QF605 Additional Examples
Session 3: Multicurve Framework and OIS Discounting

1 Questions

1. Consider the following market quotes of ***uncollateralised*** interest rate cash deposit and swaps (annual payment).

Maturity	Instrument	Rate
$6m$	CD	1.2%
$1y$	IRS	2%
$2y$	IRS	2.5%

Determine the following 6m forward LIBOR rates:

(a) $L(6m, 1y)$

(b) $L(1y, 1y6m)$

(c) $L(1y6m, 2y)$

Note: Use 30/360 day count convention. Apply linear interpolation on the discount factors if necessary.

2. The spot USD LIBOR rates are as follow:

Tenor	Rate
$1m$	1.15%
$2m$	1.20%
$3m$	1.25%
$6m$	1.40%
$9m$	1.55%
$12m$	1.75%

Suppose the overnight interest rate (Fed-Fund) is flat at 0.7%, and that we are using 30/360 day count convention. Suppose index basis swap spread is 0. Calculate:

(a) The spot $3m$ LIBOR discount factor $\tilde{D}(0, 3m)$ and the OIS discount factor $D_o(0, 3m)$.

(b) The forward LIBOR discount factor $\tilde{D}(3m, 6m)$ and the OIS discount factor $D_o(3m, 6m)$.

(c) Consider a $1y$ fixed leg paying 1.75% quarterly.

i. What is the PV of this fixed leg assuming no collateral is posted?

ii. What is the PV if USD cash collateral is posted?

(d) Consider a $1y$ floating leg paying the $3m$ LIBOR rate quarterly.

i. What is the PV of this floating leg assuming no collateral is posted?

ii. What is the PV if USD collateral is posted?

3. Consider the following market quotes of collateralised interest rate swaps (annual payment) based on $1y$ LIBOR fixing:

Maturity	Instrument	Swap Rate
$1y$	Spot LIBOR	1.5%
$2y$	Swap	2.25%
$3y$	Swap	2.55%

A collateralised forward contract pays the forward LIBOR $L(2y, 3y)$ at the end of $3y$ for a notional of \$1,000,000. Assume that collateral are posted in cash of the same denominated of the swap and that overnight interest rate is flat at 0.25%.

(a) Calculate the PV of this collateralised trade.

(b) If a bank do not take collateralisation into consideration, what would have been the PV of the forward contract?

Note: Use 30/360 day count convention. Apply linear interpolation on the discount factors if necessary.

2 Suggested Solutions

1. (a) Since this swap market is uncollateralised, we will be using LIBOR for discounting, and the $6m$ uncollateralised discount factor is given by

$$D(0, 6m) = \frac{1}{1 + 0.5 \times 0.012} = 0.994$$

Using the $1y$ IRS (annual payment), we can calculate the $1y$ uncollateralised discount factor as

$$0.02 = \frac{D(0,0) - D(0,1y)}{D(0,1y)} = \frac{1 - D(0,1y)}{D(0,1y)}$$

$$\Rightarrow \quad D(0,1y) = 0.9804$$

Hence the forward LIBOR $L(6m, 1y)$ is given by

$$L(6m, 1y) = \frac{1}{\Delta_{6m}} \frac{D(0,6m) - D(0,1y)}{D(0,1y)} = \frac{1}{0.5} \frac{0.994 - 0.9804}{0.9804} \approx 2.77\%. \quad \lhd$$

(b) Using the $2y$ IRS (annual payment), the $2y$ uncollateralised discount factor is given by

$$0.025 = \frac{D(0,0) - D(0,2y)}{D(0,1y) + D(0,2y)}$$

$$\Rightarrow \quad D(0,2y) = 0.9517$$

The $1.5y$ uncollateralised discount factor is $\frac{1}{2}\Big[D(0,1y) + D(0,2y)\Big] = 0.9661$. Hence the forward LIBOR is given by

$$L(1y, 1y6m) = \frac{1}{0.5} \frac{0.9804 - 0.9661}{0.9661} \approx 2.96\%. \quad \lhd$$

(c) Similarly

$$L(1y6m, 2y) = \frac{1}{0.5} \frac{0.966 - 0.952}{0.952} \approx 3.03\%. \quad \lhd$$

We've assumed that discount factors are linearly interpolated and that knot points are selected at the maturity of each instruments.

2. (a) The spot $3m$ LIBOR discount factor is

$$\tilde{D}(0, 3m) = \frac{1}{1 + 0.25 \times 0.0125} \approx 0.996885 \quad \lhd$$

The OIS discount factor is

$$D_o(0, 3m) = \prod_{i=1}^{90} \frac{1}{1 + \frac{1}{360} \cdot 0.007} \approx 0.99825 \quad \lhd$$

(b) The forward LIBOR discount factor $\tilde{D}(3m, 6m)$ is

$$\tilde{D}(3m, 6m) = \frac{\tilde{D}(0,6m)}{\tilde{D}(0,3m)} = \frac{\frac{1}{(1+0.5\times0.014)}}{\frac{1}{(1+0.25\times0.0125)}} \approx 0.996152 \quad \lhd$$

The forward OIS discount factor is

$$D_o(3m, 6m) = \frac{\prod_{i=1}^{180} \frac{1}{1+\frac{1}{360}\cdot 0.007}}{\prod_{i=1}^{90} \frac{1}{1+\frac{1}{360}\cdot 0.007}} \approx 099825 \quad \lhd$$

(c) i. If no collateral is posted, then we will discount the future cashflows at LIBOR. Consequently, the PV of the fixed leg is given by

$$\begin{aligned}
&\sum_{i=1}^{4} \Delta_{i-1} \cdot \tilde{D}(0,T_i) \cdot 1.75\% \\
&\qquad = 0.25 \times 0.0175 \times \left(\frac{1}{1+0.25\times 0.0125} + \frac{1}{1+0.50\times 0.014} \right. \\
&\qquad\qquad \left. + \frac{1}{1+0.75\times 0.0155} + \frac{1}{1+1.00\times 0.0175} \right) \\
&\qquad \approx 0.01733. \quad \triangleleft
\end{aligned}$$

ii. If cash collateral is posted, then we will need to discount at the collateral rate. Since USD Fed-Fund rate is given to be 0.7% flat, then the discount factors $D_o(0,3m)$, $D_o(0,6m)$, $D_o(0,9m)$, and $D_o(0,12m)$ can be calculated, and we obtain:

$$\begin{aligned}
&\sum_{i=1}^{4} \Delta_{i-1} \cdot D_o(0,T_i) \cdot 1.75\% \\
&\qquad = 0.25 \times 0.0175 \times \Big(D_o(0,3m) + D_o(0,6m) \\
&\qquad\qquad + D_o(0,9m) + D_o(0,12m) \Big) \\
&\qquad \approx 0.01742. \quad \triangleleft
\end{aligned}$$

(d) i. If no collateral is posted, then we will discount the future cashflows at LIBOR. Consequently, the expression for the PV of the floating leg is telescoping, and can be simplified into the following:

$$\sum_{i=1}^{4} \Delta_{i-1} \tilde{D}(0,T_i) L(T_{i-1},T_i) = \tilde{D}(0,0) - \tilde{D}(0,12m) \approx 0.0172. \quad \triangleleft$$

ii. If cash collateral is posted, then we will need to discount at the collateral rate. It is important to note that with collateralisation, floating leg no longer telescope — we will need to discount each cashflows one-by-one, much like how we would handle a fixed leg. First we work out the forward LIBORs using the relationship

$$L(T_i,T_{i+1}) = \frac{1}{\Delta_i}\left[\frac{\tilde{D}(0,T_i) - \tilde{D}(0,T_{i+1})}{\tilde{D}(0,T_{i+1})} \right].$$

These are given by:

$$L(3m,6m) = \frac{1}{0.25}\left[\frac{\frac{1}{1+0.25\cdot L(0,3m)} - \frac{1}{1+0.5\cdot L(0,6m)}}{\frac{1}{1+0.5\cdot L(0,6m)}}\right]$$
$$= \frac{1}{0.25}\left[\frac{\frac{1}{1+0.25\times 0.0125} - \frac{1}{1+0.5\times 0.014}}{\frac{1}{1+0.5\times 0.014}}\right] = 0.01545$$
$$L(6m,9m) = \frac{1}{0.25}\left[\frac{\frac{1}{1+0.50\cdot L(0,6m)} - \frac{1}{1+0.75\cdot L(0,9m)}}{\frac{1}{1+0.75\cdot L(0,9m)}}\right]$$
$$= \frac{1}{0.25}\left[\frac{\frac{1}{1+0.5\times 0.014} - \frac{1}{1+0.75\times 0.0155}}{\frac{1}{1+0.75\times 0.0155}}\right] = 0.01837$$
$$L(9m,12m) = \frac{1}{0.25}\left[\frac{\frac{1}{1+0.75\cdot L(0,9m)} - \frac{1}{1+1.00\cdot L(0,12m)}}{\frac{1}{1+1.00\cdot L(0,12m)}}\right]$$
$$= \frac{1}{0.25}\left[\frac{\frac{1}{1+0.75\times 0.0155} - \frac{1}{1+1.00\times 0.0175}}{\frac{1}{1+1.00\times 0.0175}}\right] = 0.02323$$

And so

$$\begin{aligned}\sum_{i=1}^{4} \Delta_{i-1}\cdot D(0,T_i)\cdot L(T_{i-1},T_i) = 0.25 \times \Big(&0.0125\times D_o(0,3m) + 0.01545\times D_o(0,6m) \\ &+ 0.01837\times D_o(0,9m) + 0.02323\times D_o(0,12m)\Big) \\ \approx{}& 0.0173. \quad \triangleleft\end{aligned}$$

3. (a) First we evaluate the overnight discount factors:

$$D_o(0,1y) = \prod_{i=1}^{360} \frac{1}{1+\frac{0.0025}{360}} = \left(\frac{1}{1+\frac{0.0025}{360}}\right)^{360} \approx 0.9975$$
$$D_o(0,2y) = \prod_{i=1}^{2\times 360} \frac{1}{1+\frac{0.0025}{360}} = \left(\frac{1}{1+\frac{0.0025}{360}}\right)^{2\times 360} \approx 0.995$$
$$D_o(0,3y) = \prod_{i=1}^{3\times 360} \frac{1}{1+\frac{0.0025}{360}} = \left(\frac{1}{1+\frac{0.0025}{360}}\right)^{3\times 360} \approx 0.9925$$

We're given that $L(0,1y) = 0.015$. The $2y$ swap (annual payment) allows us to obtain

$$\Big(D_o(0,1y) + D_o(0,2y)\Big) \times 0.0225 = D_o(0,1y)\times 0.015 + D_o(0,2y)\times L(1y,2y)$$
$$\Rightarrow L(1y,2y) = 0.03$$

Using the $3y$ swap (annual payment), we can obtain

$$\begin{aligned}&\Big(D_o(0,1y) + D_o(0,2y) + D_o(0,3y)\Big) \times 0.0255 \\ &\qquad = D_o(0,1y)\times 0.015 + D_o(0,2y)\times L(1y,2y) + D_o(0,3y)\times L(2y,3y) \\ &\Rightarrow L(2y,3y) = 0.0315\end{aligned}$$

Hence, we can evaluate the PV of the collateralised FRA trade

$$PV = D_o(0,3y)\times L(2y,3y)\times 1,000,000 = 31,263.75 \quad \triangleleft$$

(b) In the event that collateralisation is not taken into account, then all discounting will be purely based on LIBOR, so that

$$\tilde{D}(0,1y) = \frac{1}{1+0.015} = 0.985$$

Using the par swap rate formula, we can then proceed with the $2y$ swap

$$0.0225 = \frac{1-\tilde{D}(0,2y)}{\tilde{D}(0,1y)+\tilde{D}(0,2y)} \quad \Rightarrow \quad \tilde{D}(0,2y) = 0.9563$$

And finally, from the $3y$ swap

$$0.0255 = \frac{1-\tilde{D}(0,3y)}{\tilde{D}(0,1y)+\tilde{D}(0,2y)+\tilde{D}(0,3y)} \quad \Rightarrow \quad \tilde{D}(0,3y) = 0.927$$

The LIBOR rates are given by

$$\begin{aligned} L(0,1y) &= 0.015 \\ L(1y,2y) &= \frac{\tilde{D}(0,1y)-\tilde{D}(0,2y)}{\tilde{D}(0,2y)} = 0.03 \\ L(2y,3y) &= \frac{\tilde{D}(0,2y)-\tilde{D}(0,3y)}{\tilde{D}(0,3y)} = 0.0316 \end{aligned}$$

Hence, we can evaluate the PV of the FRA trade without collateralisation consideration is

$$PV = \tilde{D}(0,3y) \times L(2y,3y) \times 1,000,000 = 29,293.2 \quad \triangleleft$$