

Session 2: Interest Rate and Swap Market Tee Chyng Wen

QF605 Fixed Income Securities



Case Study: LIBOR Origin

LIBOR

Created in 1969 by Minos Zombanakis in London, who arranged an \$80 million syndicated loan for the Shah of Iran.

⇒ The first loan to charge a floating rate, split among a group of banks.

Subsequently became the benchmark for loans or bonds pricing.

Starting in 1980s, it became also the benchmark for complex derivatives.

Up until just before the 2008 crisis, LIBOR has become a rate that reflects the cost of unsecured borrowing for a specific period (generally 3m or 6m).

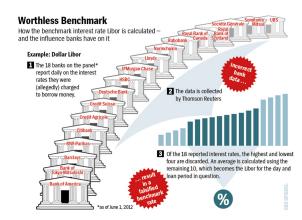


Case Study: LIBOR Scandal

Source: Der Spiegel

LIBOR

0.000000



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Case Study: How was LIBOR calculated?

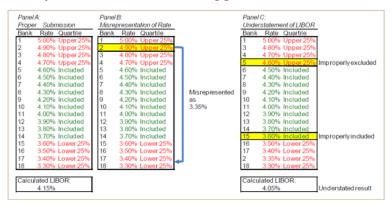
- Everyday, the Intercontinental Exchange (ICE) surveys a panel of banks asking the question
 - At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?
- Throw away the highest and lowest portion of the responses (4 each for USD LIBOR), and averages the remaining middle (10 for USD LIBOR).
- The average is reported at 11:30 am and published worldwide by Thomson Reuters.
- This process is carried out for 10 currencies for maturity up to 1 year.



LIBOR

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Case Study: How was LIBOR rigged?



If your "fair" rate is in the middle, you can raise or lower LIBOR by submitting a rate on the desired direction.

If your "fair" rate is on the upper end, you can only lower LIBOR, by submitting a low rate, and vice versa.

Case Study: Barclays' LIBOR Scandal

3 types of manipulation

LIBOR

- Altering survey's response for the benefit of own derivative positions – requests from the trading desks.
- Altering survey's reponse to protect Barclays' reputation high submission rate is seen as a sign of weakness.
- 3 Attempting to induce other banks to alter their survey responses.

In short, the mechanism (trimmed mean) put in place to remove extreme values leaves survey participants with partial power to move rates in the direction they want.



Case Study: Recent LIBOR

LIBOR

Every morning, a group of banks from around the world submits estimates of the lowest possible interest rate at which the institutions can borrow money from another bank on that day. There are five panels, each for a different currency, and every panel produces a rate for seven maturities, for a total of 35 rates a day. Below are the rates submitted on July 29 for U.S. dollar-denominated loans with a three-month ma

aturity.		
RABOBANK	0.70000	
JPMORGAN CHASE	0.70000	
SOCIETE GENERALE	0.72000	
ROYAL BANK OF CANADA	0.72000	
CITIGROUP	0.72500	
BANK OF AMERICA	0.73000	
нѕвс	0.73000	
UBS	0.73100	
BANK OF TOKYO- MITSUBISHI	0.76000	
CREDIT SUISSE	0.76000	
ROYAL BANK OF SCOTLAND	0.77500	
BARCLAYS	0.78000	
SUMITOMO MITSUI FINANCIAL GROUP	0.79000	
NORINCHUKIN	0.81000	
BNP PARIBAS	0.81000	
LLOYDS	0.84000	
CREDIT AGRICOLE	0.86250	
DEUTSCHE BANK	0.90000	

The top and bottom quarter of submissions are discarded to avoid outliers. There were 18 contributors, so the highest and lowest four were cut

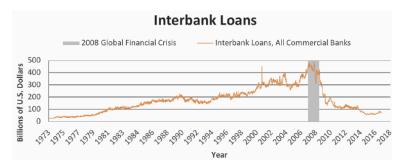
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JPMORGAN CHASE	0.70000
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ROYAL BANK OF CANADA	0.72000
CITIGROUP	0.72500
BANK OF AMERICA	0.73000
HSBC	0.73000
UBS	0.73100
BANK OF TOKYO- MITSUBISHI	0.76000
CREDIT SUISSE	0.76000
ROYAL BANK OF SCOTLAND	0.77500
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BNP PARIBAS	0.81000
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DEUTSCHE BANK	0.90000

Source: Bloomberg

Case Study: LIBOR Replacement

LIBOR

After the crisis, the market for unsecured inter-bank lending failed to recover most daily LIBOR submissions are based on "expert judgement".



US central bank data shows that in the 2^{nd} guarter in 2018, the median number of unsecured borrowing was 7.

⇒ Many may not be aware of how truly thin these markets have become.





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Central banks in different economies started to look for alternative rates to be used instead of LIBOR-style reference rates.

- USD: Secured Overnight Financing Rate (SOFR), the Treasury reporate for overnight loan. Daily average of US Treasury-collateralized overnight repurchase agreement (repo) transaction data collected by the US Federal Reserve
- GBP: Sterling Over Night Index Average (SONIA), which tracks the actual (unsecured) overnight deals in money markets.
- JPY: Tokyo Over Night Average Rate (TONAR), unsecured overnight rate published by Bank of Japan.
- CHF: Swiss Average Rate OverNight (SARON), based on CHF repo market.
- EUR: Euro Short Term Rate (€STER), based on actual money market transactions.
- SGD: Singapore Overnight Rate Average (SORA). Determined by the volume-weighted average rate of borrowing transactions in the unsecured overnight interbank SGD cash market in Singapore.



The USD LIBOR panel has ended on 30-June-2023. Some (1m, 3m, 6m) Synthetic LIBORs will continue to be published until 30-Sep-2024. The Alternative Reference Rates Committee (ARRC) recommended the US Secured Overnight Financing Rate (SOFR) to replace the USD LIBOR.

Main LIBOR transition issues to be addressed due the **term mismatch** and the **credit sensitivity**:

- ⇒ SOFR is an overnight rate while LIBOR is a term rate. SOFR must be compounded daily for a period to arrive at a simple interest rate.
- ⇒ LIBOR is paid in arrears but set in advance, so it involves expectation of future short rates.
- ⇒ Different credit exposure.

The International Swaps and Derivatives Association (ISDA) has provided collateral agreement interest rate definitions that incorporate these new risk-free rates (RFRs):

- ⇒ SOFR to replace Effective Federal Funds Rate (EFFR) for USD
- ⇒ €STR to replace the Euro Overnight Index Average (EONIA)



LIBOR

Case Study: LIBOR Replacement and Fallback

The ARRC also indicated support for the use of the CME Term SOFR Rates in areas where use of overnight or averages of SOFR has proven to be difficult:

- The CME Term SOFR Rates aim to provide a robust measure of forward-looking SOFR term rates based on market expectations implied from transactions in the derivatives markets.
- The methodology for determining CME Term SOFR Rates uses a combination of SOFR overnight indexed swaps (OIS) and one-month and three-month SOFR futures contracts.
 - 1m SOFR futures (SR1): 13 consecutive months contracts
 - 3m SOFR futures (SR3): 5 consecutive quarterly contracts

CME determines the path of overnight SOFR rates by assuming the overnight SOFR rates follow a piecewise constant step function and can only jump up or down the day after FOMC policy rate announcement dates and remains at those levels across all dates in between the FOMC policy rate announcement dates.

No expert judgement is required.

	Term SOFR	Overnight SOFR
Tenor	1m, 3m, 6m, 12m	Overnight compounded
Source	CME Group	New York Fed
Term	Forward-looking	Backward-looking rate
Fixing	Known before period starts	Determine after period ends
Pros	Similar to LIBOR	Consistent with other currencies
Cons	Missing for other currencies	Interest payable unknown when loan begins



LIBOR fallback To substitute the LIBOR fixing in the derivatives by a new quantity obtained as the sum of an adjusted RFR and an adjustment spread.

- The adjusted RFR plays the role of the floating term rate—it will depend on an overnight benchmark and be known around the date where LIBOR should have been fixed.
- The adjustment spread will be decided after the discontinuation and be seen as an adjustment to avoid value transfer between the original LIBOR fixing and the new fixing mechanism.

Historical mean/median approach The spread adjustment is based on the mean or median spot spread between the LIBOR and the adjusted RFR calculated over a signficant, static lookback period prior to the relevant announcement or publication triggering the fallback provisions.



Term Rate: LIBOR & Its Replacement

If you put your money in a money-market account for a given period, the interest earned over this period is quoted as a term rate (e.g. LIBOR). At the end of a period of length Δ , one receives an interest equal to $\Delta \times L$, where L denotes the term rate and Δ denotes the day count fraction.

Since L is always quoted as annualized rate, Δ is often referred to as the accrual fraction or day count fraction.

We have
$$1 \equiv (1 + \Delta \cdot L) \cdot D(0, \Delta)$$
.

This just states that the present value today of the notional plus the interest earned at the end of the Δ period is equal to the notional.

Note that in real markets Δ is not exactly equal to 0.25, 0.5, or 1, but is calculated according to a specific method for a given market, known as the day count convention (e.g. Act/365, Act/360 etc.)



Term Rate (LIBOR) Market

LIBOR

A forward term rate (e.g. forward LIBOR) is the interest rate one can contract for at time t to put money in a money-market account for the time period $[T, T + \Delta]$. Then we have

$$D(t,T) = (1 + \Delta \cdot L) \cdot D(t,T + \Delta)$$

At the time when the forward LIBOR is fixed, it is then called a spot LIBOR. Note that LIBOR is fixed at the beginning of each period T_i , and paid at the end of the period T_{i+1} . We call this "fixed in advance, paid in arrears."

In most markets, only forward LIBOR rates of one specific tenor are actively traded, which is usually 3m (e.g. USD) or 6m (e.g. SGD, GBP, EUR).

Everyday we have a large number of forward LIBOR rates with this specific tenor, we can denote them as

$$i = 1, 2, \cdots, N : L(T_i, T_{i+1})$$

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Forward Rate Agreements (FRA)

Suppose you agree to an interest rate today, which you will obtain on an amount of money invested at later date, that is returned with interest at an even later date.

This kind of agreement is called a forward rate agreement (FRA).

This level of the agreed rate that makes the value of entering into a forward rate agreement equal to zero is called the **forward rate**.

You have cashflow liability in the future (e.g need to borrow or deposit cash) but do not want to be exposed to interest rate risks.

Under a FRA contract:

LIBOR

- **Buyer** is obligated to **borrow** money at a pre-determined or fixed interest rate on the date when the FRA expires.
- Seller is obligated to lend at the fixed FRA rate. Thus both counterparties lock in a rate in advance and the actual lending and borrowing is done at the FRA expiration date.



Forward Rate Agreements (FRA)

FRA allows you to lock-in an interest rate today at which you can borrow or deposit in the future.

- ⇒ For borrowers, if the market rate ends up higher, then it is good for you. The opposite is true for lenders.
- ⇒ For borrowers, if the market rate ends up lower, then it is bad for you. The opposite is true for lenders.



The FRA is quoted as $A \times B$ in terms of months. For example, a 3×9 FRA means the forward contract expires in 3 months, and the underlying interest rate is a 6-month LIBOR.

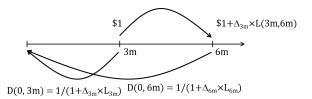


Forward Rate Agreements (FRA)

\$1 in 3m's time is worth D(0,0.25) today. If you invest that amount for 3m at the forward LIBOR L(3m, 6m), then you earn an interest

$$\begin{split} D(0,3m) &= D(0,6m)(1+\Delta_{3m}L(3m,6m))\\ \Rightarrow \quad L(3m,6m) &= \frac{1}{\Delta_{3m}}\frac{D(0,3m)-D(0,6m)}{D(0,6m)} \end{split}$$

In words, we can work out the forward LIBOR rate based on spot LIBOR rates and discount factors.

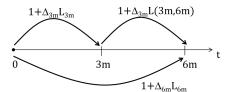


Forward Rate Agreements (FRA)

We can obtain the same result using either discount factors or spot LIBOR rates:

$$(1 + \Delta_{3m}L_{3m})(1 + \Delta_{3m}L(3m, 6m)) = 1 + \Delta_{6m}L_{6m}$$

$$\Rightarrow L(3m, 6m) = \frac{1}{\Delta_{3m}} \frac{(1 + \Delta_{6m}L_{6m}) - (1 + \Delta_{3m}L_{3m})}{1 + \Delta_{3m}L_{3m}}$$



Example If the 3m LIBOR rate is 4% and the 6m LIBOR rate is 6%, what is the in-3m-for-3m forward rate F(3m, 6m)?

Forward Rate Agreements (FRA)

Example Suppose we have the following discretely compounded zero rates:

Term Rate

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Maturity	Zero Rate
$\overline{1y}$	3.96%
2y	5.47%
3y	6.14%

Determine the forward rates L(0,1y), L(1y,2y), and L(2y,3y).

ans.:
$$L(1y, 2y) = 7\%$$
, $L(2y, 3y) = 7.5\%$.

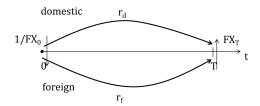


Interest Rate Parity

LIBOR

Interest rate parity is a no-arbitrage condition stating that it is impossible for investors to gain higher returns by borrowing and lending in the money market account of 2 economies.

This gives rise to a relationship between the interest rates and exchange rates of the 2 economies



Interest rate parity:

$$FX_T = FX_0 \times \left(\frac{1+r_d}{1+r_f}\right)^T$$

SOR is a **synthetic rate** in the Singapore market.

E. Calculation Methodology: Singapore Dollar Swap Offer Rate (SOR)

Benchmark	Singapore Dollar Swap Offer Rate (SOR)
Description	The synthetic rate for deposits in Singapore Dollars (SGD), which represents the effective cost of borrowing the Singapore Dollars synthetically by borrowing US Dollars (USD) for the same maturity, and swap out the US Dollars in return for the Singapore Dollars.
Calculation Methodology	The Administrator shall calculate and determine the Rate, for each maturity matching each Tenor specified below (each a "calculation period"), on each Business Day as follows:



Case Study: Swap Offer Rate (SOR) in Singapore



Case Study: MIFOR Rate in India

Mumbai Interbank Forward Offered Rate (MIFOR) in the India market.

Banks to provide a quotation of their offered side of INR/USD forward points for the forward sale of INR against USD for settlement on the last day of a period equivalent to the Designated Maturity and commencing on the Reset Date and the forward points so determined by the Calculation Agent shall be the "Forward Points" for purposes of the following formula. The Calculation Agent will then determine the rate for that Reset Date by applying the following formula:

```
Floating Rate =
       {[(Spot Rate + Forward Points) / Spot Rate * (1+ LIBOR * N1)] - 1} * N2*100
```

where:

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"Spot Rate" means the Reserve Bank of India's published USD/INR spot rate (expressed as a number of INR per one USD) which appears on Reuters Screen "RBIB" as of 13:00 p.m. India Standard Time on that Reset Date (if such rate is not available the Calculation Agent will ask each of the Reference Banks to provide a quotation of such rate);

"LIBOR" means USD-LIBOR-BBA for a period of the Designated Maturity commencing on the Reset Date;

"N1" means the number of days in the Calculation Period divided by 360;

"N2" means 365 divided by the number of days in the Calculation Period.



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THBFIX—the synthetic rate for deposits in Thai Baht, representing the effective cost of borrowing the Thai Baht synthetically by borrowing USD and swapping out the USD in return for THB.

$$\textit{THBFIX} = \left\{ \!\! \left[\!\! \left(\! \frac{\textit{Spot Rate} + Forward Points}{\textit{Spot Rate}} \right) \times \left(1 + \frac{\textit{USD Rate} \times \# days}{360} \right) \right] - 1 \!\! \right\} \\ \times \frac{365}{\# days} \times 100$$

Where:

"USD Rate" means US dollar interest rate (USD rate) in each tenor that is published on Reuters screen LIBOR01 (or other screens that shows USD LIBOR) at 11.55 a.m. London time during that business day.

"#days" means the actual number of days in each tenor (which may differ on each business day).

"Spot Rate" means USDTHB spot rate which is calculated via a volume weighted average method excluding transactions with unusually large volume or unusually high or low spot rates), in relation to all USDTHB FX Spot Qualified Transactions on each business day.

"Forward Points" means Forward points which is calculated via a volume weighted average method (excluding transactions with unusually large volume or unusually high or low forward points), in relation to all USDTHB FX Swap Qualified Transactions on each business day.

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Interest Rate Risk

LIBOR

- A standard company is usually financed by a combination of equity and debt. Consequently, interest rate risk is part and parcel of operating a company.
- A company might raise money from investors by issuing shares, and then borrows further to provide the rest of the funding.
- Clearly, the company will need to pay interest on the debt it issued, and
 eventually repay the debt in its entirety. If the interest rate <u>fluctuates</u>
 according to the prevailing rates in the market, it is said to be **floating**.
- The interest payments is typically a sizable burden, and could cripple a company if it rises too much. It is therefore better for a company to go for a fixed-rate loan.
- This obviously adds on to the complexity of the problem—if rates fall, it will be beneficial for the company to **refinance**.



Interest Rate Swaps

LIBOR

Interest rate swap is a contract where two parties agree to exchange a set of floating interest rate payments for a set of fixed interest rate payments.

- The set of floating interest rate payments is based on LIBOR rates and is called the floating leg.
- The set of fixed payments is called the **fixed leg**.

The naming convention for swaps is based on the fixed side: payer swap means you pay the fixed leg, while receiver swap means you received the fixed leg

- The times at which the floating interest rates are set are called reset dates
- The times at which the interest rate payments are made are called payment dates.
- The time between payment dates are called **settlement period**.
- The length of the swap is called the tenor of the swap (from start till maturity).
- Typically, the parties agree to exchange only the net amount owed from one party to the other party. This practice is called **netting**.
- If payments are not in the same currency, netting is not appropriate.

Interest Rate Swaps – Rationale

Interest rate swaps are useful because they allow you to turn a fixed-rate asset into a floating rate asset, and vice versa. For example:

- A bank funds itself via a mix of short-term deposits (floating rate) and fixed-rate bonds. It gives variable and fixed rate mortgages to its customers. When it raises financing, it does not know the quantities of variable and fixed rate mortgages that it will give.
- If it ends up giving more fixed rate mortgages than anticipated, it can enter into swaps to pay fixed and receive floating to hedge out interest rate risk.
- If it ends up giving more variable rate mortgages than anticipated, it can enter into swaps to receive fixed and pay floating to hedge out interest rate risk.

Interest rate swaps allow the bank to raise money first, then invest it, and fix any resulting interest rate risk exposure later.



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Interest Rate Swaps – Fixed Leg

The process of pricing the swap involves finding the swap rate such that the value of the swap contract is **worth par** at trade initiation (i.e. NPV=0).

To this end, we proceed by determining the PV of the fixed and floating leg separately.

The PV of a fixed leg is straightforward. Consider a single fixed rate payment of K at time T, with an accrual fraction of Δ . The PV of this payment is

$$D(0,T)\times K\times \Delta$$

Therefore, for a series of payments starting at T_1 and ending at T_n , the leg of fixed rate payment is

$$K\sum_{i=1}^{n} D(0,T_i) \times \Delta_{i-1}$$



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Interest Rate Swaps - Floating Leg

Recall that forward rate is related to discount factors:

$$L(3m,6m) = \frac{1}{\Delta_{3m}} \frac{D(0,3m) - D(0,6m)}{D(0,6m)}.$$

Given a set of payment dates T_i , we can express the forward LIBOR rate as

$$L(T_{i-1}, T_i) = \frac{1}{\Delta_{i-1}} \frac{D(0, T_{i-1}) - D(0, T_i)}{D(0, T_i)}.$$

The PV of a single floating rate payment at T_i can therefore be expressed as:

$$D(0,T_i) \times \Delta_{i-1} \times L(T_{i-1},T_i) = D(0,T_{i-1}) - D(0,T_i).$$

Generalizing, we see that if the floating leg comprises of \boldsymbol{n} payments, then its PV is given by

$$\sum_{i=1}^{n} D(0,T_i) \times \Delta_{i-1} \times L(T_{i-1},T_i) = 1 - D(0,T_n).$$



Par Swap Rate

LIBOR

In a swap multiple payments are exchanged:

- The value of a payer swap at time t that starts at T_0 and ends at T_n .
- At the start date T_0 the first LIBOR rate is fixed.
- Actual payments are exchanged at T_1, \ldots, T_n .
- The swap tenor is $T_n T_0$.

The present value of a payer swap is given by

$$\sum_{i=1}^{n} V_i^{flt}(t) - \sum_{i=1}^{n} V_i^{fix}(t) = (D(0, T_0) - D(0, T_n)) - K \sum_{i=1}^{n} \Delta_{i-1} D(0, T_i).$$

Similarly, the present value of a receiver swap is given by

$$K\sum_{i=1}^{n} \Delta_{i-1}D(0,T_i) - (D(0,T_0) - D(0,T_n)).$$

In the market, swaps are not quoted as prices for different fixed rates K, but only the fixed rate K is quoted for each swap such that the present value of the swap is equal to 0.

Par Swap Rate

LIBOR

This particular rate is called the **par swap rate**. We denote the par swap rate for the $[T_0,T_n]$ swap with S. Solving, we obtain

$$S = \frac{D(0, T_0) - D(0, T_n)}{\sum_{i=1}^{n} \Delta_{i-1} D(0, T_i)}.$$

The term in the denominator is also called the **present value of a basis point** (PVBP)

$$P_{1,n} = \sum_{i=1}^{n} \Delta_{i-1} D(0, T_i).$$

Pricing Interest Rate Swaps

Example Suppose we observe the following LIBOR discount factors in the market:

Discount Factor	Value
D(0, 0.25)	0.9876
D(0, 0.5)	0.9753
D(0, 0.75)	0.9632
D(0, 1)	0.9512

Determine the par swap rate for a 1y interest rate swap with quarterly payment under 30/360 day count convention.

ans.: 5.03%

LIBOR



Pricing Interest Rate Swaps

LIBOR

Example Consider a 1y interest rate swap with semi-annual payments under 30/360 day count convention. The continuously compounded zero rates are as follows:

Maturity	Rate
-6m	2.5%
12m	2.75%

- **1** Calculate the forward LIBOR rate L(6m, 12m).
- 2 Calculate the par swap rate.

ans.: 3.023%, 2.767%



Valuation of Interest Rate Swaps

Example Continuing from the previous example, suppose we long a payer swap at the par swap rate. 2m later, the zero rate is as follows:

Maturity	Rate
-4m	2.68%
10m	2.85%

What is the value of our swap position?

ans.:
$$L(4m, 10m) = 2.985\%$$
. $PVfix = 0.02722$. $PVflt = 0.02704$

Interest Rate Risk of IRS

LIBOR

Interest rate (IR) delta is the change in the NPV of the swap when interest rate increases:

- ⇒ Payer swap's PV increases when rates move upward
- ⇒ Payer swap's PV decreases when rates move downward
- ⇒ Receiver swap's PV decreases when rates move upward
- ⇒ Receiver swap's PV increases when rates move downward

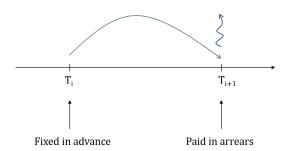
Understanding:

- Imagine you have a payer swap in your portfolio. You are paying fixed rate, and receiving floating rate.
- If LIBOR rates move up, it is good for you since you are paying fixed and receiving floating LIBOR.



LIBOR

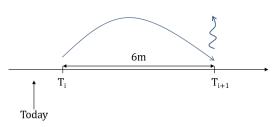
A forward LIBOR payment, accrued from T_i to T_{i+1}





LIBOR

E.g. consider a 6m forward LIBOR payment

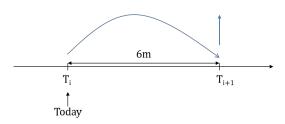


- We won't know that the LIBOR rate will be yet.
- But we can work out the no-arbitrage forward LIBOR rate if we know the spot LIBORs $L(0, T_i)$ and $L(0, T_{i+1})$
- $F(T_i, T_{i+1})$ needs to be calculated



LIBOR

E.g. consider a 6m forward LIBOR payment



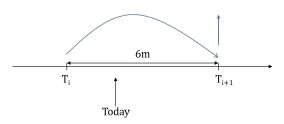
The 6m LIBOR we will be paid is fixed today, it will be the spot 6m LIBOR rate in the market.



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LIBOR

E.g. consider a 6m forward LIBOR payment

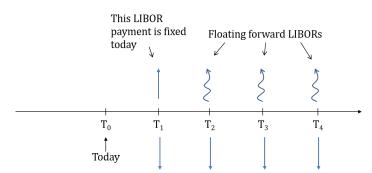


- The LIBOR payment at T_{i+1} will not change anymore since it's already been fixed
- Accrue period started at T_i and will end at T_{i+1}
- The day count fraction is always $T_{i+1} T_i$



LIBOR

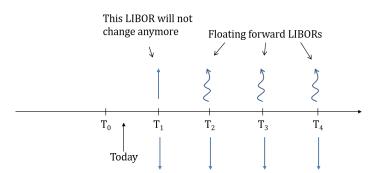
E.g. consider a spot starting interest rate swap





LIBOR

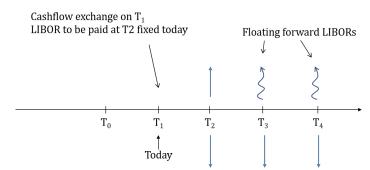
E.g. consider a spot starting interest rate swap





LIBOR

E.g. consider a spot starting interest rate swap





LIBOR

E.g. consider a forward starting interest rate swap

