Advanced Risk Management

Model Validation of Characteristics of Seasonal Fluctuations in Wheat Prices

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1 Executive Summary

This paper examines the validation of the characteristics of seasonal fluctuations in wheat prices. As we recalculated the result using the same data set by the model, we find that the results in the paper of the model is correct based on its assumption. However, one must makes a judgement call to decide the accuracy of the seasonal pattern estimated.

We reproduced the figures in the paper using the same data set with two degree of fourier expansion. We get a similar result as the paper's result. Then we improve the accuracy by using ten degree of fourier expansion. We find that the patterns are similar as before but more details in the patterns are captured.

A stress test shows that there are unusual market shocks could cause inaccuracy of the regular seasonal pattern. Therefore, a modification of separating the regular seasonal fluctuation and conditional seasonal fluctuation will help to get a more accurate regular seasonal pattern.

The model is easy to implement and understand. But users should first examine and eliminate the unusual shock from the data set to get a more close to truth regular seasonal fluctuation pattern.

2 Overview

Technical specification is summarized in Section 3. The analysis of the model assumption is listed in Section 4. Test of model pricing for season fluctuation is in Section 5. Since before the linear regression method is implemented, we need to decide how large will k be. The choice of k will have an impact on the result and we will analysis the impact in Section 6. Meanwhile, the possible modification of the model is examined in Section 6 as well. We conclude the validation with the model strengths and weaknesses in Section 7.

All the calculations are done in EXCEL and the most relevant techniques we are using is Fourier Transform and Linear Regression.

3 Technical Specification

Since wheat is harvested once a year and consumed all year round, there is a seasonal component which represents a regular and predictable variable in the wheat price.

$$p_0(t) = s(t) + p(t)$$

where $p_0(t)$ is the observed price at time t, $s(t)$ is the seasonal component at time t and $p(t)$ is the deseasonalized component at time t.

The seasonal component is periodic so we can use fourier transform to write it as:

$$s(t) = \sum_{k=1}^{\infty}[a_k \sin(k \cdot 360 \cdot t) + b_k \cos(k \cdot 360 \cdot t)]$$

where $a_k = c_k \cdot \cos(k \cdot 360 \cdot \tau_k)$, $b_k = c_k \cdot \sin(k \cdot 360 \cdot \tau_k)$, c_k is the amplitude of the k^{th} sine function and τ_k is the phase shift of that function expressed in units of time.

If we have $N + 1$ observations of price at times $t_0, t_1, ..., t_N$, we will have:

$$p_0(t_i) - p_0(t_{i-1}) = s(t_i) - s(t_{i-1}) + p(t_i) - p(t_{i-1})$$

where $i = 1, 2, ..., N$, i.e.

$$\begin{aligned} p_0(t_i) - p_0(t_{i-1}) = &\sum_{k=1}^{\infty} a_k[\sin(k \cdot 360 \cdot t_i) - \sin(k \cdot 360 \cdot t_{i-1})] \\ &+ \sum_{k=1}^{\infty} b_k[\cos(k \cdot 360 \cdot t_i) + b_k \cos(k \cdot 360 \cdot t_{i-1})] \\ &+ p(t_i) - p(t_{i-1}) \end{aligned} \tag{1}$$

where $i = 1, 2, ..., N$.

Then, implementing linear regression method in EXCEL would get the coefficients a_k and b_k with some finite k chosen beforehand.

4 Assumptions Review

The assumptions of the seasonal fluctuation model are the following:

- The observed price is a combination of a seasonal component and a deseasonalized component.
- The deseasonalized component is a stochastic component following a Gaussian random walk with zero drift.
- The seasonal component is regular and it is non-stochastic. It is a continuous and periodic function of time with a periodicity of one year and with an average value of zero over the course of a year:

$$s(t + \Delta t) = s(t)$$

$$\int_t^{t+\Delta t} s(\tau)\, d\tau = 0$$

where $\Delta t = 1$ in our case.

The first assumption about an combination of seasonal and deseasonalized components is reasonable. Since seasonal fluctuation is most likely be a constant impact on the wheat price so it would be uncorrelated with the deseasonalized component.

Since deseasonalized component is mostly driven by elements such as weather or geography which are changing variables through time. The assumption of it being stochastic is reasonable. The deseasonalized component is out of the range of our model validation here, so we would not have any comment on the distribution it follows here. And as shown in the technical specification part, the deseasonalized component has been detrended in the evaluation, it would not have impact on our seasonal fluctuation model.

As what we have talked, the seasonal fluctuation is most likely to be a constant impact on the price, the assumption about it being non-stochastic is reasonable. Since wheat is harvested once a year and consumed all year round so the assumptions of periodicity being one year and average being zero are reasonable.

However, for some years, there are not only regular seasonal fluctuations but as well conditioned seasonal fluctuations, namely unusual market shocks. The conditional component will have periodicity not has to be one year, usually longer than one year, so that it does not fit into the assumption of the regular seasonal component. This conditional seasonal component will offset the regular seasonal impact in the price. In this cases, this model is not accurate. Thus, we need to eliminate the unusual market shock before calculating the regular seasonal pattern. And we will talked about some possible modification to consider the unusual shocks in Section 6.

5 Testing of pricing

We try to use the same data set as the paper used to recover the seasonal pattern of the five kinds of wheat by applying the theory talked about in the Technical Specification section. The data set is from Jan 5, 1978 to Dec 26, 1991. All the exact calculation is preformed in the excel.

- We first plot the raw data set to check the data.

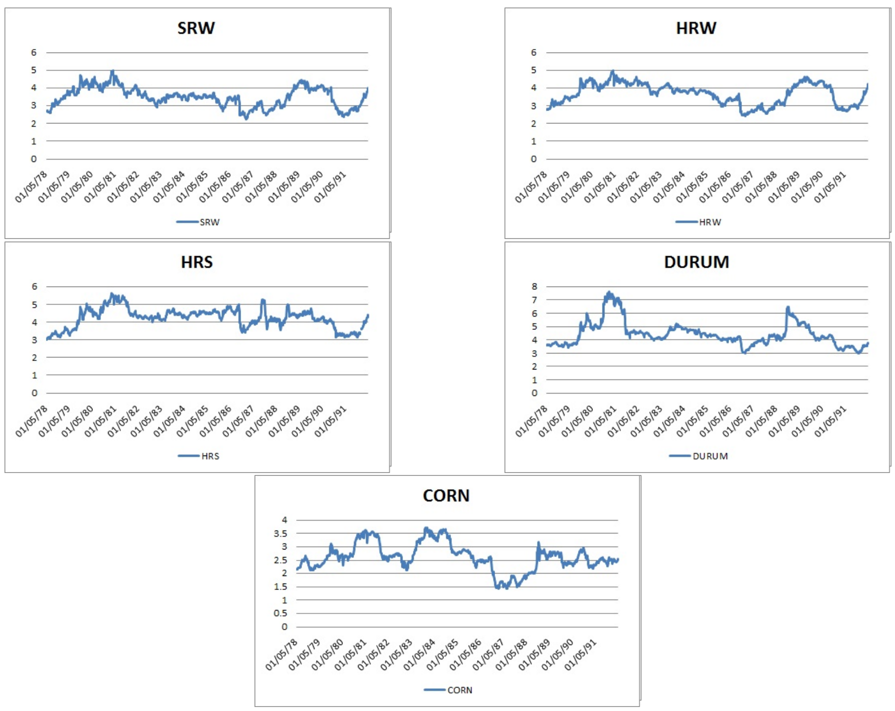

Figure 1: Raw data of each wheat

We see that the raw data shows the same as given in the paper.

- We first need to eliminate the unusual market shock in the data set to get a accurate regular seasonal pattern. Thus, we would do the regression for the Durum wheat with 1980 and 1988 data excluded. This is because as we can see from the above figure of the raw data of Durum, there are unusual impact in 1980 and 1988 which we will examine in Section 6.1.

- We will use linear regression on the difference of the *sin* functions to get the coefficients a_k and b_k, where $k = 1, 2$. For different wheat, we get the following regressors:

Regressor	SRW	HRW	HRS	DURUM	CORN
intercept	0	0	0	0	0
a_1	0.055404504	0.066217125	0.069362618	-0.014130994	0.074984558
b_1	0.096455359	0.080056881	-0.037870778	-0.006183283	-0.120389664
a_2	-0.024523617	-0.056512019	-0.098620166	-0.034216236	-0.01403909
b_2	-0.007587829	-0.012191389	0.025173787	-0.015018862	0.026281889

We notice a slightly off of the parameters as given in the exel of the model calculation because of a little error occurrence for date 04/27/89 **and** 05/04/89 **in the original data calculation. But as we can see the figures are roughly the same.**

- Then, we apply the regressors on the *sin* functions of the on year length to get the corresponding seasonal change of the wheat prices. We get the following figure of the seasonal change of the wheats:

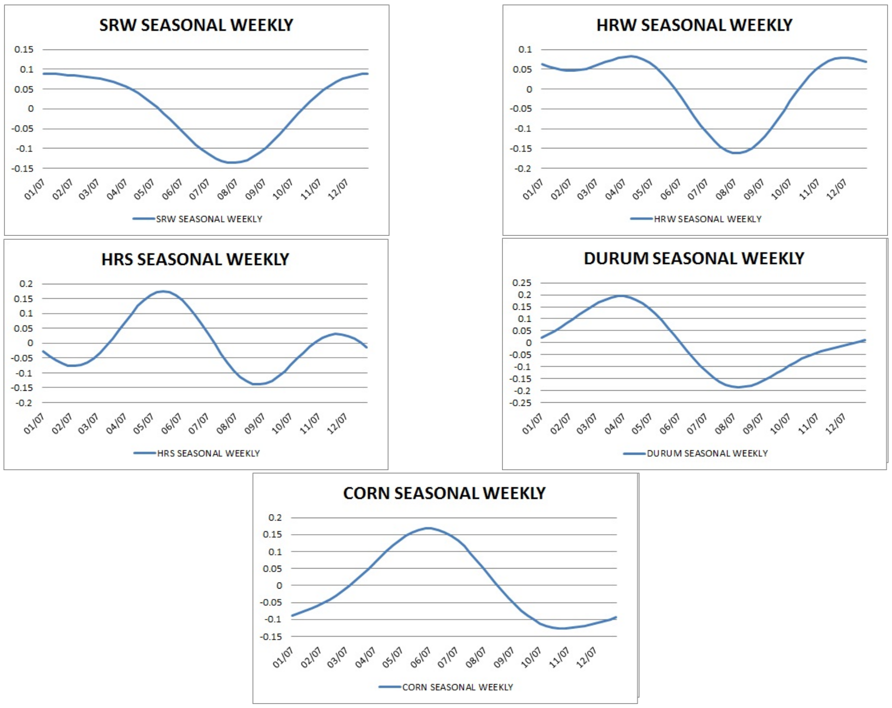

Figure 2: Seasonal weekly changes for each wheat with $k = 2$

From the figures shown above, we see the model is validate for this set of data since the figures are roughly the same as the ones in the model paper. The slightly off is due to the calculation error as talked above.

6 Sensitivity Analysis

6.1 Impact of number of Fourier expansion terms in numerical calculation

Here, we try to test the sensitivity of the seasonal fluctuation against the number of fourier expansion used in the linear regression estimation.

We try to add more effective a_k and b_k into the regression to get more accurate data. We extend $k = 1, 2, ..., 10$ and repeat the same steps as above to get the following figures:

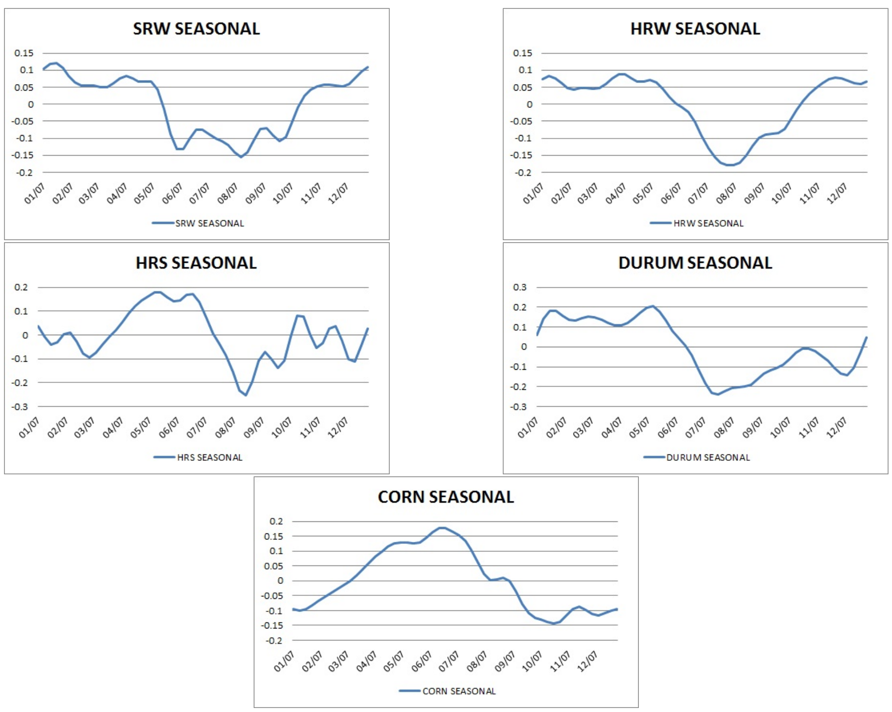

Figure 3: Seasonal weekly changes for each wheat with $k = 10$

We can see from the figures that the shapes of the curves are the same pattern but they are more accurate now that they capture more details with more terms in the fourier transform.

Users can choose the number of fourier expansion base on their need. If only a rough pattern is needed, two expansion is necessary since we can see from the comparison that when $k = 2$, the evaluation does exhibit the pattern. However, if you want to hedge out the seasonal fluctuation of the wheat price, you might want larger k to have a more accurate hedging position.

6.2 Possible Modification of Model: Unconditional seasonal fluctuation

The model assumes that the seasonal fluctuation is single regular term as we calculated above. However, sometimes unusual market shocks will happen which will cause the linear regression for the single regular seasonal component inaccurate. The unusual market shock may happen when there is bad weather such as floods, drought which has a characteristic of being a cycle.

We illustrate this point of view by recalculate the regular seasonal fluctuation of Durum using the model with whole set of data(from Jan 5, 1978 to Dec 26, 1991) instead of excluding data from 1980 and 1988. Now, considering the unusual impact of the market shock, we get the following seasonal pattern:

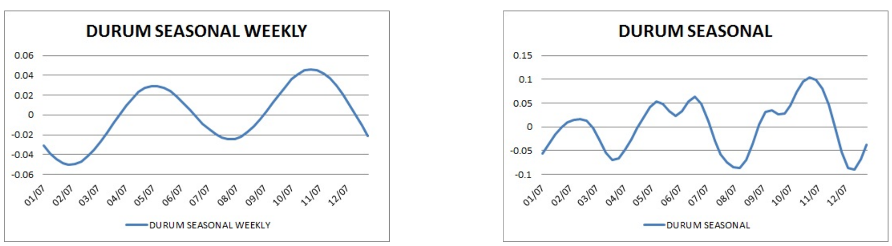

Figure 4: The seasonal pattern of Durum considering unusual market shock. (Left)The seasonal pattern with $k = 2$. (Right)The seasonal pattern with $k = 10$.

Comparing with the seasonal pattern gained in Section 5, we see there is a large difference between them. This is because of the unusual market shock. Therefore, we need some improvement of the model to get more accurate regular seasonal fluctuation.

One possible modification is to separate the one regular seasonal component into two:

- Normal years component
- Component for years in which yields fell significantly below trend with periodicity not has to be one.

This requires more close examination and modeling so we would not expand the talk here.

7 Model Strengths and Weaknesses

As seen in our discussion above, the most strength of the model is easy to calculate. As what we do, implementing the linear regression in EXCEL is fairly easy and straightforward. And the framework is easy to replicate if needed to analyze the seasonal fluctuation of prices of other kinds of wheat or even other commodities.

While the adaption of the model is easy, there are certain limits when we come to numerical calculation. Since we cannot have infinite number of fourier expansion term in the calculation, we have to make assumptions about the number of fourier expansion. This is based on the needs of the accuracy of the model result.

On the other hand, the assumption of the seasonal component could be improved so that we can separate out the conditional seasonal pattern from the unconditional seasonal pattern.

In conclusion, the model is easy to understand and calculate but it needs judgement call to decide the accuracy of the result. Moreover, there could be improvement to separate conditional and unconditional seasonal fluctuation for the wheat price.

8 Reference

- Kenneth C. Abbott,1993,'Characteristics of Fluctuations in Wheat Prices' *Topics in Money and Securities Markets*
- Kenneth D. Garbade,1991,'Characteristics of Fluctuations in the Prices of Crude Oil and Its Refined Fuels', *Topics in Money and Securities Markets*