Homework 3: Actuators

24-775 Robot Design & Experimentation

Name: Peize Hong

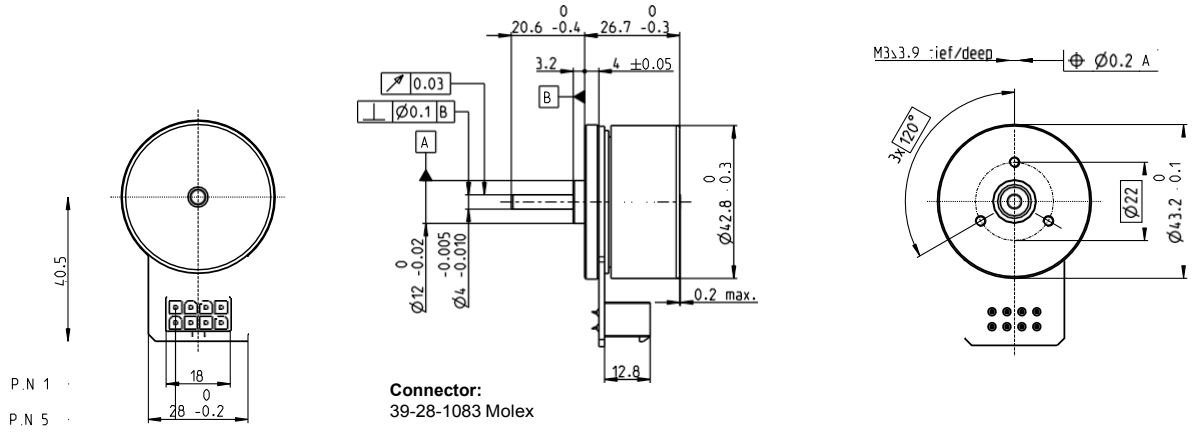
You are designing an electric wheelchair for a friend, and he has asked for you to make one that can win a drag race. To do this you must select the motor and gearbox so that the wheelchair can travel the farthest in 5 seconds. The specifications are:

Mass of wheelchair + occupant: 100 kg
Radius of wheel: 0.5 m
Battery voltage: 30 V

You may ignore friction, damping, inductance, thermodynamics, and aerodynamics to only consider the inertial terms. Upload your code or hand calculations with your answers.

a) Write out the equations of motion for this system, based on the acceleration of the wheelchair and the basic motor model.
b) Either algebraically, symbolically, or numerically solve for the forward displacement of the wheelchair with the following conditions:
   Motor: Maxon EC45-Flat 70W 30V, part # 402685 (data sheet attached)
   Gear ratio: 1:1
   Initial Conditions: $x = 0$, $dx/dt = 0$
   How far did the wheelchair travel? You may use Matlab, Python, or other software.
   Hint: In Matlab, instructions on solving differential equations symbolically:
   https://www.mathworks.com/help/symbolic/solve-a-single-differential-equation.html
   and numerically:
   https://www.mathworks.com/help/matlab/math/solve-nonstiff-odes.html
c) Plot the final displacement for all gear ratios from 1:1 to 100:1. What is the optimal gear ratio?
d) Now consider the motor selection question. Of the four EC45-Flat 70W motors in the attached datasheet, which will move the wheelchair the farthest in 5 seconds? (Pair each motor with its own optimal gear ratio)
e) Gears only come in certain ratios. Based on the options for GP 42 C gearboxes in the attached datasheet, what is the optimal motor+gearbox combination?

EC 45 flat ∅42.8 mm, brushless, 70 Watt



maxon EC motor

Stock program
Standard program
Special program (on request)

Part Numbers

	with Hall sensors	397172	402685	402686	402687
Motor Data (provisional)					
	Values at nominal voltage				
1	Nominal voltage	V	24	30	36	48
2	No load speed	rpm	6110	6230	6330	3440
3	No load current	mA	234	194	166	48.1
4	Nominal speed	rpm	4860	4990	5080	2540
5	Nominal torque (max. continuous torque)	mNm	128	112	108	134
6	Nominal current (max. continuous current)	A	3.21	2.36	1.93	0.936
7	Stall torque	mNm	1460	1170	1100	915
8	Stall current	A	39.5	25.8	20.7	6.97
9	Max. efficiency	%	85	84	83	84
	Characteristics					
10	Terminal resistance phase to phase	Ω	0.608	1.16	1.74	6.89
11	Terminal inductance phase to phase	mH	0.463	0.691	0.966	5.85
12	Torque constant	mNm / A	36.9	45.1	53.3	131
13	Speed constant	rpm / V	259	212	179	72.7
14	Speed / torque gradient	rpm / mNm	4.26	5.44	5.85	3.82
15	Mechanical time constant	ms	8.07	10.3	11.1	7.24
16	Rotor inertia	gcm²	181	181	181	181

Specifications

Thermal data

17	Thermal resistance housing-ambient	3.56 K/W
18	Thermal resistance winding-housing	4.1 K/W
19	Thermal time constant winding	29.6 s
20	Thermal time constant motor	178 s
21	Ambient temperature	-40 ... +100°C
22	Max. winding temperature	+125°C

Mechanical data (preloaded ball bearings)

23	Max. speed		10000 rpm
24	Axial play at axial load	< 4.0 N	0 mm
		> 4.0 N	0.14 mm
25	Radial play		preloaded
26	Max. axial load (dynamic)		3.8 N
27	Max. force for press fits (static)		50 N
	(static, shaft supported)		1000 N
28	Max. radial load, 5 mm from flange		21 N

Other specifications

29	Number of pole pairs	8
30	Number of phases	3
31	Weight of motor	141 g

Values listed in the table are nominal.

Connection

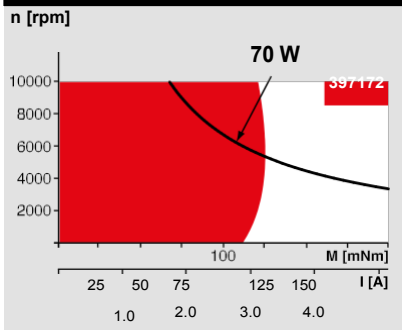
Pin 1	Hall sensor 1*
Pin 2	Hall sensor 2*
Pin 3	V_{Hall} 4.5 ... 18 VDC
Pin 4	Motor winding 3
Pin 5	Hall sensor 3*
Pin 6	GND
Pin 7	Motor winding 1
Pin 8	Motor winding 2

*Internal pull-up (7 … 13 kΩ) on pin 3
Wiring diagram for Hall sensors see p. 43

Cable

Connection cable Universal, L = 500 mm	**339380**
Connection cable to EPOS, L = 500 mm	**354045**

Operating Range



Comments

Continuous operation
In observation of above listed thermal resistance (lines 17 and 18) the maximum permissible winding temperature will be reached during continuous operation at 25°C ambient.
= Thermal limit.

Short term operation
The motor may be briefly overloaded (recurring).

Assigned power rating

maxon Modular System
Overview on page 28–36

Planetary Gearhead
∅42 mm
3 - 15 Nm
Page 347

Spur Gearhead
∅45 mm
0.5 - 2.0 Nm
Page 349

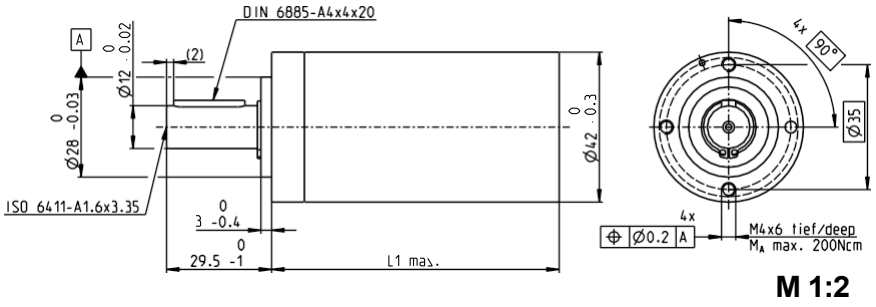
Encoder MILE
256 - 2048 CPT,
2 channels
Page 388

Recommended Electronics:

Notes	Page 32
ESCON 36/3 EC	427
ESCON Mod. 50/4 EC-S	427
ESCON Module 50/5	427
ESCON 50/5	428
DEC Module 50/5	430
EPOS2 Module 36/2	434
EPOS2 24/5, 50/5	435
EPOS2 P 24/5	438
EPOS4 Module/CB 50/5	442
MAXPOS 50/5	447

Planetary Gearhead GP 42 C ∅42 mm, 3–15 Nm

Ceramic Version



M 1:2

Technical Data

Planetary Gearhead				straight teeth
Output shaft				stainless steel
Bearing at output				preloaded ball bearings
Radial play, 12 mm from flange				max. 0.06 mm
Axial play at axial load	< 5 N			0 mm
	> 5 N			max. 0.3 mm
Max. axial load (dynamic)				150 N
Max. force for press fits				300 N
Direction of rotation, drive to output				=
Max. continuous input speed				8000 rpm
Recommended temperature range				-40…+100°C
Number of stages	1	2	3	4
Max. radial load, 12 mm from flange	120 N	240 N	360 N	360 N

Stock program
Standard program
Special program (on request)

Part Numbers	203113	203119	203120	203129

1	Reduction		3.5:1	12:1	26:1	43:1	81:1	156:1	150:1	285:1	441:1	756:1
2	Absolute reduction		7/2	49/4	26	343/8	2197/27	156	2401/16	15379/54	441	756
10	Mass inertia	gcm²	14	15	9.1	15	9.4	9.1	15	15	14	14
3	Max. motor shaft diameter	mm	10	10	8	10	8	8	10	10	10	10
	Part Numbers		**203114**	**203116**	260552*	203121	203125	260553*	203130	203134	203138	203142
1	Reduction		4.3:1	15:1	36:1	53:1	91:1	216:1	186:1	319:1	488:1	936:1
2	Absolute reduction		13/3	91/6	36/1	637/12	91	216/1	4459/24	637/2	4394/9	936
10	Mass inertia	gcm²	9.1	15	5.0	15	15	5.0	15	15	9.4	9.1
3	Max. motor shaft diameter	mm	8	10	4	10	10	4	10	10	8	8
	Part Numbers		260551*	203117		203122	203126		**203131**	203135	**203139**	260554*
1	Reduction		6:1	19:1		66:1	113:1		230:1	353 :1	546:1	1296:1
2	Absolute reduction		6/1	169/9		1183/18	338/3		8281/36	28561/81	546	1296/1
10	Mass inertia	gcm²	4.9	9.4		15	9.4		15	9.4	14	5.0
3	Max. motor shaft diameter	mm	4	8		10	8		10	8	10	4
	Part Numbers			203118		**203123**	203127		203132	203136	203140	
1	Reduction			21:1		74:1	126:1		257:1	394:1	676:1	
2	Absolute reduction			21		147/2	126		1029/4	1183/3	676	
10	Mass inertia	gcm²		14		15	14		15	15	9.1	
3	Max. motor shaft diameter	mm		10		10	10		10	10	8	
4	Number of stages		1	2	2	3	3	3	4	4	4	4
5	Max. continuous torque	Nm	3.0	7.5	7.5	15.0	15.0	15.0	15.0	15.0	15.0	15.0
6	Max. intermittent torque at gear output	Nm	4.5	11.3	11.3	22.5	22.5	22.5	22.5	22.5	22.5	22.5
7	Max. efficiency	%	90	81	81	72	72	72	64	64	64	64
8	Weight	g	260	360	360	460	460	460	560	560	560	560
9	Average backlash no load	°	0.6	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
11	Gearhead length L1**	mm	41.0	55.5	55.5	70.0	70.0	70.0	84.5	84.5	84.5	84.5

*no combination with EC 45 (150/250 W) and EC-i 40
**for EC 45 flat L1 is -3.6 mm

maxon Modular System

+ Motor	Page	+ Sensor	Page	Brake	Page	Overall length [mm] = Motor length + gearhead length + (sensor/brake) + assembly parts									
RE 35, 90 W	130					112.1	126.6	126.6	141.1	141.1	141.1	155.6	155.6	155.6	155.6
RE 35, 90 W	130	MR	405			123.5	138.0	138.0	152.5	152.5	152.5	167.0	167.0	167.0	167.0
RE 35, 90 W	130	HED_ 5540	413/415			132.8	147.3	147.3	161.8	161.8	161.8	176.3	176.3	176.3	176.3
RE 35, 90 W	130	DCT 22	421			130.2	144.7	144.7	159.2	159.2	159.2	173.7	173.7	173.7	173.7
RE 35, 90 W	130			AB 28	458	148.2	162.7	162.7	177.2	177.2	177.2	191.7	191.7	191.7	191.7
RE 35, 90 W	130	HED_ 5540	413/415	AB 28	458	165.4	179.9	179.9	194.4	194.4	194.4	208.9	208.9	208.9	208.9
RE 40, 150 W	132					112.1	126.6	126.6	141.1	141.1	141.1	155.6	155.6	155.6	155.6
RE 40, 150 W	132	MR	405			123.5	138.0	138.0	152.5	152.5	152.5	167.0	167.0	167.0	167.0
RE 40, 150 W	132	HED_ 5540	413/416			132.8	147.3	147.3	161.8	161.8	161.8	176.3	176.3	176.3	176.3
RE 40, 150 W	132	HEDL 9140	419			166.2	180.7	180.7	195.2	195.2	195.2	209.7	209.7	209.7	209.7
RE 40, 150 W	132			AB 28	458	148.2	162.7	162.7	177.2	177.2	177.2	191.7	191.7	191.7	191.7
RE 40, 150 W	132			AB 28	459	156.2	170.7	170.7	185.2	185.2	185.2	199.7	199.7	199.7	199.7
RE 40, 150 W	132	HED_ 5540	413/416	AB 28	458	165.4	179.9	179.9	194.4	194.4	194.4	208.9	208.9	208.9	208.9
RE 40, 150 W	132	HEDL 9140	419	AB 28	459	176.7	191.2	191.2	205.7	205.7	205.7	220.2	220.2	220.2	220.2
EC 40, 170 W	219					121.1	135.6	135.6	150.1	150.1	150.1	164.6	164.6	164.6	164.6
EC 40, 170 W	219	HED_ 5540	414/416			144.5	159.0	159.0	173.5	173.5	173.5	188.0	188.0	188.0	188.0
EC 40, 170 W	219	Res 26	422			148.3	162.8	162.8	177.3	177.3	177.3	191.8	191.8	191.8	191.8
EC 40, 170 W	219			AB 32	460	163.8	178.3	178.3	192.8	192.8	192.8	207.3	207.3	207.3	207.3
EC 40, 170 W	219	HED_ 5540	414/416	AB 32	460	182.2	196.7	196.7	211.2	211.2	211.2	225.7	225.7	225.7	225.7
EC 45, 150 W	220					152.3	166.8	166.8	181.3	181.3	181.3	195.8	195.8	195.8	195.8
EC 45, 150 W	220	HEDL 9140	419			167.9	182.4	182.4	196.9	196.9	196.9	211.4	211.4	211.4	211.4
EC 45, 150 W	220	Res 26	422			152.3	166.8	166.8	181.3	181.3	181.3	195.8	195.8	195.8	195.8
EC 45, 150 W	220			AB 28	459	159.7	174.2	174.2	188.7	188.7	188.7	203.2	203.2	203.2	203.2
EC 45, 150 W	220	HEDL 9140	419	AB 28	459	176.7	191.2	191.2	205.7	205.7	205.7	220.2	220.2	220.2	220.2
EC 45, 250 W	221					185.1	199.6	199.6	214.1	214.1	214.1	228.6	228.6	228.6	228.6
EC 45, 250 W	221	HEDL 9140	419			200.7	215.2	215.2	229.7	229.7	229.7	244.2	244.2	244.2	244.2
EC 45, 250 W	221	Res 26	422			185.1	199.6	199.6	214.1	214.1	214.1	228.6	228.6	228.6	228.6
EC 45, 250 W	221			AB 28	459	192.5	207.0	207.0	221.5	221.5	221.5	236.0	236.0	236.0	236.0
EC 45, 250 W	221	HEDL 9140	419	AB 28	459	209.5	224.0	224.0	238.5	238.5	238.5	253.0	253.0	253.0	253.0

Option
With Cable and Connector
(Ambient temperature -20 ... +100°C)

(a)

V_s : source voltage V_R : resistance voltage i : current K_V : Speed Constant J : MoI
r : wheel radius V_e : E.M.F voltage τ : Input torque w : Rotational speed f : efficiency
k_t : torque constant R : Resistance τ_t : Output torque K_e : electric constant

EOM Electrical : $V_s - V_R - V_e = 0$

$$fV_s - Ri - k_e w = 0$$

EOM Mechanical $\tau_t = \tau \longrightarrow \tau_t = J\dot{w}$, $\tau = k_t i$

$$\therefore k_t i = J\dot{w}$$

$$\therefore i = \frac{fV_s - k_e w}{R} \text{ (from EOM electrical)} \quad (2)$$

Plug (2) to (1), $$k_t \cdot \frac{fV_s - k_e w}{R} = J\dot{w}$$

Combined EOM: $\overbrace{\frac{k_t V_s f}{R}}^{\text{Input}} - \frac{k_t k_e w}{R} = J\dot{w}$ ← output

With Gear ratio : $$\frac{k_t V_s f}{\frac{J}{N^2} R} - \frac{k_t k_e}{\frac{J}{N^2} R}\dot{\theta} = \ddot{\theta}$$

(b)

$$\frac{1}{K_v} = k_e \qquad k_v = 212$$

$$k_e = \frac{1}{212 \cdot 0.105}$$

$$k_e = 0.0449$$

$$k_t = 0.0451$$

$$\frac{0.0451 \cdot 30 \cdot 0.84}{(181\times10^{-7}+25)\cdot 1.16} - \frac{0.0451 \cdot 0.0449}{(181\times10^{-7}+25)\cdot 1.16}\dot{\theta} = \ddot{\theta}$$

$$\begin{cases} x(0)=0 \\ \dot{x}(0)=0 \end{cases}$$

$$0.0391 - 6.98\times10^{-5}\dot{\theta} = \ddot{\theta}$$

$$t=5 \quad x = 0.245\ m$$

(c)

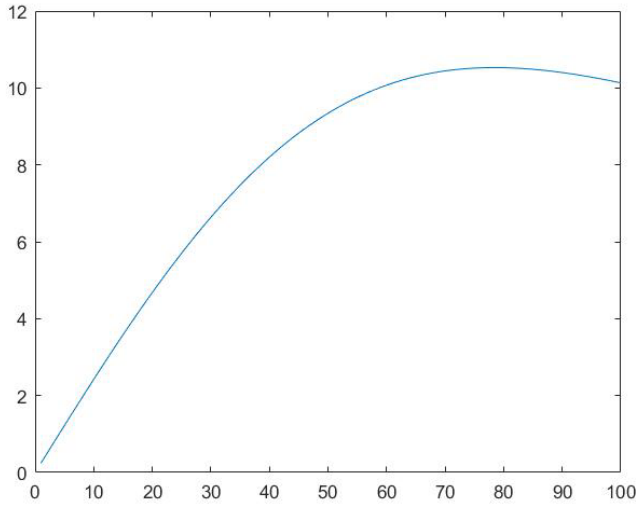
$$\frac{k_t V_s}{\frac{J}{N^2 R}} - \frac{k_t k_e}{\frac{J}{N^2 R}}\dot{\theta} = \ddot{\theta}$$

$$r \cdot \theta \cdot \frac{1}{N} = x$$

$$N = 1, 2, 3, \ldots 100$$

Optimal $N = 78$ $\quad 78:1$ is the best

Maximum distance run : $10.53\ m$

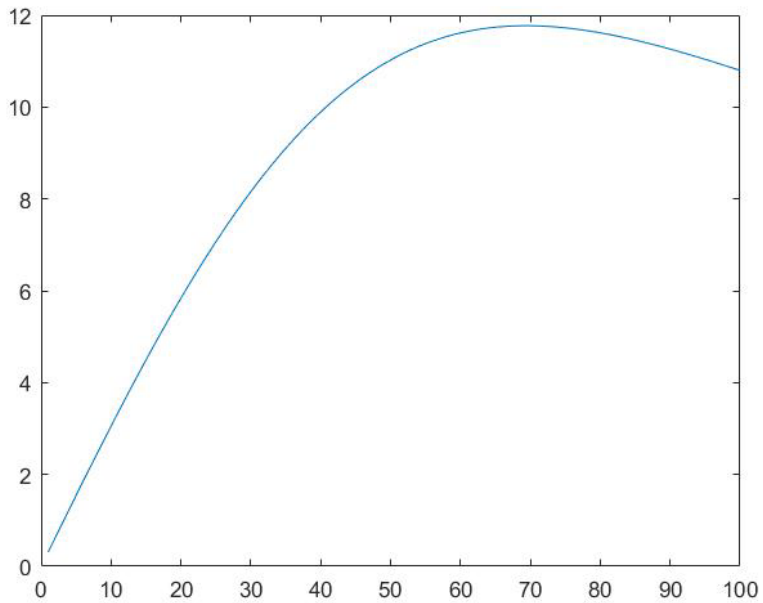


(d) Motor 397172: $K_t = 36.9 \times 10^{-3} = 0.0369$

$K_e = \frac{1}{259 \cdot 0.105} = 0.0368 \approx K_t$

$R = 0.608$

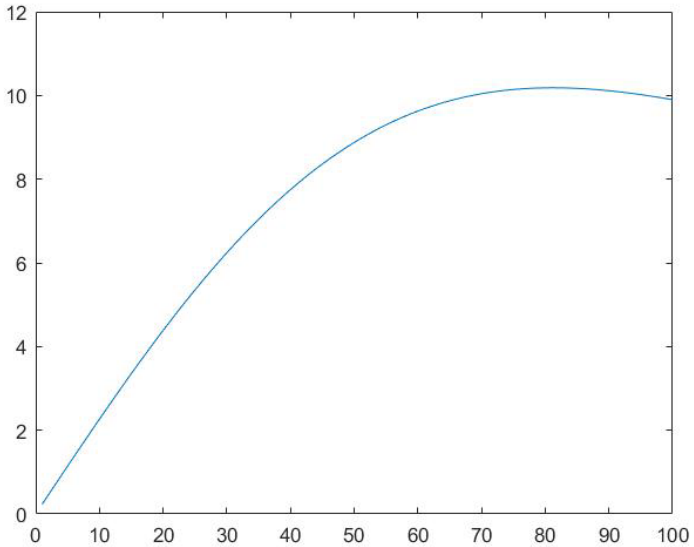
Optimal $N = 69$ Max dist = 11.77 m



Motor 402686 $K_t = 0.0533$ $K_e = K_t = 0.0533$

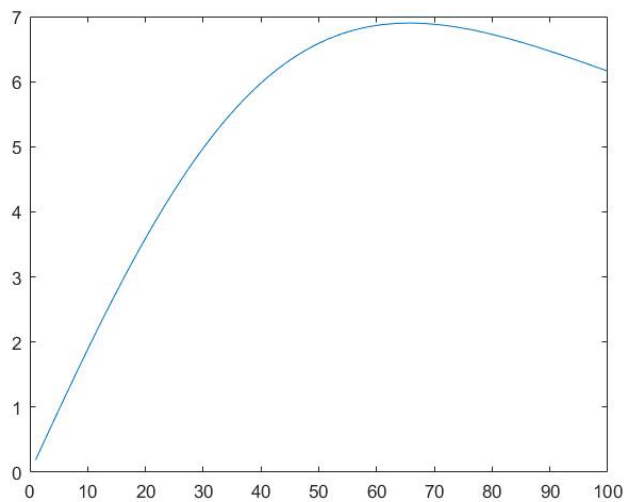
$R = 1.74$

Optimal $N = 81$ Max-dist $= 10.18$ m



Motor 402687 $K_t = 0.131 = K_e$ $R = 6.89$

Opt $N = 66$ Max dist $= 6.90$ m



(e) ① 397172 N = 66 Dist = 11.75 m
Pair with #203122

② 402685 N = 81 Dist = 10.52 m

Pair with part no with gear ratio 81

③ 402686 N=81, Dist = 10.18 m
pair with part no. with gear ratio 81

④ 402687 N = 66 Dist = 8.90 m
pair with #203122

∴ Best is option 1, #397172 + #203122