BIOSTAT M280, Homework 1

Due Thu Jan 21 @ 11:59PM, 2016

Some R exercises. You can also do this homework in Julia.

1. Register an account on `bitbucket.org` using your UCLA email and create a private repository `biostat-m280-2016-winter`. Add teaching assistant Max Tolkoff (`mtolkoff`) and instructor Hua Zhou (`HuaZhou`) as your collaborators. Top directories should be `hw1`, `hw2`, ... Create two branches `master` and `develop`. The `develop` branch will be your own playground, the place where you develop solution (code) to homework problems and write up report. The `master` branch will be your presentation area. Put your homework submission files (html or pdf from RMarkdown, code to reproduce results, ...) in this branch. *No handwritten homework reports are accepted for this course.* After each homework due date, teaching assistant and instructor will check out your `master` branch for grading. Tag each of your homework submissions with tag names `biostatm280hw1`, `biostatm280hw2`, ...

   Efficient and abundant use of Git, e.g., frequent and well-documented commits, is an important criterion for evaluating your homework.

2. Let $a = 0.7$, $b = 0.2$, and $c = 0.1$.
   (a) Derive the internal floating-point representation of these numbers and verify your answers in R.
   (b) Test whether $(a + b) + c$ equals 1.
   (c) Test whether $a + (b + c)$ equals 1.
   (d) Test whether $(a + c) + b$ equals 1.
   (e) Explain what you found.

3. Create the vector $\boldsymbol{v} = (969, 971, 972, \ldots, 1022, 1023)$ of 54 elements.
   (a) Compute the sum $\sum_{i=1}^{54} 2^{v_i}$.
   (b) Compute the sum $\sum_{i=2}^{54} 2^{v_i}$.
   (c) Compute the sum $2^{v_1} + \sum_{i=2}^{54} 2^{v_i}$.
   (d) Explain what you found.

4. Create the vector $\boldsymbol{x} = (0.988, 0.989, 0.990, \ldots, 1.010, 1.011, 1.012)$.
   (a) Plot the polynomial $y = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ at points x_i in $\boldsymbol{x}$.
   (b) Plot the polynomial $y = (x - 1)^7$ at points x_i in $\boldsymbol{x}$.
   (c) Explain what you found.

5. Let $\boldsymbol{u} = (1, 2, 3, 3, 2, 1)^{\mathsf{T}}$.

(a) Compute $\boldsymbol{U} = \boldsymbol{I} - (2/d)\boldsymbol{u}\boldsymbol{u}^{\mathsf{T}}$ where $d = \boldsymbol{u}^{\mathsf{T}}\boldsymbol{u}$. (This type of matrix is known as an 'elementary reflector' or a 'Householder transformation.')

(b) Let $\boldsymbol{C} = \boldsymbol{U}\boldsymbol{U}$, the matrix product of $\boldsymbol{U}$ and itself. Find the largest and smallest off-diagonal elements of $\boldsymbol{C}$.

(c) Find the largest and smallest diagonal elements of $\boldsymbol{C}$.

(d) Compute $\boldsymbol{U}\boldsymbol{u}$. (matrix times vector)

(e) Compute the scalar $\max_i \sum_j |U(i,j)|$

(f) Print the third row of $\boldsymbol{U}$.

(g) Print the elements of the second column below the diagonal.

(h) Let $\boldsymbol{A}$ be the first three columns of $\boldsymbol{U}$. Compute $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}$.

(i) Show that $\boldsymbol{P}$ is idempotent by recomputing (e) with $\boldsymbol{P}\boldsymbol{P} - \boldsymbol{P}$.

(j) Let $\boldsymbol{B}$ be the last three columns of $\boldsymbol{U}$. Compute $\boldsymbol{Q} = \boldsymbol{B}\boldsymbol{B}^{\mathsf{T}}$.

(k) Show that $\boldsymbol{Q}$ is idempotent by recomputing (e) with $\boldsymbol{Q}\boldsymbol{Q} - \boldsymbol{Q}$.

(l) Compute $\boldsymbol{P} + \boldsymbol{Q}$.

6. Read in the matrix in the file 'oringp.dat' on the failure of O-rings leading to the Challenger disaster. The columns are flight number, date, number of O-rings, number failed, and temperature at launch. Compute the correlation between number of failures and temperature at launch, deleting the last, missing observation (the disaster).

7. Let the $n \times n$ matrix $\boldsymbol{A}$ have elements $A(i,j) = 1/(i+j-1)$.

   (a) Compute and print $\boldsymbol{A}$ for $n = 10$.

   (b) Compute and print the Cholesky factorization for $\boldsymbol{A}$ for $n = 10$.

   (c) Compute the Cholesky factorization for $n = 20$. Does it fail? If not, find the determinant.