\lambda = \Upsilon \\
&A\ddot{q} + \dot{A}\dot{q} = 0
\end{aligned}
$$

Please compute and save them in the symbolic variables `Mbar`, `Cbar`, `Nbar`, `Y`, `A`, `Nbar`, `dA`, and `EOM` respectively in the script. `EOM` should be a 5 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0]. It is obtained by subtracting Υ on both sides of the equations.

(Hint: Start by writing down the definition of each quantity and develop a list of what kinematic quantities (what rigid transforms g_{ab} and Jacobains J^b_{ab}) are needed. Then, fill in the details for this problem. Remember to use $\bar{G}^T = G^T J^b_{po}$ when multiplying $\dot{x}$.)

Solution: (25 points) Much of this homework set builds on the previous topics and homeworks, so a large part of the difficulty arises from managing all the details in writing the Jacobians, adjoints, transformations, constraint equations, and general calculations. As such, this solution document will leave most of these details to the accompanying commented MATLAB scripts. This document will highlight some of the more high level aspects of solving these problems.

For problem 1.1, each matrix can be found according to the manipulation equations listed in class. The $\hat{M}$ matrix, which maps twist coordinates to kinetic energy, is a block diagonal matrix that is a function of the generalized inertia matrix of each link and the body Jacobians of those links. The $\bar{M}$ matrix maps local coordinates $\dot{q} = [\theta, \dot{x}]^T$ to kinetic energy (rather than twist coordinates), and thus requires $\hat{M}$ as well as the Jacobian J^b_{po}, which maps the local coordinates for the object velocity $\dot{x}$ into the body velocity of the object relative to the stationary palm, V^b_{po}. We have $V^b_{po} = J^b_{po}\dot{x}$.

The $\bar{C}$ and $\bar{N}$ matrices are found as in the equations in the course notes. Υ only includes applied torques τ_1 and τ_2. The A matrix can be found with $A = [-J_h, \bar{G}^T]$. Note that $\bar{G}^T = G^T J^b_{po}$ so that $A\dot{q} = 0$ is equivalent to satisfying the closed loop equation in local coordinates, $J_h\dot{\theta} = G^T J^b_{po}\dot{x}$.

1.2) What is the expression of the instantaneous acceleration ($\ddot{q}$) of the object and joints and what is the numerical value of it if we apply torque of $\tau_1 = 200$ Nm and $\tau_2 = 0$ Nm? Assume $l = 0.1$ m, $m_l = 1$ kg, $I_l = 8.33 \times 10^{-4}$ kg·m^2, $w = 0.2$ m, $m_o = 24$ kg, $I_o = 0.16$ kg·m^2, $g = -9.81$ m/s^2, with initial conditions $q = [90°, -90°, 0.1, 0.2, 0°]^T$ and $\dot{q} = 0$?

Please compute and save them in the symbolic variable `ddq_massive` and numerical variable `ddq_eval_massive`.

(Hint: substitute these values in before inverting any matrices.)

Solution: (15 points) We can solve for the accelerations of this system by substituting each of these values, inverting the block matrix of $\bar{M}$ and A, and extracting the accelerations to get

$$\ddot{q} = \begin{bmatrix} 1829.3 \\ -1927.9 \\ -73.2 \\ -9.9 \\ -1097.6 \end{bmatrix}$$

1.3) If we assume that both links are massless, so $m_l = I_l = 0$, what are the new equations of motion?

Please compute and save it in the symbolic variable `EOM_massless`. `EOM_massless` should be a 5 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0]. It is obtained by subtracting Υ on both sides of the equations.

Solution: (15 points) To find out the EOM for massless dynamics, you only need to substitute m_l and I_l with zeros for the EOM in Problem 1.1. The assumption of massless links greatly simplifies the equations of motion, which reduce to only terms with m_o, λ, or τ. See the MATLAB script for the exact equations.

1.4) What is the instantaneous acceleration ($\ddot{q}$) of the object and joints if we take the links to be massless ($m_l = I_l = 0$)? Use the same values as in Problem 1.2 for all other quantities. What is the percent error ($\frac{\text{abs}(\ddot{q}-\ddot{q}_{\text{massless}})}{\ddot{q}} \times 100$) in each acceleration term from this massless assumption?

Please compute and save them in the symbolic variable `ddq_massive` and numerical variables `ddq_eval_massless` and `error_from_massless`.

Solution: (15 points) We have

$$\ddot{q}_{massless} = \begin{bmatrix} 2083.3 \\ -2181.4 \\ -83.3 \\ -9.8 \\ -1250.0 \end{bmatrix}$$

The error is:

$$err = \begin{bmatrix} 13.8885\% \\ 13.1422\% \\ 13.8885\% \\ 0.6804\% \\ 13.8885\% \end{bmatrix}$$

1.5) If instead the contact was taken as frictionless (e.g. after a stick-slip transition), what are the new A, $\dot{A}$, and equations of motion?

Please compute and save them in the symbolic variables `A_frictionless`, `dA_frictionless`, and `EOM_frictionless` respectively in the script. `EOM_frictionless` should be a 5 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0]. It is obtained by subtracting Υ on both sides of the equations.

Solution: (15 points) Frictionless contact only has the contact normal in its wrench basis, so the A matrix is reduced to one row, which maps the contact normal force into generalized forces on each of the coordinates. Every other element of the manipulator equation is unchanged.

1.6) In the frictionless case with massless links, what's the new EOM? What's the rank of the block matrix

$$\begin{bmatrix} \bar{M} & A^T \\ A & 0 \end{bmatrix}$$

Explain why we cannot compute the instantaneous acceleration. What could be taken as massless that would still allow us to compute the instantaneous acceleration?

Please compute and save them in the symbolic variable `EOM_massless_frictionless` and numerical variable `rank_block` respectively in the script. `EOM_massless_frictionless` should be a 5 by 1 symbolic matrix that is equal to [0; 0; 0; 0; 0]. It is obtained by subtracting Υ on both sides of the equations.

Solution: (15 points) If we assume both massless link and frictionless contact, the block matrix

$$\begin{bmatrix} M & A^T \\ A & 0 \end{bmatrix}$$

becomes singular. Physically, the combination of these assumptions is invalid because there is no inertia seen by the fingertip in the contact tangential direction, and so accelerations in this direction are unbounded. If we assume that the first link is massless, motion of this link will have to move the inertia of the other link and thus the accelerations will become bounded again.