Project — Part I (Bootstrapping Swap Curves)

1. In the `IR Data.xlsm` spreadsheeet, OIS data is provided. Bootstrap the OIS discount factor $D_o(0,T)$ and plot the discount curve for $T \in [0,30]$.

2. Using the IRS data provided, bootstrap the LIBOR discount factor $D(0,T)$, and plot it for $T \in [0,30]$.

   $\Rightarrow$ Assume that the swap market is collateralized in cash and overnight interest is paid on collateral posted.

3. Calculate the following forward swap rates:

   - $1y \times 1y$, $1y \times 2y$, $1y \times 3y$, $1y \times 5y$, $1y \times 10y$
   - $5y \times 1y$, $5y \times 2y$, $5y \times 3y$, $5y \times 5y$, $5y \times 10y$
   - $10y \times 1y$, $10y \times 2y$, $10y \times 3y$, $10y \times 5y$, $10y \times 10y$

Use linear interpolation on discount factors when necessary.

Project — Part II (Swaption Calibration)

Under the `Swaption` tab of `IR Data.xlsm`, swaption implied volatilities (lognormal) are provided.

1. Calibrate the displaced-diffusion model to the swaption market data, and document
   - a table of σ parameters
   - a table of β parameters
2. Calibrate the SABR model to the swaption market data using $\beta = 0.9$, and document
   - a table of α parameters
   - a table of ρ parameters
   - a table of ν parameters
3. Price the following swaptions using the calibrated displaced-diffusion and SABR model:
   - payer $2y \times 10y \quad K = 1\%, 2\%, 3\%, 4\%, 5\%, 6\%, 7\%, 8\%$
   - receiver $8y \times 10y$ $K = 1\%, 2\%, 3\%, 4\%, 5\%, 6\%, 7\%, 8\%$

Project — Part III (Convexity Correction)

1. Using the SABR model calibrated in the previous question, value the following constant maturity swap (CMS) products:
   - PV of a leg receiving CMS10y semi-annually over the next 5 years
   - PV of a leg receiving CMS2y quarterly over the next 10 years

2. Compare the forward swap rates with the CMS rate:
   - $1y \times 1y$, $1y \times 2y$, $1y \times 3y$, $1y \times 5y$, $1y \times 10y$
   - $5y \times 1y$, $5y \times 2y$, $5y \times 3y$, $5y \times 5y$, $5y \times 10y$
   - $10y \times 1y$, $10y \times 2y$, $10y \times 3y$, $10y \times 5y$, $10y \times 10y$

   Discuss the effect of maturity and tenor on convexity correction (difference between forward swap rates and CMS rates).

Project — Part IV (Decompounded Options)

1. A *decompounded option* pays the following at time $T = 5y$:

$$\text{CMS } 10\text{y}^{1/p} - 0.04^{1/q}$$

where $p = 4$ and $q = 2$. Use static replication to value the PV of this payoff.

2. Supose the payoff is now

$$\left(\text{CMS } 10\text{y}^{1/p} - 0.04^{1/q}\right)^{+}$$

Use static replication to value the PV of this payoff.

Project Report

Deadline: 9-Apr-25 (Wednesday) noon.

Please submit

- Project report (no more than 10 pages, including title page and appendix)
- Python codes (1 file for each part, 4 files overall)