exercise3

2

Electrons hit a ciruclar plate with unti radius. Let X be the random vairbale representing the distance of a particle strike from the centre of the plate. Assuming that a particle is equally likely to strike anywhere on the plate

(a)

$P(x \leq r)$ **and hence write down the full cumulative distribution function of** X, F_X

we can calculate the probability using the area of strike

$P(x \leq r) = \frac{\pi * r^2}{\pi * 1^2} = r^2$

so $F_X(r) = P(x \leq r) = r^2$, where r is only defined in [0,1]

therefore, the cdf of X is

$$F(r) = \begin{cases} 0 & r \leq 0 \\ r^2 & 0 < r < 1 \\ 1 & r \geq 1 \end{cases}$$

(b)

find $P(r < X \leq s)$**, where** $0 < r < s < 1$

$P(r < X \leq s) = P(X \leq s) - P(X \leq r) = F_X(s) - F_X(r) = s^2 - r^2$

(c)

would the expression of $P(r \leq X \leq s)$ **be different? Explain**

no, geometrically a ring with radius r and no width has area 0, so $P(X = r) = 0$, however, in probability, this does not mean that the event is not possible to happen, instead, it only means the possibility of $P(X = r)$ is so low that it is negligable

(d)

find the probability density function for X, f_X

so $f_X(r) = F'_X = \frac{dr^2}{dr} = 2r$

since in this question r can only be in [0,1]

$$f_X(r) = \begin{cases} 2r & 0 \leq r \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(e)

calculate the mean distance of a particle strike from the origin

in this question, the scope of x can only be [0,1]
$E(x) = \int_0^1 x f_x(x) dx = \int_0^1 x * 2x dx = \int_0^1 2x^2 dx = \frac{2}{3}x^3|_0^1 = \frac{2}{3}$