Validation of CDS Pricing Model by Meshulam Ross

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Contents

1. Executive Summary

This paper examines the validity of Meshulam Ross' Excel-based model for pricing single-name credit default swaps. We find that it does not work well in its current state, and that further calibration and enhancements need to be implemented.

Based on the model validation results and the fact that the outputs are different from the benchmark model's by approximately constant number, we suggest that **Meshulam's CDS pricer could be utilized as a quick-and-dirty CDS valuation tool, provided necessary calibration to the model is done, and under assumption that Bloomberg model produces valid outputs.**

2. Overview

In accordance with the OCC 2011-12 specifications, this paper lays out model objectives, technical specifications, core assumptions, logic and implementation. In the section "Technical Specifications", we go over CDS derivative security. We briefly explain purpose of the security and how it's being priced. We then explain theoretical part of it. In the next section, Review of Assumption, we analyze input, processing, and output of the CDS pricing model. For each of the criteria we analyze with details, implementation versus theory and point out what assumptions are being used in the pricing model. Sections 5 and 6 perform sensitivity and benchmark tests to determine robustness, reliability and validity of the model. In sensitivity section we tried to price CDS by changing one input variable at the time and keep all other constant. We performed sensitivity tests by changing: maturity, yield curve, survival probabilities and recovery rate. In Benchmark section we choose a Bloomberg CDS prices as benchmark model. By fetching same inputs we compared output of two models. In the last section, we perform a Stress Testing of our model to explore model's limits. We use two different hypothetical stress test scenarios, and analyze the performance of our model under stress conditions.

3. Technical specifications

Credit Default Swap (CDS) is an over-the-counter (OTC) agreement between two parties A and C. A bond holder, party A, can buy insurance from party C against default of a bond issuer B. For the protection, the buyer of CDS agrees to pay premium payments to insurer, who agrees to make up for losses on debt of a particular issuer. Premium is paid with some frequency in arrears. If default happens between premium payments, premium accrues since the last payment until the moment of default.

At the time contract is signed, the value of CDS should be equal to 0, so that neither protection buyer nor protection seller has an advantage: V_premium = V_protection, where V_premium is present value (PV) of all premium payments, V_protection is present value of protection that is paid in case of event of default or other credit event.

The objective of the CDS pricier is to compute fair spread such that V_deal = V_premium – V_protection = 0, given that all other variables are known! Let's elaborate on V_premium and V_protection in order to determine what parameters should be known in order to compute fair spread:

V_premium =

$$V_{premium} = \sum_{i=1}^{n} N \frac{s}{m} P_{t_i} P(\tau > t_i) dt + \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} Ns(t - t_{i-1}) P_t P(\tau = t) dt$$

, where

$\sum_{i=1}^{n} N \frac{s}{m} P_{t_i} P(\tau > t_i) dt$ is expected PV of premium payments given credit even is greater time t;

$\sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} Ns(t - t_{i-1}) P_t P(\tau = t) dt$ is expected PV of accrued premium, in case credit even happen between premium payments.

s – CDS par spread; N – notional, Ƭ – time of the event, t – time, m – frequency of premium payments per year.

V_protection =

$$V_{protection} = N(1 - R) \int_{0}^{T} P_t P(\tau = t) dt$$

, where entire term is expected PV of credit loss, N – notional, R is recovery rate.

CDS pricing model computes CDS fair spread "s", such that V_premium = V_protection:

$s^* = \frac{L_T^*}{A_{m,T}^*}$, where

risky annuity is: $$A_{m,T}^* = \sum_{i=1}^{n} \frac{1}{m} P_{t_i} P(\tau > t_i) dt + \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} (t - t_{i-1}) P_t P(\tau = t) dt$$,

Expected present value of loss given default: $$L_T^* = (1 - R) \int_{0}^{T} P_t P(\tau = t) dt$$

4. Review of Assumption

Let's analyze three aspects of the model: input, processing, and output.

Parameters/Input

As mentioned in previous section, the model assumes certain parameters are known in advance in order to compute par CDS spread. Let's extract all parameters from formulas and compare if they match the input parameters to the model.

Terms those are known according the formulas for premium and protection legs:

1. Notional
2. Risk-free interest rate curve
3. Frequency of premium payments
4. Recovery rate
5. Survival probabilities
6. Time to maturity

Input parameters in CDS pricing model:

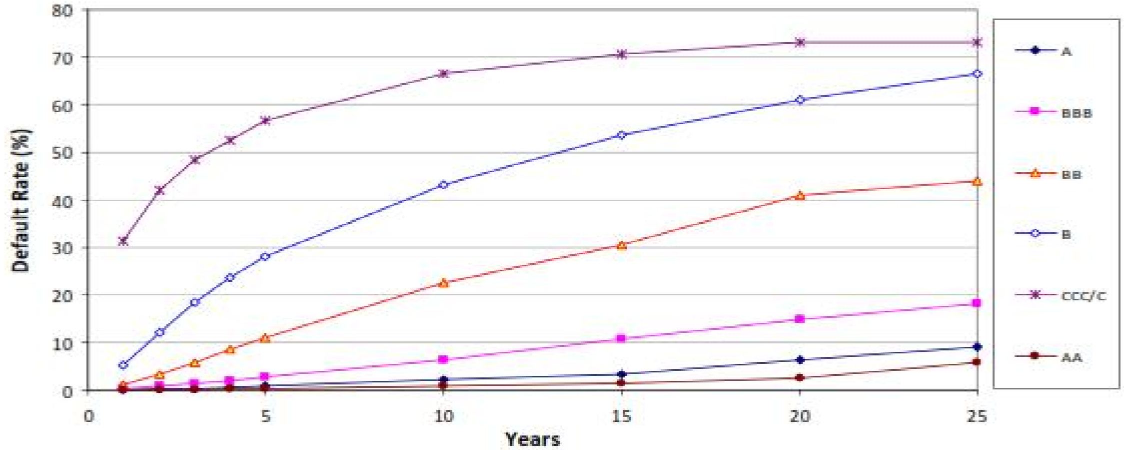
1. N – notional
2. T – time to maturity
3. S – CDS spread (note that it's not a fair spread)
4. M – frequency of premium payments
5. R – recovery rate
6. Discount factors (based on risk-free rates) term structure
7. Survival probabilities (based on hazard rates) term structure

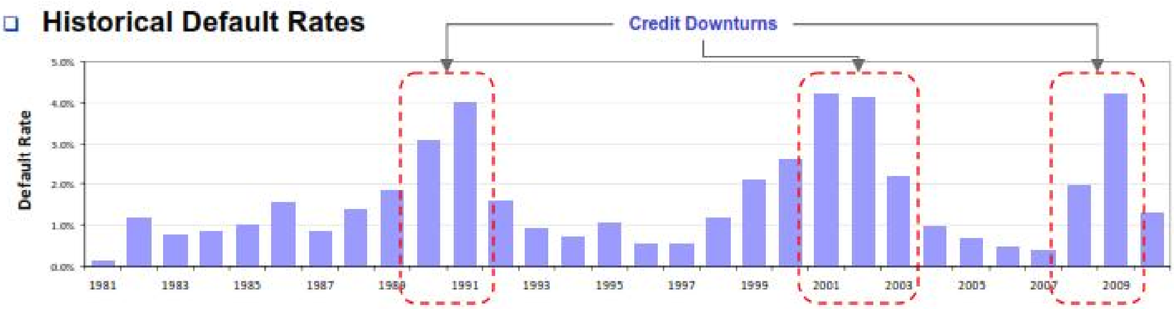
The lists are the same except for a few points. CDS spread "s" – does not affect CDS par spread computation and used only in computing extra values such as V_premium and V_deal. As we mentioned before, fair CDS spread should lead to V_premium – V_protection = 0. If S > S_par, value of deal will be positive, meaning that protection buyer overpays, if S < S_par, value of the deal is negative and protection buyer underpays. We flipped equation from V_deal = V_proteciton – V_premium to V_deal = V_premium – V_protection since it's more intuitive to look at the price from buyer's perspective.

There are two big assumptions the model makes: default distribution is known, and recovery rate is fixed to 40%.

1. There are several components that effects default rate of the company such as its credit rating, sector it belongs to, economy of the country, business model and financial statements (see graphs below). Therefore, default probabilities distribution has to be estimated by another model.
2. Recovery rate is treated as a constant in the pricer and fixed to 0.4 (40%). This is oversimplified assumption. Intuitively we can think that recovery rate is correlated to default probability (negatively correlated) and depends on many factors (for example industry characteristics, economy, etc). Therefore, its distribution has to be estimated by another model as well.

Default probability term structure of different credit ratings and historical default rates:





Processing: interpolation,

The CDS pricer interpolates survival probabilities and risk-free rates using piecewise constant forwards method. The method assumes that a forward rate between two adjacent points is constant. Resulting spot rates from constant forward method lack smooth and continuous derivative.

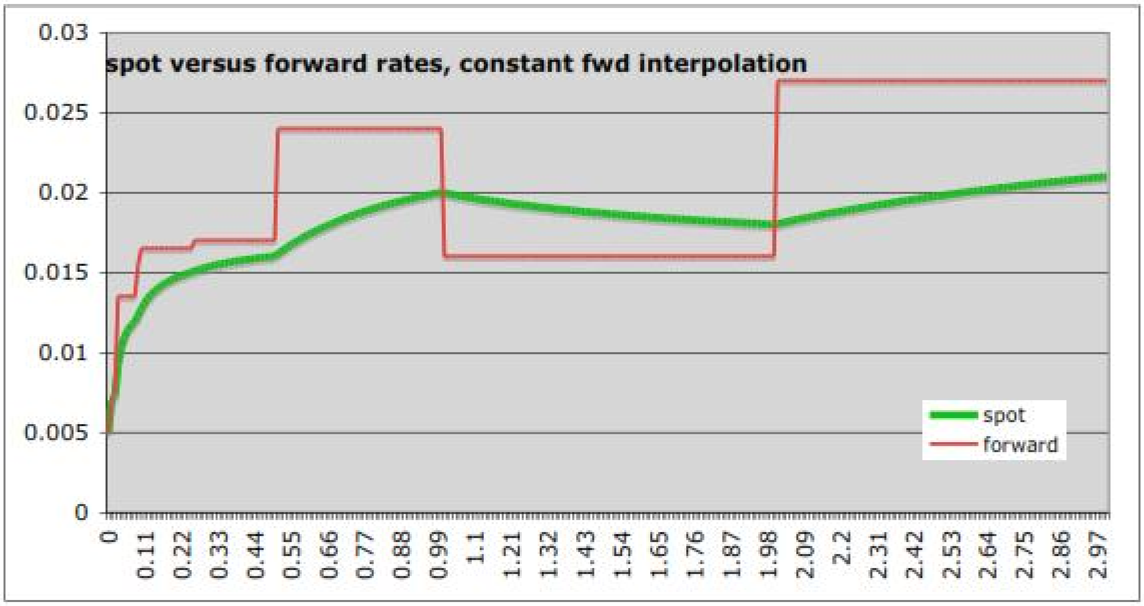
If we are interested in calculating discount factor between known factors at t_i and t_i+1, we can interpolate using formula below:

$$\ln P_t = \frac{\ln P_{t_{i+1}} - \ln P_{t_i}}{t_{i+1} - t_i}(t - t_i) + \ln P_{t_i}$$

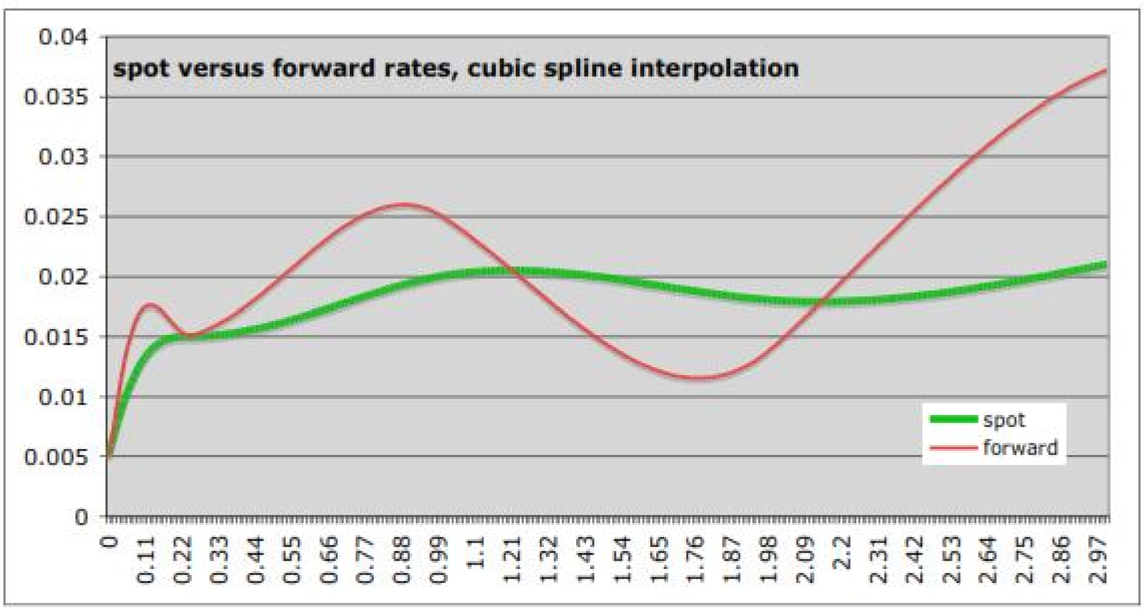
, where Pt_i, Pt_i+1 are known adjacent discount factors, and t_i, t_i+1 corresponding times. There are other methods to interpolate: quadratic, cubic and quartic. Quadratic will have zigzags, however, cubic and quartic are smooth and probably the method we would want to use.

Examples of the how term structure of spot rates resulted from interpolation against spot rate from the market are shown in the following plots:

Piecewise constant forward interpolation:



Cubic Spline Interpolation:



The pricing model we are validating is using analytical formula to calculate fair CDS spread. Numerical method such as Newton's or Secant methods can be used to approximate fair spread. We conducted a test to compare two methods. We used t-test to check if they are significantly different. We see that both methods are not significantly different.

Output

We think that the CDS pricer can be used either by protection buyer or seller. Bond holder needs to know how much of a premium he needs to pay to protection seller for a particular bond. Likewise, protection seller needs to know how much to charge for insurance. Our pricing model also computes V_deal based on given CDS spread. It can be beneficial to determine in which party's favor the deal is.

5. Sensitivity Analysis

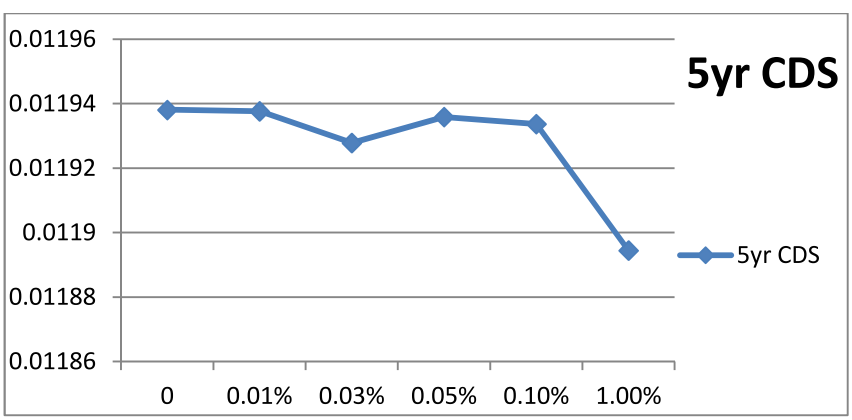
In order to perform the sensitivity analysis, we examine the possible implications to the model output if different input parameters undergo changes. We incrementally change one of the parameters while leaving the others constant.

We test and graph the model outputs while changing the following four input parameters:

1) Discount rate curve (incrementing risk-free rate up and down)
2) Default probability curve (incrementing default probabilities up and down)
3) Recovery rate
4) Maturity

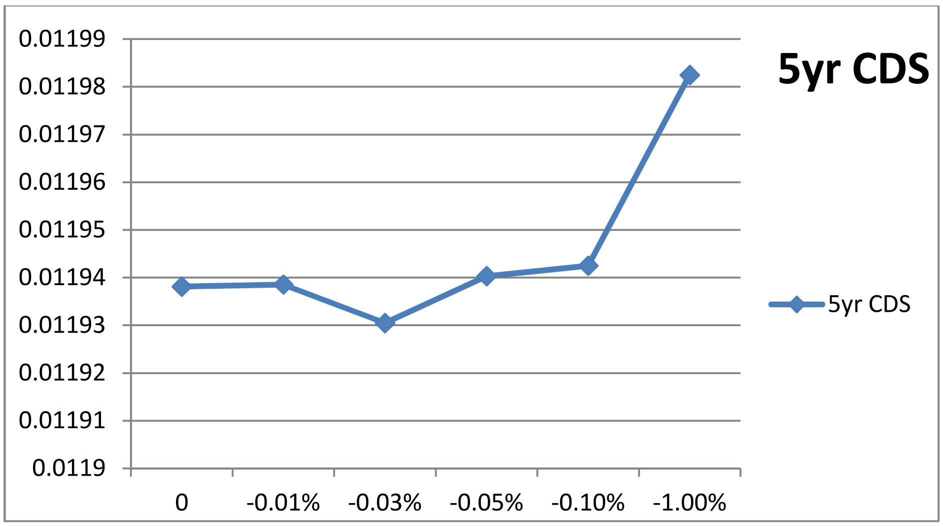
1a. Discount rate curve sensitivity (upward increments) – 5-yr CDS contract:
When applying positive shocks to our model, we observe the irregular pattern of changes to the CDS price (fair spread) – it falls gradually until 0.03% shock, where it dips, then it goes up and continues gradual decrease.

Percentage changes	Fair spread
0	0.011938112
0.01%	0.011937672
0.03%	0.011927868
0.05%	0.011935913
0.10%	0.011933716
1.00%	0.011894481



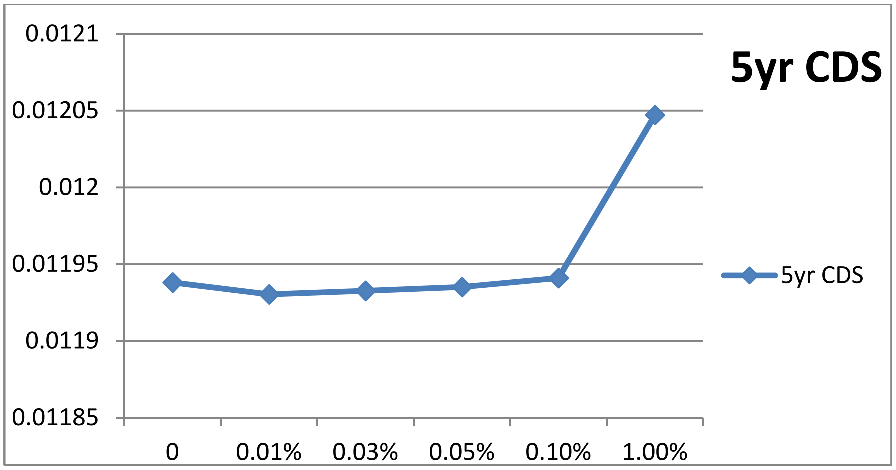
1b. Discount rate curve sensitivity (downward increments) – 5-yr CDS contract:
While applying negative shocks, we observe similar picture (nearly symmetric image) but in the other direction. There is a similar “inverted hump” (at 0.03% shock) and same gradual change.

Percentage changes	Fair spread
0	0.011938112
-0.01%	0.011938552
-0.03%	0.011930476
-0.05%	0.011940313
-0.10%	0.011942515
-1.00%	0.011982481



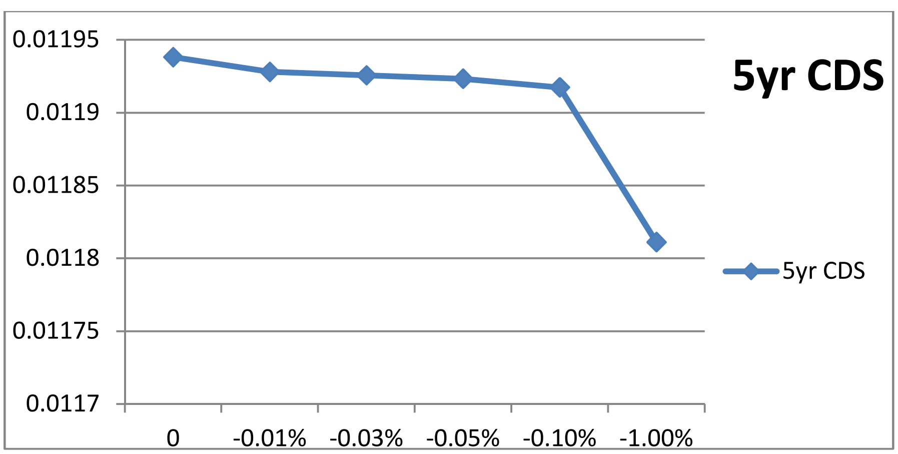
2a. Default probability curve sensitivity (upward increments) – 5-yr CDS contract:
While applying positive shocks, we observe the price going slightly down at 0.01% shock, then changing direction as we continue to apply increasing shocks.

Percentage changes	Fair spread
0	0.011938112
0.01%	0.011930352
0.03%	0.011932713
0.05%	0.011935074
0.10%	0.011940975
1.00%	0.012047196



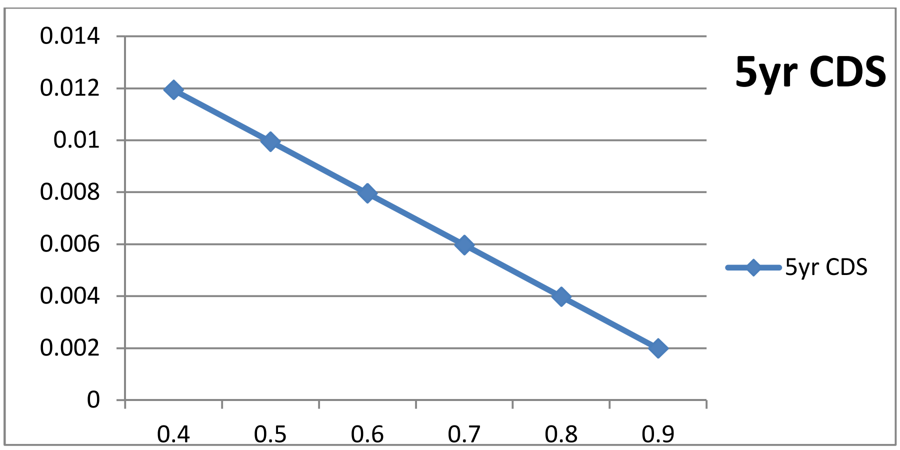
2b. Default probability curve sensitivity (downward increments) – 5-yr CDS contract:
With negative shocks, we observe a different behavior – there is monotonic decrease in price as we apply decreasing shocks. No changes of direction or sudden drops or spikes.

Percentage changes	Fair spread
0	0.011938112
-0.01%	0.011927991
-0.03%	0.011925631
-0.05%	0.01192327
-0.10%	0.011917368
-1.00%	0.011811123



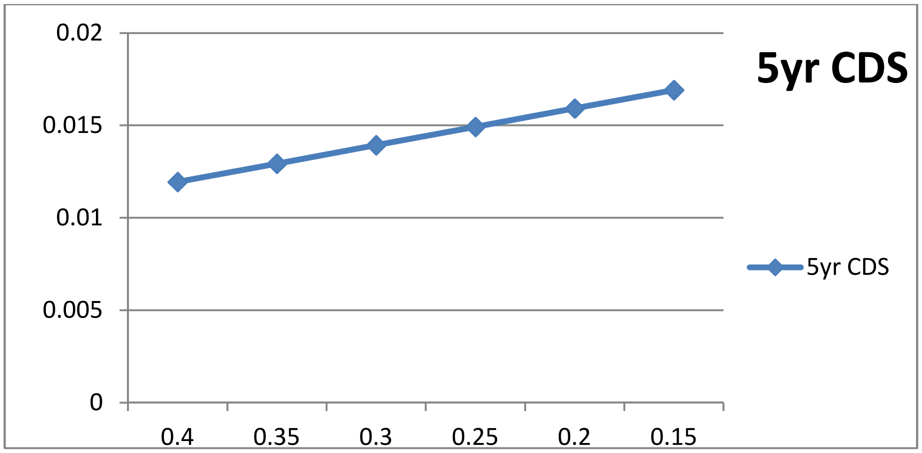
3a. Recovery Rates (upward increase) – 5yr CDS:
We observe linear decrease in CDS price, as our recovery rate increases. This can be explained by the model (as the seller can recover large proportion of defaulted debt, the lower the price (s)he would accept) and this is exactly what is expected.

Recovery Rates	Fair spread
0.4	0.011938112
0.5	0.009948427
0.6	0.007958741
0.7	0.005969056
0.8	0.003979371
0.9	0.001989685



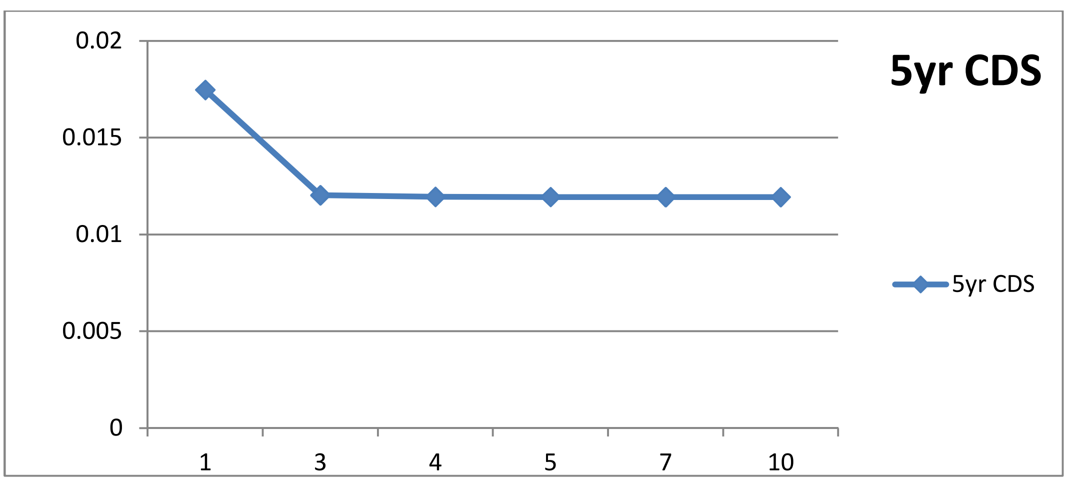
3b. Recovery Rates (downward decrease) – 5yr CDS:
Similar to previous case, there is an increase in price as the recovery rate decreases – seller needs to be compensated more as the percentage of defaulted debt recovery decreases.

Recovery Rates	Fair spread
0.4	0.011938112
0.35	0.012932955
0.3	0.013927797
0.25	0.01492264
0.2	0.015917483
0.15	0.016912325



4. Maturity (changes in both directions) – 5yr CDS:
With increasing maturity, and while keeping all other parameters constant, we see that the price decreases abruptly when maturity changes from 1 year to 3 years, and then settles down at approximately 1.19% as we keep increasing the maturity of the contract.

Maturity	Fair spread
1	0.017473065
3	0.012030434
4	0.011946391
5	0.011938112
7	0.01193722
10	0.011937212



6. Comparison with Benchmark Model

As required by the OCC 2011-12 guidelines, we compare CDS Pricing Model against a benchmark model. The benchmark chosen is the Bloomberg Terminal's Bloomberg Fair Value Model (B) (abbrev. CDSW). The Bloomberg CDS pricing model (Bloomberg Fair Value Model (B)) features several more parameters than our model, but for consistency, we will ignore those.

Bloomberg Fair Value CDS pricing model (Bloomberg 2015) values single name and index credit default swaps as a function of their schedule, currency, deal spread, notional, CDS curve and benchmark yield curve. Internally, the CDS pricer takes as an input a schedule, a default probability function and a discount function and produce he present value of the default leg (also known as protection leg) for a unit loss given default which is then scaled by the notional amount.

We chose IBM bond (Senior debt) as a Reference Entity in our benchmarking of the CDS Pricing model and its default probability and discount rates term structures provided by Bloomberg (USD ISDA Standard Curve). We set the trade date of the CDS to be 3/20/2015 in order to streamline the computations and avoid accruals calculations.

Below are the test results of the benchmark process. We produced the following output results using both our model and Bloomberg model – CDS prices for 7-yr, 5-yr, 4-yr, and 3-yr maturity derivatives. Default probability term structure was provided by Bloomberg with the exception that for the t=0 node we had to modify the value provided by Bloomberg (at t=0.5) in order to account for the missing data point. We took the average between value 0 and value at t=0.5 in Bloomberg Model's default probability curve.

The benchmark rates curve to compute the discount rates term structure is provided by Bloomberg as of 3/20/2015:

Benchmark Curve Rates

Benchmark S260 (USD ISDA Standard Curve - Mid)
Date 03/20/2015

Term	Rate	Term	Rate	Term	Rate
1 Month	0.1733	12 Year	2.1385		
2 Month	0.2188	15 Year	2.2230		
3 Month	0.2646	20 Year	2.3010		
6 Month	0.3939	25 Year	2.3320		
12 Month	0.6869	30 Year	2.3515		
2 Year	0.8750				
3 Year	1.2010				
4 Year	1.4405				
5 Year	1.6180				
6 Year	1.7495				
7 Year	1.8500				
8 Year	1.9295				
9 Year	1.9960				
10 Year	2.0495				

1) Apply Close

Summary test results:

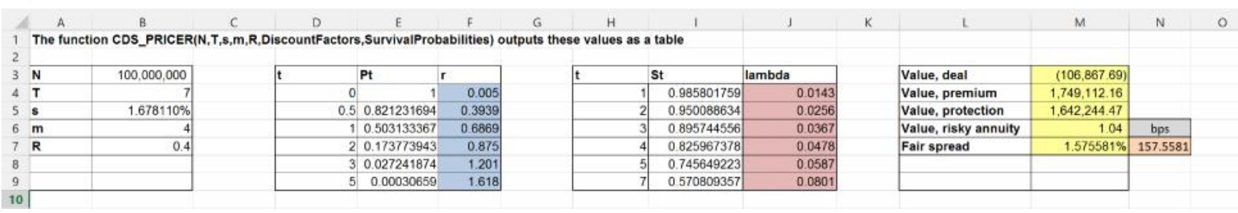
N (notional)	T (maturity in years)	m (coupon frequency)	R (recovery rate)	Meshulam	Bloomberg	Rel. Difference
100,000,000	7	4	0.4	157.5581	68.1131	-56.7695345%
100,000,000	5	4	0.4	119.3811	51.2626	-57.0597021%
100,000,000	4	4	0.4	90.29664	38.5739	-57.2809132%
100,000,000	3	4	0.4	62.79124	27.5655	-56.0997681%

We observe here a significant discrepancy of the tested model's outputs as compared with the benchmarked model (upwards of 50%). This error is consistent across several maturities, which indicates that the tested model requires calibration.

We suggest that after the necessary calibrations to the model are implemented, this discrepancy can be reduced to a certain degree.

Screenshots:

7-yr test:



5-yr test:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table													
2														
3	N	100,000,000		t	Pt	r		t	St	lambda		Value, deal	(506,818.56)	
4	T	5		0	1	0.005		1	0.989258111	0.0108		Value, premium	1,756,140.69	
5	s	1.678110%		0.5	0.821231694	0.3939		2	0.962135486	0.0193		Value, protection	1,249,322.13	
6	m	4		1	0.503133367	0.6869		3	0.91998308	0.0278		Value, risky annuity	1.05	bps
7	R	0.4		2	0.173773943	0.875		4	0.865195315	0.0362		Fair spread	1.193811%	119.3811
8				3	0.027241874	1.201		5	0.800515007	0.0445				
9				5	0.00030659	1.618								
10														

4-yr test:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table													
2														
3	N	100,000,000		t	Pt	r		t	St	lambda		Value, deal	(813,132.15)	
4	T	4		0	1	0.005		1	0.991932717	0.0081		Value, premium	1,760,351.12	
5	s	1.678110%		0.5	0.821231694	0.3939		2	0.971222201	0.0146		Value, protection	947,218.97	
6	m	4		1	0.503133367	0.6869		3	0.938943474	0.021		Value, risky annuity	1.05	bps
7	R	0.4		2	0.173773943	0.875		4	0.896551089	0.0273		Fair spread	0.902966%	90.29664
8				3	0.027241874	1.201								
9				5	0.00030659	1.618								
10														

3-yr test:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table													
2														
3	N	100,000,000		t	Pt	r		t	St	lambda		Value, deal	(1,097,058.00)	
4	T	3		0	1	0.005		0.5	0.99825153	0.0035		Value, premium	1,752,987.97	
5	s	1.678110%		0.5	0.821231694	0.3939		1	0.994216788	0.0058		Value, protection	655,929.97	
6	m	4		1	0.503133367	0.6869		2	0.979414828	0.0104		Value, risky annuity	1.04	bps
7	R	0.4		2	0.173773943	0.875		3	0.955997482	0.015		Fair spread	0.627912%	62.79124
8				3	0.027241874	1.201								
9				5	0.00030659	1.618								
10														

7. Stress Testing

In order to observe how the model performs under extreme input values, we implemented the stress test scenarios, which will provide insight into how well or how bad model performs when exposed to hypothetical unrealistic input values.

We decided to implement the following two hypothetical scenarios:

1. Extreme values of discount rates – large upward parallel shift of the curve and extreme curve steepening
2. Extreme values of default probabilities – large upward parallel shift of the curve and curve steepening

1a. Discount Rates – large upward parallel shift of the curve (+500 bps):

We observe a relatively small change (increase) in the resultant price of the CDS and the model does not go out of bounds.

The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table

N	100,000,000	t	Pt	r	Parallel shift (+500 bps)	t	St	lambda	Value, deal	(25,509.70)	
T	5	0	1	0.005	5.005	1	0.989258111	0.0108	Value, premium	145,175.54	
s	1.678110%	0.5	0.067410802	0.3939	5.3939	2	0.962135486	0.0193	Value, protection	119,665.84	
m	4	1	0.003390086	0.6869	5.6869	3	0.91998308	0.0278	Value, risky annuity	0.09	bps
R	0.4	2	7.88932E-06	0.875	5.875	4	0.865195315	0.0362	Fair spread	1.383239%	138.3239
		3	8.33335E-09	1.201	6.201	5	0.800515007	0.0445			
		5	4.2579E-15	1.618	6.618						

1b. Discount Rates – extreme curve steepening (+250 to +1000 bps long-end of the curve):

The extreme curve steepening causes the output price to decrease significantly. This result also stays within bounds and model does not break.

The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table

N	100,000,000	t	Pt	r	Extreme steepening (+250 to +1000 bps)	t	St	lambda	Value, deal	(433,937.86)	
T	5	0	1	0.005	0.005	1	0.989258111	0.0108	Value, premium	819,126.27	
s	1.678110%	0.5	0.821231694	0.3939	0.3939	2	0.962135486	0.0193	Value, protection	385,188.41	
m	4	1	0.041299702	0.6869	3.1869	3	0.91998308	0.0278	Value, risky annuity	0.49	bps
R	0.4	2	7.88932E-06	0.875	5.875	4	0.865195315	0.0362	Fair spread	0.789120%	78.91197
		3	4.60905E-12	1.201	8.701	5	0.800515007	0.0445			
		5	5.91335E-26	1.618	11.618						

2a. Default Probabilities – large upward parallel shift of the curve (+500 bps):

We observe an extremely large increase in resultant price and this suggests that model produces unrealistic output.

The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table

N	100,000,000	t	Pt	r	t	St	lambda	Parallel shift (+0.5)	Value, deal	23,581,096.61	
T	5	0	1	0.005	1	0.600015374	0.0108	0.5108	Value, premium	1,231,550.94	
s	1.678110%	0.5	0.821231694	0.3939	2	0.353949865	0.0193	0.5193	Value, protection	24,812,647.55	
m	4	1	0.503133367	0.6869	3	0.205275972	0.0278	0.5278	Value, risky annuity	0.73	bps
R	0.4	2	0.173773943	0.875	4	0.117091453	0.0362	0.5362	Fair spread	33.809694%	3380.969
		3	0.027241874	1.201	5	0.065710273	0.0445	0.5445			
		5	0.00030659	1.618							

2b. Default Probabilities – curve steepening:

The curve steepening scenario results in large increase as well, but it is not as extreme as with the parallel shift scenario.

The function CDS_PRICER(N,T,s,m,R,DiscountFactors,SurvivalProbabilities) outputs these values as a table

N	100,000,000	t	Pt	r	t	St	lambda	Curve steepening	Value, deal	7,479,336.86	
T	5	0	1	0.005	1	0.941011423	0.0108	0.0608	Value, premium	1,629,040.37	
s	1.678110%	0.5	0.821231694	0.3939	2	0.787729911	0.0193	0.1193	Value, protection	9,108,377.23	
m	4	1	0.503133367	0.6869	3	0.205275972	0.0278	0.5278	Value, risky annuity	0.87	bps
R	0.4	2	0.173773943	0.875	4	0.043075539	0.0362	0.7862	Fair spread	9.302740%	938.274
		3	0.027241874	1.201	5	0.008892918	0.0445	0.9445			
		5	0.00030659	1.618							

Overall, in the above limited stress testing we observe how the model performs under extreme conditions (parameter inputs) and whether it "breaks." Under discount rates stress scenario, the model behaves as expected producing relatively small changes in the output results, whereas under default probabilities stress we see the model producing unrealistic result figures.

8. References

1. Robert Spruill, Lecture notes from Baruch MFE class: Intro to Pricing Financial Instruments.
2. John Hull, "Options, futures, and other derivatives", Sixth edition.
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4. Fleasker et al., The Bloomberg CDS Model, Bloomberg L.P. (April 21, 2015).