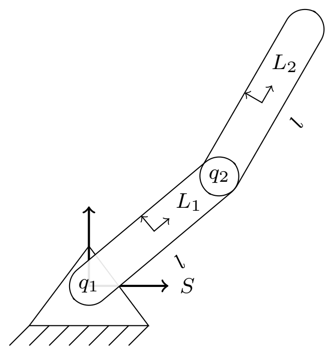
Homework 12: Trajectory Optimization

24-760 Robot Dynamics & Analysis
Fall 2021

Name: Solutions

Problem 1) Two-link Robot



Consider a planar two link robot, as shown above. Each link has length $l = 1$, mass $m_l = 1$, and inertia $I_l = 1/12$. Each joint i of the robot is actuated with some torque τ_i. The gravity vector points in the $-y$ direction in the world frame. You may use code from prior homeworks to help with the calculations.

1.1) Write out in equations a direct collocation trajectory optimization problem for this system that finds a trajectory from an initial state $q(t_0) = [-\frac{\pi}{2}, 0]^T$ and $\dot{q}(t_0) = [0, 0]^T$ to final state $q(t_f) = [\frac{\pi}{2}, 0]^T$ and $\dot{q}(t_f) = [0, 0]^T$. Use fixed timesteps of 20 ms, a total duration of 1.5 s, and an objective function to minimize the thermal cost of actuation, given by $\int \tau^T \tau dt$. Use linear interpolation for control and acceleration dynamics, and quadratic interpolation for velocity and state trajectories. How many decision variables and how many constraints are there?

Solution: (30 points) The trajectory optimization problem can be written as

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=0}^{N-1} h \frac{\tau^T(t_{k+1})\tau(t_{k+1}) + \tau^T(t_k)\tau(t_k)}{2} \\
\text{with respect to} \quad & q(t_k), \dot{q}(t_k), \tau(t_k), \ \forall\, k \in [1, N] \\
\text{such that} \quad & \dot{q}(t_{k+1}) = \dot{q}(t_k) + h\left(\frac{f(t_{k+1}) + f(t_k)}{2}\right) \\
& q(t_{k+1}) = q(t_k) + h\left(\frac{\dot{q}(t_{k+1}) + \dot{q}(t_k)}{2}\right) \\
& [q(1), \dot{q}(1)]^T = \left[-\frac{\pi}{2}, 0, 0, 0\right]^T \\
& [q(N), \dot{q}(N)]^T = \left[\frac{\pi}{2}, 0, 0, 0\right]^T
\end{aligned}$$

where $h = 0.02$ and $N = 76$ are the time step and number of collocation points, respectively. There are a total of 456 decision variables: the position, velocity, and control input for each of the two joints are described by 6 variables at each collocation point. Defining each of these variables for 76 collocation points amounts to 456 total variables. There are a total of 308 constraints: position and velocity collocation constraints for each joint are described by 4 constraints for each of the 75 finite elements, which amounts to 300 constraints. There are 8 total boundary constraints (initial and final position and velocity for each joint) to make 308 total equality constraints. For this problem there are no inequality constraints.

1.2) Now implement this problem in Matlab using `fmincon`. Write separate functions to handle the objective and constraints. Attached is a helper function that returns the $M, C, N,$ and Υ matrices for a given q, $\dot{q}$, and τ, as well as a function that will animate the results. Note that `fmincon` may take several minutes to complete. We recommend using the following options (though you may need to change `MaxFunctionEvaluations`):

```
options = optimoptions('fmincon','Display', 'iter', 'MaxFunctionEvaluations', 1e5);
```

These tutorials may also help:

https://www.mathworks.com/help/optim/ug/fmincon.html

https://www.mathworks.com/help/optim/ug/nonlinear-equality-and-inequality-constraints.html

https://www.mathworks.com/help/optim/ug/example-nonlinear-constrained-minimization.html

Solution: (40 points) See attached code.

1.3) Run the optimization for both the original problem and again with the added constraint that $q_i \in (-\frac{3\pi}{4}, \frac{3\pi}{4})$. How do the results change, in terms of trajectory and cost? Submit a figure of each trajectory using the animation script.

Solution: (30 points) See attached code and Fig. 1 and 2 . The trajectory with inequality constraints on the joint angles cannot take advantage of the singular position as effectively as the unconstrained solution. The end effector remains further from the origin over the course of the trajectory and as a result yields a higher thermal cost (119.76 versus 79.04).

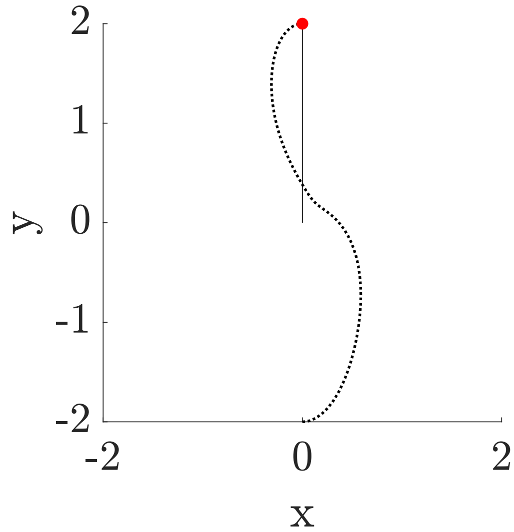


Figure 1: Optimized swing up trajectory with no joint limits.

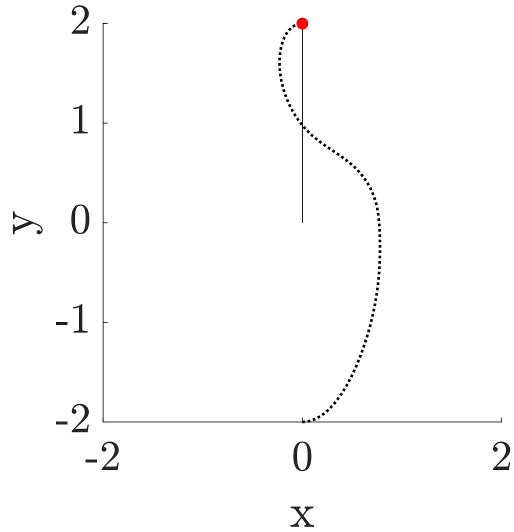


Figure 2: Optimized swing up trajectory with joint limits.