Chapter 1 Decision-Making Tools

Session 1 Decision Strategy and Decision Table

We make decisions every day, to do homework or to watch a game. We also make decisions in an important life stage, I know some of you are in the four year and you are making one of the most important decisions in your life. I remember when I got my master degree, only 3 years ago. I planned to apply for a PhD program in Business School.

UMD/ UConn/ GSU - Alternatives

Alternatives are something subjective, they are decided by ourselves.

You also have something that is objective, what you cannot control, the number of competitors, the policy from the US government.

To make it simple: say we only have two scenarios: Good application environment, Bad application environment

We called this **state-of-nature**

Alternatives: A course of action or strategy that may be chosen by a decision maker (e.g. applying for UConn; taking umbrella tomorrow)

State of nature: An occurrence or a situation over which the decision maker has little or no control (e.g. bad application market-too many competitors and very few positions; tomorrow's weather)

Example

We want to determine the best real estate investment project given the following table of payoffs for three possible interest rate scenarios.

	Interest Rates (%)		
Projects	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office building	1.5	1.9	2.4
Warehouse	1.7	1.4	1.0
Shopping center	0.7	2.4	3.6
Apartment	3.2	1.5	0.6

S1.1 Decision making under uncertainty

The **optimistic** decision criteria. (Maximax)

Find the highest interest rate regardless of different states of nature.

	Interest Rates		
Projects	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office building	1.5	1.9	2.4
Warehouse	1.7	1.4	1.0
Shopping center	0.7	2.4	3.6
Apartment	3.2	1.5	0.6

Step 1: take the Maximum of each row

Step 2: take the Maximum of the column of maxima

The **pessimistic** (but also most safe) decision criteria. (Maximin)

Which project's lowest interest rate is highest?

Step 1: take the Minimum of each row

Step 2: take the Maximum of the column of minima

	Interest Rates		
Projects	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office building	1.5	1.9	2.4
Warehouse	1.7	1.4	1.0
Shopping center	0.7	2.4	3.6
Apartment	3.2	1.5	0.6

Equally likely: take average of each row and choose the optimal one(s).

	Interest Rates			
Projects	Decline	Stable	Increase	Average
Office park	0.5	1.7	4.5	2.23
Office building	1.5	1.9	2.4	1.93
Warehouse	1.7	1.4	1.0	1.37
Shopping center	0.7	2.4	3.6	2.23
Apartment	3.2	1.5	0.6	1.77

Extension

Which one is generally the best? Taking average.

Max/min can be easily affected by outliers (extreme values).

e.g. We have a series of number, Kobe Bryant's scores per game (SPG) in each season:

7.6 15.4 19.9 22.5 28.5 …

min: 7.6 (96-97) max: 35.4 (05-06) average: 25.0 (whole career)

If we use one number to describe Kobe Bryant as an attacker, the min and max are not reasonable.

S1.2 Decision making under risk

Expected Value

How to calculate expected value?

Expectation is the weighted average of all values.

e.g. Roll a fair die:

X	1	2	3	4	5	6
Prob.	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Expected Value: $EX = \frac{1}{6} \times (1 + 2 + 3 + 4 + 5 + 6) = 3.5$

Expected value is equal to the sum of each value times its corresponding probability.

If we have probabilities for each state of nature, we can use them to calculate an expected value (EV), also called expected monetary value (EMV) when it is about money.

	Interest Rates		
	Decline	Stable	Increase
Prob. of Scenarios	p_D	p_S	p_I
Office park	0.5	1.7	4.5
Office building	1.5	1.9	2.4
Warehouse	1.7	1.4	1.0
Shopping center	0.7	2.4	3.6
Apartment	3.2	1.5	0.6

Suppose $p_D = p_S = p_I = \frac{1}{3}$, which means we have equal likelihood for each case, then we simply take average of each row, and find the optimal value.

<u>What is the difference between this case and “equally likely”?</u>

The first one is under uncertainty, the second one is under risk. The decisions are the same, but the conditions are different. Uncertainty means we have no information about the probability of state of nature, the best we can do is to take average, thus making use of the information as much as possible; however, in the second case, the probability of each state of nature happens to be the same, so we use this information and get the expected value.

	Interest Rates			
	Decline	Stable	Increase	
Prob. of Scenarios	$p_D = \frac{1}{3}$	$p_S = \frac{1}{3}$	$p_I = \frac{1}{3}$	Expected Value
Office park	0.5	1.7	4.5	2.23
Office building	1.5	1.9	2.4	1.93
Warehouse	1.7	1.4	1.0	1.37
Shopping center	0.7	2.4	3.6	2.23
Apartment	3.2	1.5	0.6	1.77

We will select either the Office park or the Shopping center.

Suppose $p_D = 0.5, p_S = 0.35, p_I = 0.15$, then we need to take **weighted** average of each row, and find the optimal value.

	Interest Rates			
	Decline	Stable	Increase	
Prob. of Scenarios	$p_D = 0.5$	$p_S = 0.35$	$p_I = 0.15$	Expected Value
Office park	0.5	1.7	4.5	1.52
Office building	1.5	1.9	2.4	1.775
Warehouse	1.7	1.4	1.0	1.49
Shopping center	0.7	2.4	3.6	1.73
Apartment	3.2	1.5	0.6	2.215

Extension

Why do we call it "under risk"? Risk is related to expected value.

Gamble: you pay \$100, then we flip a coin. If head, you will get \$200; if tail, you will get nothing. The expected payoff is exactly \$100, which is equal to the cost you pay. If we repeat this game for millions of times, you will lose nothing.

If you refuse to play this game – risk-averse

If you do not care – risk-neutral

If you prefer – risk-lover

For risk-averse, risk is actually a type of cost/loss, people need extra money to make up their feeling of risk.

S1.3 Decision making under certainty

Expected Value under Certainty (EVUC)

What would we do if we knew which scenario would come true? For example, if we know it will be a decline scenario, then we will choose Apartment. Similarly, if we know it will be stable, we will choose shopping center; if increase, then office park.

<u>Definition</u> EVUC

Pick the decision that has the highest value for that column, and weight these values by the corresponding probabilities. We call this result the Expected Value under Certainty (EVUC)

$$EVUC = 0.5 \times 3.2 + 0.35 \times 2.4 + 0.15 \times 4.5 = 3.115$$

The number is larger than EV (=2.215), because we assume we know "something", which is actually the **information**. But how much does such information worth? Compare EVUC and EV, the difference is the value of information, we call this:

Expected Value of Perfect Information (EVPI)

$$EPVI = EVUC - EV = 3.115 - 2.215 = 0.9$$

Summary

<u>Under uncertainty</u>

The Optimistic Decision Criteria-Maximax

The Pessimistic Decision Criteria-Maximin

Equally likely

<u>Under risk</u>

Expected Value (EV) or Expected Monetary Value

<u>Under certainty</u>

Expected Value under Certainty (EVUC)

Expected Value of Perfect Information (EVPI) = EVUC – EV

Session 2 Decision Tree

S2.1 Elements of Decision Tree

Announcement for quiz and problems set, taking laptop.

Review on **Decision Table**

How to understand decision under **certainty**.

How to understand EVUC, let's start from EV, the second scenario.

Start from the second scenario, suppose we repeat this investment 100 times, among which, in expectation, we will have 50 times decline, 35 times stable, and 15 times increase. Our decision will always be "Apartment", the total payoff is 50*3.2+35*1.5+15*0.6=221.5.

Our decisions are not **perfect**? Why?

If we knew the probability, our decision would be … (if we know it will be a decline scenario, then we will choose Apartment. Similarly, if we know it will be stable, we will choose shopping center; if increase, then office park.)

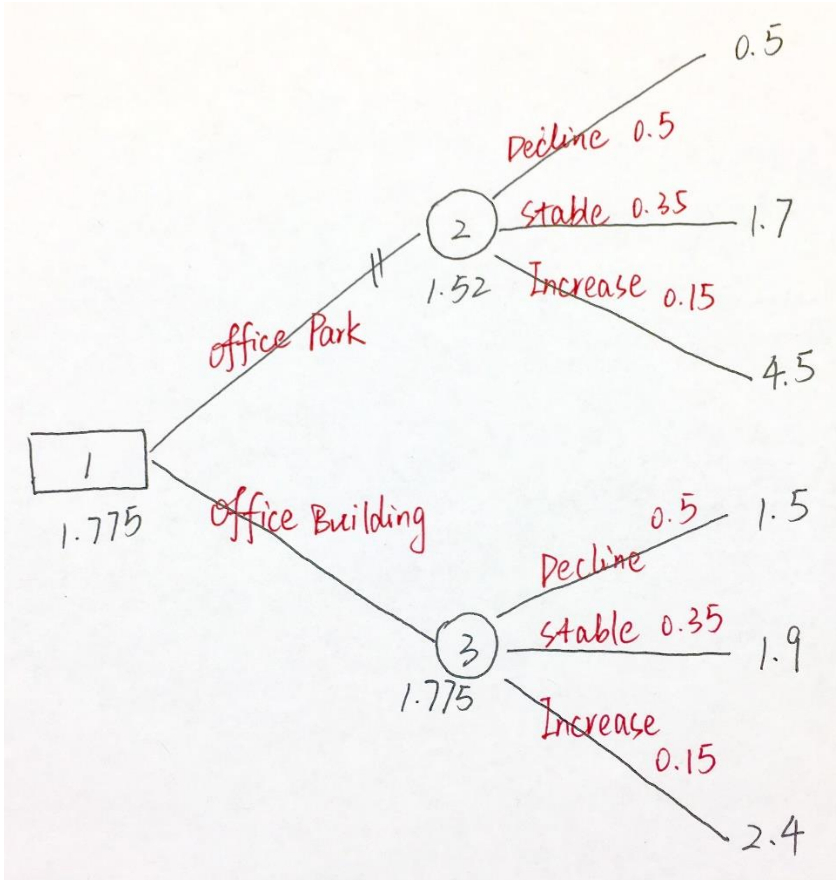
subjunctive mood (If I were a bird, I would be able to fly in the air. I wish I could pass the examination.)

This is exactly the third scenario: under certainty.

First, is there any **order** for human's decision and nature's decision? Which happens first?

Example from last class:

	Decline	Stable	Increase	
Prob. of Scenarios	$p_D = 0.5$	$p_S = 0.35$	$p_I = 0.15$	Expected Value
Office park	0.5	1.7	4.5	1.52
Office building	1.5	1.9	2.4	1.775



EV of Node 2: $0.5 \times 0.5 + 1.7 \times 0.35 + 4.5 \times 0.15 = 1.52$

EV of Node 3: $1.5 \times 0.5 + 1.9 \times 0.35 + 2.4 \times 0.15 = 1.775$

EV of Node 1: choose the larger one, i.e. 1.775

Decision: we should choose the office building.

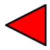
The overall expected value is 1.775.

<u>How to draw a decision tree</u>

1. Two types of nodes

State-of-nature node: **circle** 

Alternative node: **rectangle** 

End node: triangle  (not commonly used)

<u>Remark:</u>

There exists probability only for **states of nature**. e.g., how the weather will be, or which football team will win a game. Each state of nature may happen with corresponding probability.

Alternative is an option without likelihood. For example, we choose among alternatives A, B, and C. If we choose A, then the “probability” of choosing A is “1”, the probability of choosing B and C are “0”. Thus, it is meaningless to use probability when talking about **alternatives**. Because reasonable people always choose the optimal option, so alternatives have no “probability”. Never write “probability” on the line out of “rectangles”, just choose the optimal one.

2. Each Node contains a single value (write ordinal numbers on each node)

3. End node contains a given value.

4. One possible state of nature will happen out of a circle (state-of-nature node) and we need to take expectation to calculate the EV of a state-of-nature node; Alternative node contains an optimal value (maximum or minimum).

5. The root of the tree (the first rectangle) returns the **overall expected value**.

So far, it seems decision tree and decision table work the same. The probability of states of nature are the same for all the alternatives. Is it possible that the probability will change among different alternatives?

S2.2 Example

Date: Feb. 4, 2018

Time: About 6:30 p.m. ET

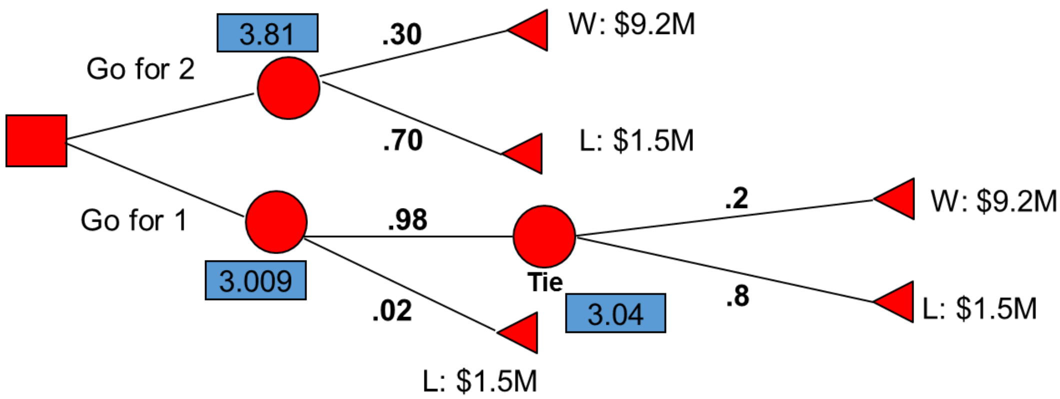
Location: U.S. Bank Stadium, Minneapolis, Minn.

TV: NBC

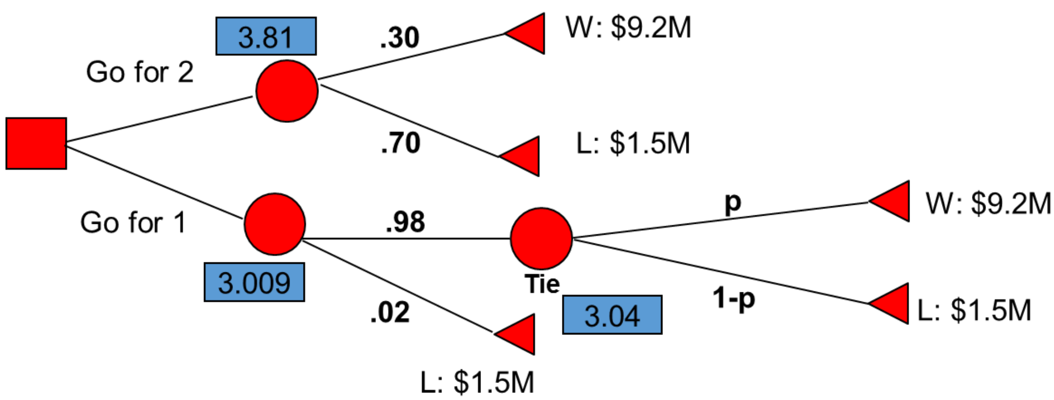
Should the team go for two points (for the win) or one point (for the tie) with no time remaining?

Two-point success rate: 30%; One-point success rate: 98%; Chances in overtime: 20%

Payoffs Win: Sugar Bowl = 9.2M, Lose: Gator Bowl = 1.5M



What must the probability of winning in overtime be to make them indifferent between going for one and going for two?



$$EV\ of\ \text{Tie}: 9.2 \times p + 1.5 \times (1-p) = 1.5 + 7.7p$$

$$EV\ of\ \text{Go for 1}: 0.98 \times (1.5 + 7.7p) + 0.02 \times 1.5 = 1.5 + 7.546p$$

$$EV\ of\ \text{Go for 2} = 3.81$$

We want EV of "Go for 1" equals to EV of "Go for 2":

So we need: $1.5 + 7.546p = 3.81$

$$\Rightarrow p = 0.306$$

S2.3 A Sequence of Decisions

Now, let's simplify the first example, and add one more potential decision process.

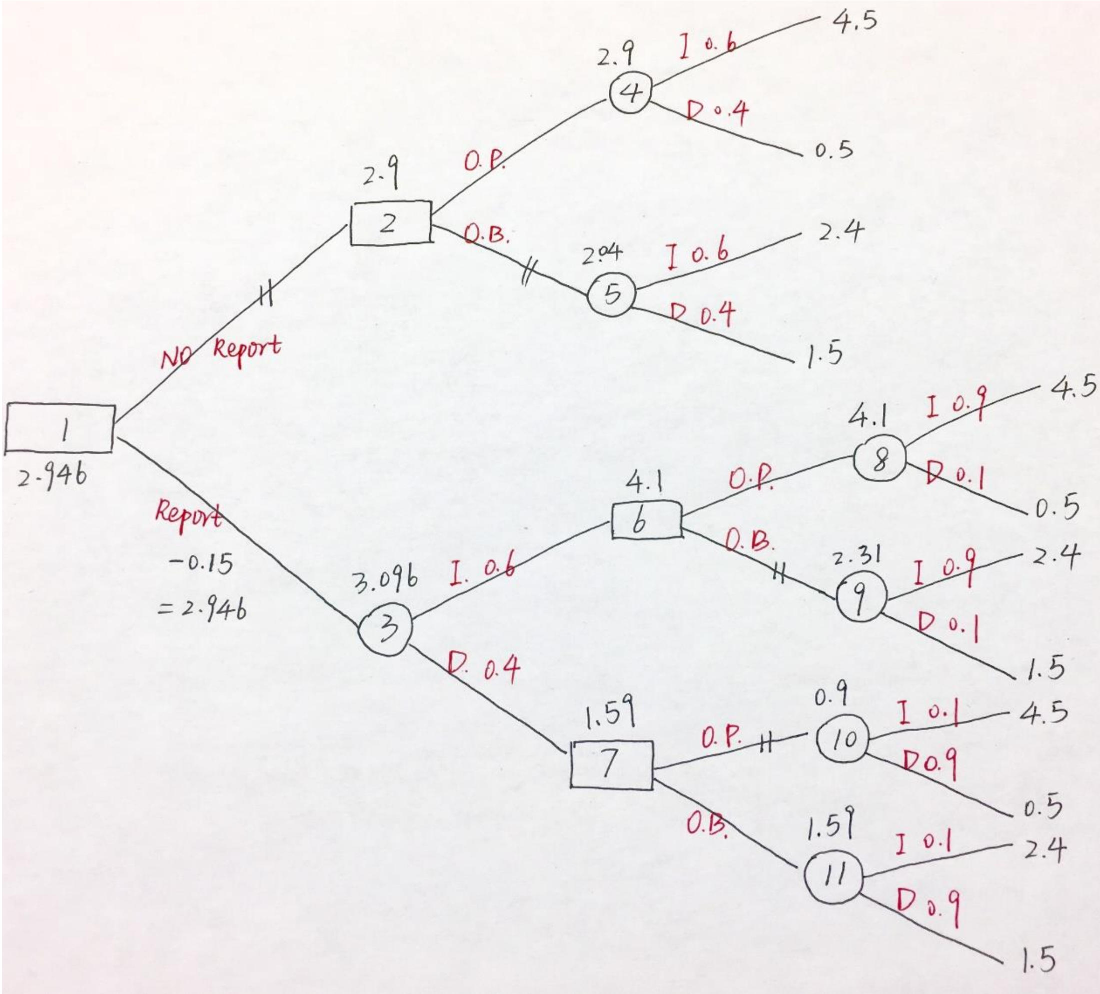
	Decline	Increase
Prob. of Scenarios	$p_D = 0.6$	$p_I = 0.4$
Office park	0.5	4.5
Office building	1.5	2.4

Suppose before choosing from office park and office building, we could either make the decision right away, or pay 0.15% rate of benefits to a consulting firm for a report, and then decide what action to take.

The report is 90% accurate. (If the report says Decline, then the probability of decline will change to 0.9. If the report says Increase, then the probability of Increase will change to 0.9.)

There is a 60% chance of an Increase prediction from the report.

Now this problem becomes much more complicated, and can hardly be solved by a decision table.



<u>When **a sequence of decisions** must be made, decision trees are more powerful.</u>

How to describe decisions? You need to tell **all** the information of a sequence of decisions.

First, we need to ask for the report; then, we choose the Office park.

Quiz 1

Quiz 1 will include one problem context. From this context, you need to be able to identify the alternatives and states of nature. You may also need to calculate the expected values of each state of nature, and the overall expected value of the root. Of course, you need to draw a decision tree.

Intermission A Simple Review on Excel

Make a copy to a new sheet.

1. Sheet and File, name a sheet

2. View – Freeze Panes

3. “Drag” and double click, ctrl+up,

Fix a column/row $

4. Simple calculation

Every time when you calculate something, it should begin with an equal sign “=”.

$+ - \times \div; a^b;$

5. Function (case-insensitive)

Using “Tab” to finish one function.

Every time when you use a function, it should begin with an equal sign “=”.

= average (); = sum (); =sumproduct() (expected value); =abs()