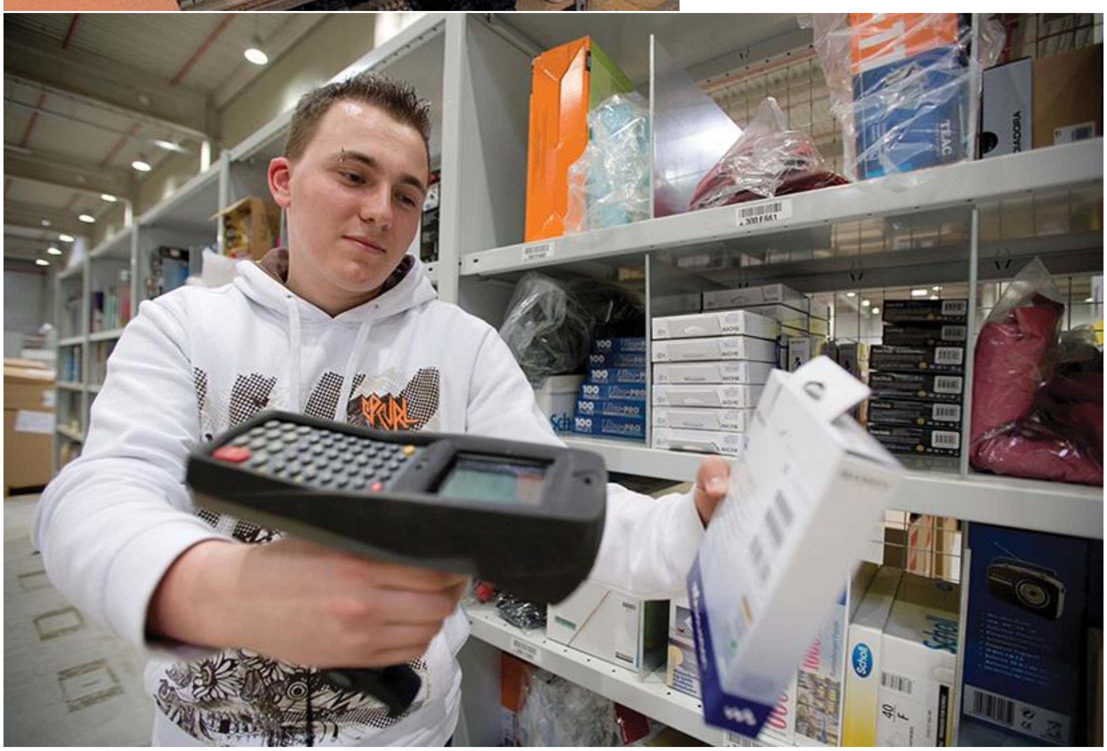
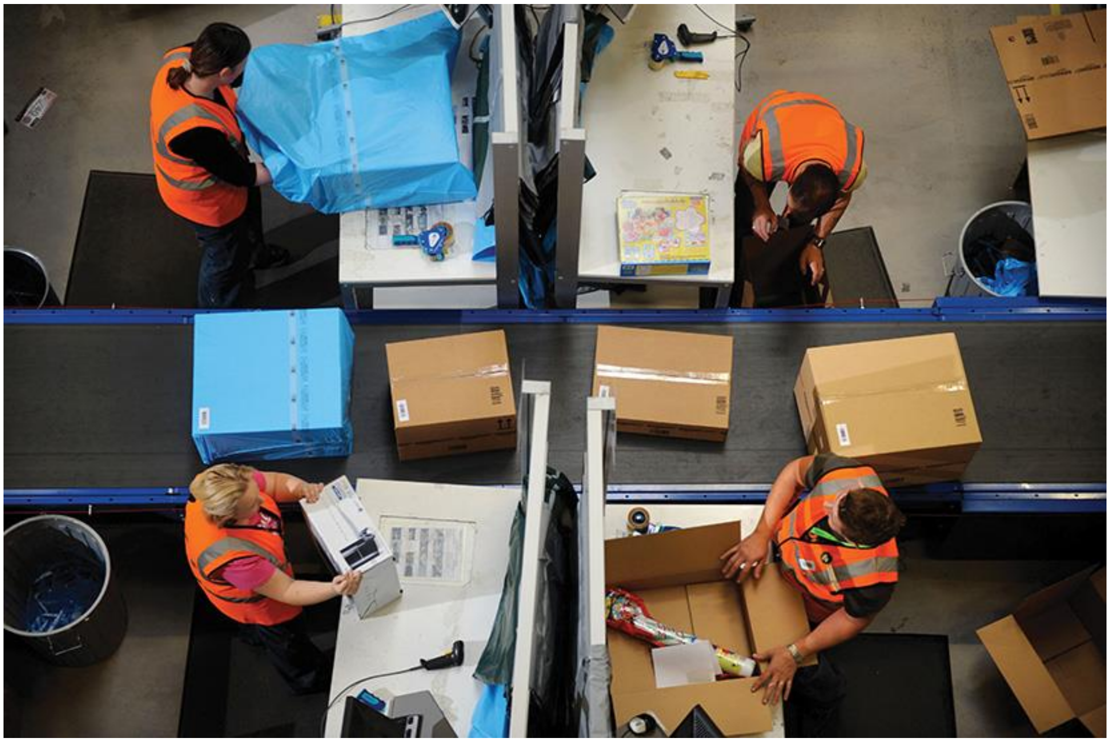
Chapter 7 Inventory Management





Suppose you are the manager of a supermarket and you want to sell apples to satisfy your customers. Also suppose the suppliers of apples are fruit farmers, and the apple trees are located a rather long distance from your supermarket.

Every time you need to use a truck to take a certain amount of apples from the production place to your supermarket. One ideal case for the supermarket is that we can produce and sell at the same time and location, which is almost impossible, especially for a supermarket that is selling numbers of types of goods.

1) Therefore, the supermarket need to ship apples from the farmers.

2) In addition, they cannot simply order one apple at one time, but they need to ship a certain amount of apples at one time. Therefore, they need to find a way to keep these apples and prevent from going bad.

Both of these two steps will cost money.

What you want is to minimize your cost during the whole process.

How to solve this problem?

Business Problem ⟹ (abstracted into) Math Problem⟹ Solve it with math tools (Operations Research) ⟹ Put the result back into the business background

*The ability to "**Abstract**" a business problem into a math problem is one of business students' core competences. We have business problem – usually we have a trade off, something goes up, something will go down (inventory, if you want to stock, holding cost will increase…but the order cost/setup cost will decrease, then you will have quadratic curve) – we need to find a balance point to maximize the profits or minimize the cost. Then we abstract this business problem to a math problem, and solve it, this is what OM does. How to solve it? It is about math, usually we call it OR*

1. Transfer a business context to a pure math problem – Formulation

The first step is the most important and usually most difficult.

2. Solve the math.

We usually have some regular/conventional method in this step. Nowadays' research focuses on this step because problems can be too difficult to solve. Researchers will think of some method to find a sub-optimal solution (the optimal solution will take hundreds of years)

Let's focus on the first step.

Let's discuss several things about this problem.

Can we just wait for the customers' request? We need to have some inventory - stocked goods!

0. Why do we need inventory?

Reasons to Hold Inventory

- Meet unexpected demand
- Smooth seasonal or cyclical demand
- Meet variations in customer demand
- Take advantage of price discounts
- Hedge against price increases
- Quantity discounts

1. What is the focal/decision variable in this problem?

focal/decision variable: in business context, focal variable is something policy makers decide to optimize the process. (either maximize the profits or minimize the cost)

You want to decide how many apples in one order to purchase from fruit farmers.

2. What kind of cost you need to consider in this process and how the cost will change as the order amount changes.

a) We need to hire some workers/rent some machines to pick the apples

b) We need to arrange some trucks/train to ship the apples to the supermarket

c) We may need a room/house and fridge to keep the apples

d) Of course, the apples themselves also cost

Comparison the unit cost:

10 apples in one order vs. 100 apple in one order

Inventory Costs

Carrying Cost/Holding Cost

Cost of holding an item in inventory,

May include cost of obsolescence

An overall inventory carrying cost of less than 15% is very unlikely, but this cost can exceed 40%, especially in high-tech and fashion industries.

Ordering Cost/Setup Cost

Cost of replenishing inventory

Shortage Cost

Temporary or permanent loss of sales when demand cannot be met

Math Fundamentals:

Inequality of geometric means

$\sqrt[n]{x_1 \cdot x_2 \cdots x_n}$ (geometric mean)

we have that for any list of n nonnegative real numbers $x_1 \cdot x_2 \cdots x_n$,

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

and that equality holds if and only if $x_1 = x_2 = \cdots = x_n$.

I will prove when n=2, and we only need the case for n=2.

First order condition (use derivate for optimization)

First-order condition (FOC) The necessary condition for a relative extremum (maximum or minimum) is that the first-order derivative be zero

http://users.etown.edu/p/pauls/ec309/lectures/lec04_unconst.html

Session 1 EOQ (Economic Order Quantity) Model

Assumptions:

1. Demand for an item is known, reasonably constant, and independent of decisions for other items.

(How do we know the future demand?) – Forecasting, simulation

2. Lead time—that is, the time between placement and receipt of the order—is known and consistent, constant.

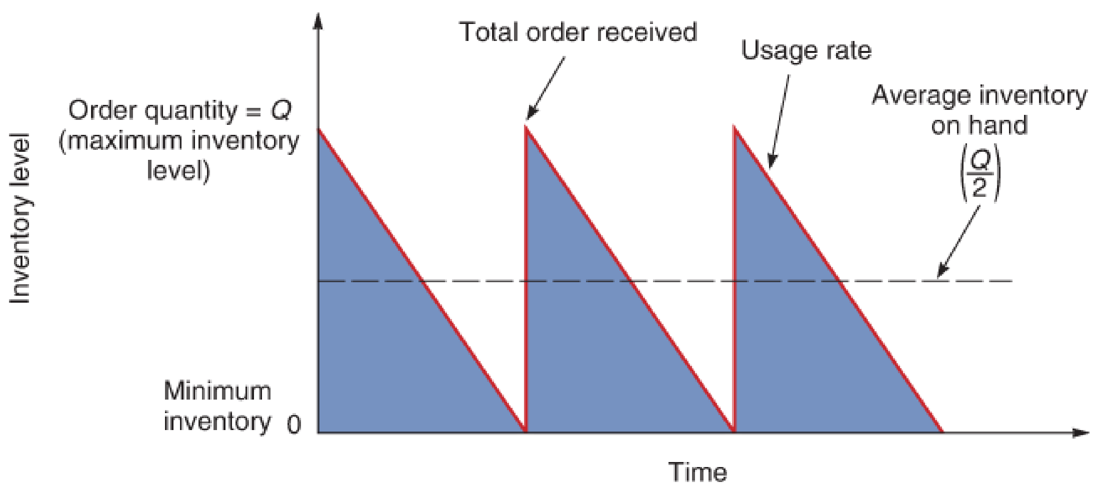
3. Receipt of inventory is instantaneous and complete. In other words, the inventory from an order arrives in one batch at one time.

4. Quantity discounts are not possible. (More orders, more cheap)

5. The only variable costs are the cost of setting up or placing an order (setup or ordering cost) and the cost of holding or storing inventory over time (holding or carrying cost).

6. Stockouts (shortages) can be completely avoided if orders are placed at the right time.

With these assumptions, the graph of inventory usage over time has a sawtooth shape. In the following Figure, Q represents the amount that is ordered. If this amount is 500 dresses, all 500 dresses arrive at one time (when an order is received). Thus, the inventory level jumps from 0 to 500 dresses. In general, an inventory level increases from 0 to Q units when an order arrives. Because demand is constant over time, inventory drops at a uniform rate over time. (Refer to the sloped lines in Figure) Each time the inventory is received, the inventory level again jumps to Q units (represented by the vertical lines). This process continues indefinitely over time.

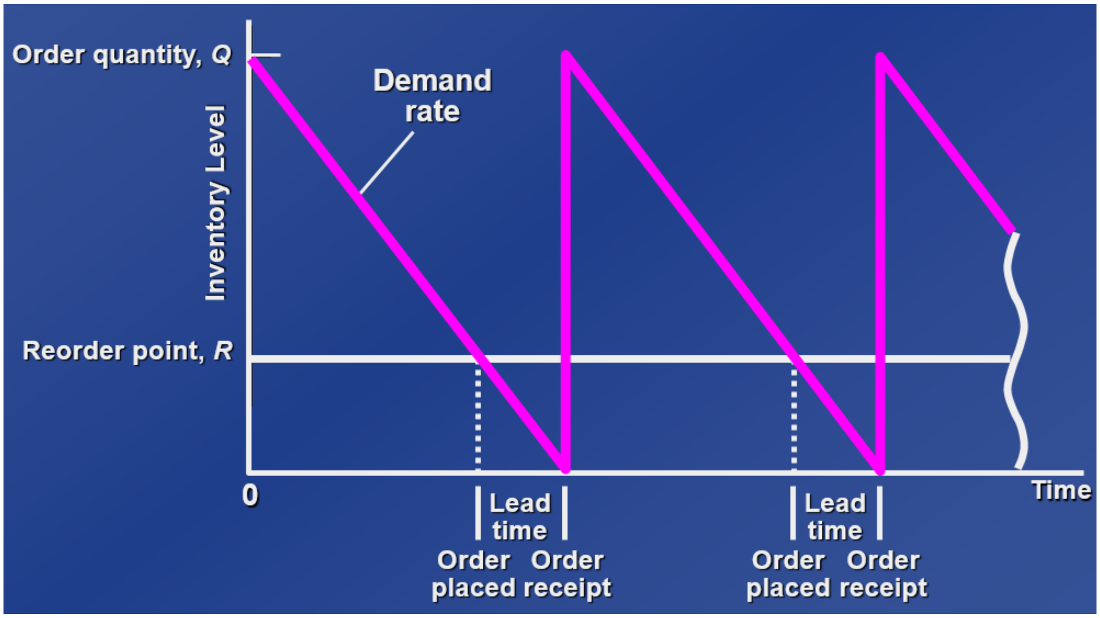


Average inventory

similar example,

A car with an initial speed 10m/s, with a fixed acceleration a = -2m/s2

v0 = 10, vt = 0



When to order

Reorder Point is the level of inventory at which a new order is placed

R = dL

where

  ***d* = demand rate per period**

  ***L* = lead time**

Example

Demand = 10,000 yards/year

Store open 311 days/year

Daily demand = 10,000 / 311 = 32.154 yards/day

Lead time = L = 10 days

R = dL = (32.154)(10) = 321.54 yards

Next, let's solve the optimal Q.

Notations:

Q = Number of units per order

Q* = Optimum number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Notice: Q is the decision variable (unknown), our task is to determine the minimum cost with a suitable Q, given D, S, and H.

1. Develop an expression for setup or ordering cost.

We know the unit setup cost (S), we also need to know how many orders we will place in the whole year.

We know the demand for the whole year is D, if each order contains Q units

then Number of orders placed per year = D/Q

Annual setup cost = (Number of orders placed per year) × (Setup or order cost per order)

$= \frac{D}{Q} \times S$

2. Develop an expression for holding cost.

Similarly, we know the unit holding cost (H), we also need to know how many orders we hold in the inventory, however, this number variate all the time.

Annual holding cost = (Average inventory level) × (Holding cost per unit per year)

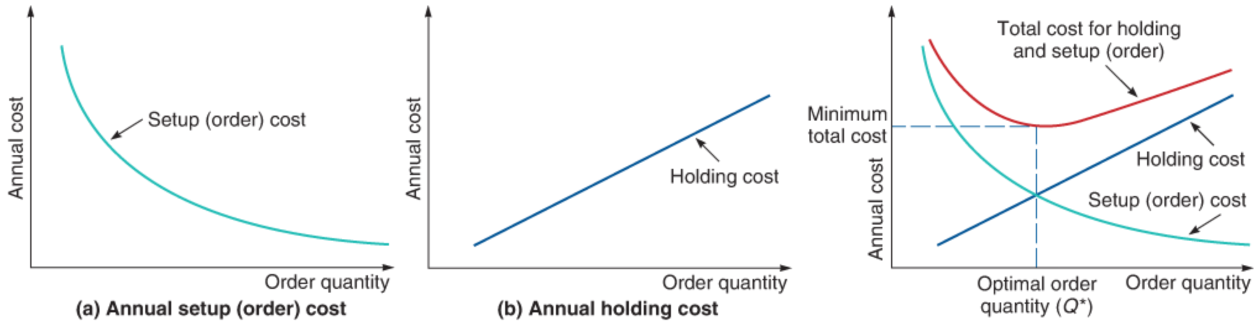
$= \frac{Q}{2} \times H$

$$\mathrm{TC} = \frac{D}{Q} \times S + \frac{Q}{2} \times H \geq 2\sqrt{\frac{D}{Q} \times S \ \times \frac{Q}{2} \times H} = \sqrt{2\mathrm{D}SH}$$

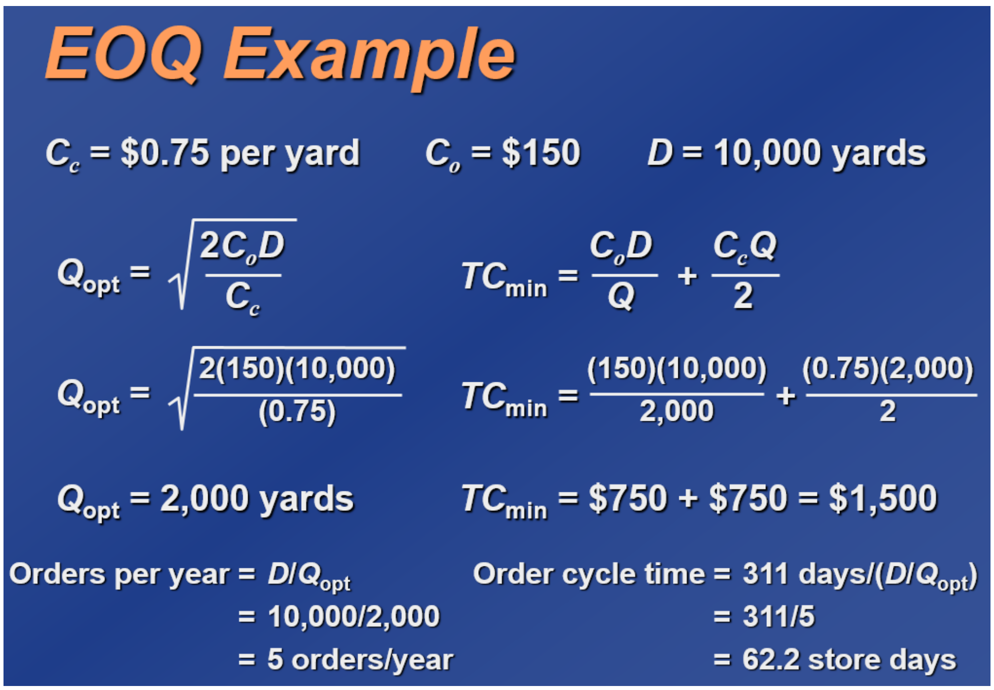
The equal sign is achieved when $\frac{D}{Q} \times S = \frac{Q}{2} \times H \Longrightarrow Q^* = \sqrt{\frac{2\mathrm{D}S}{H}}$

Or use derivative to get the minimum (First-order condition)

$$\mathrm{TC}_{min} = \sqrt{2\mathrm{D}SH}$$



(a) Annual setup (order) cost (b) Annual holding cost (c) Total costs



Example

Finding the Optimal Order Size at Sharp, Inc. Sharp, Inc., a company that markets painless hypodermic needles to hospitals, would like to reduce its inventory cost by determining the optimal number of hypodermic needles to obtain per order. APPROACH The annual demand is 1,000 units; the setup or ordering cost is $10 per order; and the holding cost per unit per year is $.50. (Directly use the formula, 200)

LEARNING EXERCISE

If D increases to 1,200 units, what is the new Q*? [Answer: Q* = 219 units.]

<u>Robust Model</u>

A benefit of the EOQ model is that it is robust. By robust we mean that it gives satisfactory answers even with substantial variation in its parameters. As we have observed, determining accurate ordering costs and holding costs for inventory is often difficult. Consequently, a robust model is advantageous. The total cost of the EOQ changes little in the neighborhood of the minimum. The curve is very shallow. This means that variations in setup costs, holding costs, demand, or even EOQ make relatively modest differences in total cost.

EOQ is a Robust Model

Example

Management in the Sharp, Inc., examples underestimates total annual demand by 50% (say demand is actually 1,500 needles rather than 1,000 needles) while using the same Q. How will the annual inventory cost be impacted?

We will solve for annual costs twice. First, we will apply the wrong EOQ; then we will re-compute costs with the correct EOQ.

If demand is actually 1,500 needles rather than 1,000, but management uses an order quantity of Q = 200 (when it should be Q = 244.9 based on D = 1,500), the sum of holding and ordering cost increases to $125:

$$\text{Annual cost} = \frac{D}{Q} \times S + \frac{Q}{2} \times H = \frac{1500}{200} * 10 + \frac{200}{2} * 0.5 = 125 \text{ (True cost)}$$

However, had we known that the demand was for 1,500 with an EOQ of 244.9 units, we would have spent $122.47, as shown:

$$\text{Annual cost} = \frac{D}{Q} \times S + \frac{Q}{2} \times H = \frac{1500}{244.9} * 10 + \frac{244.9}{2} * 0.5 = 122.47 \text{ (True cost)}$$

Note that the expenditure of $125.00, made with an estimate of demand that was substantially wrong, is only 2% ($2.52/$122.47) higher than we would have paid.

LEARNING EXERCISE

Demand at Sharp remains at 1,000, H is still $.50, and we order 200 needles at a time (as in Example 5). But if the true order cost is $15 (rather than $10), what is the annual cost? [Answer: Annual order cost increases to $75, and annual holding cost stays at $50. So the total cost = $125.]

$$\text{Expected number of orders} = \text{N} = \text{Demand/Order quantity} = \text{D/Q} *$$

$$\text{Expected time between orders } = \text{T} = \text{Number of working days per year/N}$$

Why robust? Think about math

Session 2 POQ (Production Order Quantity) Model

In the previous inventory model, we assumed that the entire inventory order was received at one time. There are times, however, when the firm may receive its inventory over a period of time. Such cases require a different model, one that does not require the instantaneous-receipt assumption. This model is applicable under two situations:

(1) when inventory continuously flows or builds up over a period of time after an order has been placed or

(2) when units are produced and sold simultaneously.

Under these circumstances, we take into account daily production (or inventory-flow) rate and daily demand rate. Figure shows inventory levels as a function of time (and inventory dropping to zero between orders).

Because this model is especially suitable for the production environment, it is commonly called the production order quantity model.

or EOQ with Noninstantaneous Receipt

It is useful when inventory continuously builds up over time, and traditional economic order quantity assumptions are valid. We derive this model by setting ordering or setup costs equal to holding costs and solving for optimal order size, Q*. Using the following symbols, we can determine the expression for annual inventory holding cost for the production order quantity model :

Q = Number of units per order

H = Holding cost per unit per year

p = Daily production rate

d = Daily demand rate, or usage rate

t = Length of the production run in days

$\text{Annual inventory holding cost} = \text{Average inventory level} \times \text{Holding cost per unit per year}$

$(\text{Average inventory level}) = \frac{\text{Maximum inventory level}}{2}$

$(\text{Maximum inventory level}) =$

(Total production during the production run) − (Total used during the production run) =

$\text{pt} - \text{dt}$

However, $\text{Q} = \text{total produced} = \text{pt}$, and thus $\text{t} = \frac{\text{Q}}{\text{p}}$. Therefore:

$\text{Maximum inventory level} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q - \frac{d}{p} \times Q = Q\,(1 - \frac{d}{p})$

Annual inventory holding cost (or simply holding cost) =

$$\frac{\text{Maximum inventory level}}{2} \times H = \frac{Q}{2}[1 - \frac{d}{p}]H$$

The setup cost does not change ($\frac{D}{Q}S$), Holding cost = $\frac{Q}{2}[1 - \frac{d}{p}]H$

$$Q_p^* = \sqrt{\frac{2DS}{H[1 - d/p]}}$$

EOQ is a special case of POQ. When we relax some assumptions, we will have a more generalized version, which will be more close to the reality, and of course will be more complicated.

Mathematically solvable may not be pragmatic. Limitation of computing

Session 3 Quantity Discount Models

Total annual cost=Annual setup (ordering) cost+Annual holding cost+Annual product cost,

or

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + P_Q D$$

where

$Q =$ Quantity ordered

$D =$ Annual demand in units

$S =$ Setup or ordering cost per order

$P =$ Price per unit, P is a function of Q, therefore, we cannot take derivative to find the optimal quantity

$I =$ Holding cost per unit per year expressed as a percent of price P

Example

Quantity Discount Model

Price Range	Quantity Ordered	Price Per Unit P
Initial Price	1–119	$100
Discount Price 1	120–1,499	$98
Discount Price 2	1,500 And Over	$96

Chris Beehner Electronics stocks toy remote control flying drones. Recently, the store has been offered a quantity discount schedule for these drones. This quantity schedule was shown in Table 12.2. Furthermore, setup cost is $200 per order, annual demand is 5,200 units, and annual inventory carrying charge as a percent of cost, I, is 28%. What order quantity will minimize the total inventory cost?

$$\mathrm{D} = 5200, \mathrm{S} = 200, \mathrm{I} = 0.28$$

<u>SOLUTION</u>

First we calculate the Q* for the lowest possible price of $96, as we did earlier:

$$\mathrm{Q}^*_{\$96} = \sqrt{\frac{2\mathrm{DS}}{\mathrm{IP}}} = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$96)}} = 278 \text{ flying drones per order}$$

Because $278 < 1{,}500$, this EOQ is infeasible for the $96 price. So now we calculate Q* for the next-higher price of $98:

$$\mathrm{Q}^*_{\$98} = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$98)}} = 275 \text{ flying drones per order}$$

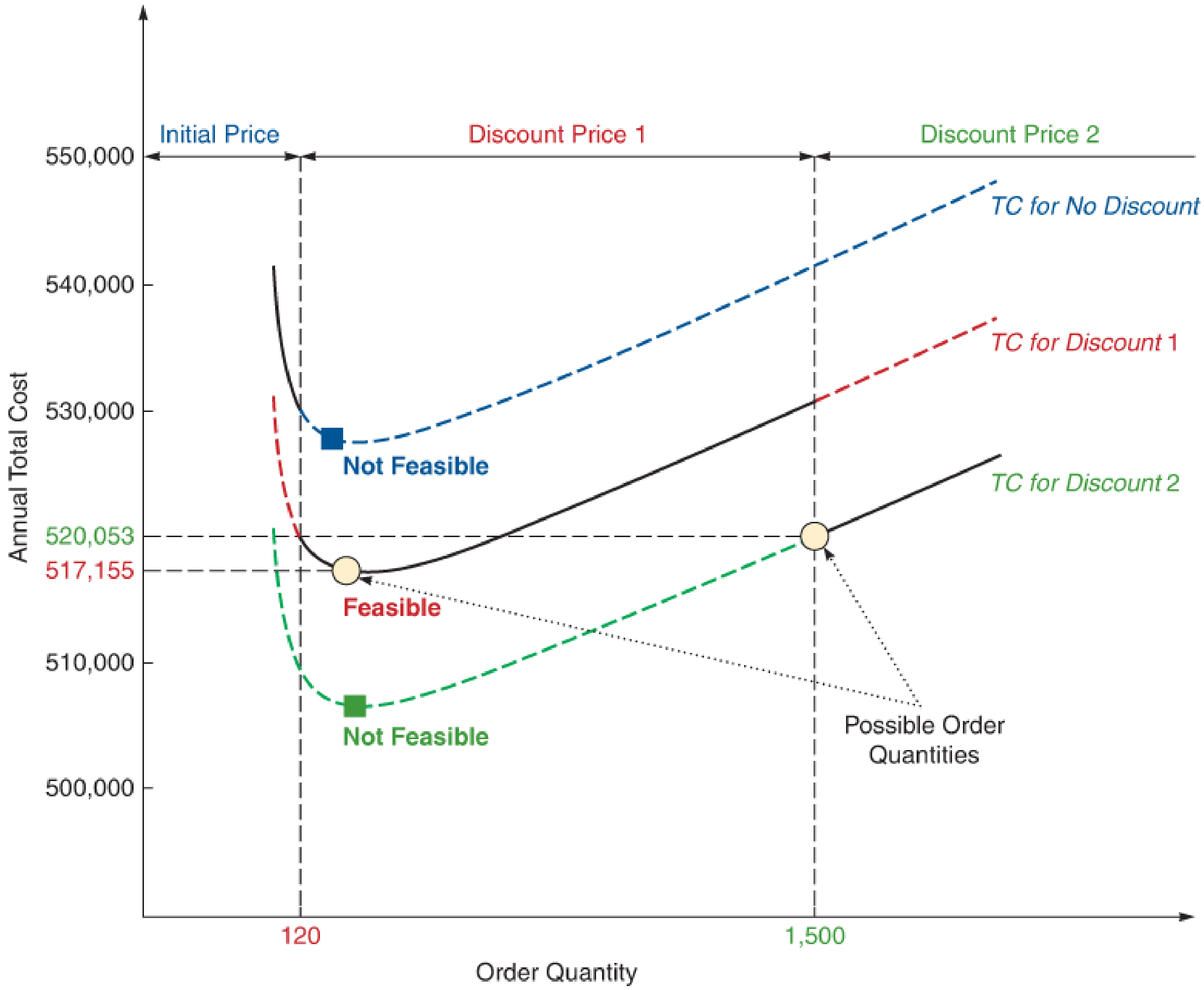
Because 275 is between 120 and 1,499 units, this EOQ is feasible for the $98 price. Thus, the possible best order quantities are 275 (the first feasible EOQ) and 1,500 (the price-break quantity for the lower price of $96). We need not bother to compute Q* for the initial price of $100 because we found a feasible EOQ for a lower price.

Step 2 uses Equation

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + P_Q D$$

to compute the total cost for each of the possible best order quantities. This step is taken with the aid of the following Table.

Order Quantity	Unit Price	Annual Ordering Cost	Annual Holding Cost	Annual Product Cost	Total Annual Cost
275	$98	$3,782	$3,773	$509,600	$517,155
1,500	$96	$693	$20,160	$499,200	$520,053



Because the total annual cost for 275 units is lower, 275 units should be ordered.

INSIGHT

Even though Beehner Electronics could save more than $10,000 in annual product costs, ordering 1,500 units (28.8% of annual demand) at a time would generate even more than that in increased holding costs. So in this example it is not in the store's best interest to order enough to attain the lowest possible purchase price per unit. On the other hand, if the price-break quantity for the $96 had been 1,000 units rather than 1,500 units, then total annual costs would have been $513,680, which would have been cheaper than ordering 275 units at $98.

Summary: Solution Procedure

Step 1: Starting with the lowest possible purchase price in a quantity discount schedule and working toward the highest price, keep calculating Q^* from Equation $Q^* = \sqrt{\frac{2DS}{IP}}$ until the first feasible EOQ is found. The first feasible EOQ is a possible best order quantity, along with all price-break quantities for all lower prices.

Step 2: Calculate the total annual cost TC using Equation

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + P_Q D$$

for each of the possible best order quantities determined in Step 1. Select the quantity that has the lowest total cost.

Note that no quantities need to be considered for any prices greater than the first feasible EOQ found in Step 1. This occurs because if an EOQ for a given price is feasible, then the EOQ for any higher price cannot lead to a lower cost (TC is guaranteed to be higher).

Business Intuition - Assumption

Assumption exists anywhere in science, not only business.

Reasons why we introduce assumptions:

1) Information not available but necessary to solve the problem-assumptions are must;

2) Information is available but too complicated to solve-simplify problems;

3) Results are still practical and applicable

1. More assumptions, farther away from reality

2. Progress of research sometimes is to relax assumption

3. Not solvable may also means cannot be solved with current technology (speed of computation is too slow)

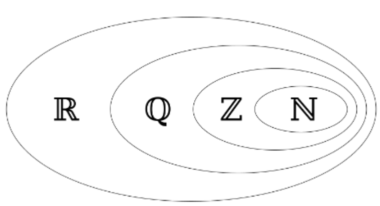
4. Make proper and reasonable assumptions can help humans solve some very difficult problems.

5. Sometimes assumptions can be verified but most assumptions can never be proved.

Math Example:

Natural Number (not close for subtraction) →Integer (not close for division) → Rational Number (not close for taking square root) →Real Number (not close if taking square root for a negative number)→ A complex number is a number that can be expressed in the form a + bi, where a and b are real numbers, and i is a solution of the equation x2 = −1.

The operations of addition, subtraction, multiplication, and division



Classical examples:

1. Linear relationship. We usually assume the relationship between two things are linear.

2. Standard distribution. Why is distribution important?

3. Why is normal distribution very popular? Disturbance. e.g. We all know price can influence demand, but there will be other reasons such as weather, or some factors which cannot be observed (people' mood). We usually assume the impact from the disturbance is normally distributed, with specific variance and zero mean.

4. Why are classical economics criticized? One big assumption: human beings are reasonable.

Economics is a science based on strong assumptions on human beings.