QF605 Fixed-Income Securities
Assignment 4, Due Date: 26-Mar-2025

1. Let S_t denote a forward swap rate at time t. Suppose a CMS product has the following payoff on maturity T:

$$g(S_T) = \begin{cases} 0, & S_T < K_1 \\ S_T - K_1 & K_1 \leq S_T \leq K_2 \\ K_2 - K_1 & S_T > K_2 \end{cases}$$

   Starting with

$$\int_0^\infty h(K) \frac{\partial^2 V^{pay}(K)}{\partial K^2} \, dK$$

   where $h(K) = \frac{g(K)}{\text{IRR}(K)}$, derive the static replication formula for this payoff.

2. The Ho-Lee interest rate model is given by

$$dr_t = \theta(t)dt + \sigma dW_t^*,$$

   where W_t^* is a standard Brownian motion under the measure $\mathbb{Q}^*$. Determine the mean and variance of the integral

$$\int_0^T r_u \, du.$$

3. Suppose we use a discrete ($\Delta t = 1y$) binomial-tree approximation of the Ho-Lee model, where at every step the rate can move up or down by 0.5%, and the risk-neutral probabilities of an up or down move are both 0.5. We observe the following discount factors:

Instrument	Value
$D(0, 1y)$	0.9656
$D(0, 2y)$	0.9224
$D(0, 3y)$	0.8903

   Draw the Ho-Lee binomial tree and determine the no-arbitrage values for θ_0 and θ_1.