# MTH9845 Risk Management

## Model Validation Report

*with*

# ¡Need-to-chose Pricing Model¿ Continue-the-Title

Submitted by


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# Contents

# List of Figures

# Chapter 1

# Executive Summary

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# Chapter 2

# Overview

## 2.1 Background and Recent Research

### 2.1.1 ¡any sub section here¿

### 2.1.2 Literature Survey

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#### ¡Sub-subsection title¿

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## 2.2 Motivation

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# Chapter 3

# Technical Specification

The Vanna-Volga pricing method is a technique used to price first generation exotic options in foreign exchange market. This method derives from the trader's idea that the difference between market price and Black-Scholes price is the volatility smile impact, which could be adjusted with costs incurred by hedging three main risks associated to the volatility of the option: the Vega, the Vanna and the Volga.

## 3.1 Greeks

The foreign exchange spot process is considered to follow Geometric Brownian motion (GBM). Thus we find the results we are able to obtain in equity markets hold in the case of FX options as well.
Then the Black-Scholes value of call option is:

$$V_{call} = Se^{-r_f T} N(d_1) - Ke^{-r_d T} N(d_2) \tag{3.1}$$
$$V_{put} = Ke^{-r_d T} N(-d_2) - Se^{-r_f T} N(-d_1) \tag{3.2}$$

where

$$d_1 = \frac{ln\frac{S}{K} + \left(r_d - r_f + \frac{\sigma^2}{2}T\right)}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

and $r_d$ and $r_f$ are the domestic and foreign risk free rate, respectively. $T$ is the time to maturity. $N$ denotes as the cumulative density function of standard normal distribution. Below we will discuss the Greeks in the context of Black-Scholes model.

### 3.1.1 Vega

Vega $\nu$ is the first derivative of the option value with respect to the volatility $\sigma$.
By taking the derivative we have

$$\begin{aligned}\nu_C = \frac{\partial C}{\partial \sigma} &= Se^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} \\ &= Se^{-r_f T}\Phi(d_1)\frac{d_1 - d_2}{\sigma} \\ &= Se^{-r_f T}\Phi(d_1)\sqrt{T}\end{aligned} \tag{3.3}$$

$$\begin{aligned}\nu_P = \frac{\partial P}{\partial \sigma} &= -Ke^{-r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} + Se^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= \nu_C\end{aligned} \tag{3.4}$$

### 3.1.2 Vanna

Vanna is the second order derivative of the option value, once to the volatility $\sigma$ and once to the initial spot price.
By taking the derivative we have:

$$\begin{aligned}Vanna_C = \frac{\partial^2 C}{\partial S \partial \sigma} = \frac{\partial \Delta_C}{\partial \sigma} &= e^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= e^{-r_f T}\Phi(d_1)\left(\sqrt{T} - \frac{d_1}{\sigma}\right) \\ &= -\frac{d_2}{S\sigma\sqrt{T}}\nu_C\end{aligned} \tag{3.5}$$

$$\begin{aligned}Vanna_P = \frac{\partial^2 P}{\partial S \partial \sigma} = \frac{\partial \Delta_P}{\partial \sigma} &= e^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= Vanna_C\end{aligned} \tag{3.6}$$

### 3.1.3 Volga

Volga is the second order derivative of the option value with respect to the volatility $\sigma$ twice.

By taking the derivative we have:

$$\begin{aligned} Volga_C &= \frac{\partial^2 C}{\partial^2 \sigma} = \frac{\partial \nu_C}{\partial \sigma} = e^{-r_f T} S\sqrt{T} \frac{\partial \Phi(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} \\ &= e^{-r_f T} S\sqrt{T} \Phi(d_1) \frac{d_1 d_2}{\sigma} \\ &= \frac{\nu_C d_1 d_2}{\sigma} \end{aligned} \tag{3.7}$$

$$\begin{aligned} Volga_P &= \frac{\partial^2 P}{\partial^2 \sigma} = \frac{\partial \nu_P}{\partial \sigma} = \frac{\partial \nu_C}{\partial \sigma} \\ &= Volga_C \end{aligned} \tag{3.8}$$

## 3.2 Model Framework and Equations

### 3.2.1 Model Assumption and Justification

Vanna-Volga pricing method is a mathematical method for pricing options in foreign exchange market. Thus, it follows some basic assumptions for the FX options. Here are some assumptions about the market and the options:

- The considered underlying asset $S_t$ is an FX rate quoted in foreign/domestic format. For example, EUR/USD open today is 1.0868, which means 1 EUR is worth 1.0868 USD and in this case EUR is foreign currency and USD is the domestic currency.
- The underlying asset, FX rate, is assumed to follow Geometric Brownian motion (GBM)

$$dS_t = (r_d - r_f)\, S_t dt + \sigma_t S_t dB_t \tag{3.9}$$

- The FX option is European style, which could only be executed at maturity time T.
- Volatility $\sigma$ is considered as a stochastic process which is obtained from the market at time $t$ for all $t$ before maturity $T$.
- The market is liquid and efficient and the transaction cost is not considered.

The log-normal distribution assumption for FX rate is reasonable. Although the log-normality of FX rate is not generally observed, but it provides a good approximation.
One most significant assumption under Black-Scholes model is the volatility.

While in real market, volatility can be relatively constant in very short term, it is never constant in longer term. In the FX option market, the options are priced depending on their delta. Each time when exchange rate moves, the delta of option would change accordingly and a new implied volatility need to be plugged in the pricing formula. Unlike sophisticated stochastic volatility/local volatility/jumps models, the Vanna-Volga pricing method calculated volatility smile impact using relative constant volatility captured in market.
Just like Black-Scholes model, here we assume that any amount of options could be transacted in the market. Also the transaction cost is neglected, which is not realistic in real market.

### 3.2.2 The Simplified Vanna-Volga Equation

The simplified formulation of the Vanna-Volga pricing method could be found in several publications: Wystup (2006), Castagna and Mercurio (2006) and Bossens et al. (2010). The equation is given by:

$$X^{VV} = X^{BS} + \frac{Vanna(X)}{Vanna(RR)} RR_{cost} + \frac{Volga(X)}{Volga(BF)} BF_{cost} \tag{3.10}$$

where

$$\begin{aligned} RR_{cost} &= \left[Call\left(K_C, \sigma\left(K_C\right)\right) - Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &\quad - \left[Call\left(K_C, \sigma_0\right) - Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.11}$$

$$\begin{aligned} BF_{cost} &= \frac{1}{2}\left[Call\left(K_C, \sigma\left(K_C\right)\right) + Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &\quad - \frac{1}{2}\left[Call\left(K_C, \sigma_0\right) + Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.12}$$

and $X^{BS}$ denotes the Black-Scholes price of the vanilla option, Vanna and Volga of option $X$ are calculated with ATM volatility.
It is worth noting that in this version of the Vanna-Volga pricing model, a small but non-zero fraction of Volga carried by RR and a small fraction of Vanna carried by BF are not taken into account. The risk associated with Vega is also neglected here.

### 3.2.3 The Exact Vanna-Volga method

As mentioned in last section, the simplified method is a good approximation but not exact. A modified Vanna-Volga method has been proposed (e.g. Carr et al. (2006), Fisher (2007)) and proved (Shkolnikov (2009)). It has been shown that the following proposition is true for any contract.

**Proposition 1** *Under the assumption that $S$ follows Geometric Brownian motion with stochastic but strike-independent implied volatility, there exists a unique self-financing portfolio $\Pi^{MK} = X^{MK} - \Delta^{MK} S - \sum_{i=1}^{3} x_i C_i^{MK}$ such that $\Pi^{MK} = \Pi^{BS}$ for any $0 \leq t \leq T$. It follows that the Vanna-Volga price is given by:*

$$X_{VV} = X^{BS} + \sum_{i=1}^{3} x_i \left( C_i - C_i^{BS} \right) \tag{3.13}$$

It is worth noting that in the exact formula, the pivot calls and pivot puts could be used interchangeably due to the put-call parity. Using puts would change the value of delta but the coefficient vector $x$ would not be affected. In this report, we would focus on the simplified Vanna-Volga method which has easy implementation and simple calculation.

### 3.2.4 Model Inputs

As shown above, the key inputs are:

- The foreign exchange rate. Published real-time in Bloomberg terminal.
- The interest rate from each country.
- The volatility matrix in the bid/ask format in terms of ATM, $25\Delta$ and $10\Delta$ butterflies and risk reversals.
- The maturity time.

### 3.2.5 Model Outputs

The outputs would be:

- Option price calculated using Black-Scholes formula
- $25\Delta$ and $10\Delta$ butterflies cost
- $25\Delta$ and $10\Delta$ risk reversals cost
- Greeks. Vanna for option and risk reversals strategy, Volga for option and butterflies strategy.
- Vanna-Volga method corrected option price.

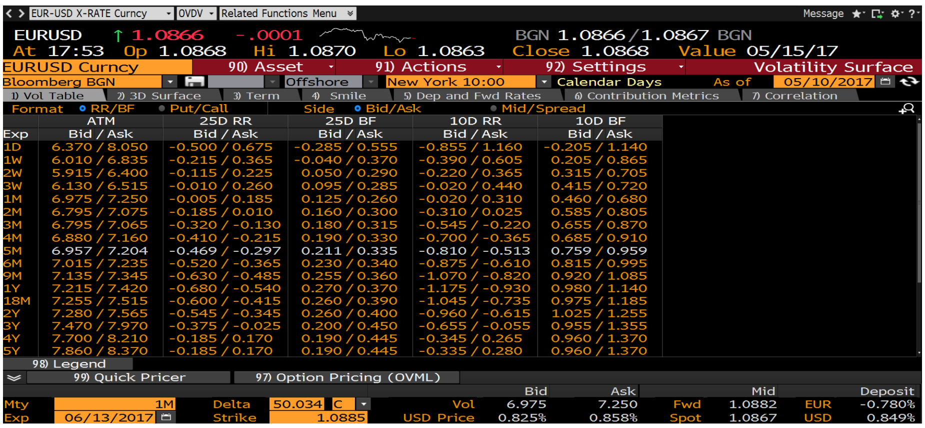
# Chapter 4

# Test of Vanna-Volga Pricing

## 4.1 Data Source

### 4.1.1 Volatility Matrix

Volatity matrix data in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR) with three FX derivatives for Vanna-Volga models was sourced from Bloomberg. An example of volatility matrix data of EUR/USD observed on May 10, 2017 with 1M maturity was showing below.




| | ATM | 25D RR | 25D BF | 10D RR | 10D BF |
|---|---|---|---|---|---|
| Exp | Bid / Ask | Bid / Ask | Bid / Ask | Bid / Ask | Bid / Ask |
| 1D | 6.370 / 8.050 | -0.500 / 0.675 | -0.285 / 0.555 | -0.855 / 1.160 | -0.205 / 1.140 |
| 1W | 6.010 / 6.835 | -0.215 / 0.365 | -0.040 / 0.370 | -0.390 / 0.605 | 0.205 / 0.865 |
| 2W | 5.915 / 6.400 | -0.115 / 0.225 | 0.050 / 0.290 | -0.220 / 0.365 | 0.315 / 0.705 |
| 3W | 6.130 / 6.515 | -0.010 / 0.260 | 0.095 / 0.285 | -0.020 / 0.440 | 0.415 / 0.720 |
| 1M | 6.975 / 7.250 | -0.005 / 0.185 | 0.125 / 0.260 | -0.020 / 0.310 | 0.460 / 0.680 |
| 2M | 6.795 / 7.075 | -0.185 / 0.010 | 0.160 / 0.300 | -0.310 / 0.025 | 0.585 / 0.805 |
| 3M | 6.795 / 7.065 | -0.320 / -0.130 | 0.180 / 0.315 | -0.545 / -0.220 | 0.655 / 0.870 |
| 4M | 6.880 / 7.160 | -0.410 / -0.215 | 0.190 / 0.330 | -0.700 / -0.365 | 0.685 / 0.910 |
| 5M | 6.957 / 7.204 | -0.469 / -0.297 | 0.211 / 0.335 | -0.810 / -0.513 | 0.759 / 0.959 |
| 6M | 7.015 / 7.235 | -0.520 / -0.365 | 0.230 / 0.340 | -0.875 / -0.610 | 0.815 / 0.995 |
| 9M | 7.135 / 7.345 | -0.630 / -0.485 | 0.255 / 0.360 | -1.070 / -0.820 | 0.920 / 1.085 |
| 1Y | 7.215 / 7.420 | -0.680 / -0.540 | 0.270 / 0.370 | -1.175 / -0.930 | 0.980 / 1.140 |
| 18M | 7.255 / 7.515 | -0.600 / -0.415 | 0.260 / 0.390 | -1.045 / -0.735 | 0.975 / 1.185 |
| 2Y | 7.280 / 7.565 | -0.545 / -0.345 | 0.260 / 0.400 | -0.960 / -0.615 | 1.025 / 1.255 |
| 3Y | 7.470 / 7.970 | -0.375 / -0.025 | 0.200 / 0.450 | -0.655 / -0.055 | 0.955 / 1.355 |
| 4Y | 7.700 / 8.210 | -0.185 / 0.170 | 0.190 / 0.445 | -0.345 / 0.265 | 0.960 / 1.370 |
| 5Y | 7.860 / 8.370 | -0.185 / 0.170 | 0.190 / 0.445 | -0.335 / 0.280 | 0.960 / 1.370 |

Figure 4.1: Volatility matrix data in the bid/ask format in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR), observed on May 10, 2017. Source: *Bloomberg*

In order to verify the price calculated by using Vanna-Volga model, two benchmark sources (Bloomberg pricing model and *investing.com* with price provided by *Sentry Derivatives*) for all three FX derivatives had been put in *Appendix 1.* Since Vanna-Volga is an analytically derived correction to Black-Scholes model, the price calucluated by Black-Scholes model also had been included for analysis.

The interest rates used for Vanna-Volga model and Black-Scholes model was obtained from *www.tradingeconomics.com* and listed in the following table.

Table 4.1: FX interest rates observed on May 10, 2017

| Symbol | USD | EUR |
|---|---|---|
| Rates | 1.00% | 0.00% |

### 4.1.2 Model Implementation

As the volatility matrix obtianed from Bloomberg is in the bid/ask format, the averaged mid volatility was used for Black-Scholes model and Vanna-Volga model. The mid volatility matrix data of three FX derivatives were list below. And we consider that live exchange rate as the initial price of FX options $S_0$.

Table 4.2: Mid volatiloty matrix

| FX derivatives | ATM | 25D RR | 25D BF | 10D RR | 10D BF | $S_0$ |
|---|---|---|---|---|---|---|
| EUR/USD | 7.1125 | 0.09 | 0.1925 | 0.145 | 0.57 | 1.0866 |

With the conventions and definitions specified in **Technical Specification**, the implementation of Black-Scholes model and Vanna-Volga model of FX derivatives had been coded in jupyter notebook with Python 3.5 in *Appendix B.*

## 4.2 Testing Results

In order to compare the prices calculated from Vanna-Volga to the benchmark prices easily, we define the price to be % FOR (foreign currency). For example, we use % EUR to be the price form of Vanna-Volga model for EUR/USD options.

The results from the implemented codes with EUR/USD call option had been put in the following table. The details of the prices of four methods corresponding to the strike could be found in *Appendix B*

Table 4.3: EUR/USD call option prices

| Strike | Bloomberg | investing.com | BS | Vanna-Volga |
|---|---|---|---|---|
| 1.065 | 0.023241 | 0.0233 | 0.024682 | 0.023363 |
| 1.070 | 0.019381 | 0.0195 | 0.020663 | 0.019671 |
| 1.075 | 0.015802 | 0.0160 | 0.016970 | 0.016277 |
| 1.080 | 0.012571 | 0.0128 | 0.013649 | 0.013224 |
| 1.085 | 0.009770 | 0.0101 | 0.010733 | 0.010544 |

From the table, we could find that the prices of Vanna-Volga were close to the prices of Bloomberg and investing.com. In particular, Vanna-Volga prices were very close to the prices used in investing.com. In order to visulize the results, a figure contained all the prices had been put in the below.

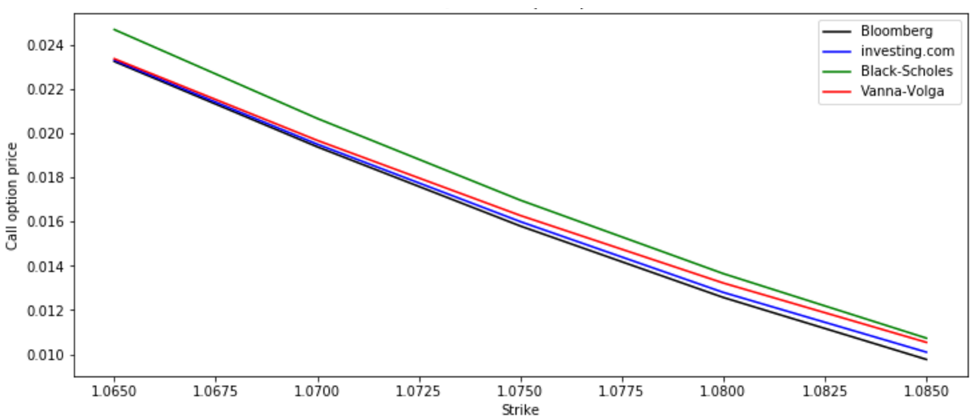



Figure 4.2: Prices of EUR/USD call options with data observed on May 10, 2017

The figure clearly showed that the prices calculated through Vanna-Volga model were very close to the benchmark prices. Besides, as we might see, the Black-Scholes prices (green line) were far away from the benchmark prices compared to Vanna-Volga prices. This figure was consistent with the purpose of the Vanna-Volga model, which is that Vanna-Volga model is an analytically derived correction by capturing the greeks of vanna and volga to Black-Scholes model. Therefore, we consider the model is valid and approprite.

# Chapter 5

# Model Strengths and Weaknesses

## 5.1 Strengths

As we all known that Black-Scholes model is most often used to price vanilla options. However, the parameters used in Black-Scholes model are far from market quotations. The main reason is the unrealistic assumption that the volatility remain constant throught the lifetime of the vanilla options. Besides, the volatility surfaces of FX derivatives tend to be smile shaped or skewed. Thus, Black-Scholes model is insufficient in FX market.

There are models, such as Heston model and local volatility model, could capture and well replicate the smile shaped or skewed volatility surface of FX derivatives. However, none of them is easy to implement and require delicate calibration. Therefore, compare to other models used for FX derivatives, Vanna-Volga model has following strengths:

- Vanna-Volga is easy to implement, comparing to other models
- Vanna-Volga is simple and no or few calibration is needed
- Vanna-Volga is very efficient in computation, i.e., the calculation speed is significantly better than Heston model or local volatility model
- The instruments used for constructing the Vanna-volga model are very liquid in FX market. Typically, people are using straddle, risk reversal, and butterfly to construct Vanna-Volga framework

- Vanna-Volga is an analytically derived correction by capturing the greeks of vanna and volga to Black-Scholes model, i.e., by using vega, vanna and volga of the options. Therefore, it is easy to understand intiuitively.

## 5.2 Weaknesses

Even though Vanna-Volga model also known as *trader's rule of thmb* and has some features listed above, it dose have some drawbacks or conditions need to be understood before using it. Typical weaknesses of Vanna-Volga model are following:

- Vanna-Volga is precise when the maturity of options is up to 1 year, since the model assumes constant interest rates which does not lead to significantly mispricing for short maturity options in FX market.
- The application of Vanna-Volga model is limited to plain vanilla options and first-generation exotic options, such as barrier options, since it cannot fully replicate the volatility surface. However, many of the options in FX market is vanilla or first-generation exotic options.
- Vanna-Volga model perform well when the volatility surface is standard (such as smile shaped, typical skewed) of FX derivatives.

# Chapter 6

# Work Done

## 6.1 ¡Section title¿

### 6.1.1 ¡Sub-section title¿

### 6.1.2 ¡Sub-section title¿

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### 6.1.3 ¡Sub-section title¿

### 6.1.4 ¡Sub-section title¿

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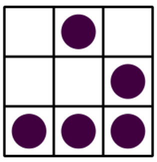

Figure 6.1: ¡Caption here¿

### 6.1.5 ¡Sub-section title¿

## 6.2 ¡Section title¿

# Chapter 7

# Future Work

¡Future work here¿

# Chapter 8

# Conclusion

¡Conclusion here¿

# Acknowledgments

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# Appendix A

## A.1 Data Sources

## A.2 Definitions

25 and 10 Δ-Risk-reversal (RR) volatility:

$$\sigma_{RR25} = \sigma_{25\Delta C} - \sigma_{25\Delta P} \tag{A.1}$$
$$\sigma_{RR10} = \sigma_{10\Delta C} - \sigma_{10\Delta P} \tag{A.2}$$

25 and 10 Δ-Butterfly (BF) volatility:

$$\sigma_{BF25} = \frac{1}{2}\left[\sigma_{25\Delta C} + \sigma_{25\Delta P}\right] - \sigma_{ATM} \tag{A.3}$$
$$\sigma_{BF10} = \frac{1}{2}\left[\sigma_{10\Delta C} + \sigma_{10\Delta P}\right] - \sigma_{ATM} \tag{A.4}$$

These yields to the following equations:

$$\sigma_{25\Delta C} = \sigma_{ATM} + \sigma_{BF25} + \frac{1}{2}\sigma_{RR25} \tag{A.5}$$
$$\sigma_{25\Delta P} = \sigma_{ATM} + \sigma_{BF25} - \frac{1}{2}\sigma_{RR25} \tag{A.6}$$
$$\sigma_{10\Delta C} = \sigma_{ATM} + \sigma_{BF10} + \frac{1}{2}\sigma_{RR10} \tag{A.7}$$
$$\sigma_{10\Delta P} = \sigma_{ATM} + \sigma_{BF10} - \frac{1}{2}\sigma_{RR10} \tag{A.8}$$

Strike retrived from deltas can be calculated through following equation:

$$K = S_0 e^{(r_d - r_f)T - \phi\sigma\sqrt{T}N^{-1}(\phi\Delta) + \frac{1}{2}\sigma^2 T} \tag{A.9}$$

where $\phi = 1$ for call, and $\phi = -1$ for put.

# References

[1] ¡Name of the reference here¿, <urlhere>

[2] ¡Name of the reference here¿, <urlhere>