RodiCS: a finite element solver of Kirchhoff rods under fluid flow and more

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1   Summary

Elastic rods are long and thin structures, and are the object of many dynamical systems in nature, from bending and swaying plants [Gosselin, 2019] to swimming flagellated and ciliated bacteria [Brennen and Winet, 1977, Lauga, 2016] and propulsing carangiform fish [Cheng et al., 1998]. In industry as well, elastic rods serve as models to study drilling risers in offshore sites and tall chimney stacks in constructions that may vibrate in presence of water or air flows [Païdoussis et al., 2010], and even to investigate the cooking process of spaghetti [Goldberg and O'Reilly, 2020]. With the prevalence of rods in mechanical problems, it is important to have a simple to use efficient numerical tool reproducing faithfully the observed dynamics. RodiCS is a Python code that fulfills this task, and simulates the motion of elastic rods under defined forces in general, and their interaction with fluid flows more specifically. While numerous researchers [Violette et al., 2007, Lei and Nepf, 2019, Gosselin et al., 2010, Hassani et al., 2016, Leclercq and de Langre, 2018, Leclercq and de Langre, 2018, Leclercq and de Langre, 2016, Luhar and Nepf, 2016, Buchak et al., 2010, Goldberg and O'Reilly, 2020, Mukundan et al., 2009] spend valuable time and effort in writing *ad hoc* codes with limited verification and validation, RodiCS comes as a versatile and practical open source library that scientific projects can benefit from and build upon.

2   Description

With the FEniCS platform [Alnæs et al., 2015], RodiCS solves the Kirchhoff equations of elastic rods [Landau and Lifshitz, 1970, Audoly and Pomeau, 2010] using finite elements. The first step in each simulation is to import the mesh of the rod from an '.xml' file. The mesh consists of one-dimensional first order Lagrange elements embedded in a three-dimensional space. Then, external forces applied on the rod need to be listed. In FEniCS, the Unified Form Language (UFL) [Alnæs et al., 2014] ensures a convenient syntax to write mathematical expressions. Algebraic, vectorial, and differential operators, as well as tensor algebra and index notation, are all included in UFL, so users can enter force expressions exactly as they figure in equations without any difficulty. After applying boundary conditions, Kirchhoff equations are solved with the Newton method. In the case of transient simulations, these equations are discretised in time using the backward Euler scheme. Figure 1(a) shows the static reconfiguration of a 2D beam under a semi-empirical non-linear
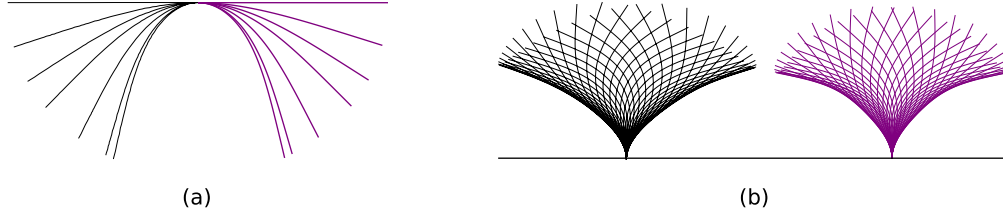
Figure 1: Deformation profiles of experiments on thin plates (black) from Gosselin et al. [2010] and Leclercq and de Langre [2018] compared to numerical simulations on elastic rods using RodiCS (purple). (a) Static deformation under drag. (b) Dynamic deformation under an oscillatory flow.

drag formulation for several load magnitudes, whereas Figure 1(b) records the dynamic deformation of a similar beam under an oscillatory flow over a period. Numerical results match well experiments on thin plates, which are also governed by Kirchhoff equations, conducted by Gosselin et al. [2010] and Leclercq and de Langre [2018]. A more detailed description of Kirchhoff equations, solving method, and code verification and validation can be found in chapter 7 of Boudina [2020].

3   Statement of Need

The traditional method to simulate the motion of elastic rods in FEniCS is to solve the elasticity equation in three dimensions. This method takes a mesh with a large number of volumetric elements, hence is costly, time-consuming, and particularly inappropriate for slender structures. A more efficient option is to consider a rod theory that describes deformation based only on the curvilinear coordinate of the rod, which transposes numerically into using one-dimensional elements in three-dimensional space. In FEniCS, such a code was proposed by Bleyer [2018] solving Timoshenko beams. However, it is restricted to static deformations, and is valid only for small displacements due to the linearity of equations. In RodiCS, we simulate both static and dynamic deformations of Kirchhoff rods, which are more representative of thin rods, and we account for large displacements owing to its nonlinear character. Therefore, RodiCS fills a need in open source finite element codes for a way to solve the one-dimensional Kirchhoff rod equation embedded in a three-dimensional space.

Another elastic rod solver worth to mention is the Discrete Elastic Rods (DER) code of Jawed et al. [2014]. Because the DER code is written in C++ and has an intricate file structure, only programmer users are able to modify the source code, excluding thus users new in coding or having basic knowledge in programming. Moreover, it is not cross-platform and has several dependencies, limiting again access to a number of users. Last but not least, Jawed et al. [2014] acknowledge themselves that they cannot maintain the code or provide support anymore. These drawbacks were incentives for the authors of the present paper to think of writing their own code based on the FEniCS project, an open source finite element solver with an active community, continuously developed, and cross-platform; all significant advantages. Also as an essential modular feature, FEniCS imposes a standard in the mesh definition, making possible either the mesh generation with external meshing software, or the combination of elements in RodiCS with other types of finite elements. Finally, written in Python, RodiCS is inclusive and accessible for users of all programming levels.

RodiCS is intended to solve elastic rod dynamics subjected to any kind of external forces, should they be constant, time-dependent, space-dependent, or a mix of them. In fact, the original idea of designing such
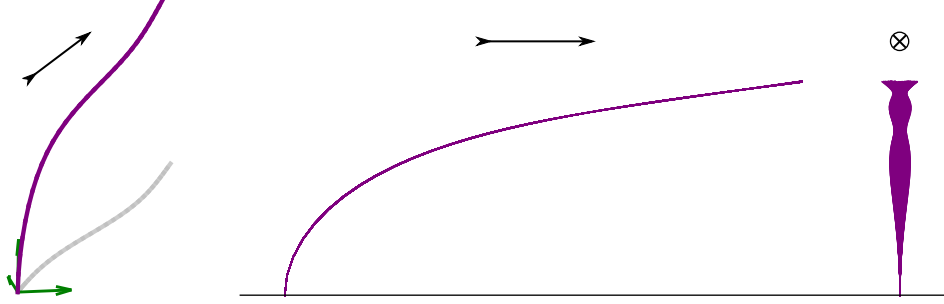
Figure 2: An elastic rod undergoing fluid-dynamic drag along with vortex-induced vibrations. Views from left to right: perspective, lateral, and frontal. Arrows indicate the direction of the flow.

a code came from the shortage in commercial software considering customised forces. Even in the handful of those enabling customised entries, the only way is to write subroutines coded in low-level, user-unfriendly languages. Not only does it imply a familiarity with these languages, but it also requires users to be well acquainted with the available user-defined function environment in the software, which is rarely documented in a practical manner. Coded in a high-level language, RodiCS does away with this difficulty and offers users, in academia and industry alike, a user-friendly and easy-to-implement framework to simulate elastic rod deformation pertaining to a wide range of mechanical problems.

Complex Dynamics

Furthermore, RodiCS can tackle complex mechanical phenomena coupling elastic rod motion with dynamical variables. In fluid-structure interaction, an example of such phenomena is vortex-induced vibrations (VIV), arising due to the periodic vortex shedding downstream from the rod. One of the most successful phenomenological models of VIV is the wake-oscillator model proposed by Facchinetti et al. [2004], where the vortex-induced lift has no explicit expression, but is rather a self-excited variable governed by the van der Pol equation with a feedback proportional to the rod acceleration. In RodiCS, the lift is an additional unknown of the problem, and the van der Pol equation with the acceleration feedback is included in the variational formulation. RodiCS can even combine vortex-induced lift with other forces and make simulations closer to reality. To reproduce the motion of a vibrating soft coral branch, Boudina et al. [2020] included the added mass force to account for inertial effects, as well as the semi-empirical non-linear fluid-dynamic drag so that the rod bends with large amplitude and undergoes VIV at the same time, as shown in Figure 2.

Finally, we encourage mechanicians and programmers to contribute in developing RodiCS, whether by incorporating more simulation cases with specific forces, optimising computational methods, or even improving visualisation tools, thence bringing valuable benefit in research and education.

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References

Frédérick P. Gosselin. Mechanics of a plant in fluid flow. *Journal of Experimental Botany*, 70(14):3533–3548, July 2019. ISSN 0022-0957, 1460-2431. doi: 10.1093/jxb/erz288. URL `https://academic.oup.com/jxb/article/70/14/3533/5518960`.

Christopher Brennen and Howard Winet. Fluid mechanics of propulsion by cilia and flagella. *Annual Review of Fluid Mechanics*, 9(1):339–398, 1977. doi: 10.1146/annurev.fl.09.010177.002011. Publisher: Annual Reviews 4139 El Camino Way, PO Box 10139, Palo Alto, CA 94303-0139, USA.

Eric Lauga. Bacterial Hydrodynamics. *Annual Review of Fluid Mechanics*, 48(1):105–130, 2016. doi: 10.1146/annurev-fluid-122414-034606. URL `https://doi.org/10.1146/annurev-fluid-122414-034606`. _eprint: https://doi.org/10.1146/annurev-fluid-122414-034606.

J.–Y. Cheng, T. J. Pedley, and J. D. Altringham. A continuous dynamic beam model for swimming fish. *Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences*, 353(1371):981–997, June 1998. doi: 10.1098/rstb.1998.0262. URL `https://royalsocietypublishing.org/doi/abs/10.1098/rstb.1998.0262`. Publisher: Royal Society.

M.P. Païdoussis, S.J. Price, and Emmanuel de Langre. *Fluid-Structure Interactions: Cross-Flow-Induced Instabilities*. Cambridge University Press, 2010. ISBN 978-1-139-49190-7.

Nathaniel N. Goldberg and Oliver M. O'Reilly. Mechanics-based model for the cooking-induced deformation of spaghetti. *Physical Review E*, 101(1):013001, January 2020. doi: 10.1103/PhysRevE.101.013001. URL `https://link.aps.org/doi/10.1103/PhysRevE.101.013001`.

R. Violette, E. de Langre, and J. Szydlowski. Computation of vortex-induced vibrations of long structures using a wake oscillator model: Comparison with DNS and experiments. *Computers & Structures*, 85 (11):1134–1141, June 2007. ISSN 0045-7949. doi: 10.1016/j.compstruc.2006.08.005. URL `http://www.sciencedirect.com/science/article/pii/S004579490600229X`.

Jiarui Lei and Heidi Nepf. Wave damping by flexible vegetation: Connecting individual blade dynamics to the meadow scale. *Coastal Engineering*, 147:138–148, May 2019. ISSN 03783839. doi: 10.1016/j.coastaleng.2019.01.008. URL `https://linkinghub.elsevier.com/retrieve/pii/S0378383918300905`.

Frédérick Gosselin, Emmanuel de Langre, and Bruno A. Machado-Almeida. Drag reduction of flexible plates by reconfiguration. *Journal of Fluid Mechanics*, 650:319, May 2010. ISSN 0022-1120, 1469-7645. doi: 10.1017/S0022112009993673. URL `http://www.journals.cambridge.org/abstract_S0022112009993673`.

Masoud Hassani, Njuki W. Mureithi, and Frédérick P. Gosselin. Large coupled bending and torsional deformation of an elastic rod subjected to fluid flow. *Journal of Fluids and Structures*, 62:367–383, April 2016. ISSN 08899746. doi: 10.1016/j.jfluidstructs.2015.12.009. URL `http://linkinghub.elsevier.com/retrieve/pii/S0889974616000062`.

T. Leclercq and Emmanuel de Langre. Reconfiguration of elastic blades in oscillatory flow. *Journal of Fluid Mechanics*, 838:606–630, March 2018. ISSN 0022-1120, 1469-7645. doi: 10.1017/jfm.2017.910. URL `https://tinyurl.com/y27jg5wv`.

Tristan Leclercq and Emmanuel de Langre. Vortex-induced vibrations of cylinders bent by the flow. *Journal of Fluids and Structures*, 80:77–93, July 2018. ISSN 0889-9746. doi: 10.1016/j.jfluidstructs.2018.03.008. URL http://www.sciencedirect.com/science/article/pii/S0889974617307764.

Tristan Leclercq and Emmanuel de Langre. Drag reduction by elastic reconfiguration of non-uniform beams in non-uniform flows. *Journal of Fluids and Structures*, 60:114–129, January 2016. ISSN 08899746. doi: 10.1016/j.jfluidstructs.2015.10.007. URL http://linkinghub.elsevier.com/retrieve/pii/S088997461500242X.

M. Luhar and H.M. Nepf. Wave-induced dynamics of flexible blades. *Journal of Fluids and Structures*, 61:20–41, February 2016. ISSN 08899746. doi: 10.1016/j.jfluidstructs.2015.11.007. URL http://linkinghub.elsevier.com/retrieve/pii/S0889974615002613.

P. Buchak, C. Eloy, and P. M. Reis. The Clapping Book: Wind-Driven Oscillations in a Stack of Elastic Sheets. *Physical Review Letters*, 105(19):194301, November 2010. doi: 10.1103/PhysRevLett.105.194301. URL https://link.aps.org/doi/10.1103/PhysRevLett.105.194301.

H. Mukundan, Y. Modarres-Sadeghi, J. M. Dahl, F. S. Hover, and M. S. Triantafyllou. Monitoring VIV fatigue damage on marine risers. *Journal of Fluids and Structures*, 25(4):617–628, May 2009. ISSN 0889-9746. doi: 10.1016/j.jfluidstructs.2009.03.003. URL http://www.sciencedirect.com/science/article/pii/S0889974609000383.

Martin Alnæs, Jan Blechta, Johan Hake, August Johansson, Benjamin Kehlet, Anders Logg, Chris Richardson, Johannes Ring, Marie E. Rognes, and Garth N. Wells. The FEniCS Project Version 1.5. *Archive of Numerical Software*, 3(100), December 2015. ISSN 2197-8263. doi: 10.11588/ans.2015.100.20553. URL https://journals.ub.uni-heidelberg.de/index.php/ans/article/view/20553. Number: 100.

L. D. Landau and E. M. Lifshitz. *Theory of Elasticity*. Pergamon Press, January 1970.

Basile Audoly and Yves Pomeau. *Elasticity and Geometry: From hair curls to the nonlinear response of shells*. Oxford University Press, Oxford ; New York, August 2010. ISBN 978-0-19-850625-6.

Martin S. Alnæs, Anders Logg, Kristian B. Ølgaard, Marie E. Rognes, and Garth N. Wells. Unified form language: A domain-specific language for weak formulations of partial differential equations. *ACM Transactions on Mathematical Software*, 40(2):9:1–9:37, March 2014. ISSN 0098-3500. doi: 10.1145/2566630. URL https://doi.org/10.1145/2566630.

Mouad Boudina. On the mechanics of flow-induced vibration of soft corals and particle interception. Master's thesis, Polytechnique Montréal, August 2020.

Jeremy Bleyer. Numerical Tours of Computational Mechanics with FEniCS. 2018. doi: 10.5281/zenodo.1287832. Publisher: Zenodo.

Mohammad K Jawed, Fang Da, Jungseock Joo, Eitan Grinspun, and Pedro M Reis. Coiling of elastic rods on rigid substrates. *Proceedings of the National Academy of Sciences*, page 201409118, 2014. doi: 10.1073/pnas.1409118111.

M. L. Facchinetti, Emmanuel de Langre, and F. Biolley. Coupling of structure and wake oscillators in vortex-induced vibrations. *Journal of Fluids and Structures*, 19(2):123–140, February 2004. ISSN 0889-9746. doi: 10.1016/j.jfluidstructs.2003.12.004. URL `http://www.sciencedirect.com/science/article/pii/S0889974603001853`.

Mouad Boudina, Frédérick P. Gosselin, and Stéphane Etienne. Vortex-induced vibrations: a soft coral feeding strategy? *Journal of Fluid Mechanics (submitted)*, December 2020.