ST758, Homework 7

Due Dec 6, 2013

1. Read carefully the following material about how to properly design monte carlo studies for comparing statistical methods. These are available on course website.

   - WH Swallow and JF Monahan (1984), Monte Carlo comparison of ANOVA, MIVQUE, REML, and ML estimators of variance components, *Technometrics*, 26(1):47–57
   - JF Monahan, A guide for simulation studies in statistics, *unpublished note*.
   - M Davidian, Simulation studies in statistics, *slides for ST810A Spring 2005*.

   Your report will be evaluated based on these guidelines.

2. The paper by Swallow and Monahan (1984) concerns the estimation of the variance component σ_a^2 in a linear mixed model

   $$\boldsymbol{y} \sim \text{Normal}(\mu\mathbf{1}, \sigma_a^2\boldsymbol{V}_1 + \sigma_e^2\boldsymbol{I}).$$

   In this project, we consider the problem of testing the significance of the variance component σ_a^2, i.e., testing $H_0 : \sigma_a^2 = 0$ vs $H_A : \sigma_a^2 > 0$. Perform a simulation experiment to study the performance (size and power) of following tests

   - Likelihood ratio test (LRT) using the $0.5\chi_0^2 + 0.5\chi_1^2$ asymptotic null distribution
   - LRT using the exact finite sample null distribution derived in
     CM Crainiceanu and D Ruppert (2004). Likelihood ratio tests in linear mixed models with one variance component. *Journal of the Royal Statistical Society, Series B*, 66(1): 165–185.
   - Restricted likelihood ratio test (RLRT) using the exact finite sample null distribution derived in the same paper.

   Your simulation design must contain at least the following factors

   - n-patterns: see Swallow and Monahan (1984)
   - σ_a^2/σ_e^2: see Swallow and Monahan (1984)

   Hints on computation: `lme()` function in `R` fits linear mixed model. The LRT and RLRT proposed by Crainiceanu and Ruppert (2004) are implemented in the `RLRsim` package in `R`.

3. Summarize your results in a written report. Your report should follow

   - statement of problem
   - questions to be addressed
   - description of the design of the experiment
   - computational details

- analysis of results
- conclusions
- of course, include your code (with comments) and output