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Discrete Probability Problem Set

Please work all problems, showing work and answers. I suggest photocopying your worksheet before handing it in , so that you will be able to follow our discussions without trying to remember what you wrote.

$p=0.85$

$n=11$

1. The probability that any individual passes a course is 0.85. In a course with 11 students, let X represent the number who pass. Please compute, and round to three decimals,
   a. $P(X \geq 1)$ b. $P(X \leq 9)$ c. $P(X=0)$

a. $P(X \geq 1)$
$= 1 - P(X=0)$
$= 1 - C(11,0)\, 0.85^0 \cdot 0.15^{11}$
$\approx 1 - 1 \cdot 1 \cdot 0.000$
$\approx 1 - 0$
$= 1.000$

b. $P(X \leq 9)$
$= 1 - P(X=11) - P(X=10)$
$= 1 - C(11,11) \cdot 0.85^{11} \cdot 0.15^0 - C(11,10) \cdot 0.85^{10} \cdot 0.15^1$
$\approx 1 - 1 \cdot 0.1673 \cdot 1 - \frac{P(11,10)}{10!} \cdot 0.1969 \cdot 0.15$
$\approx 1 - 0.1673 - 11 \cdot 0.1969 \cdot 0.15$
$\approx 1 - 0.1673 - 0.3248$
$= 0.508$

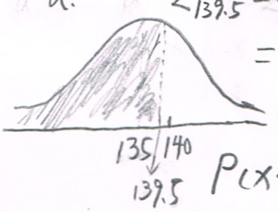
c. $P(X=0)$
$= C(11,0) \cdot 0.85^0 \cdot 0.15^{11}$
$= 1 \cdot 1 \cdot 0.000$
≈ 0

2. The probability that any job applicant is granted an interview, after submitting a resume, is 0.30. If there are 450 applicants, please compute
   a. $P(X < 140)$ b. $P(X \geq 120)$

$p = 0.30$
$n = 450$

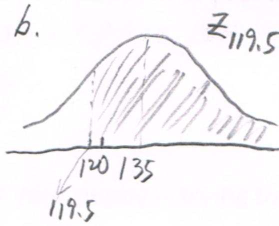
$\mu = np = 135$. $\sigma = \sqrt{np(1-p)}$
$= \sqrt{450 \cdot 0.30 \cdot 0.70}$
≈ 9.7211

a. $Z_{139.5} = \frac{139.5 - \mu}{\sigma}$
$= \frac{139.5 - 135}{9.7211}$
≈ 0.4629



$P(X<140) = 0.677$

b. $Z_{119.5} = \frac{119.5 - 135}{9.7211}$
≈ -1.5945



$P(X \geq 120) = 1 - 0.0559$
≈ 0.944

improvement 0.75
no change 0.20
deterioration 0.05
$n = 10$.

3. After taking a certain drug, the probability of improvement is 0.75; of no change, 0.20; of deterioration, 0.05. Among 10 patients who were given the drug, please compute
   a. The probability that no patient experienced a change
   b. The probability that everyone deteriorated
   c. The probability that half improved and half had no change.

a. $P(\text{10 no change}) = \frac{10!}{10!0!0!} \cdot 0.20^{10} \cdot 0.75^0 \cdot 0.05^0 \approx 0.0000001024 \approx 0.000$

b. $P(\text{10 deterioration}) = \frac{10!}{10!0!0!}\, 0.75^0 \cdot 0.20^0 \cdot 0.05^{10} = 0.00000000000009765625 \approx 0.000$

c. $P(\text{5 improved, 5 no change}) = \frac{10!}{5!0!5!} \cdot 0.75^5 \cdot 0.20^5 \cdot 0.05^0 = 252 * 0.75^5 * 0.20^5 * 1 \approx 0.019$

4. Suppose your professor needs a little prodding to respond to emails. The probability that she will respond to any given email is 0.85. What is the probability that the first email she responds to is the third one you send?

$p=0.85$

$P(X=3)=0.15^2\cdot 0.85 \approx 0.019$

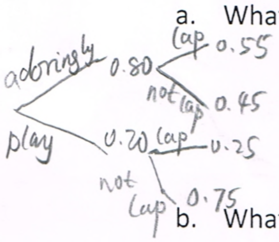
5. A textbook has an average of 3 typos per chapter. What is the probability that a given chapter has at least one typo? At most two typos? $\mu=3$.

$P(X\geq 1)=1-P(X<1)$
$=1-P(X=0)$
$=1-\frac{e^{-3}*3^0}{0!}$
$=1-0.0498$
≈ 0.950

$P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)$
$=0.0498+\frac{e^{-3}*3^1}{1!}+\frac{e^{-3}*3^2}{2!}$
$\approx 0.0498+0.1494+0.2240$
≈ 0.423

6. Your puppy either stares at you adoringly (0.80) or is trying to get you to play (0.20). If she is staring adoringly, she is either in your lap (0.55) or not; and if she is trying to get you to play, she also is either in your lap (0.25) or not.

a. What is the probability that your puppy is in your lap?



$P(\text{lap})=0.80*0.55+0.20*0.25$
$=0.49$

b. What is the probability that your puppy is trying to get you to play, given that she is not in your lap?

$P(\text{not lap while playing})=0.20*0.75=0.15$

7. Toss two dice. What is the probability of getting

a. Both faces odd

b. Sum of faces less than 5

c. Sum of faces greater than or equal to 9

$n=6*6=36$

Sum = 2~12

S = {
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

a. P(both odd) = {(1,1), (3,1), (5,1), (1,3), (3,3), (5,3), (1,5), (3
(5,5)} $=\frac{9}{36}=0.25$

b. P(sum<5) = {(1,1), (2,1), (3,1), (1,2), (2,2), (1,3)} $=\frac{6}{36}$

c. P(sum≥9) = {(6,3), (5,4), (6,4), (4,5), (5,5), (6,5)
(3,6), (4,6), (5,6), (6,6)}
$=\frac{10}{36}=\frac{5}{18}$

Statistics Problem Set #2

1. When discussing ANOVA, we said that ~~that~~ the samples must pass normality tests. Then we described the problem that would arise if this condition were not met. What is that problem?

It is nonnormality. Because one of ANOVA assumptions is the sample must be normal.

2. What is the benefit of using the Scheffe test instead of the Tukey, as an ANOVA post-hoc test?

The scheffé test is customarily used with unequal sample size, although it could be used with equal sample size.
The Tukey test is only usable when the sample sizes are the same.

3. In what sense are the Poisson and Binomial distributions special cases of each other?

The ordinary binomial distribution is a special case of the Poisson binomial distribution, when all success probabilities are the same, that is $p_1 = p_2 = \cdots = p_n$.

4. If a distribution is skewed, can it have kurtosis? If it has kurtosis, can it be skewed?

Yes. only to determine if skewness and kurtosis are significatly non-normal.
skewness interval $(-2\sqrt{\frac{6}{N}}, 2\sqrt{\frac{6}{N}})$
kurtosis interval $(-2\sqrt{\frac{24}{N}}, 2\sqrt{\frac{24}{N}})$

5. Explain why we construct confidence intervals for slope coefficients as part of one-way ANOVA. Regression

A confidence interval with a particular confidence level is intended to give the assurance that, if the statistical model is correct then taken over all the data that might have been obtained, the procedure for constructing the interval would deliver a confidence interval that included the true value of the parameter the proportion of the time set by the confidence lev

6. Since R^2 is never negative, then how can it detect inverse relationships?

only r can determine the relationship.

7. In the context of hypothesis tests for μ, please describe the trade-off between confidence and precision for fixed n. What if n is not fixed?

increased precision comes at the cost of decreased confidence.
more n, the sample becomes more precise, confidence interval will decrease in width.

8. In One-Way ANOVA, can η^2 be big while F_{calc} is small? Answer also in the reverse.

$F_{calc} = \frac{MS\ between}{MS\ within}$, $\eta^2 = \frac{MS\ between}{MS\ total}$.

$MS\ total = MS\ between + MS\ within$.

$\therefore F_{calc} = \frac{\eta^2}{1-\eta^2} = -1 + \frac{1}{1-\eta^2}$

$\Rightarrow \eta^2$ be big while Fcalc is big.

9. In the context of Two Way ANOVA, please define the term interaction.

There is an interaction between two factors if the effect of one factor depends on the levels of the second factor. When the two factors are identified as A and B, the interaction is identified as the A × B interaction.

10. Can a data set's variance ever be smaller than its standard deviation?

Yes. $S = \sqrt{variance}$

11. In the context of hypothesis tests, please define the term power.

The power in hypothesis testing is the probability of correctly rejecting the value stated in the null hypothesis.
The power of a hypothesis test is affected by three factors:
* Sample size (n)
* Significance level (α)
* The "true" value of the parameter being tested.