exercise2

13.

Molly the dog is very particular about her food, so she created a start-up to make cartons of her favourite food, which she called MollyBix. These cartons are produced in three plants, in Lancashire, Derbyshire and Yorkshire, and supplied to customers through various distribution centres. MollyBix production is not an exact science, and not every carton is approved by Molly. She likes 95%, 40% and 25% of the MollyBix made in Lancashire, Derbyshire and Yorkshire, each plant of which produces 50%, 20% and 30% of cartons, respectively. Molly's food-orders come in boxes of 500 cartons that are sourced randomly from all three plants.

(a)

If a carton of MollyBix is selected at random from a box, what is the probability that Molly will like it?

95% *50% + 40%* 20% + 25% * 30% = 47.5% + 8% + 7.5% = 63% = 0.630

(b)

Molly doesn't like a randomly selected carton. What are the probabilities that it was produced at each of the plants?

$P(\text{Lancashire}|\text{doesnt like}) = \frac{P(\text{Lancashire}\cap\text{doesnt like})}{P(\text{doesnt like})} = \frac{(1-95\%)*50\%}{1-63\%} = \frac{5}{74} = 0.0676$

$P(\text{Derbyshire}|\text{doesnt like}) = \frac{P(\text{Derbyshire}\cap\text{doesnt like})}{P(\text{doesnt like})} = \frac{(1-40\%)*20\%)}{1-63\%} = \frac{12}{37} = 0.324$

$P(\text{Yorkshire}|\text{doesnt like}) = \frac{P(\text{Yorkshire}\cap\text{doesnt like})}{P(\text{doesnt like})} = \frac{(1-25\%)*30\%}{1-63\%} = \frac{45}{74} = 0.608$

(c)

In a particular box, assume that the numbers of cartons produced at each plant are exactly in proportion to the production percentages given, e.g. 250 were produced at the Lancashire plant. What are the expectation and standard deviation of the number of cartons in the box that Molly will like? (Hint: Let Xi be the number of cartons that Molly likes out of those produced at plant i. Model Xi as a Binomial distribution.)

so this a combination of 3 binomial distributions

Lancashire:

Binomial(250,0.95)

- n = 250, p = 0.95
- mean: $E(X_{\text{Lancashire}}) = np = 250 * 0.95 = 237.5$
- variance: $Var(X_{\text{Lancashire}}) = np(1-p) = 250 * 0.95 * (1-0.95) = 11.875$

Derbyshire

Binomial(100,0.4)

- n = 100, p = 0.4
- mean: $E(X_{\text{Derbyshire}}) = np = 100 * 0.4 = 40.0$
- variance: $Var(X_{\text{Derbyshire}}) = np(1-p) = 100 * 0.4 * 0.6 = 24$

Yorkshire

Binomial(150,0.25)

- n = 150, p = 0.25
- mean: $E(X_{\text{Yorkshire}}) = np = 150 * 0.25 = 37.5$
- variance: $Var(X_{Yorkshire}) = np(1-p) = 150 * 0.25 * 0.75 = 28.125$
- skew: 0.09

combined

since the events of the three places are obviouly independent

we denote S_X as the three events combined
$E(S_X) = E(X_{\text{Lancashire}}) + E(X_{\text{Derbyshire}}) + E(X_{\text{Yorkshire}}) = 237.5 + 40.0 + 37.5 = 315$
$Var(S_X) = Var(X_{\text{Lancashire}}) + Var(X_{\text{Derbyshire}}) + Var(X_{\text{Yorkshire}}) = 11.875 + 24 + 28.125 = 64 = \sigma(S_X)^2$
so $\sigma(S_X) = \sqrt{Var(S_X)} = \sqrt{64} = 8$

14.

A network consists of the links shown in the diagram below. Each node represents a router.The directed edges indicate the links that a packet can use between each node pair. Consider a packet that needs to traverse the network from A to B. Each edge is annotated with the probability hij that the link $i \to j$ is healthy, e.g., $H_{AD} = 0.75$. Links fail independently of each other. A path is said to be faulty if there exist at least one faulty link along the path. A path without faulty links is said to be healthy. Compute the

probability that there exist at least a healthy path between node A and node B. Detail and justify your steps. (Hint: consider first the same question for the two sub-paths that start from C)

then

$$\begin{aligned} P(\text{faulty } CEB) &= 1 - P(\text{healthy } CEB) \\ &= 1 - H_{CE} * H_{EB} \\ &= 1 - 0.8 * 0.9 \\ &= 0.28 \end{aligned}$$

$$\begin{aligned} P(\text{faulty } CFB) &= 1 - P(\text{healthy } CFB) \\ &= 1 - H_{CF} * H_{FB} \\ &= 1 - 0.85 * 0.95 \\ &= 0.1925 \end{aligned}$$

therefore

$$\begin{aligned} P(\text{at least one healthy path between } C \text{ and } B) &= 1 - P(\text{faulty } CB) \\ &= 1 - P(\text{faulty } CEB \cap \text{faulty } CFB) \\ &= 1 - P(\text{faulty } CEB) * P(\text{faulty } CFB) \\ &= 1 - 0.28 * 0.1925 \\ &= 0.9461 \end{aligned}$$

and

$$\begin{aligned} P(\text{faulty } ACB) &= 1 - P(ACB) \\ &= 1 - P(AC) * P(\text{at least one healthy path in } CB) \\ &= 1 - 0.9 * 0.9461 \\ &= 0.14851 \end{aligned}$$

$$\begin{aligned} P(\text{faulty } ADB) &= 1 - P(ADB) \\ &= 1 - H_{AD} * H_{DB} \\ &= 1 - 0.75 * 0.95 \\ &= 0.2875 \end{aligned}$$

therefore,

$$\begin{aligned} P(\text{at least one healthy path in } A \text{ and } B) &= 1 - P(\text{no healthy path between } A \text{ and } B) \\ &= 1 - P(\text{faulty } ACB \cap \text{faulty } ADB) \\ &= 1 - 0.14851 * 0.2875 \\ &= 0.957 \end{aligned}$$

in conclusion, the probability that there is at least one healthy path between A and B is 0.957 or 95.7%