MTH9845 Risk Management

Model Validation Report

with

¡Need-to-chose Pricing Model¿ Continue-the-Title

Submitted by

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Chapter 1

Executive Summary

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Chapter 2

Overview

2.1 Background and Recent Research

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2.1.2 Literature Survey

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2.2 Motivation

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Chapter 3

Technical Specification

The Vanna-Volga pricing method is a technique used to price first generation exotic options in foreign exchange market. This method derives from the trader's idea that the difference between market price and Black-Scholes price is the volatility smile impact, which could be adjusted with costs incurred by hedging three main risks associated to the volatility of the option: the Vega, the Vanna and the Volga.

3.1 Greeks

The foreign exchange spot process is considered to follow Geometric Brownian motion (GBM). Thus we find the results we are able to obtain in equity markets hold in the case of FX options as well.
Then the Black-Scholes value of call option is:

$$V_{call} = Se^{r_f T}N(d_1) - Ke^{r_d T}N(d_2) \tag{3.1}$$

$$V_{put} = Ke^{r_d T}N(-d_2) - Se^{r_f T}N(-d_1) \tag{3.2}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where r_d and r_f are the domestic and foreign risk free rate, respectively. T is the time to maturity. N denotes as the cumulative density function of standard normal distribution. Below we will discuss the Greeks in the context of Black-Scholes model.

3.1.1 Vega

Vega ν is the first derivative of the option value with respect to the volatility σ.
By taking the derivative we have

$$\begin{aligned}\nu_C = \frac{\partial C}{\partial \sigma} &= Se^{r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} \\ &= Se^{r_f T}\Phi(d_1)\frac{d_1 - d_2}{\sigma} \\ &= Se^{r_f T}\Phi(d_1)\sqrt{T}\end{aligned} \tag{3.3}$$

$$\begin{aligned}\nu_P = \frac{\partial P}{\partial \sigma} &= -Ke^{r_d T}\Phi(d_2)\frac{\partial d_2}{\partial \sigma} + Se^{r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= \nu_C\end{aligned} \tag{3.4}$$

3.1.2 Vanna

Vanna is the second order derivative of the option value, once to the volatility σ and once to the initial spot price.
By taking the derivative we have:

$$\begin{aligned}Vanna_C = \frac{\partial^2 C}{\partial S \partial \sigma} = \frac{\partial \Delta_C}{\partial \sigma} &= e^{r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= e^{r_f T}\Phi(d_1)\left(\sqrt{T} - \frac{d_1}{\sigma}\right) \\ &= -\frac{d_2}{S\sigma\sqrt{T}}\nu_C\end{aligned} \tag{3.5}$$

$$\begin{aligned}Vanna_P = \frac{\partial^2 P}{\partial S \partial \sigma} = \frac{\partial \Delta_P}{\partial \sigma} &= e^{r_f T}\Phi(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= Vanna_C\end{aligned} \tag{3.6}$$

3.1.3 Volga

Volga is the second order derivative of the option value with respect to the volatility σ twice.

By taking the derivative we have:

$$\begin{aligned} Volga_C &= \frac{\partial^2 C}{\partial^2 \sigma} = \frac{\partial \nu_C}{\partial \sigma} = e^{r_f T} S\sqrt{T} \frac{\partial \Phi(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} \\ &= e^{r_f T} S\sqrt{T} \Phi(d_1) \frac{d_1 d_2}{\sigma} \\ &= \frac{\nu_C d_1 d_2}{\sigma} \end{aligned} \tag{3.7}$$

$$\begin{aligned} Volga_P &= \frac{\partial^2 P}{\partial^2 \sigma} = \frac{\partial \nu_P}{\partial \sigma} = \frac{\partial \nu_C}{\partial \sigma} \\ &= Volga_C \end{aligned} \tag{3.8}$$

3.2 Model Framework and Equations

3.2.1 Model Assumption and Justification

3.2.2 The Equation

The standard formulation of the Vanna-Volga pricing method is given by:

$$X^{VV} = X^{BS} + \frac{Vanna(X)}{Vanna(RR)} RR_{cost} + \frac{Volga(X)}{Volga(BF)} BF_{cost} \tag{3.9}$$

where

$$\begin{aligned} RR_{cost} = &\left[Call\left(K_C, \sigma\left(K_C\right)\right) - Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &- \left[Call\left(K_C, \sigma_0\right) - Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.10}$$

$$\begin{aligned} BF_{cost} = &\frac{1}{2}\left[Call\left(K_C, \sigma\left(K_C\right)\right) + Put\left(K_p, \sigma\left(K_p\right)\right)\right] \\ &- \frac{1}{2}\left[Call\left(K_C, \sigma_0\right) + Put\left(K_p, \sigma_0\right)\right] \end{aligned} \tag{3.11}$$

and X^{BS} denotes the Black-Scholes price of the vanilla option, Vanna and Volga are calculated with ATM volatility.

Chapter 4

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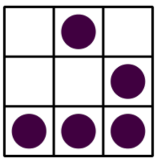

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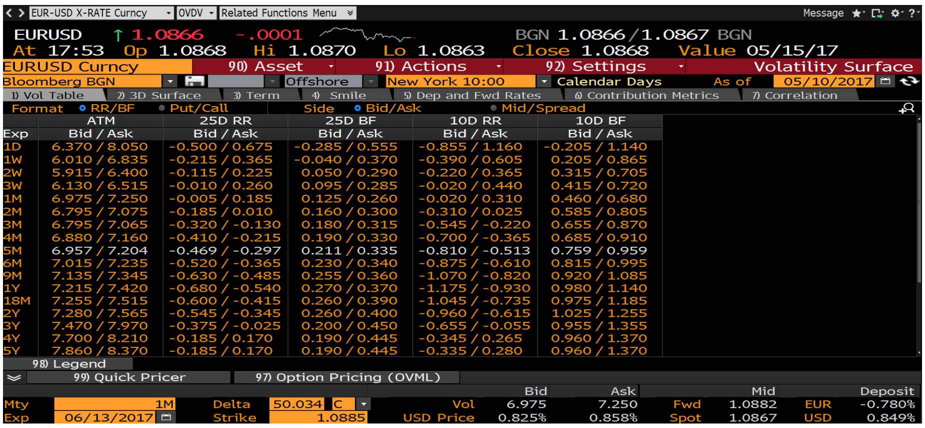
Chapter 5

Test of Vanna-Volga Pricing

5.1 Data Source

5.1.1 Volatility Matrix

Volatity matrix data in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR) with three FX derivatives for Vanna-Volga models was sourced from Bloomberg. An example of volatility matrix data of EUR/USD observed on May 10, 2017 with 1M maturity was showing below. More volatility matrix data of other derivatives had been put in *Appendix 1*.



	ATM	25D RR	25D BF	10D RR	10D BF
Exp	Bid / Ask	Bid / Ask	Bid / Ask	Bid / Ask	Bid / Ask
1D	6.370 / 8.050	-0.500 / 0.675	-0.285 / 0.555	-0.855 / 1.160	-0.205 / 1.140
1W	6.010 / 6.835	-0.215 / 0.365	-0.040 / 0.370	-0.390 / 0.605	0.205 / 0.865
2W	5.915 / 6.400	-0.115 / 0.225	0.050 / 0.290	-0.220 / 0.365	0.315 / 0.705
3W	6.130 / 6.515	-0.010 / 0.260	0.095 / 0.285	-0.020 / 0.440	0.415 / 0.720
1M	6.975 / 7.250	-0.005 / 0.185	0.125 / 0.260	-0.020 / 0.310	0.460 / 0.680
2M	6.795 / 7.075	-0.185 / 0.010	0.160 / 0.300	-0.310 / 0.025	0.585 / 0.805
3M	6.795 / 7.065	-0.320 / -0.130	0.180 / 0.315	-0.545 / -0.220	0.655 / 0.870
4M	6.880 / 7.160	-0.410 / -0.215	0.190 / 0.330	-0.700 / -0.365	0.685 / 0.910
5M	6.957 / 7.204	-0.469 / -0.297	0.211 / 0.335	-0.810 / -0.513	0.759 / 0.959
6M	7.015 / 7.235	-0.520 / -0.365	0.230 / 0.340	-0.875 / -0.610	0.815 / 0.995
9M	7.135 / 7.345	-0.630 / -0.485	0.255 / 0.360	-1.070 / -0.820	0.920 / 1.085
1Y	7.215 / 7.420	-0.680 / -0.540	0.270 / 0.370	-1.175 / -0.930	0.980 / 1.140
18M	7.255 / 7.515	-0.600 / -0.415	0.260 / 0.390	-1.045 / -0.735	0.975 / 1.185
2Y	7.280 / 7.565	-0.545 / -0.345	0.260 / 0.400	-0.960 / -0.615	1.025 / 1.255
3Y	7.470 / 7.970	-0.375 / -0.025	0.200 / 0.450	-0.655 / -0.055	0.955 / 1.355
4Y	7.700 / 8.210	-0.185 / 0.170	0.190 / 0.445	-0.345 / 0.265	0.960 / 1.370
5Y	7.860 / 8.370	-0.185 / 0.170	0.190 / 0.445	-0.335 / 0.280	0.960 / 1.370

Figure 5.1: Volatility matrix data in the bid/ask format in terms of ATM, 10Δ, and 25Δ butterflies (BF) and risk reversals (RR), observed on May 10, 2017. Source: *Bloomberg*

In order to verify the price calculated by using Vanna-Volga model, two benchmark sources (Bloomberg pricing model and *investing.com* with price provided by *Sentry Derivatives*) for all three FX derivatives had been put in *Appendix 1.* Since Vanna-Volga is an analytically derived correction to Black-Scholes model, the price calucluated by Black-Scholes model also had been included for analysis.

The interest rates used for Vanna-Volga model and Black-Scholes model was obtained from *www.tradingeconomics.com* and listed in the following table.

Table 5.1: FX interest rates observed on May 10, 2017

Symbol	USD	EUR	GBP	JPY
Rates	1.00%	0.00%	0.00%	-0.10%

5.1.2 Model Implementation

As the volatility matrix obtianed from Bloomberg is in the bid/ask format, the averaged mid volatility was used for Black-Scholes model and Vanna-Volga model. The mid volatility matrix data of three FX derivatives were list below. And we consider that live exchange rate as the initial price of FX options S_0.

Table 5.2: Mid volatiloty matrix

FX derivatives	ATM	25D RR	25D BF	10D RR	10D BF	S_0
EUR/USD	7.1125	0.09	0.1925	0.145	0.57	1.0866
GBP/USD	6.7125	-0.28	0.2275	-0.4825	0.635	1.2933
USD/JPY	8.225	-0.4125	0.2225	-0.775	0.615	114.32

With the conventions and definitions specified in **Technical Specification**, the implementation of Black-Scholes model and Vanna-Volga model of FX derivatives had been coded in jupyter notebook with Python 3.5 in *Appendix 2.*

5.2 Testing Results

USD/GBP, USD/JYP, USD/EUR, historical interest rate, historical exchange rate, bloomberg 25delta Bloomberg: OVDL EUR/USD price form: %EUR is price calculated (EUR Pips in *intesting.com*) devide by S_0: the exchange rate, i.e., $\%EUR = \frac{EURPips}{S_0}$

Chapter 6

Model Strengths and Weaknesses

6.1 Strengths

As we all known that Black-Scholes model is most often used to price vanilla options. However, the parameters used in Black-Scholes model are far from market quotations. The main reason is the unrealistic assumption that the volatility remain constant throught the lifetime of the vanilla options. Besides, the volatility surfaces of FX derivatives tend to be smile shaped or skewed. Thus, Black-Scholes model is insufficient in FX market.

There are models, such as Heston model and local volatility model, could capture and well replicate the smile shaped or skewed volatility surface of FX derivatives. However, none of them is easy to implement and require delicate calibration. Therefore, compare to other models used for FX derivatives, Vanna-Volga model has following strengths:

- Vanna-Volga is easy to implement, comparing to other models
- Vanna-Volga is simple and no or few calibration is needed
- Vanna-Volga is very efficient in computation, i.e., the calculation speed is significantly better than Heston model or local volatility model
- The instruments used for constructing the Vanna-volga model are very liquid in FX market. Typically, people are using straddle, risk reversal, and butterfly to construct Vanna-Volga framework

- Vanna-Volga is an analytically derived correction by capturing the greeks of vanna and volga to Black-Scholes model, i.e., by using vega, vanna and volga of the options. Therefore, it is easy to understand intiuitively.

6.2 Weaknesses

Even though Vanna-Volga model also known as *trader's rule of thmb* and has some features listed above, it dose have some drawbacks or conditions need to be understood before using it. Typical weaknesses of Vanna-Volga model are following:

- Vanna-Volga is precise when the maturity of options is up to 1 year, since the model assumes constant interest rates which does not lead to significantly mispricing for short maturity options in FX market.
- The application of Vanna-Volga model is limited to plain vanilla options and first-generation exotic options, such as barrier options, since it cannot fully replicate the volatility surface. However, many of the options in FX market is vanilla or first-generation exotic options.
- Vanna-Volga model perform well when the volatility surface is standard (such as smile shaped, typical skewed) of FX derivatives.

Chapter 7

Conclusion

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Appendix A

A.1 Data Sources

A.2 Definitions

25 and 10 Δ-Risk-reversal (RR) volatility:

$$\sigma_{RR25} = \sigma_{25\Delta C} - \sigma_{25\Delta P} \tag{A.1}$$
$$\sigma_{RR10} = \sigma_{10\Delta C} - \sigma_{10\Delta P} \tag{A.2}$$

25 and 10 Δ-Butterfly (BF) volatility:

$$\sigma_{BF25} = \frac{1}{2}\left[\sigma_{25\Delta C} + \sigma_{25\Delta P}\right] - \sigma_{ATM} \tag{A.3}$$
$$\sigma_{BF10} = \frac{1}{2}\left[\sigma_{10\Delta C} + \sigma_{10\Delta P}\right] - \sigma_{ATM} \tag{A.4}$$

These yields to the following equations:

$$\sigma_{25\Delta C} = \sigma_{ATM} + \sigma_{BF25} + \frac{1}{2}\sigma_{RR25} \tag{A.5}$$
$$\sigma_{25\Delta P} = \sigma_{ATM} + \sigma_{BF25} - \frac{1}{2}\sigma_{RR25} \tag{A.6}$$
$$\sigma_{10\Delta C} = \sigma_{ATM} + \sigma_{BF10} + \frac{1}{2}\sigma_{RR10} \tag{A.7}$$
$$\sigma_{10\Delta P} = \sigma_{ATM} + \sigma_{BF10} - \frac{1}{2}\sigma_{RR10} \tag{A.8}$$

Strike retrived from deltas can be calculated through following equation:

$$K = S_0 e^{(r_d - r_f)T - \phi\sigma\sqrt{T}N^{-1}(\phi\Delta) + \frac{1}{2}\sigma^2 T} \tag{A.9}$$

where $\phi = 1$ for call, and $\phi = -1$ for put.

References

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[2] ¡Name of the reference here¿, <urlhere>