

tut3

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Electrons hit a circular plate with unit radius. Let X be the random variable representing the distance of a particle strike from the centre of the plate. Assuming that a particle is equally likely to strike anywhere on the plate

(a)

$P(x \leq r)$ and hence write down the full cumulative distribution function of X , F_X

we can calculate the probability using the area of strike

$$P(x \leq r) = \frac{\pi r^2}{\pi \cdot 1^2} = r^2$$

so $F_X(r) = P(x \leq r) = r^2$, where r is only defined in $[0,1]$

(b)

find $P(r < X \leq s)$, where $0 < r < s < 1$

$$P(r < X \leq s) = P(X \leq s) - P(X \leq r) = F_X(s) - F_X(r) = s^2 - r^2$$

(c)

would the expression of $P(r \leq X \leq s)$ be different? Explain

no, geometrically a ring with radius r and no width has area 0, so $P(X = r) = 0$, however, in probability, this does not mean that the event is not possible to happen, instead, it only means the possibility of $P(X = r)$ is so low that it is negligible

(d)

find the probability density function for X , f_X

$$\text{so } f_X = F'_X = \frac{d}{dx} x^2 = 2x$$

(e)

calculate the mean distance of a particle strike from the origin

in this question, the scope of x can only be $[0,1]$

$$E(x) = \int_0^1 x f_x(x) dx = \int_0^1 x * 2x dx = \int_0^1 2x^2 dx = \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3}$$