MTH9845 Risk Management

Model Validation Report

with

Vanna-Volga Pricing Model for FX vanilla options

Submitted by

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Chapter 1

Executive Summary

This paper examines the validity of Vanna-Volga Pricing method, a technique for pricing options in Foreign Exchange (FX) markets.

The Foreign Exchange option's market is the largest and most liquid market of options in the world. Numerous shares of options, ranging from simple vanilla options to exotics options, are traded everyday. Thus, it is imperative for any pricing model to provide a rapid and accurate mark-to-market price calculation.

The most straightforward model would be Black-Scholes model, which could derive analytical prices based on several unrealistic assumptions. It is clearly wrong to assume that the interest rate and FX-spot volatility would remain constant throughout the the maturity of the option. These two factors would be assumed to follow stochastic processes in more realistic models, such as Heston model and SABR volatility model. These models are accurate and rigorous, while normally they are computationally demanding, complex to implement and need delicate calibration.

As an alternative approach, the Vanna-Volga method provide price adjustment for smile impact. It has easy implementation, efficient computation and simple or no calibration features. It takes a small amount of market quotes for liquid instruments and constructs an hedging portfolio which zeros out the sensitivity to volatility, up to second order (Vega, Vanna and Volga). Typically, the ATM options, Butterflies and Risk Reversals strategies are picked for construction.

In particular, we took the data from Bloomberg, *investing.com*, and *tradingeconomics.com* for our test. In order to verify the correction to Black-Scholes model, both Black-Shcoles model and Vanna-Volga model for pricing EUR/USD vanilla call options (we will use FX options as FX vanilla options, unless otherwise specified.) had been implemented in Python code. Industry used prices from Bloomberg pricing tools and *investing.com* had been collected to be the banchmark prices. After carefully testing, our test showed that the Vanna-Volga model was better than Black-Scholes model for FX options pricing, and it was very close to the benchmark prices we collected. As Vanna-Volga model was targed to short-dated (typically less than 1Y) maturity FX options, a test with same procedure of 1M maturity for 1Y maturity EUR/USD call option was also conducted.

Overall, the Vanna-Volga pricing method is easy to understand and provide reasonable results for FX option price. If more accurate results are required, a modified Vanna-Volga method is provided which takes into account some small but non-zero fraction of Vanna and Volga risks for strategies.

Chapter 2

Overview

In accordance with the OCC 2011-12 specifications, this paper lays out the model specification and logic, tests it against other models and finally compares its predictions with observed market prices.

The technical specification is summarized in Chapter 3. Also in this chapter the model assumptions and justifications are discussed. In Chapter 4, we showed the source of our data. The Vanna-Volga method is implemented and predictions are compared with prices from market and calculated by Bloomberg pricing tools. The model strengths and weaknesses are concluded in Chapter 5. More detailed data sources, definitions, and python code has been put in appendices. And Finally the paper is rounded off by a list of relevant literatures.

All the calculations are done in Python and they are mainly for a graphical and qualitative presentation rather than quantitative in a rigorous statistical sense. A complete validation would procure extra data and more details.

Chapter 3

Technical Specification

The Vanna-Volga pricing method is a technique used to price first generation exotic options in foreign exchange market. This method derives from the trader's idea that the difference between market price and Black-Scholes price is the volatility smile impact, which could be adjusted with costs incurred by hedging three main risks associated to the volatility of the option: the Vega, the Vanna and the Volga.

3.1 Greeks

The foreign exchange spot process is considered to follow Geometric Brownian motion (GBM). Thus we find the results we are able to obtain in equity markets hold in the case of FX options as well.
Then the Black-Scholes value of call option is:

$$V_{call} = Se^{-r_f T}N(d_1) - Ke^{-r_d T}N(d_2) \tag{3.1}$$

$$V_{put} = Ke^{-r_d T}N(-d_2) - Se^{-r_f T}N(-d_1) \tag{3.2}$$

where

$$d_1 = \frac{ln\frac{S}{K} + \left(r_d - r_f + \frac{\sigma^2}{2}T\right)}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and r_d and r_f are the domestic and foreign risk free rate, respectively. T is the time to maturity. N denotes as the cumulative density function of standard normal distribution. Below we will discuss the Greeks in the context of Black-Scholes model.

3.1.1 Vega

Vega ν is the first derivative of the option value with respect to the volatility σ.

By taking the derivative we have

$$
\begin{aligned}
\nu_C &= \frac{\partial C}{\partial \sigma} = Se^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-r_f T}\Phi(d_1)\frac{d_1 - d_2}{\sigma} \\
&= Se^{-r_f T}\Phi(d_1)\sqrt{T}
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
\nu_P &= \frac{\partial P}{\partial \sigma} = -Ke^{-r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} + Se^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\
&= \nu_C
\end{aligned}
\tag{3.4}
$$

3.1.2 Vanna

Vanna is the second order derivative of the option value, once to the volatility σ and once to the initial spot price.

By taking the derivative we have:

$$
\begin{aligned}
Vanna_C &= \frac{\partial^2 C}{\partial S \partial \sigma} = \frac{\partial \Delta_C}{\partial \sigma} = e^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\
&= e^{-r_f T}\Phi(d_1)\left(\sqrt{T} - \frac{d_1}{\sigma}\right) \\
&= -\frac{d_2}{S\sigma\sqrt{T}}\nu_C
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
Vanna_P &= \frac{\partial^2 P}{\partial S \partial \sigma} = \frac{\partial \Delta_P}{\partial \sigma} = e^{-r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\
&= Vanna_C
\end{aligned}
\tag{3.6}
$$

3.1.3 Volga

Volga is the second order derivative of the option value with respect to the volatility σ twice.

By taking the derivative we have:

$$\begin{aligned} Volga_C &= \frac{\partial^2 C}{\partial^2 \sigma} = \frac{\partial \nu_C}{\partial \sigma} = e^{-r_f T} S\sqrt{T} \frac{\partial \Phi(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} \\ &= e^{-r_f T} S\sqrt{T} \Phi(d_1) \frac{d_1 d_2}{\sigma} \\ &= \frac{\nu_C d_1 d_2}{\sigma} \end{aligned} \tag{3.7}$$

$$\begin{aligned} Volga_P &= \frac{\partial^2 P}{\partial^2 \sigma} = \frac{\partial \nu_P}{\partial \sigma} = \frac{\partial \nu_C}{\partial \sigma} \\ &= Volga_C \end{aligned} \tag{3.8}$$

3.2 Model Framework and Equations

3.2.1 Model Inputs

As shown above, the key inputs are:

- The foreign exchange rate. Published real-time in Bloomberg terminal.
- The interest rate from each country.
- The volatility matrix in the bid/ask format in terms of ATM, 25Δ and 10Δ butterflies and risk reversals.
- The maturity time.

3.2.2 The Simplified Vanna-Volga Process

The simplified formulation of the Vanna-Volga pricing method could be found in several publications: Wystup (2006), Castagna and Mercurio (2006) and Bossens et al. (2010). The equation is given by:

$$X^{VV} = X^{BS} + \frac{Vanna(X)}{Vanna(RR)} RR_{cost} + \frac{Volga(X)}{Volga(BF)} BF_{cost} \tag{3.9}$$

where

$$\begin{aligned} RR_{cost} = &\left[Call\left(K_C, \sigma\left(K_C\right)\right) - Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &- \left[Call\left(K_C, \sigma_0\right) - Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.10}$$

$$\begin{aligned} BF_{cost} = &\frac{1}{2}\left[Call\left(K_C, \sigma\left(K_C\right)\right) + Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &- \frac{1}{2}\left[Call\left(K_C, \sigma_0\right) + Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.11}$$

and X^{BS} denotes the Black-Scholes price of the vanilla option, Vanna and Volga of option X are calculated with ATM volatility.

It is worth noting that in this version of the Vanna-Volga pricing model, a small but non-zero fraction of Volga carried by RR and a small fraction of Vanna carried by BF are not taken into account. The risk associated with Vega is also neglected here.

3.2.3 Model Outputs

The outputs would be:

- 25Δ and 10Δ butterflies cost
- 25Δ and 10Δ risk reversals cost
- Greeks. Vanna for option and risk reversals strategy, Volga for option and butterflies strategy.
- Vanna-Volga method corrected option price.

Chapter 4

Review of Assumptions

Vanna-Volga pricing method is a mathematical method for pricing options in foreign exchange market. Thus, it follows some basic assumptions for the FX options. Here are some assumptions about the market and the options:

- The considered underlying asset S_t is an FX rate quoted in foreign/domestic format. For example, EUR/USD open today is 1.0868, which means 1 EUR is worth 1.0868 USD and in this case EUR is foreign currency and USD is the domestic currency.
- The underlying asset, FX rate, is assumed to follow Geometric Brownian motion (GBM)

$$dS_t = (r_d - r_f)\, S_t dt + \sigma_t S_t dB_t \tag{4.1}$$

- The FX option is European style, which could only be executed at maturity time T.
- Volatility σ is considered as a stochastic process which is obtained from the market at time t for all t before maturity T.
- The market is liquid and efficient and the transaction cost is not considered.

The log-normal distribution assumption for FX rate is reasonable. Although the log-normality of FX rate is not generally observed, but it provides a good approximation.

One most significant assumption under Black-Scholes model is the volatility. While in real market, volatility can be relatively constant in very short term, it is never constant in longer term. In the FX option market, the options are priced depending on their delta. Each time when exchange rate moves, the delta of option would change accordingly and a new implied volatility need to be plugged in the pricing formula. Unlike sophisticated stochastic volatility/local volatility/jumps models, the Vanna-Volga pricing method calculated volatility smile impact using relative constant volatility captured in market.

Just like Black-Scholes model, here we assume that any amount of options could be transacted in the market. Also the transaction cost is neglected, which is not realistic in real market.

Chapter 5

Comparison to Other Models

It is well known that Black-Scholes model assumes constant volatility which is not realistic in real market. Using Exploratory Data Analysis (EDA) techniques, we visualized the volatility data in Bloomberg Terminal and presented following figures.

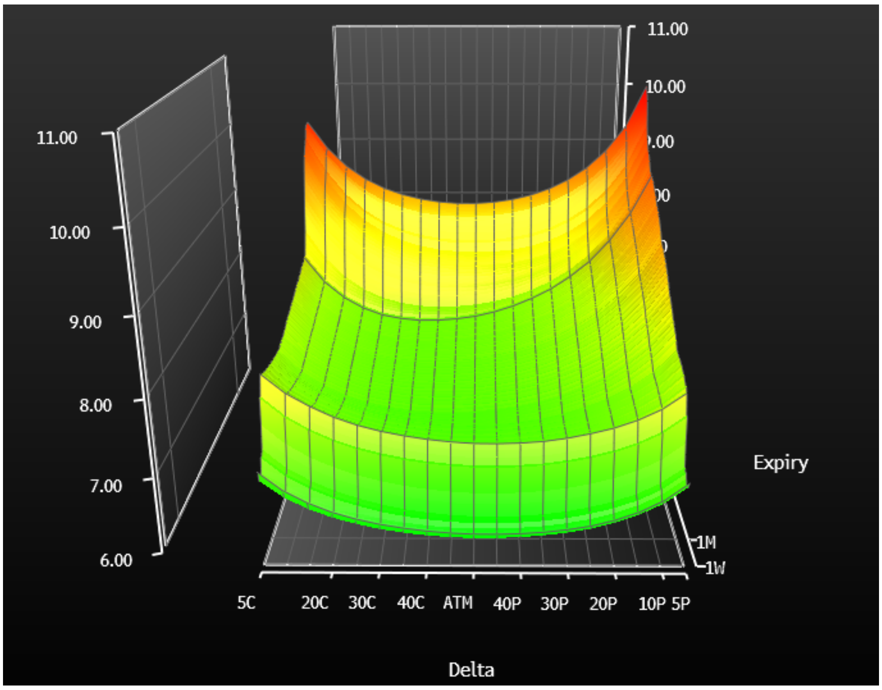


Figure 5.1: Three-dimensional volatility surface in terms of Delta and Maturity Time for EUR/USD option, observed on May 10, 2017. Source: *Bloomberg*

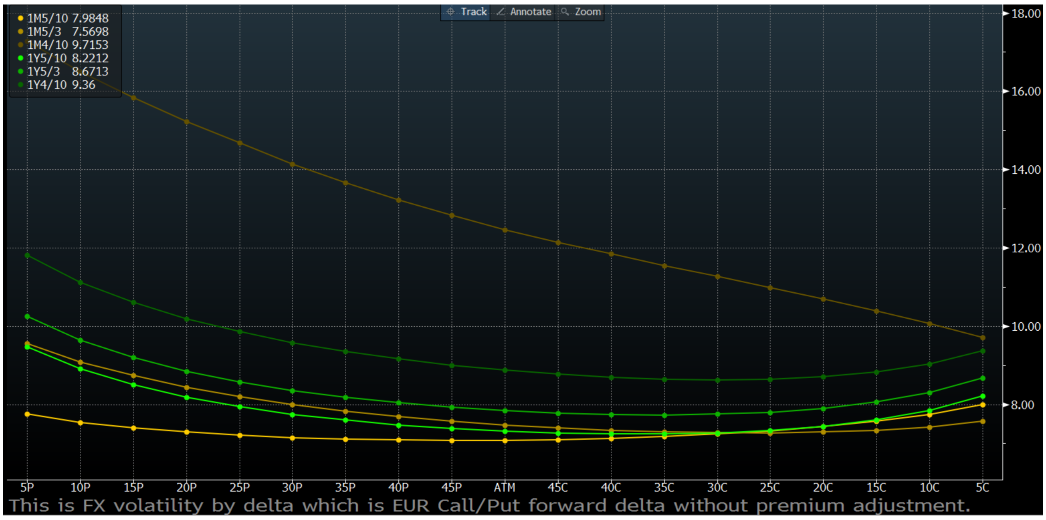


Figure 5.2: Volatility surface in terms of Delta for 1-month and 1-year EUR/USD option. Source: *Bloomberg*

From both figures we clearly see that the volatility is not constant. Figure 5.1 shows three-dimensional volatility surface for EUR/USD option. Consider a given expiration, deep in-the-money and out-of-the money options have higher implied volatility than at-the-money option. If we fix a specific strike, we could obtained that long-term maturity usually has higher implied volatility.

In Figure 5.2 we selected 1-month and 1-year EUR/USD option and showed the volatility change in terms of Delta. For 1-month option data extracted on Apr. 10, 2017, volatility skew was observed while volatility smile detected for other dates.

In addition to the simplified Vanna-Volga method we introduced in Chapter 3, several other models have been developed to mitigate the volatility smile impact. Here we will briefly introduce the exact Vanna-Volga pricing method and two stochastic volatility model: Heston model and SABR model.

5.1 The Exact Vanna-Volga Method

As mentioned, the simplified method is a good approximation while neglecting some small fraction of Vanna and Volga. A modified Vanna-Volga method, which provide more accurate result, has been proposed (e.g. Carr et al. (2006), Fisher (2007)) and proved (Shkolnikov (2009)). It has been shown that the following proposition is true for any contract.

Proposition 1 *Under the assumption that S follows Geometric Brownian motion with stochastic but strike-independent implied volatility, there exists a unique self-financing portfolio $\Pi^{MK} = X^{MK} - \Delta^{MK} S - \sum_{i=1}^{3} x_i C_i^{MK}$ such that $\Pi^{MK} = \Pi^{BS}$ for any $0 \leq t \leq T$. It follows that the Vanna-Volga price is given by:*

$$X_{VV} = X^{BS} + \sum_{i=1}^{3} x_i \left(C_i - C_i^{BS} \right) \tag{5.1}$$

It is worth noting that in the exact formula, the pivot calls and pivot puts could be used interchangeably due to the put-call parity. Using puts would change the value of delta but the coefficient vector x_i would not be affected.

5.2 Heston Model

The Heston Model is a commonly used stochastic volatility model, in which the randomness of the variance process varies as the square root of variance.

In basic Heston model, the underlying asset S_t follows a stochastic process:

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S \tag{5.2}$$

where the instantaneous variance ν_t is a CIR process:

$$d\nu_t = \kappa \left(\theta - \nu_t \right) dt + \xi \sqrt{\nu_t} dW_t^{\nu} \tag{5.3}$$

and dW_t^S, dW_t^{ν} are Wiener processes with correlation ρ.

The parameters have following meanings:

- μ is the rate of return of the asset.
- κ is the mean reversion speed.
- θ is the long term average price variance.
- ξ is the volatility of volatility which determines the variance of ν_t.

The Heston model produces more realistic dynamics and covers wide product range. It also has several extensions which could add more degrees of freedom to the original model and cover more features from the volatility surface.

5.3 SABR Model

SABR model is another widely used stochastic volatility model which attempts to capture the volatility smile in derivative markets. It is an extension of the CEV model in which the volatility parameter is assumed to follow a stochastic process. Its dynamics are explicitly given by:

$$\begin{aligned} dF_t &= \sigma_t F_t^{\beta} dW_t \\ d\sigma_t &= \alpha \sigma_t dZ_t \end{aligned} \tag{5.4}$$

and dW_t, dZ_t are Wiener processes with correlation ρ.
The single forward F_t here could be related to any asset including an index, interest rate, bond, currency or equity.

The SABR model can also have extensions by assuming the parameters to be time-dependent which furthermore complicates the calibration procedure.

5.4 Conclusion

Normally the stochastic volatility models could produce more realistic dynamics which could describe the system better. More parameters allow them to have better fit for each different situation. Meanwhile they have some disadvantages. They usually need delicate calibrations and excessive computation, which limit their use when time is the dominant factor. Also for products which depend only on terminal distributions, the fit would be poor.

The modified Vanna-Volga pricing method could provide more accurate result compared to simplified version. It also reduces the complexity of implementation versus stochastic volatility models. Thus it would be a good compromise between simplified Vanna-Volga method and stochastic volatility method.

In this report, we would focus on the simplified Vanna-Volga method which has easy implementation and simple calculation. It is very computationally efficient and provide good approximations for FX options, especially for short-term maturity options.

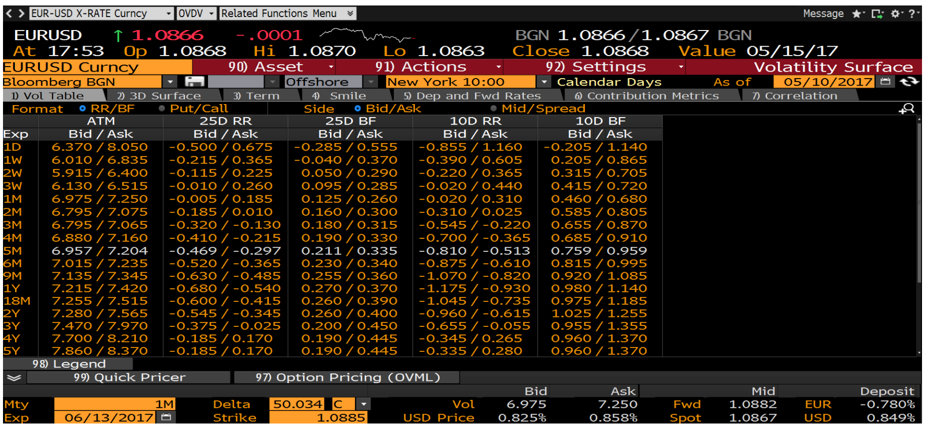
Chapter 6

Test of Vanna-Volga Pricing

6.1 Data Source

6.1.1 Volatility Matrix

Volatity matrix data in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR) with three FX derivatives for Vanna-Volga models was sourced from Bloomberg. An example of volatility matrix data of EUR/USD observed on May 10, 2017 with 1M maturity was showing below.



	ATM	25D RR	25D BF	10D RR	10D BF
Exp	Bid / Ask	Bid / Ask	Bid / Ask	Bid / Ask	Bid / Ask
1D	6.370 / 8.050	-0.500 / 0.675	-0.285 / 0.555	-0.855 / 1.160	-0.205 / 1.140
1W	6.010 / 6.835	-0.215 / 0.365	-0.040 / 0.370	-0.390 / 0.605	0.205 / 0.865
2W	5.915 / 6.400	-0.115 / 0.225	0.050 / 0.290	-0.220 / 0.365	0.315 / 0.705
3W	6.130 / 6.515	-0.010 / 0.260	0.095 / 0.285	-0.020 / 0.440	0.415 / 0.720
1M	6.975 / 7.250	-0.005 / 0.185	0.125 / 0.260	-0.020 / 0.310	0.460 / 0.680
2M	6.795 / 7.075	-0.185 / 0.010	0.160 / 0.300	-0.310 / 0.025	0.585 / 0.805
3M	6.795 / 7.065	-0.320 / -0.130	0.180 / 0.315	-0.545 / -0.220	0.655 / 0.870
4M	6.880 / 7.160	-0.410 / -0.215	0.190 / 0.330	-0.700 / -0.365	0.685 / 0.910
5M	6.957 / 7.204	-0.469 / -0.297	0.211 / 0.335	-0.810 / -0.513	0.759 / 0.959
6M	7.015 / 7.235	-0.520 / -0.365	0.230 / 0.340	-0.875 / -0.610	0.815 / 0.995
9M	7.135 / 7.345	-0.630 / -0.485	0.255 / 0.360	-1.070 / -0.820	0.920 / 1.085
1Y	7.215 / 7.420	-0.680 / -0.540	0.270 / 0.370	-1.175 / -0.930	0.980 / 1.140
18M	7.255 / 7.515	-0.600 / -0.415	0.260 / 0.390	-1.045 / -0.735	0.975 / 1.185
2Y	7.280 / 7.565	-0.545 / -0.345	0.260 / 0.400	-0.960 / -0.615	1.025 / 1.255
3Y	7.470 / 7.970	-0.375 / -0.025	0.200 / 0.450	-0.655 / -0.055	0.955 / 1.355
4Y	7.700 / 8.210	-0.185 / 0.170	0.190 / 0.445	-0.345 / 0.265	0.960 / 1.370
5Y	7.860 / 8.370	-0.185 / 0.170	0.190 / 0.445	-0.335 / 0.280	0.960 / 1.370

Figure 6.1: Volatility matrix data in the bid/ask format in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR), observed on May 10, 2017. Source: *Bloomberg*

In order to verify the price calculated by using Vanna-Volga model, two benchmark sources (Bloomberg pricing model and *investing.com* with price provided by *Sentry Derivatives*) for FX derivatives had been put in *Appendix*

A.3. Since Vanna-Volga is an analytically derived correction to Black-Scholes model, the price calucluated by Black-Scholes model also had been included for analysis.

The interest rates used for Vanna-Volga model and Black-Scholes model was obtained from *www.tradingeconomics.com* and listed in the following table.

Table 6.1: FX interest rates observed on May 10, 2017

Symbol	USD	EUR
Rates	1.00%	0.00%

6.1.2 Model Implementation

As the volatility matrix obtianed from Bloomberg is in the bid/ask format, the averaged mid volatility was used for Black-Scholes model and Vanna-Volga model. The mid volatility matrix data of three FX derivatives were list below. And we consider that live exchange rate as the initial price of FX options S_0.

Since Vanna-Volga model is limited to the short-dated (tipically less than 1Y) FX options, a test of EUR/USD call options with 1Y maturity also had been conducted with same procedure of 1M maturity.

Table 6.2: Mid volatility matrix of EUR/USD with 1M maturity

FX derivatives	ATM	25D RR	25D BF	10D RR	10D BF	S_0
EUR/USD	7.1125	0.09	0.1925	0.145	0.57	1.0866

Table 6.3: Mid volatility matrix of EUR/USD with 1Y maturity

FX derivatives	ATM	25D RR	25D BF	10D RR	10D BF	S_0
EUR/USD	7.415	-0.685	0.325	-1.175	1.065	1.0923

With the conventions and definitions specified in **Technical Specification**, the implementation of Black-Scholes model and Vanna-Volga model of FX derivatives had been coded in jupyter notebook with Python 3.5 in *Appendix A.3*.

6.2 Testing Results

In order to compare the prices calculated from Vanna-Volga to the benchmark prices easily, we define the price to be % FOR (foreign currency). For example, we use % EUR to be the price form of Vanna-Volga model for EUR/USD options.

6.2.1 Short-dated Maturity

The results from the implemented codes with EUR/USD call option had been put in the table. The details of the prices of four methods corresponding to the strike could be found in *Appendix A.3*.

Table 6.4: Prices of EUR/USD call option with 1M matiruty

Strike	Heston	investing.com	BS	Vanna-Volga
1.065	0.023241	0.0233	0.024682	0.023363
1.070	0.019381	0.0195	0.020663	0.019671
1.075	0.015802	0.0160	0.016970	0.016277
1.080	0.012571	0.0128	0.013649	0.013224
1.085	0.009770	0.0101	0.010733	0.010544

From the table, we could find that the prices of Vanna-Volga were close to the prices of Bloomberg and investing.com. In particular, Vanna-Volga prices were very close to the prices used in investing.com. In order to visulize the results, a figure contained all the prices had been put in the below.

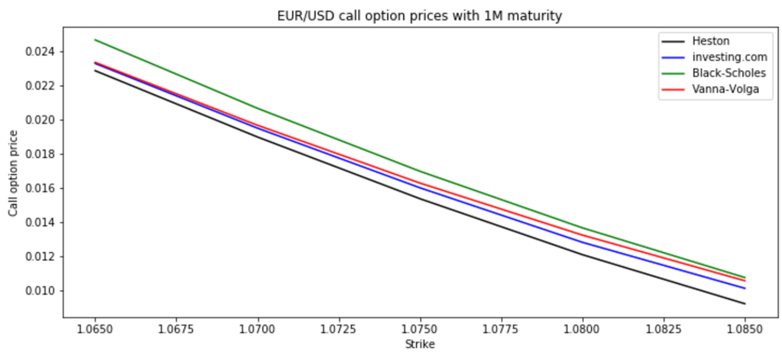


Figure 6.2: Prices of EUR/USD call options with 1M maturity. Data observed on May 10, 2017

The figure clearly showed that the prices calculated through Vanna-Volga model were very close to the benchmark prices. Besides, as we might see, the

Black-Scholes prices (green line) were far away from the benchmark prices compared to Vanna-Volga prices. This figure was consistent with the purpose of the Vanna-Volga model, which is that Vanna-Volga model is an analytically derived correction by capturing the greeks of vanna and volga to Black-Scholes model.

6.2.2 1Y Maturity

Following the same procedure, we got the result as following.

Table 6.5: Prices of EUR/USD call option with 1Y matiruty

Strike	Heston	investing.com	BS	Vanna-Volga
1.05	0.069127	0.0695	0.065060	0.058870
1.06	0.062125	0.0623	0.058001	0.052372
1.07	0.055455	0.0555	0.051381	0.046276
1.08	0.049155	0.0490	0.045220	0.040596
1.09	0.043257	0.0429	0.039532	0.035339

From the table, we found that two benchmark prices (Heston from Bloomberg pricing tools and prices from *investing.com*) are very close. However, Black-Scholes prices were much far away from the benchmark prices and the Vanna-Volga prices were even far away from the benchmark prices. In order to visulize the results, a figure contained all the prices had been put in the below.

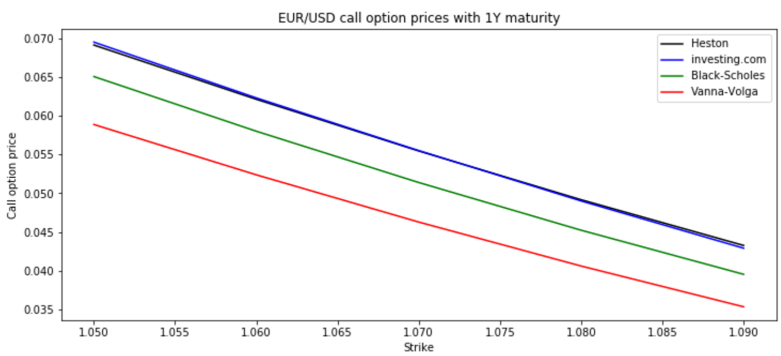


Figure 6.3: Prices of EUR/USD call options with 1Y maturity. Data observed on May 12, 2017

The figure clearly showed that both Black-Scholes prices and Vanna-Volga prices were far away from the benchmark prices, which implied that the

Black-Scholes model and Vanna-Volga model were not good for long-dated maturity FX options.

6.2.3 Test Conclusion

Therefore, we conclued that the model is valid and appropriate for short-dated (less than 1Y) maturity FX vanilla options pricing. As the limitation of the report, only FX vanilla options pricing had been tested. However, Agnieszka Janek showed that Vanna-Volga model was also good for pricing first-generation FX options, e.g., FX barrier options. His paper had been included in the references list.

Chapter 7

Model Strengths and Weaknesses

7.1 Strengths

As we all known that Black-Scholes model is most often used to price vanilla options. However, the parameters used in Black-Scholes model are far from market quotations. The main reason is the unrealistic assumption that the volatility remain constant throught the lifetime of the vanilla options. Besides, the volatility surfaces of FX derivatives tend to be smile shaped or skewed. Thus, Black-Scholes model is insufficient in FX market.

There are models, such as Heston model and local volatility model, could capture and well replicate the smile shaped or skewed volatility surface of FX derivatives. However, none of them is easy to implement and require delicate calibration. Therefore, compare to other models used for FX derivatives, Vanna-Volga model has following strengths:

- Vanna-Volga is easy to implement, comparing to other models
- Vanna-Volga is simple and no or few calibration is needed
- Vanna-Volga is very efficient in computation, i.e., the calculation speed is significantly better than Heston model or local volatility model
- The instruments used for constructing the Vanna-volga model are very liquid in FX market. Typically, people are using straddle, risk reversal, and butterfly to construct Vanna-Volga framework
- Vanna-Volga is an analytically derived correction by capturing the greeks of vanna and volga to Black-Scholes model, i.e., by using vega,

vanna and volga of the options. Therefore, it is easy to understand intiuitively.

7.2 Weaknesses

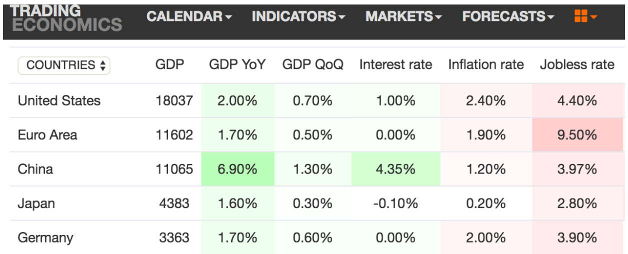
Even though Vanna-Volga model also known as *trader's rule of thmb* and has some features listed above, it dose have some drawbacks or conditions need to be understood before using it. Typical weaknesses of Vanna-Volga model are following:

- Vanna-Volga is precise when the maturity of options is up to 1 year, since the model assumes constant interest rates which does not lead to significantly mispricing for short maturity options in FX market.
- The application of Vanna-Volga model is limited to plain vanilla options and first-generation exotic options, such as barrier options, since it cannot fully replicate the volatility surface. However, many of the options in FX market is vanilla or first-generation exotic options.
- Vanna-Volga model perform well when the volatility surface is standard (such as smile shaped, typical skewed) of FX derivatives.

Appendix A

A.1 Data Sources

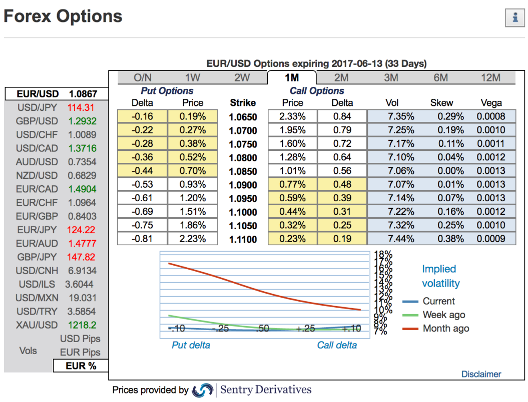
The interest rates used for Black-Scholes model and Vanna-Volga model were obtained from *www.tradingeconomics.com* and was listed below.



COUNTRIES	GDP	GDP YoY	GDP QoQ	Interest rate	Inflation rate	Jobless rate
United States	18037	2.00%	0.70%	1.00%	2.40%	4.40%
Euro Area	11602	1.70%	0.50%	0.00%	1.90%	9.50%
China	11065	6.90%	1.30%	4.35%	1.20%	3.97%
Japan	4383	1.60%	0.30%	-0.10%	0.20%	2.80%
Germany	3363	1.70%	0.60%	0.00%	2.00%	3.90%

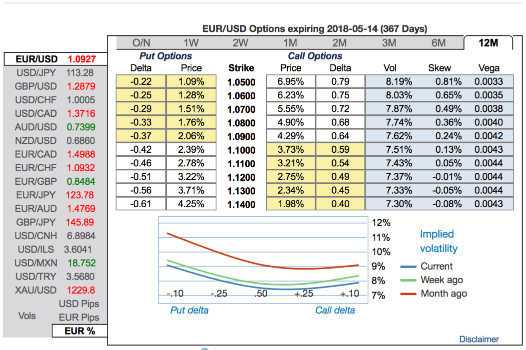
Figure A.1: Interest rates from tradingeconomics.com

The prices of EUR/USD with 1M and 1Y maturity obtained from *www.investing.com* were showing as following:



Put Options			Call Options				
Delta	Price	Strike	Price	Delta	Vol	Skew	Vega
-0.16	0.19%	1.0650	2.33%	0.84	7.35%	0.29%	0.0008
-0.22	0.27%	1.0700	1.95%	0.79	7.25%	0.19%	0.0010
-0.28	0.38%	1.0750	1.60%	0.72	7.17%	0.11%	0.0011
-0.36	0.52%	1.0800	1.28%	0.64	7.10%	0.04%	0.0012
-0.44	0.70%	1.0850	1.01%	0.56	7.06%	0.00%	0.0013
-0.53	0.93%	1.0900	0.77%	0.48	7.07%	0.01%	0.0013
-0.61	1.20%	1.0950	0.59%	0.39	7.14%	0.07%	0.0013
-0.69	1.51%	1.1000	0.44%	0.31	7.22%	0.16%	0.0012
-0.75	1.86%	1.1050	0.32%	0.25	7.32%	0.25%	0.0010
-0.81	2.23%	1.1100	0.23%	0.19	7.44%	0.38%	0.0009

Figure A.2: EUR/USD call prices from investing.com



EUR/USD Options expiring 2018-05-14 (367 Days)

Put Options Delta	Put Options Price	Strike	Call Options Price	Call Options Delta	Vol	Skew	Vega
-0.22	1.09%	1.0500	6.95%	0.79	8.19%	0.81%	0.0033
-0.25	1.28%	1.0600	6.23%	0.75	8.03%	0.65%	0.0035
-0.29	1.51%	1.0700	5.55%	0.72	7.87%	0.49%	0.0038
-0.33	1.76%	1.0800	4.90%	0.68	7.74%	0.36%	0.0040
-0.37	2.06%	1.0900	4.29%	0.64	7.62%	0.24%	0.0042
-0.42	2.39%	1.1000	3.73%	0.59	7.51%	0.13%	0.0043
-0.46	2.78%	1.1100	3.21%	0.54	7.43%	0.05%	0.0044
-0.51	3.22%	1.1200	2.75%	0.49	7.37%	-0.01%	0.0044
-0.56	3.71%	1.1300	2.34%	0.45	7.33%	-0.05%	0.0044
-0.61	4.25%	1.1400	1.98%	0.40	7.30%	-0.08%	0.0043

Figure A.3: EUR/USD call prices from investing.com

Since Bloomber has its own pricing model (chosed Heston in our report) for FX options, the prices of EUR/USD call options with different strikes are obtained in Bloomberg terminal and listed below (using strike 1.065 as example).

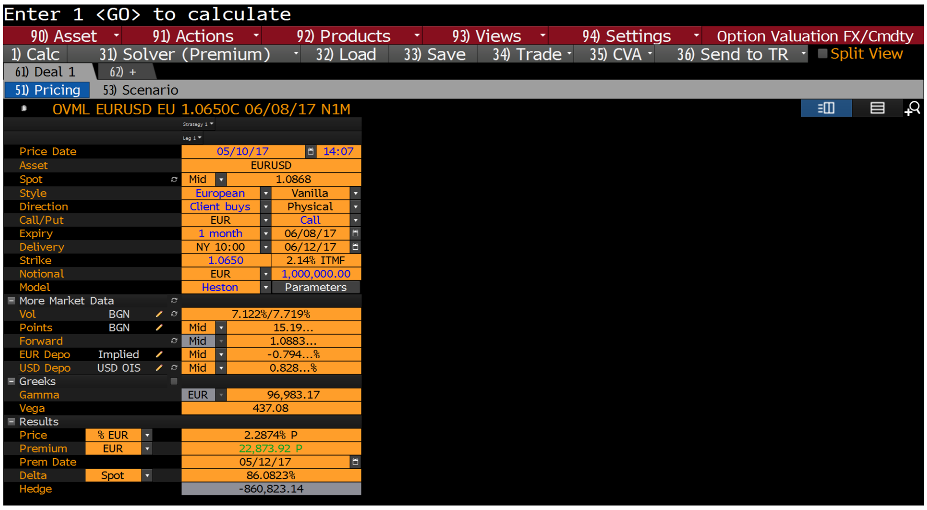


Figure A.4: EUR/USD call prices with strike 1.065 and 1M maturity

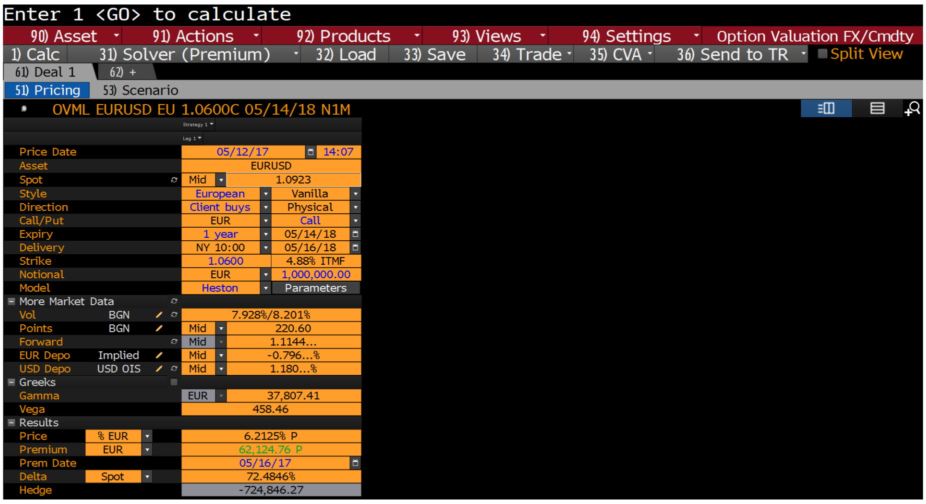


Figure A.5: EUR/USD call prices with strike 1.06 and 1Y maturity

A.2 Definitions

25 and 10 Δ-Risk-reversal (RR) volatility:

$$\sigma_{RR25} = \sigma_{25\Delta C} - \sigma_{25\Delta P} \tag{A.1}$$
$$\sigma_{RR10} = \sigma_{10\Delta C} - \sigma_{10\Delta P} \tag{A.2}$$

25 and 10 Δ-Butterfly (BF) volatility:

$$\sigma_{BF25} = \frac{1}{2}\left[\sigma_{25\Delta C} + \sigma_{25\Delta P}\right] - \sigma_{ATM} \tag{A.3}$$
$$\sigma_{BF10} = \frac{1}{2}\left[\sigma_{10\Delta C} + \sigma_{10\Delta P}\right] - \sigma_{ATM} \tag{A.4}$$

These yields to the following equations:

$$\sigma_{25\Delta C} = \sigma_{ATM} + \sigma_{BF25} + \frac{1}{2}\sigma_{RR25} \tag{A.5}$$
$$\sigma_{25\Delta P} = \sigma_{ATM} + \sigma_{BF25} - \frac{1}{2}\sigma_{RR25} \tag{A.6}$$
$$\sigma_{10\Delta C} = \sigma_{ATM} + \sigma_{BF10} + \frac{1}{2}\sigma_{RR10} \tag{A.7}$$
$$\sigma_{10\Delta P} = \sigma_{ATM} + \sigma_{BF10} - \frac{1}{2}\sigma_{RR10} \tag{A.8}$$

Strike retrived from deltas can be calculated through following equation:

$$K = S_0 e^{(r_d - r_f)T - \phi\sigma\sqrt{T}N^{-1}(\phi\Delta) + \frac{1}{2}\sigma^2 T} \tag{A.9}$$

where $\phi = 1$ for call, and $\phi = -1$ for put.

A.3 Python Code of Vanna-Volga Model

The codes of Vanna-Volga model had been implemented in jupyter notebook with kernel Python 3.5. Details of the codes could be found in the corresponding jupyter notebook with name as "MTH9845_final_project_codes.ipynb". The jupyter notebook also had been converted to a pdf file with name as "MTH9845_final_project_codes.pdf". Please see the details in the seperate files.

The Black-Scholes prices and Vanna-Volga prices calculated by implemented codes had been put in the following figure. The prices from Bloomberg and *inversting.com* were also included.

Out[10]:

	EURUSD Strike	Heston Price	investing.com Price	BS Price	VV Price
0	1.065	0.022874	0.0233	0.024682	0.023363
1	1.070	0.018981	0.0195	0.020663	0.019671
2	1.075	0.015359	0.0160	0.016970	0.016277
3	1.080	0.012075	0.0128	0.013649	0.013224
4	1.085	0.009195	0.0101	0.010733	0.010544

Figure A.6: Combined prices of EUR/USD with 1M maturity and different strikes

Out[13]:

	EURUSD Strike	Heston Price	investing.com Price	BS Price	VV Price
0	1.05	0.069127	0.0695	0.065060	0.058870
1	1.06	0.062125	0.0623	0.058001	0.052372
2	1.07	0.055455	0.0555	0.051381	0.046276
3	1.08	0.049155	0.0490	0.045220	0.040596
4	1.09	0.043257	0.0429	0.039532	0.035339

Figure A.7: Combined prices of EUR/USD with 1Y maturity and different strikes

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