## ST552, Homework 3

## Due Wednesday, Oct 2, 2013

- 1. JM 3.13 (p68)
- 2. JM 3.17 (p68)
- 3. If X and Y are two independent random variables, then they are uncorrelated. Give a concrete counter example to show that the converse is not true in general. That is to find two random variables X and Y such that they are uncorrelated but not independent. (Hint: it does not have to be complicated. An example where X and Y can take only three values suffices.)
- 4. Mutual independence implies pairwise independence. Give a concrete example to show that the converse is not true in general. Hint: let X and Y be uniformly distributed on  $\{0,1,2\}$ . You may consider random variables  $Z_n = (X + nY) \mod 2$ , n = 0, 1, 2. Show that  $Z_0, Z_1, Z_2$  are pairwise independent but not mutually independent.
- 5. Under the Gauss-Markov model, the covariance matrix of the least squares estimator of an estimable function  $\Lambda b$  is  $Var(\Lambda \hat{b}) = \sigma^2 \Lambda (X^T X)^- \Lambda^T$ . Show that  $\Lambda (X^T X)^- \Lambda^T$  is invariant to the choice of generalized inverse  $(X^T X)^-$ .
- 6. Let  $\Omega_1$  and  $\Omega_2$  be two positive semidefinite matrices.
  - (a) Prove that if  $\Omega_2 \succeq \Omega_1$ , then  $\operatorname{tr}(\Omega_2) \geq \operatorname{tr}(\Omega_1)$ .
  - (b) Is the converse true? That is, is it true that  $tr(\Omega_2) \ge tr(\Omega_1)$  implies  $\Omega_2 \succeq \Omega_1$ ?

This shows that any best (minimum variance) affine unbiased estimator is an affine minimum-trace unbiased estimator, but the converse is not true.

- 7. JM A.72 (p268)
- 8. JM 4.4 (p91)
- 9. JM 4.5 (p91)