ST552, Homework 3

Due Wednesday, Oct 2, 2013

1. JM 3.13 (p68)

2. JM 3.17 (p68)

3. If X and Y are two independent random variables, then they are uncorrelated. Give a concrete counter example to show that the converse is not true in general. That is to find two random variables X and Y such that they are uncorrelated but not independent. (Hint: it does not have to be complicated. An example where X and Y can take only three values suffices.)

4. Mutual independence implies pairwise independence. Give a concrete example to show that the converse is not true in general. Hint: let X and Y be uniformly distributed on $\{0, 1, 2\}$. You may consider random variables $Z_n = (X + nY) \bmod 3$, $n = 0, 1, 2$. Show that Z_0, Z_1, Z_2 are pairwise independent but not mutually independent.

5. Under the Gauss-Markov model, the covariance matrix of the least squares estimator of an estimable function $\boldsymbol{\Lambda b}$ is $\text{Var}(\boldsymbol{\Lambda}\hat{\boldsymbol{b}}) = \sigma^2 \boldsymbol{\Lambda}(\boldsymbol{X}^T\boldsymbol{X})^- \boldsymbol{\Lambda}^T$. Show that $\boldsymbol{\Lambda}(\boldsymbol{X}^T\boldsymbol{X})^- \boldsymbol{\Lambda}^T$ is invariant to the choice of generalized inverse $(\boldsymbol{X}^T\boldsymbol{X})^-$.

6. Let $\boldsymbol{\Omega}_1$ and $\boldsymbol{\Omega}_2$ be two positive semidefinite matrices.

   (a) Prove that if $\boldsymbol{\Omega}_2 \succeq \boldsymbol{\Omega}_1$, then $\text{tr}(\boldsymbol{\Omega}_2) \geq \text{tr}(\boldsymbol{\Omega}_1)$.

   (b) Is the converse true? That is, is it true that $\text{tr}(\boldsymbol{\Omega}_2) \geq \text{tr}(\boldsymbol{\Omega}_1)$ implies $\boldsymbol{\Omega}_2 \succeq \boldsymbol{\Omega}_1$?

   This shows that any best (minimum variance) affine unbiased estimator is an affine minimum-trace unbiased estimator, but the converse is not true.

7. JM A.72 (p268)

8. JM 4.4 (p91)

9. JM 4.5 (p91)