MATH240 LATEX : Continuous functions.

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Abstract

This document introduces continuous functions. We give the definition of continuity and state some results concerning continuous functions. Further, we list some well-known functions along with their derivatives and plot exp(x), sin(x) and cos(x).

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1 introduction

In Section 2 we define a continuous function and give examples of such functions. In Section 3 we give a table of derivatives and plot some basic functions.

$$example : y = x, y = x^2 \tag{1}$$

2 mathematics

Definition 1 *the$\epsilon - \delta$ definition of a continuous function is defined as the limit of f(x) at x=c is L if for any $\epsilon > 0$ there's a δ¿0 such that if the distance of x from c is less than ϵ, then the distance of f(x) from L is less than δ.*

[]

the symbol for this definition is $(\forall x_0 \in \mathbf{R})$ $(\forall \epsilon > 0)(\exists \delta > 0)(\forall x \in \mathbf{R})(0 < |x - x_0| < \delta) \longrightarrow |f(x) - f(x_0)| < \epsilon)$

2:use example to show the expoential function is an example of continuous function

Example 1 *exp(x)=*$\sum\limits_{n=0}^{\infty} \frac{x^n}{n!}$.
as$x \rightarrow x_0$ *f(x)=L if and only if* $\forall \epsilon > 0)(\exists \delta > 0)(\forall x \in \mathbf{R})(0 < |x - x_0| < \delta) \longrightarrow |f(x) - f(x_0)| < \epsilon$
proves e^x *is continuous*

Proof 1 *3.prove:let f,g be continuous function then f+g is still continuous known f and g are continuous at a gives for any* ϵ *values* δ_1,δ_2 *such that for* $\|x - a\| < \delta_2$ *it is true that* $\|f_1(x) - f_1(a)\| < \epsilon$ *and for* $\|x - a\| < \delta_2$ *its true that* $\|f_2(x) - f_2(a)\| < \epsilon$ *it follows that for* δ=*min*(δ_1, δ_2)
$2\epsilon > \|f_1(x) - f_1(a)\| + \|f_2(x) - f_2(a)\| \geq \|f_1(x) - f_1(a) + f_2(x) - f_2(a)\| (traingleinequality)$
$= \|(f_1 + f_2)(x) - (f_1 + f_2)(a)$
$if \|x - a\| < \delta$

so we can find any δ satisify the criterion for any value 2ϵ and those for any ϵ so f_1 and f_2 is continuous at a
4.

Definition 2 *the hdeavisde function , H:* $\mathbf{Z} \rightarrow \{0, 1\}$*is defined to be*

$$H[n] = \begin{cases} 0 & for\ n\ < \ 0, \\ 1 & for\ n \geq \ 0. \end{cases}$$

	function	derivative
1.	x	1
2.	x^n	nx^{n-1}
3.	exp(x)	exp(x)
4.	$\sin\theta$	$\cos\theta$
5.	$\cos\theta$	-$\sin\theta$
6.	$\tan\theta$	$\sec^2\theta$
7.	$\cosh x$	$\sinh x$
8.	$\sinh x$	$\cosh x$
9.	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
10.	$\frac{1}{x-1}$	$\frac{1}{x^2-1}$

Table 1: this tables contains a list of functions and their derivatives

the heaviside function is discontinuous because according to the graph of the function there should be a gap between 1+ and 1- so its discontinuoous

Theorm 1 *the intermediate value theorm is:*
consider the interval **I**= *[a,b] in the real numbers* **R***and a continuous function* **F** :*I*$\mapsto$ **R** *the image set f(***I***) is also an interval and it contains [min(f(a),f(b)),max(f(a),f(b))]*

Proof 2 *let S be the set of all* $x \in [a,b] such that f(x) \leq$ *u.Then* **S** *is non-empty since a is an element of* **S**, *and* **S***is bounded above by b, hence by completeness, the supremum c=sup* **S** *exists. that is, c is the smallest number that is greater than or equal to every member of* **S**. *we claim that f(c)=u*
fix some $\epsilon > 0$ *, since f is continuous , there is a* $\delta such that |f(x) - f(c)| < \epsilon whenever |x-c| < \delta$ *this means that*
f(x)-$\epsilon < f(c) < f(x) + \epsilon$

3 tables and figures

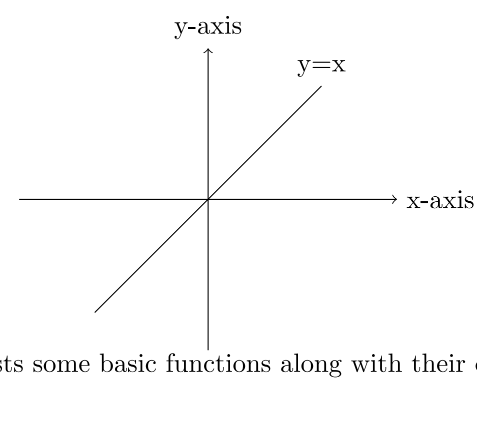
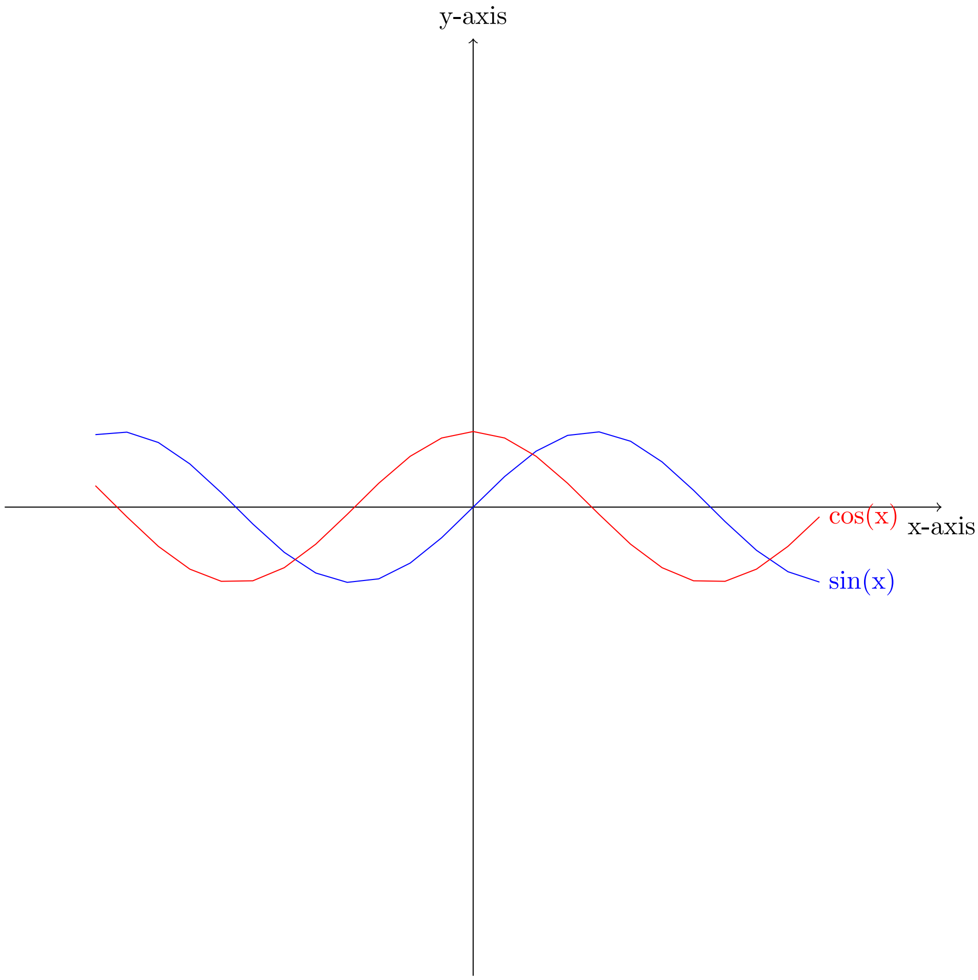
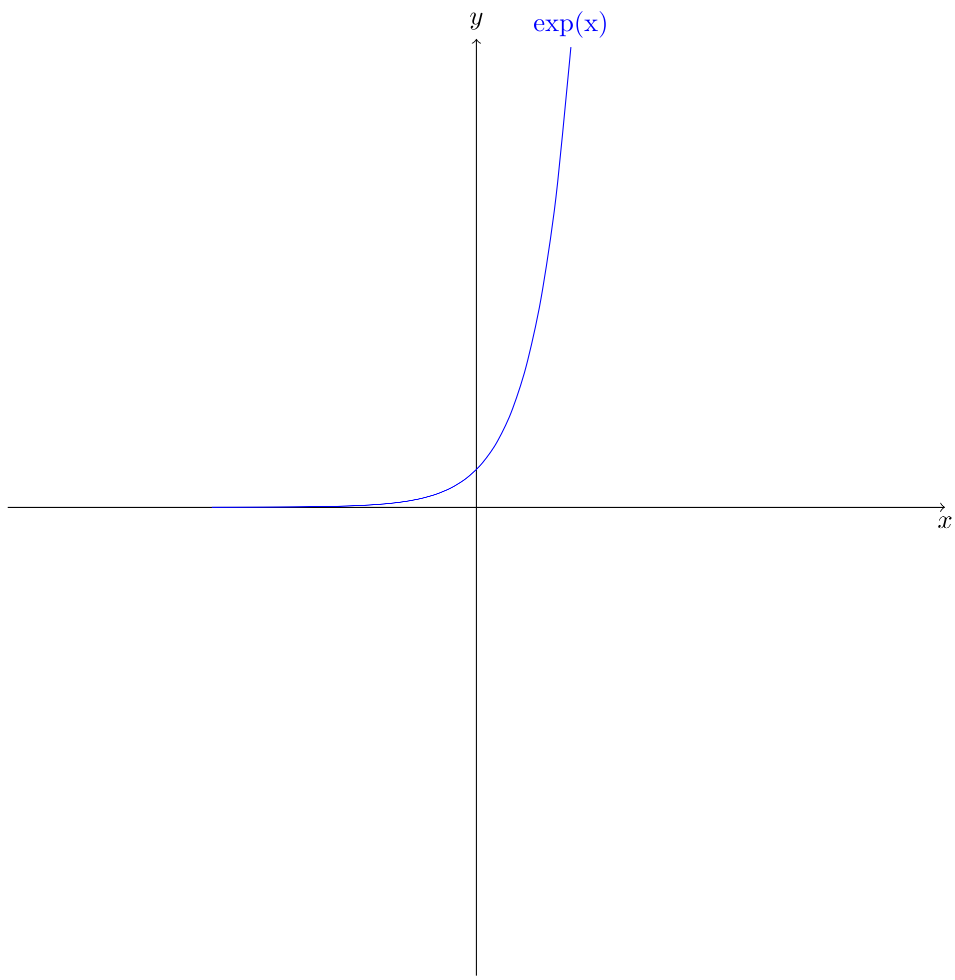


Table 3 lists some basic functions along with their derivatives.”



From Figure 3 we see that both sin and cos are continuous.

angry cat



3 we see that the exponential function is continuous.

4 book

this is a book about the Watershed continuous function [2]and there si a note about the interrelation of subsets of independent variables of a continuous function with continuous first derivatives[1]and math is very fun.

References

[1] Wassily Leontief. A note on the interrelation of subsets of independent variables of a continuous function with continuous first derivatives. *Bulletin of the American mathematical Society*, 53(4):343–350, 1947.

[2] Laurent Najman and Michel Schmitt. Watershed of a continuous function. *Signal Processing*, 38(1):99–112, 1994.