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A New Method for Target Tracking with Debiased Consistent Converted Measurements in Direction Cosines*

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Abstract — In tracking applications, target dynamics is usually modeled in the Cartesian coordinates, while target measurements are got in the sensor coordinates, such as polar, spherical and direction cosine coordinates. Many well-known measurement-conversion techniques for polar and spherical measurements in mechanical rotating/scanning radars have been developed. However, they are not applicable to direction cosine measurements in electronic scanning radars, such as phased array radars. A new method directed to direction cosines measurements for target tracking with debiased consistent conversion (abb. CMKFDcos) is proposed. The converted measurements bias and covariance are explicitly derived. The consistency of converted measurements errors and the covariance is outlined. The simulation results validate that the converted measurements completely capture the true mean and covariance of the original measurements reported in direction cosines. The performance of CMKFDcos is superior to Extended Kalman filter (EKF) and Linear converted measurements KF (CMKFL) in accuracy and consistency for all practical situations. The proposed procedure can be employed in the cross-range errors being significantly large relative to the range errors with the efficiency and modest computational load.

Key words — Target tracking, Phased array radar, Direction cosine, Converted measurement.

I. Introduction

In tracking applications, target dynamics is usually modeled in the Cartesian coordinates (CAR), while target measurements are reported in polar or spherical coordinates in mechanical rotating/scanning radars or Direction cosines (DCOS[1], *i.e.* $R, \alpha = x/R$, $\beta = y/R$) in electronic scanning radars. Generally, this nonlinear filtering problem of target tracking can be handled in two ways. One method is using EKF in mixed coordinates. The other approach is converting the measurements to the CAR and use a linear Kalman filter on converted measurements (CMKFL). With the EKF[2], the prediction is done in CAR and the measurement prediction is the nonlinear mapping of the prediction in CAR. And the predicted measurement covariance is the conversion of state predicted covariance by the Jacobian. The measurements are directly utilized in their DCOS form, so EKF gain is a first-order approximation due to the use of the Jacobian.

The CMKFL is proposed in Ref.[3]. In this case the Cartesian components of the errors in the converted measurements are correlated. The CMKFD (debiased converted measurements KF) presented in Refs.[4, 5] outperforms the EKF and CMKFL in terms of accuracy and consistency. But the result is only for polar and spherical measurements. The BLUE (Best linear unbiased filtering) outlined in Refs.[6-8] is theoretically optimal in the sense of minimizing the means-square error among all linear unbiased filters in CAR. Its characteristics is converting the polar measurements into the CAR in a pseudo-linear form[6,8], and there is no the fundamental limitations of the measurement-conversion approach. The special expression is not suitable for converting DCOS measurements into CAR. So the measurement-conversion approach should be accounted for DCOS measurements for target tracking in CAR. In addition, as we known that the converting measurements accuracy depends on the geometry (range and two direction cosines) and the inaccuracies of these measurements as well as the ratio of the cross-range errors relative to the range errors[4]. In view of this, the severity of the converting measurements reported by phased array radars in DCOS is plainly standout and it can be seen from several factors as follows. First, phased array radar is widely used for many fields and longer range is desired. Second, sometimes, the beam width is up to several degrees required by rapidly searching a volume of space and locating the position of targets, therefore the angle accuracy is poor. In addition, the range accuracy can be made more easily good than angle for all radars. Thus these factors listed above result in the cross-range errors being significantly large relative to the range errors.

The rest of the paper is organized as follows. The converted measurements bias and covariance are derived in Section II. It is the main contribution of this work. A new method directed to DCOS measurements for target tracking with debi-

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ased consistent conversion (CMKFDcos) is proposed in Section III. Some simulation and comparison are done in Section IV. The conclusions are given in Section V.

II. Converted Measurements Errors and Covariance

Phased array radar measurements are given by $Z_{m\cos} = (R_m\alpha_m\beta_m)^T$, $R\alpha\beta$ are assumed to be the true position of target, we have

$$R = R_m + \tilde{R}, \alpha = a_m + \tilde{\alpha}, \beta = \beta_m + \tilde{\beta} \tag{1}$$

where the measurement error $V_k = (\tilde{R}\tilde{\alpha}\tilde{\beta})^T$ is assumed to be white Gaussian with zero means and covariance $R_{\cos}(\sigma_R^2, \sigma_\alpha^2, \sigma_\beta^2)$, the DCOS-to-CAR is

$$Z_m = (x_m y_m z_m)^T = (R_m\alpha_m R_m\beta_m R_m\gamma_m)^T \tag{2}$$

Similarly, we have

$$\begin{aligned} x &= x_m + \tilde{x} = (\alpha_m + \tilde{\alpha})(R_m + \tilde{R}) \\ y &= y_m + \tilde{y} = (\beta_m + \tilde{\beta})(R_m + \tilde{R}) \\ z &= z_m + \tilde{z} = (\gamma_m + \tilde{\gamma})(R_m + \tilde{R}) \end{aligned} \tag{3}$$

The converted measurement errors can be given by expanding Eq.(3)

$$\begin{aligned} &\tilde{x} = R_m\tilde{\alpha} + \alpha_m\tilde{R} + \tilde{\alpha}\tilde{R}, \quad \tilde{y} = R_m\tilde{\beta} + \beta_m\tilde{R} + \tilde{\beta}\tilde{R} \\ &\tilde{z} = R_m\tilde{\gamma} + \gamma_m\tilde{R} + \tilde{\gamma}\tilde{R} \end{aligned} \tag{4}$$

which is not independent, the errors depend on the range and direction cosines as well as the measurement accuracies, moreover, the statistics of $\tilde{\gamma}$ is unknown and the relationship of γ and α, β is

$$\gamma = z/R = \cos(\varepsilon) = \sqrt{1 - \alpha^2 - \beta^2} \tag{5}$$

where ε is known as scan angle[1], taking the first-order terms of a Taylor series expansion (TSE) for γ around α_m, β_m, the error is

$$\tilde{\gamma} = \tilde{\alpha}\gamma'_\alpha + \tilde{\beta}\gamma'_\beta \triangleq \tilde{\gamma}_1 \tag{6}$$

where $\gamma'_\alpha, \gamma'_\beta$ are the first derivative of γ relative to α, β, respectively (It is no longer explained for the similar expressions later). The noises $\tilde{\alpha}$ and $\tilde{\beta}$ are assumed to be independent of each other and of α_m, β_m, and be Gaussian with zero means and standard dev. σ_α and σ_β, respectively. The mean of Eq.(6) conditioned on measurement is $E[\tilde{\gamma}_1/\alpha_m\beta_m] = 0$ (for brevity, let $E1[\tau]$ and $\text{cov1}(\tau)$ stand for $E[\tau/\cdot]$ and $\text{cov}(\tau/\cdot)$, respectively, below). So the converted measurement error is $\tilde{Z}_{m1} = (\tilde{x}\tilde{y}\tilde{z}_1)^T$, where $\tilde{z}_1 = R_m\tilde{\gamma}_1 + \gamma_m\tilde{R} + \tilde{\gamma}_1\tilde{R}$, conditioned on measurement, the mean of $\tilde{Z}_{m1}$ is zero and the covariance $R_{m1} = \text{cov1}(\tilde{Z}_{m1})$ is as follows

$$\begin{aligned} (R_{m1})_{11} =& \alpha_m^2\sigma_R^2 + R_m^2\sigma_\alpha^2 + \sigma_\alpha^2\sigma_R^2 \\ (R_{m1})_{12} =& \alpha_m\beta_m\sigma_R^2 \\ (R_{m1})_{13} =& \alpha_m\gamma_m\sigma_R^2 - \alpha_m R_m^2\sigma_\alpha^2/\gamma_m - \alpha_m\sigma_\alpha^2\sigma_R^2/\gamma_m \\ (R_{m1})_{22} =& \beta_m^2\sigma_R^2 + R_m^2\sigma_\beta^2 + \sigma_\beta^2\sigma_R^2 \\ (R_{m1})_{23} =& \beta_m\gamma_m\sigma_R^2 - \beta_m R_m^2\sigma_\beta^2/\gamma_m - \beta_m\sigma_\beta^2\sigma_R^2/\gamma_m \\ (R_{m1})_{33} =& \gamma_m^2\sigma_R^2 + (\alpha_m^2 R_m^2\sigma_\alpha^2 + \beta_m^2 R_m^2\sigma_\beta^2 \\ &+ \alpha_m^2\sigma_\alpha^2\sigma_R^2 + \beta_m^2\sigma_\beta^2\sigma_R^2)/\gamma_m^2 \end{aligned} \tag{7}$$

The use of zero-mean $\tilde{Z}_{m1}$ does not account for the converted measurements bias, even though the transformation Eq.(2) is biased. Taking the second-order terms of a Taylor series expansion for γ around α_m, β_m, the error is

$$\tilde{\gamma}_2 = \tilde{\gamma}_1 + c + \tilde{b}, \quad c = (\tilde{\alpha}^2\gamma''_\alpha + \tilde{\beta}^2\gamma''_\beta)/2, \quad \tilde{b} = \tilde{\alpha}\tilde{\beta}\gamma''_{\alpha\beta} \tag{8}$$

The additional c will generated the mean of $\tilde{\gamma}_2$, c and $\tilde{b}$ will increase the covariance, as one would expect. Let $\sigma_\alpha = \sigma_\beta$, thus

$$E1[\tilde{\gamma}_2] = -\sigma_\alpha^2(\gamma^2 + 1)/(2\gamma^3) \tag{9}$$

Here, the converted measurement error is $\tilde{Z}_{m2} = (\tilde{x}\tilde{y}\tilde{z}_2)^T$, where

$$\begin{aligned} &\tilde{z}_2 = R_m\tilde{\gamma}_2 + \gamma_m\tilde{R} + \tilde{\gamma}_2\tilde{R} = \tilde{z}_1 + \tilde{z}_d, \\ &\tilde{z}_d = (R_m + \tilde{R})(c + \tilde{b}) \end{aligned} \tag{10}$$

Since $\tilde{R}\tilde{\alpha}\tilde{\beta}$ are independent of each other, $(\tilde{R}, \tilde{\alpha}, \tilde{\beta})$ and (R_m, α_m, β_m) are also independent, thus $E1[\tilde{Z}_{m2}] = (0, 0, R_m \cdot E1[\tilde{\gamma}_2])^T$ and the $\text{cov1}(\tilde{Z}_{m2})$ is

$$R_{m2} = \text{cov1}(\tilde{Z}_{m2}) = \begin{pmatrix} (R_{m1})_{11} & (R_{m1})_{12} & (R_{m2})_{13} \\ (R_{m1})_{12} & (R_{m1})_{22} & (R_{m2})_{23} \\ (R_{m2})_{13} & (R_{m2})_{23} & (R_{m2})_{33} \end{pmatrix} \tag{11}$$

where

$$\begin{aligned} (R_{m2})_{13} =& \text{cov1}(\tilde{x}, \tilde{z}_2) \\ =& \text{cov1}(\tilde{x}, \tilde{z}_1 + \tilde{z}_d) \\ =& \text{cov1}(\tilde{x}, \tilde{z}_1) + \text{cov1}(\tilde{x}, \tilde{z}_d) \\ =& (R_{m1})_{13} + \alpha_m E1[\tilde{\gamma}_2]\sigma_R^2, \\ (R_{m2})_{23} =& \text{cov1}(\tilde{y}, \tilde{z}_2) = (R_{m1})_{23} + \beta_m E1[\tilde{\gamma}_2]\sigma_R^2, \\ (R_{m2})_{33} =& \text{cov1}(\tilde{z}_2) \\ =& \text{cov1}(\tilde{z}_1) + 2\text{cov1}(\tilde{z}_1, \tilde{z}_d) + \text{cov1}(\tilde{z}_d) \\ =& (R_{m1})_{33} + 2\gamma_m E1[\tilde{\gamma}_2]\sigma_R^2 + \sigma_R^2(E1[\tilde{\gamma}_2])^2 \\ &+ (R_m^2 + \sigma_R^2)\{2(E1[\tilde{\gamma}_2])^2 - \sigma_\alpha^2\sigma_\beta^2/\gamma_m^4\} \end{aligned} \tag{12}$$

The simulation results show that the converted measurements based on the second-order TSE approximation provide considerable improvement in accuracy and consistency over linear approximation and are sufficient to match the first two moments of original measurements reported in direction cosines. In a theoretical aspect, the converted measurements based on the 4th-order TSE approximation are given below. Taking the fourth-order terms of a Taylor series expansion for γ around α_m, β_m, the error is

$$\begin{aligned} \tilde{\gamma}_4 =& \tilde{\gamma}_2 + e + \tilde{s} \\ e =& (\tilde{\alpha}^4\gamma_\alpha^{''''} + \tilde{\beta}^4\gamma_\beta^{''''} + 6\tilde{\alpha}^2\tilde{\beta}^2\gamma_{\alpha\alpha\beta\beta}^{''''})/4! \\ \tilde{s} =& (\tilde{\alpha}^3\gamma_\alpha^{m} + \tilde{\beta}^3\gamma_\beta^{'''} + 3\tilde{\alpha}^2\tilde{\beta}\gamma_{\alpha\alpha\beta}^{'''})/3! \\ &+ (\tilde{\alpha}\tilde{\beta}^3\gamma_{\alpha\beta\beta\beta}^{''''} + \tilde{\alpha}^3\tilde{\beta}\gamma_{\alpha\alpha\alpha\beta}^{''''})/6 \end{aligned} \tag{13}$$

The e will generate the additional mean relative to the mean of $\tilde{\gamma}_2$, e and $\tilde{s}$ will farther increase the covariance. Let $\sigma_\alpha = \sigma_\beta$, thus $E1[\tilde{\gamma}_4] = E1[\tilde{\gamma}_2] + E1[e]$, where

$$E1[e] = -\sigma_\alpha^4[15 - \gamma_m^2(\gamma_m^2 + 6)]/(8\gamma_m^7) \tag{14}$$

Here the converted measurement error is $\tilde{Z}_{m4} = (\tilde{x}\tilde{y}\tilde{z}_4)^T$, where

$$\tilde{z}_4 = R_m\tilde{\gamma}_4 + \gamma_m\tilde{R} + \tilde{\gamma}_4\tilde{R} = \tilde{z}_2 + \tilde{z}_g,$$
$$\tilde{z}_g = (e+\tilde{s})(R_m + \tilde{R}) \tag{15}$$

Similarly, $E1[\tilde{Z}_{m4}] = (00R_mE1[\tilde{\gamma}_4])^T$ and $\mathrm{cov1}(\tilde{Z}_{m4})$ is

$$R_{m4} = \mathrm{cov1}(\tilde{Z}_{m4}) = \begin{pmatrix} (R_{m1})_{11} & (R_{m1})_{12} & (R_{m4})_{13} \\ (R_{m1})_{12} & (R_{m1})_{22} & (R_{m4})_{23} \\ (R_{m4})_{13} & (R_{m4})_{23} & (R_{m4})_{33} \end{pmatrix} \tag{16}$$

where

$$\begin{aligned}(R_{m4})_{13} =&\mathrm{cov1}(\tilde{x},\tilde{z}_4)\\ =&\mathrm{cov1}(\tilde{x},\tilde{z}_2+\tilde{z}_g)\\ =&(R_{m2})_{13} + \alpha_m\sigma_R^2E1[e]\\ &- \alpha_m(R_m^2+\sigma_R^2)\sigma_\alpha^4(3+\gamma_m^2)/\gamma_m^5/2\end{aligned} \tag{17}$$

$$\begin{aligned}(R_{m4})_{23} =&\mathrm{cov1}(\tilde{y},\tilde{z}_4)\\ =&\mathrm{cov1}(\tilde{y},\tilde{z}_2+\tilde{z}_g)\\ =&(R_{m2})_{23} + \beta_m\sigma_R^2E1[e]\\ &- \beta_m(R_m^2+\sigma_R^2)\sigma_\beta^4(3+\gamma_m^2)/(2\gamma_m^5)\end{aligned} \tag{18}$$

$$\begin{aligned}(R_{zm4})_{33} =&\mathrm{cov1}(\tilde{z}_4)\\ =&\mathrm{cov1}(\tilde{z}_1+\tilde{z}_d+\tilde{z}_g)\\ =&(R_{zm2})_{33} + 2\mathrm{cov1}(\tilde{z}_1,\tilde{z}_g)\\ &+ 2\mathrm{cov1}(\tilde{z}_d,\tilde{z}_g) + \mathrm{cov1}(\tilde{z}_g)\end{aligned} \tag{19}$$

Where the $\mathrm{cov1}(\tilde{z}_d,\tilde{z}_g)$ is the conditional cross-covariance between $\tilde{z}_d$ and $\tilde{z}_g$, it is negligible because of being direct ratio to $\sigma_\alpha^6(\sigma_\alpha=\sigma_\beta)$. The $\mathrm{cov1}(\tilde{z}_g)$ is dealt with similarly.

Since the moment E_n of any r.v. x assumed Gaussian with zero mean and cov. σ is zero while n is odd, and $E_n = 1\cdot3\cdot5\cdots(n-1)\sigma^n = (n-1)!!\sigma^n$ while $n\geq 2$ is even, and let $\sigma_\alpha=\sigma_\beta$, we have

$$\mathrm{cov1}(\tilde{z}_1,\tilde{z}_g) = (R_m^2+\sigma_R^2)\sigma_\alpha^4(\gamma_m^2+3)(1-\gamma_m^2)/(2\gamma_m^6)+\gamma_m\sigma_R^2E1[e] \tag{20}$$

The new DCOS-TO-CAR unbiased consistent conversion is

$$Z_{car} = (\alpha_mR_m\ \beta_mR_m\ \gamma_mR_m)^T + \mu \tag{21}$$

where $\mu = (0\ 0\ R_mE1[\tilde{\gamma}_2])^T$. With the bias μ and covariance R_{m2}, Eq.(21) is the only one matching KF with correct feedback control relationship.

III. Target Tracking in DCOS

1. Converted measurement filter

Assume that the measurement function is

$$Z_{\cos} = (R\quad\alpha\quad\beta)^T = h(X),$$
$$h(X) = (\sqrt{x^2+y^2+z^2}x/Ry/R)^T \tag{22}$$

let $X = (x\quad\dot{x}\quad y\quad\dot{y}\quad z\quad\dot{z})^T$, the predicted measurement is $\hat{Z}_{k+1|k} = H\hat{X}_{k+1|k}$, the predicted state and its cov. are

$$\hat{X}_{k+1|k} = \boldsymbol{\Phi}\hat{X}_{k|k}, P_{k+1|k} = \boldsymbol{\Phi}P_{k|k}\Phi^T + Q_k \tag{23}$$

where Φ is the diagonal matrix made of $\Phi_1 = \begin{pmatrix}1 & T\\ 0 & 1\end{pmatrix}$, $H = \begin{pmatrix}1&0&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&1&0\end{pmatrix}$, Q_k is process noise. The rest are as follows

$$K_{k+1} = P_{k+1|k}H^T[HP_{k+1|k}H^T + R_{car(k+1)}]^{-1} \tag{24}$$

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}(Z_{car(k+1)} - \hat{Z}_{k+1|k}) \tag{25}$$

$$\begin{aligned}P_{k+1|k+1} =&P_{k+1|k} - K_{k+1}(HP_{k+1|k}H^T\\ &+ R_{car(k+1)})K_{k+1}^T\end{aligned} \tag{26}$$

where $Z_{car(k+1)}$ is the debiased conversion Eq.(21) corrected for μ in our proposed CMKFDcos, and does not account for the bias in the CMKFL. $R_{car(k+1)}$ is R_{m2} or R_{m1} according to CMKFDcos and CMKFL, respectively.

2. EKF in DCOS measurements

With the EKF, the predicted measurement is $\hat{Z}_{\cos k+1|k} = h(\hat{X}_{k+1|k})$. The gain and update state are

$$\begin{aligned}&K_{k+1} = P_{k+1|k}H_{k+1}^T(H_{k+1}P_{k+1|k}H_{k+1}^T + R_{\cos(k+1)})^{-1},\\ &\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}(Z_{\cos(k+1)} - \hat{Z}_{\cos k+1|k})\end{aligned} \tag{27}$$

The predicted state and its covariance are same as Eqs.(23). The Jacobian is

$$H_k = \begin{pmatrix} xR^2 & 0 & yR^2 & 0 & zR^2 & 0\\ (R^2-x^2) & 0 & -xy & 0 & -xz & 0\\ -xy & 0 & (R^2-y^2) & 0 & -yz & 0\end{pmatrix}\Big/R^3 \tag{28}$$

where the index m, for brevity, is dropped.

IV. Simulation and Comparison

1. Consistency of converted measurements

NES[4] is used to investigate the consistency of the converted measurements.

$$NES = \frac{1}{MC}\sum_{i=1}^{MC}\tilde{Z}_i^TP_{\tilde{Z}\tilde{Z}}^{-1}\tilde{Z}_i \tag{29}$$

where $\tilde{Z}_i$ is the vector to be tested, $P_{\tilde{Z}\tilde{Z}}$ is the covariance of $\tilde{Z}_i$. Assume that original sensor noises are Gaussian with zero means and $\sigma_R = 15$m, $\sigma_\alpha=\sigma_\beta = 0.2\sim 2\pi/180$, $MC = 1000$ runs. The true target position is assumed that its range is 100/500/1000/2000km, resp., with fixed $x = 70/\sqrt{2}$km and $y = 70$km thus the corresponding scan angle[1] is $\varepsilon\approx 59/10/5/2.5^0$, resp., defined above. If the errors and the cov. match each other, the NES will be within the chi-square probability bounds to be assumed 0.99. The covariance R_{m2} of CMKFDcos is consistent with the errors corrected by μ over all practical situations (shown in Fig.1). But the covariance R_{m1} of the linear approximation is consistent only for rather smaller errors at 100km (shown in Fig.2). It is also seen that longer ranges require more accurate measurements for using a CMKFL significantly.

2. Consistency of nonlinear transformation

The performance of the three filters will be further disclosed by investigating the nonlinear transformation as follows.

The nonlinear transformation in EKF is performed by using Jacobian. The predicted state is directly described in CAR and the predicted measurement in DCOS is the nonlinear mapping of the predicted state. Let the state in CAR be white Gaussian with mean $(x, y, z) = (70\sqrt{7/2}, 70, 210\sqrt{3/2})$km and cov. $R_{car}(\sigma_x = \sigma_y = 30\text{km}, \sigma_z = 10\text{km})$. It is known that the distribution in DCOS is approximated with mean $(R\alpha\beta)$ and cov. $H_k \cdot R_{car} \cdot H_k^T$ mapping from CAR into DCOS. Using 500 runs, the Gaussian distribution in CAR is shown in Fig.3 with $*$ and that in DCOS with o. It can be seen that the transformation from (xyz) to $(R\alpha\beta)$ is biased (shown in Fig.4) and the distribution in DCOS is not also Gaussian.

With the CMKFL, the measurements reported in DCOS are converted to CAR by linear transformation (CML). Assume that the distribution of measurements in DCOS is white Gaussian with mean $(R, \alpha, \beta) = (210\sqrt{2}\text{km}, \sqrt{7}/6, 1/(3\sqrt{2}))$ and cov. $R_{\cos}$ $(\sigma_R = 15\text{m}, \sigma_\alpha = \sigma_\beta = 1.5\pi/180)$. The distribution in CAR is approximated with mean (xyz) and cov. R_{m1} in CMKFL. The Gaussian distribution in DCOS is shown in Fig.5 with * and that in CAR with o. It can be seen that the transformation from $(R\alpha\beta)$ to (xyz) is biased (shown in Fig.6), and the distribution in CAR is not also Gaussian.

In our proposed CMKFDcos, the measurements reported in DCOS are converted to CAR by accounting for μ and R_{m2} (CMDcos). The test condition is same as CML. The distribution in DCOS and CAR are shown in Fig.7 with * and o, respectively. Fig.8 presents the corresponding mean and cov.. It can be seen that the approximation to the distribution and the first two moments of original measurements in CMDcos is much better than Jacobian and CML. So the performance of CMKFDcos is superior to EKF and CMKFL in terms of accuracy and consistency (shown in simulations later).

EKF, CMKFL, CMKFDcos are examples of point (as opposed to density) estimation approximation techniques of nonlinear dynamic systems. Most real-world systems are nonlinear. Nonlinearity destroys Gaussianity of original measurements[9,10]. This is the reason for the fundamental limitations of point estimation approximation. The problem is how to get at least approximately estimations by using the flexible and simple point estimation approximation techniques.

3. Simulation and analysis of target tracking

The scenario is designed to examine the effects of CMKFDcos on target tracking for different accuracies and all practical geometries. Assume that a target is moving at $(\dot{x}, \dot{y}, \dot{z}) = (0, 0, 2000)$m/s with the initial location of $x = z = 70/\sqrt{2}$, $y = 70$km, $T = 1$s, 150 measurements, 50 runs, $\sigma_R = 15$m, case 1: $\sigma_{\alpha 1} = \sigma_{\beta 1} = 0.5\pi/180$. case 2: $\sigma_{\alpha 2} = \sigma_{\beta 2} = 1.5\pi/180$, and the rest is same as case 1.

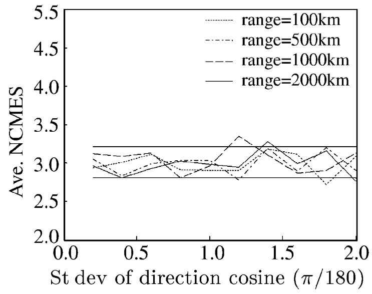

Fig. 1. Average NES for CMKFDcos

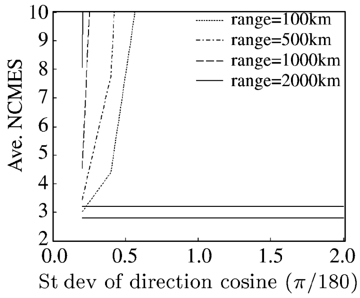

Fig. 2. Average NES for CMKFL

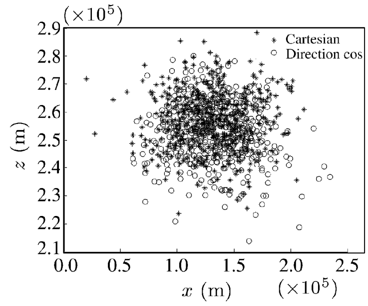

Fig. 3. Distribution (Jacobian)

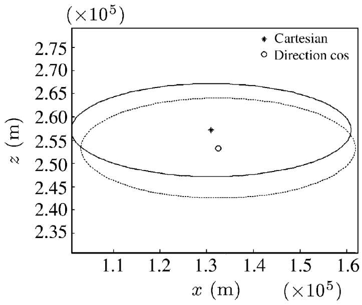

Fig. 4. Mean & cov. (Jacobian)

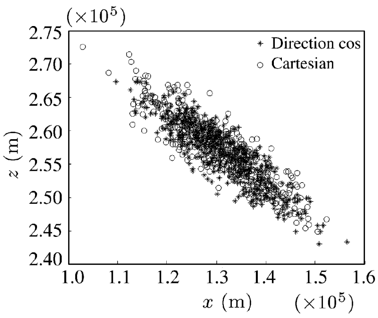

Fig. 5. Distribution (CML)

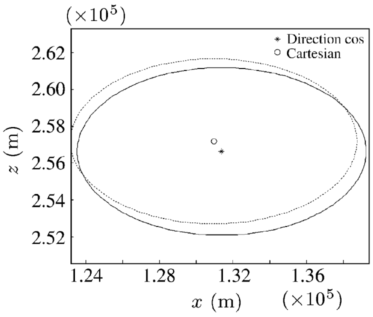

Fig. 6. Mean & cov. (CML)

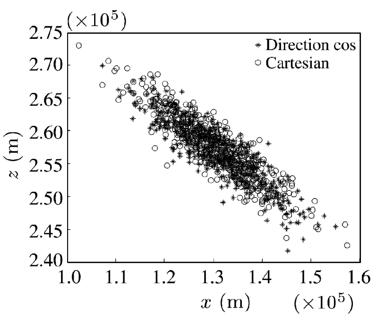

Fig. 7. Distribution (CMDcos)

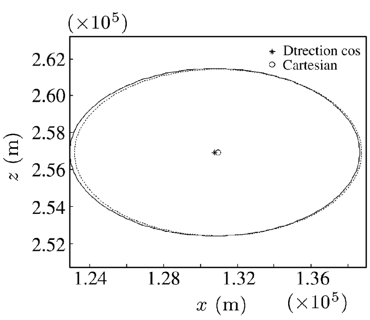

Fig. 8. Mean & cov. (CMDcos)

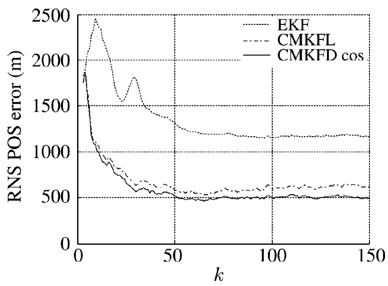

Fig. 9. RMSE of pos. (case 1)

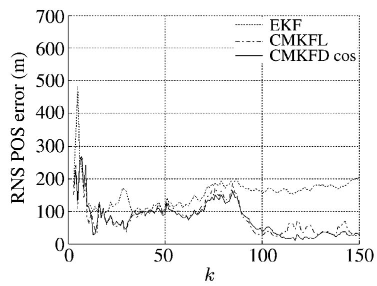

Fig. 10. RSSE of pos. (case 1)

In case 1, the position rms and rss error, the position and velocity ANEES[8] (Average normalized estimation error square) for the three filters are shown in Figs.9–12. It can be seen CMKFDcos provides the best accuracy and consistency due to the corresponding bias μ and covariance R_{m2} correction derived in Section II over all ranges. But CMKFL is consistent only within the first some tracking dots for such small measurement errors. The EKF is wholly inconsistent and its performance is inferior to CMKFL. Figs.13–16 present the results with case 2. It is seen that CMKFL and EKF are wholly inconsistent and CMKFDcos is still very credible and much more accurate than the other two.

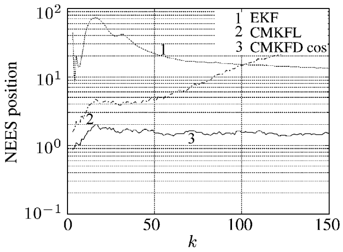

Fig. 11. ANEES of pos. (case 1)

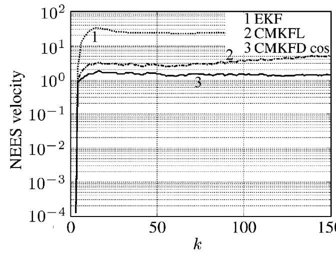

Fig. 12. ANEES of vel. (case 1)

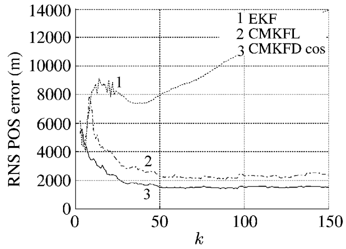

Fig. 13. RMSE of pos. (case 2)

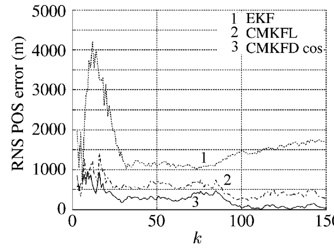

Fig. 14. RSSE of pos. (case 2)

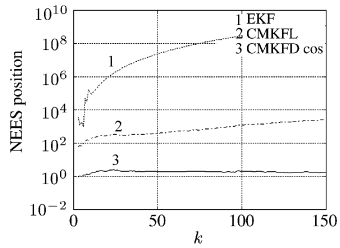

Fig. 15. ANEES of pos. (case 2)

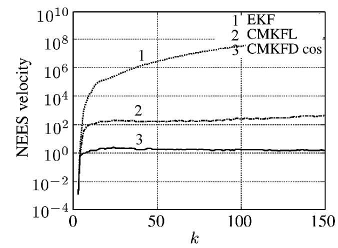

Fig. 16. ANEES of vel. (case 2)

V. Conclusions

The mean and covariance of converted direction cosines measurements are derived. The converted measurements completely capture the true mean and covariance of the original direction-cosine measurements reported in phased array radars, at least approximately estimation since the errors are not Gaussian. The consistency of the conversion is valid for any target range or any direction cosine measurements accuracy and for all practical phased array radar work regions. The proposed procedure clearly outperforms EKF and CMKFL in accuracy and consistency, and computational load is modest. So the filter can be employed in the cross-range errors being significantly large relative to the range errors for all electronic scanning radars, while EKF and CMKFL result in consistent characteristics only for a significantly narrower range of measurement errors.

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