# **BayesPy Documentation**

Release 0.2.3

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### INTRODUCTION

BayesPy provides tools for Bayesian inference with Python. The user constructs a model as a Bayesian network, observes data and runs posterior inference. The goal is to provide a tool which is efficient, flexible and extendable enough for expert use but also accessible for more casual users.

Currently, only variational Bayesian inference for conjugate-exponential family (variational message passing) has been implemented. Future work includes variational approximations for other types of distributions and possibly other approximate inference methods such as expectation propagation, Laplace approximations, Markov chain Monte Carlo (MCMC) and other methods. Contributions are welcome.

# 1.1 Project information

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BayesPy including the documentation is licensed under Version 3.0 of the GNU General Public License. See LICENSE file for a text of the license or visit http://www.gnu.org/copyleft/gpl.html.

- Documentation:
  - http://bayespy.org
  - PDF file
  - RST format in doc directory
- Repository: https://github.com/bayespy/bayespy.git
- Bug reports: https://github.com/bayespy/bayespy/issues
- Mailing list: bayespy@googlegroups.com
- IRC: #bayespy @ freenode
- Author: Jaakko Luttinen jaakko.luttinen@iki.fi
- Latest release:
- Build status:
- Unit test coverage:

# 1.2 Similar projects

VIBES (http://vibes.sourceforge.net/) allows variational inference to be performed automatically on a Bayesian network. It is implemented in Java and released under revised BSD license.

Bayes Blocks (http://research.ics.aalto.fi/bayes/software/) is a C++/Python implementation of the variational building block framework. The framework allows easy learning of a wide variety of models using variational Bayesian learning. It is available as free software under the GNU General Public License.

Infer.NET (http://research.microsoft.com/infernet/) is a .NET framework for machine learning. It provides message-passing algorithms and statistical routines for performing Bayesian inference. It is partly closed source and licensed for non-commercial use only.

PyMC (https://github.com/pymc-devs/pymc) provides MCMC methods in Python. It is released under the Academic Free License.

OpenBUGS (http://www.openbugs.info) is a software package for performing Bayesian inference using Gibbs sampling. It is released under the GNU General Public License.

Dimple (http://dimple.probprog.org/) provides Gibbs sampling, belief propagation and a few other inference algorithms for Matlab and Java. It is released under the Apache License.

Stan (http://mc-stan.org/) provides inference using MCMC with an interface for R and Python. It is released under the New BSD License.

PBNT - Python Bayesian Network Toolbox (http://pbnt.berlios.de/) is Bayesian network library in Python supporting static networks with discrete variables. There was no information about the license.

### 1.3 Contributors

The list of contributors:

- · Jaakko Luttinen
- · Hannu Hartikainen

Each file or the git log can be used for more detailed information.

# 1.4 Version history

### 1.4.1 Version 0.2.3 (2014-12-03)

- Fix matplotlib compatibility broken by recent changes in matplotlib
- · Add random sampling for Binomial and Bernoulli nodes
- Fix minor bugs, for instance, in plot module

### 1.4.2 Version 0.2.2 (2014-11-01)

- Fix normalization of categorical Markov chain probabilities (fixes HMM demo)
- Fix initialization from parameter values

#### 1.4.3 Version 0.2.1 (2014-09-30)

- · Add workaround for matplotlib 1.4.0 bug related to interactive mode which affected monitoring
- Fix bugs in Hinton diagrams for Gaussian variables

### 1.4.4 Version 0.2 (2014-08-06)

- · Added all remaining common distributions: Bernoulli, binomial, multinomial, Poisson, beta, exponential.
- Added Gaussian arrays (not just scalars or vectors).
- Added Gaussian Markov chains with time-varying or swithing dynamics.
- Added discrete Markov chains (enabling hidden Markov models).
- · Added joint Gaussian-Wishart and Gaussian-gamma nodes.
- Added deterministic gating node.
- Added deterministic general sum-product node.
- Added parameter expansion for Gaussian arrays and time-varying/switching Gaussian Markov chains.
- Added new plotting functions: pdf, Hinton diagram.
- Added monitoring of posterior distributions during iteration.
- · Finished documentation and added API.

### 1.4.5 Version 0.1 (2013-07-25)

- Added variational message passing inference engine.
- · Added the following common distributions: Gaussian vector, gamma, Wishart, Dirichlet, categorical.
- Added Gaussian Markov chain.
- · Added parameter expansion for Gaussian vectors and Gaussian Markov chain.
- Added stochastic mixture node.
- Added deterministic dot product node.
- · Created preliminary version of the documentation.

1.4. Version history

**CHAPTER** 

**TWO** 

### **USER GUIDE**

### 2.1 Installation

BayesPy is a Python 3 package and it can be installed from PyPI or the latest development version from GitHub. The instructions below explain how to set up the system by installing required packages, how to install BayesPy and how to compile this documentation yourself. However, if these instructions contain errors or some relevant details are missing, please file a bug report at https://github.com/bayespy/bayespy/issues.

### 2.1.1 Installing BayesPy

BayesPy can be installed easily by using Pip if the system has been properly set up. If you have problems with the following methods, see the following section for some help on installing the requirements.

#### For users

First, you may want to set up a virtual environment. Using virtual environment is optional but recommended. To create and activate a new virtual environment, run (in the folder in which you want to create the environment):

```
virtualenv -p python3 --system-site-packages ENV
source ENV/bin/activate
```

The latest release of BayesPy can be installed from PyPI simply as

```
pip install bayespy
```

If you want to install the latest development version of BayesPy, use GitHub instead:

```
pip install https://github.com/bayespy/bayespy/archive/master.zip
```

#### For developers

If you want to install the development version of BayesPy in such a way that you can easily edit the package, follow these instructions. Get the git repository:

```
git clone https://github.com/bayespy/bayespy.git
cd bayespy
```

Create and activate a new virtual environment (optional but recommended):

```
virtualenv -p python3 --system-site-packages ENV
source ENV/bin/activate
```

Install BayesPy in editable mode:

```
pip install -e .
```

#### Checking installation

If you have problems installing BayesPy, read the next section for more details. It is recommended to run the unit tests in order to check that BayesPy is working properly. Thus, install Nose and run the unit tests:

```
pip install nose
nosetests bayespy
```

### 2.1.2 Installing requirements

BayesPy requires Python 3.2 (or later) and the following packages:

- NumPy (>=1.8.0),
- SciPy (>=0.13.0)
- matplotlib (>=1.2)
- h5py

Ideally, Pip should install the necessary requirements and a manual installation of these dependencies is not required. However, there are several reasons why the installation of these dependencies needs to be done manually in some cases. Thus, this section tries to give some details on how to set up your system. A proper installation of the dependencies for Python 3 can be a bit tricky and you may refer to http://www.scipy.org/install.html for more detailed instructions about the SciPy stack. Detailed instructions on installing recent SciPy stack for various platforms is out of the scope of these instructions, but we provide some general guidance here. There are basically three ways to install the dependencies:

- 1. Install a Python distribution which includes the packages. For Windows, Mac and Linux, there are several Python distributions which include all the necessary packages: http://www.scipy.org/install.html#scientific-python-distributions. For instance, you may try Anaconda or Enthought.
- 2. Install the packages using the system package manager. On Linux, the packages might be called something like python-scipy or scipy. However, it is possible that these system packages are not recent enough for BayesPy.
- 3. Install the packages using Pip: pip install "numpy>=1.8.0" "scipy>=0.13.0" "matplotlib>=1.2" h5py. However, this may require that the system has the libraries needed for compiling (e.g., C compiler, Python development files, BLAS/LAPACK). For instance, on Ubuntu (>= 12.10), you may install the required system libraries for each package as:

```
sudo apt-get build-dep python3-numpy
sudo apt-get build-dep python3-scipy
sudo apt-get build-dep python3-matplotlib
sudo apt-get build-dep python-h5py
```

Then installation using Pip should work. Also, make sure you have recent enough version of Distribute (required by Matplotlib): pip install "distribute>=0.6.28".

### 2.1.3 Compiling documentation

This documentation can be found at http://bayespy.org/ in HTML and PDF formats. The documentation source files are also readable as such in reStructuredText format in doc/source/ directory. It is possible to compile the documen-

tation into HTML or PDF yourself. In order to compile the documentation, Sphinx is required and a few extensions for it. Those can be installed as:

```
pip install "sphinx>=1.2.3" sphinxcontrib-tikz sphinxcontrib-bayesnet sphinxcontrib-bibtex "numpydoc
```

In order to visualize graphical models in HTML, you need to have ImageMagick or Netphm installed. The documentation can be compiled to HTML and PDF by running the following commands in the doc directory:

```
make html
make latexpdf
```

You can also run doctest to test code snippets in the documentation:

```
make doctest
```

or in the docstrings:

```
nosetests --with-doctest bayespy
```

# 2.2 Quick start guide

This short guide shows the key steps in using BayesPy for variational Bayesian inference by applying BayesPy to a simple problem. The key steps in using BayesPy are the following:

- · Construct the model
- Observe some of the variables by providing the data in a proper format
- · Run variational Bayesian inference
- Examine the resulting posterior approximation

To demonstrate BayesPy, we'll consider a very simple problem: we have a set of observations from a Gaussian distribution with unknown mean and variance, and we want to learn these parameters. In this case, we do not use any real-world data but generate some artificial data. The dataset consists of ten samples from a Gaussian distribution with mean 5 and standard deviation 10. This dataset can be generated with NumPy as follows:

```
>>> import numpy as np
>>> data = np.random.normal(5, 10, size=(10,))
```

### 2.2.1 Constructing the model

Now, given this data we would like to estimate the mean and the standard deviation as if we didn't know their values. The model can be defined as follows:

$$p(\mathbf{y}|\mu,\tau) = \prod_{n=0}^{9} \mathcal{N}(y_n|\mu,\tau)$$
$$p(\mu) = \mathcal{N}(\mu|0, 10^{-6})$$
$$p(\tau) = \mathcal{G}(\tau|10^{-6}, 10^{-6})$$

where  $\mathcal{N}$  is the Gaussian distribution parameterized by its mean and precision (i.e., inverse variance), and  $\mathcal{G}$  is the gamma distribution parameterized by its shape and rate parameters. Note that we have given quite uninformative priors for the variables  $\mu$  and  $\tau$ . This simple model can also be shown as a directed factor graph: This model can be constructed in BayesPy as follows:

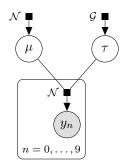


Figure 2.1: Directed factor graph of the example model.

```
>>> from bayespy.nodes import GaussianARD, Gamma
>>> mu = GaussianARD(0, 1e-6)
>>> tau = Gamma(1e-6, 1e-6)
>>> y = GaussianARD(mu, tau, plates=(10,))
```

This is quite self-explanatory given the model definitions above. We have used two types of nodes GaussianARD and Gamma to represent Gaussian and gamma distributions, respectively. There are much more distributions in bayespy.nodes so you can construct quite complex conjugate exponential family models. The node y uses keyword argument plates to define the plates  $n=0,\ldots,9$ .

### 2.2.2 Performing inference

Now that we have created the model, we can provide our data by setting y as observed:

```
>>> y.observe(data)
```

Next we want to estimate the posterior distribution. In principle, we could use different inference engines (e.g., MCMC or EP) but currently only variational Bayesian (VB) engine is implemented. The engine is initialized by giving all the nodes of the model:

```
>>> from bayespy.inference import VB
>>> Q = VB(mu, tau, y)
```

The inference algorithm can be run as long as wanted (max. 20 iterations in this case):

```
>>> Q.update(repeat=20)

Iteration 1: loglike=-6.020956e+01 (... seconds)

Iteration 2: loglike=-5.820527e+01 (... seconds)

Iteration 3: loglike=-5.820290e+01 (... seconds)

Iteration 4: loglike=-5.820288e+01 (... seconds)

Converged at iteration 4.
```

Now the algorithm converged after four iterations, before the requested 20 iterations. VB approximates the true posterior  $p(\mu, \tau | \mathbf{y})$  with a distribution which factorizes with respect to the nodes:  $q(\mu)q(\tau)$ .

### 2.2.3 Examining posterior approximation

The resulting approximate posterior distributions  $q(\mu)$  and  $q(\tau)$  can be examined, for instance, by plotting the marginal probability density functions:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.subplot(2, 1, 1)
<matplotlib.axes...AxesSubplot object at 0x...>
>>> bpplt.pdf(mu, np.linspace(-10, 20, num=100), color='k', name=r'\mu')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.subplot(2, 1, 2)
<matplotlib.axes...AxesSubplot object at 0x...>
>>> bpplt.pdf(tau, np.linspace(1e-6, 0.08, num=100), color='k', name=r'\tau')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.tight_layout()
>>> bpplt.pyplot.show()
                                             q(\mu)
  0.12
  0.10
  0.08
  0.06
  0.04
  0.02
  0.00
                                 0
                                               5
                                                            10
                                                                         15
                                                                                       20
                                               \mu
                                             q(\tau)
  160
  140
  120
  100
    80
    60
    40
    20
    مار
0.00
             0.01
                      0.02
                                0.03
                                         0.04
                                                  0.05
                                                           0.06
                                                                    0.07
                                                                             0.08
                                                                                      0.09
```

This example was a very simple introduction to using BayesPy. The model can be much more complex and each phase contains more options to give the user more control over the inference. The following sections give more details about the phases.

# 2.3 Constructing the model

In BayesPy, the model is constructed by creating nodes which form a directed network. There are two types of nodes: stochastic and deterministic. A stochastic node corresponds to a random variable (or a set of random variables) from a specific probability distribution. A deterministic node corresponds to a deterministic function of its parents. For a list of built-in nodes, see the *User API*.

#### 2.3.1 Creating nodes

Creating a node is basically like writing the conditional prior distribution of the variable in Python. The node is constructed by giving the parent nodes, that is, the conditioning variables as arguments. The number of parents and their meaning depend on the node. For instance, a Gaussian node is created by giving the mean vector and the precision matrix. These parents can be constant numerical arrays if they are known:

```
>>> from bayespy.nodes import Gaussian
>>> X = Gaussian([2, 5], [[1.0, 0.3], [0.3, 1.0]])
```

or other nodes if they are unknown and given prior distributions:

```
>>> from bayespy.nodes import Gaussian, Wishart
>>> mu = Gaussian([0, 0], [[1e-6, 0], [0, 1e-6]])
>>> Lambda = Wishart(2, [[1, 0], [0, 1]])
>>> X = Gaussian(mu, Lambda)
```

Nodes can also be named by providing name keyword argument:

```
>>> X = Gaussian(mu, Lambda, name='x')
```

The name may be useful when referring to the node using an inference engine.

For the parent nodes, there are two main restrictions: non-constant parent nodes must be conjugate and the parent nodes must be mutually independent in the posterior approximation.

#### Conjugacy of the parents

In Bayesian framework in general, one can give quite arbitrary probability distributions for variables. However, one often uses distributions that are easy to handle in practice. Quite often this means that the parents are given conjugate priors. This is also one of the limitations in BayesPy: only conjugate family prior distributions are accepted currently. Thus, although in principle one could give, for instance, gamma prior for the mean parameter mu, only Gaussian-family distributions are accepted because of the conjugacy. If the parent is not of a proper type, an error is raised. This conjugacy is checked automatically by BayesPy and NoConverterError is raised if a parent cannot be interpreted as being from a conjugate distribution.

#### Independence of the parents

Another a bit rarely encountered limitation is that the parents must be mutually independent (in the posterior factorization). Thus, a node cannot have the same stochastic node as several parents without intermediate stochastic nodes. For instance, the following leads to an error:

```
>>> from bayespy.nodes import Dot
>>> Y = Dot(X, X)
Traceback (most recent call last):
...
ValueError: Parent nodes are not independent
```

The error is raised because X is given as two parents for Y, and obviously X is not independent of X in the posterior approximation. Even if X is not given several times directly but there are some intermediate deterministic nodes, an error is raised because the deterministic nodes depend on their parents and thus the parents of Y would not be independent. However, it is valid that a node is a parent of another node via several paths if all the paths or all except one path has intermediate stochastic nodes. This is valid because the intermediate stochastic nodes have independent posterior approximations. Thus, for instance, the following construction does not raise errors:

```
>>> from bayespy.nodes import Dot
>>> Z = Gaussian(X, [[1,0], [0,1]])
>>> Y = Dot(X, Z)
```

This works because there is now an intermediate stochastic node Z on the other path from X node to Y node.

#### 2.3.2 Effects of the nodes on inference

When constructing the network with nodes, the stochastic nodes actually define three important aspects:

- 1. the prior probability distribution for the variables,
- 2. the factorization of the posterior approximation,
- 3. the functional form of the posterior approximation for the variables.

#### Prior probability distribution

First, the most intuitive feature of the nodes is that they define the prior distribution. In the previous example, mu was a stochastic GaussianARD node corresponding to  $\mu$  from the normal distribution, tau was a stochastic Gamma node corresponding to  $\tau$  from the gamma distribution, and y was a stochastic GaussianARD node corresponding to y from the normal distribution with mean  $\mu$  and precision  $\tau$ . If we denote the set of all stochastic nodes by  $\Omega$ , and by  $\pi_X$  the set of parents of a node X, the model is defined as

$$p(\Omega) = \prod_{X \in \Omega} p(X|\pi_X),$$

where nodes correspond to the terms  $p(X|\pi_X)$ .

#### Posterior factorization

Second, the nodes define the structure of the posterior approximation. The variational Bayesian approximation factorizes with respect to nodes, that is, each node corresponds to an independent probability distribution in the posterior approximation. In the previous example, mu and tau were separate nodes, thus the posterior approximation factorizes with respect to them:  $q(\mu)q(\tau)$ . Thus, the posterior approximation can be written as:

$$p(\tilde{\Omega}|\hat{\Omega}) \approx \prod_{X \in \tilde{\Omega}} q(X),$$

where  $\tilde{\Omega}$  is the set of latent stochastic nodes and  $\hat{\Omega}$  is the set of observed stochastic nodes. Sometimes one may want to avoid the factorization between some variables. For this purpose, there are some nodes which model several variables jointly without factorization. For instance, GaussianGammaISO is a joint node for  $\mu$  and  $\tau$  variables from the normal-gamma distribution and the posterior approximation does not factorize between  $\mu$  and  $\tau$ , that is, the posterior approximation is  $q(\mu,\tau)$ .

### Functional form of the posterior

Last, the nodes define the functional form of the posterior approximation. Usually, the posterior approximation has the same or similar functional form as the prior. For instance, Gamma uses gamma distribution to also approximate the posterior distribution. Similarly, GaussianARD uses Gaussian distribution for the posterior. However, the posterior approximation of GaussianARD uses a full covariance matrix although the prior assumes a diagonal covariance matrix. Thus, there can be slight differences in the exact functional form of the posterior approximation but the rule of thumb is that the functional form of the posterior approximation is the same as or more general than the functional form of the prior.

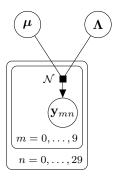
### 2.3.3 Using plate notation

#### **Defining plates**

Stochastic nodes take the optional parameter plates, which can be used to define plates of the variable. A plate defines the number of repetitions of a set of variables. For instance, a set of random variables  $y_{mn}$  could be defined as

$$\mathbf{y}_{mn} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}), \qquad m = 0, \dots, 9, \quad n = 0, \dots, 29.$$

This can also be visualized as a graphical model:



The variable has two plates: one for the index m and one for the index n. In BayesPy, this random variable can be constructed as:

```
>>> y = Gaussian(mu, Lambda, plates=(10,30))
```

**Note:** The plates are always given as a tuple of positive integers.

Plates also define indexing for the nodes, thus you can use simple NumPy-style slice indexing to obtain a subset of the plates:

```
>>> y_0 = y[0]
>>> y_0.plates
(30,)
>>> y_even = y[:,::2]
>>> y_even.plates
(10, 15)
>>> y_complex = y[:5, 10:20:5]
>>> y_complex.plates
(5, 2)
```

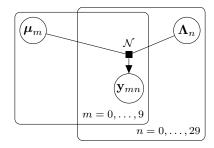
Note that this indexing is for the plates only, not for the random variable dimensions.

#### Sharing and broadcasting plates

Instead of having a common mean and precision matrix for all  $y_{mn}$ , it is also possible to share plates with parents. For instance, the mean could be different for each index m and the precision for each index n:

$$\mathbf{y}_{mn} \sim \mathcal{N}(\boldsymbol{\mu}_m, \boldsymbol{\Lambda}_n), \quad m = 0, \dots, 9, \quad n = 0, \dots, 29.$$

which has the following graphical representation:



This can be constructed in BayesPy, for instance, as:

```
>>> from bayespy.nodes import Gaussian, Wishart
>>> mu = Gaussian([0, 0], [[1e-6, 0], [0, 1e-6]], plates=(10,1))
>>> Lambda = Wishart(2, [[1, 0], [0, 1]], plates=(1,30))
>>> X = Gaussian(mu, Lambda)
```

There are a few things to notice here. First, the plates are defined similarly as shapes in NumPy, that is, they use similar broadcasting rules. For instance, the plates (10,1) and (1,30) broadcast to (10,30). In fact, one could use plates (10,1) and (30,1) to get the broadcasted plates (10,30) because broadcasting compares the plates from right to left starting from the last axis. Second, X is not given plates keyword argument because the default plates are the plates broadcasted from the parents and that was what we wanted so it was not necessary to provide the keyword argument. If we wanted, for instance, plates (20,10,30) for X, then we would have needed to provide plates=(20,10,30).

The validity of the plates between a child and its parents is checked as follows. The plates are compared plate-wise starting from the last axis and working the way forward. A plate of the child is compatible with a plate of the parent if either of the following conditions is met:

- 1. The two plates have equal size
- 2. The parent has size 1 (or no plate)

Table below shows an example of compatible plates for a child node and its two parent nodes:

node	plates							
parent1		3	1	1	1	8	10	
parent2			1	1	5	1	10	
child	5	3	1	7	5	8	10	

#### Plates in deterministic nodes

Note that plates can be defined explicitly only for stochastic nodes. For deterministic nodes, the plates are defined implicitly by the plate broadcasting rules from the parents. Deterministic nodes do not need more plates than this because there is no randomness. The deterministic node would just have the same value over the extra plates, but it is not necessary to do this explicitly because the child nodes of the deterministic node can utilize broadcasting anyway. Thus, there is no point in having extra plates in deterministic nodes, and for this reason, deterministic nodes do not use plates keyword argument.

#### Plates in constants

It is useful to understand how the plates and the shape of a random variable are connected. The shape of an array which contains all the plates of a random variable is the concatenation of the plates and the shape of the variable. For instance, consider a 2-dimensional Gaussian variable with plates (3, ). If you want the value of the constant mean vector and constant precision matrix to vary between plates, they are given as (3, 2)-shape and (3, 2, 2)-shape arrays, respectively:

```
>>> import numpy as np
>>> mu = [[0,0], [1,1], [2,2]]
>>> Lambda = [ [[1.0, 0.0],
                [0.0, 1.0]],
               [[1.0, 0.9],
. . .
                [0.9, 1.0]],
               [[1.0, -0.3],
                [-0.3, 1.0]]
>>> X = Gaussian(mu, Lambda)
>>> np.shape(mu)
(3, 2)
>>> np.shape(Lambda)
(3, 2, 2)
>>> X.plates
(3,)
```

Thus, the leading axes of an array are the plate axes and the trailing axes are the random variable axes. In the example above, the mean vector has plates (3,) and shape (2,2), and the precision matrix has plates (3,) and shape (2,2).

#### **Factorization of plates**

It is important to undestand the independency structure the plates induce for the model. First, the repetitions defined by a plate are independent a priori given the parents. Second, the repetitions are independent in the posterior approximation, that is, the posterior approximation factorizes with respect to plates. Thus, the plates also have an effect on the independence structure of the posterior approximation, not only prior. If dependencies between a set of variables need to be handled, that set must be handled as a some kind of multi-dimensional variable.

#### Irregular plates

The handling of plates is not always as simple as described above. There are cases in which the plates of the parents do not map directly to the plates of the child node. The user API should mention such irregularities.

For instance, the parents of a mixture distribution have a plate which contains the different parameters for each cluster, but the variable from the mixture distribution does not have that plate:

```
>>> from bayespy.nodes import Gaussian, Wishart, Categorical, Mixture
>>> mu = Gaussian([[0], [0], [0]], [[[1]], [[1]], [[1]]])
>>> Lambda = Wishart(1, [ [[1]], [[1]], [[1]]])
>>> Z = Categorical([1/3, 1/3, 1/3], plates=(100,))
>>> X = Mixture(Z, Gaussian, mu, Lambda)
>>> mu.plates
(3,)
>>> Lambda.plates
(3,)
>>> Z.plates
(100,)
>>> X.plates
(100,)
```

The plates (3,) and (100,) should not broadcast according to the rules mentioned above. However, when validating the plates, Mixture removes the plate which corresponds to the clusters in mu and Lambda. Thus, X has plates which are the result of broadcasting plates () and (100,) which equals (100,).

Also, sometimes the plates of the parents may be mapped to the variable axes. For instance, an automatic relevance determination (ARD) prior for a Gaussian variable is constructed by giving the diagonal elements of the precision

matrix (or tensor). The Gaussian variable itself can be a scalar, a vector, a matrix or a tensor. A set of five  $4 \times 3$  -dimensional Gaussian matrices with ARD prior is constructed as:

```
>>> from bayespy.nodes import GaussianARD, Gamma
>>> tau = Gamma(1, 1, plates=(5,4,3))
>>> X = GaussianARD(0, tau, shape=(4,3))
>>> tau.plates
(5, 4, 3)
>>> X.plates
(5,)
```

Note how the last two plate axes of tau are mapped to the variable axes of X with shape (4,3) and the plates of X are obtained by taking the remaining leading plate axes of tau.

### 2.3.4 Example model: Principal component analysis

Now, we'll construct a bit more complex model which will be used in the following sections. The model is a probabilistic version of principal component analysis (PCA):

$$\mathbf{Y} = \mathbf{C}\mathbf{X}^T + \text{noise}$$

where  $\mathbf{Y}$  is  $M \times N$  data matrix,  $\mathbf{C}$  is  $M \times D$  loading matrix,  $\mathbf{X}$  is  $N \times D$  state matrix, and noise is isotropic Gaussian. The dimensionality D is usually assumed to be much smaller than M and N.

A probabilistic formulation can be written as:

$$p(\mathbf{Y}) = \prod_{m=0}^{M-1} \prod_{n=0}^{N-1} \mathcal{N}(y_{mn} | \mathbf{c}_m^T \mathbf{x}_n, \tau)$$

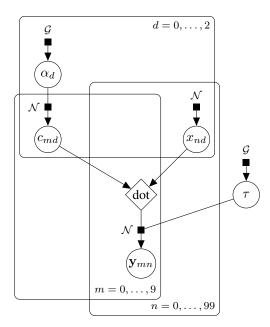
$$p(\mathbf{X}) = \prod_{n=0}^{N-1} \prod_{d=0}^{D-1} \mathcal{N}(x_{nd} | 0, 1)$$

$$p(\mathbf{C}) = \prod_{m=0}^{M-1} \prod_{d=0}^{D-1} \mathcal{N}(c_{md} | 0, \alpha_d)$$

$$p(\boldsymbol{\alpha}) = \prod_{d=0}^{D-1} \mathcal{G}(\alpha_d | 10^{-3}, 10^{-3})$$

$$p(\tau) = \mathcal{G}(\tau | 10^{-3}, 10^{-3})$$

where we have given automatic relevance determination (ARD) prior for C. This can be visualized as a graphical model:



Now, let us construct this model in BayesPy. First, we'll define the dimensionality of the latent space in our model:

```
>>> D = 3
```

Then the prior for the latent states **X**:

```
>>> X = GaussianARD(0, 1,
... shape=(D,),
... plates=(1,100),
... name='X')
```

Note that the shape of X is (D,), although the latent dimensions are marked with a plate in the graphical model and they are conditionally independent in the prior. However, we want to (and need to) model the posterior dependency of the latent dimensions, thus we cannot factorize them, which would happen if we used plates=(1,100,D) and shape=(). The first plate axis with size 1 is given just for clarity.

The prior for the ARD parameters  $\alpha$  of the loading matrix:

```
>>> alpha = Gamma(1e-3, 1e-3, ... plates=(D,), ... name='alpha')
```

The prior for the loading matrix C:

Again, note that the shape is the same as for X for the same reason. Also, the plates of alpha, (D, ), are mapped to the full shape of the node C, (10, 1, D), using standard broadcasting rules.

The dot product is just a deterministic node:

```
>>> F = Dot(C, X)
```

However, note that Dot requires that the input Gaussian nodes have the same shape and that this shape has exactly one axis, that is, the variables are vectors. This the reason why we used shape (D, ) for X and C but from a bit different perspective. The node computes the inner product of D-dimensional vectors resulting in plates (10,100) broadcasted from the plates (1,100) and (10,1):

```
>>> F.plates (10, 100)
```

The prior for the observation noise  $\tau$ :

```
>>> tau = Gamma(1e-3, 1e-3, name='tau')
```

Finally, the observations are conditionally independent Gaussian scalars:

```
>>> Y = GaussianARD(F, tau, name='Y')
```

Now we have defined our model and the next step is to observe some data and to perform inference.

# 2.4 Performing inference

Approximation of the posterior distribution can be divided into several steps:

- Observe some nodes
- Choose the inference engine
- Initialize the posterior approximation
- Run the inference algorithm

In order to illustrate these steps, we'll be using the PCA model constructed in the previous section.

### 2.4.1 Observing nodes

First, let us generate some toy data:

```
>>> c = np.random.randn(10, 2)
>>> x = np.random.randn(2, 100)
>>> data = np.dot(c, x) + 0.1*np.random.randn(10, 100)
```

The data is provided by simply calling observe method of a stochastic node:

```
>>> Y.observe(data)
```

It is important that the shape of the data array matches the plates and shape of the node Y. For instance, if Y was Wishart node for  $3 \times 3$  matrices with plates (5, 1, 10), the full shape of Y would be (5, 1, 10, 3, 3). The data array should have this shape exactly, that is, no broadcasting rules are applied.

#### Missing values

It is possible to mark missing values by providing a mask which is a boolean array:

```
>>> Y.observe(data, mask=[[True], [False], [False], [True], [True], ... [False], [True], [True], [True], [False]])
```

True means that the value is observed and False means that the value is missing. The shape of the above mask is (10,1), which broadcasts to the plates of Y, (10,100). Thus, the above mask means that the second, third, sixth and tenth rows of the  $10 \times 100$  data matrix are missing.

The mask is applied to the *plates*, not to the data array directly. This means that it is not possible to observe a random variable partially, each repetition defined by the plates is either fully observed or fully missing. Thus, the mask is

applied to the plates. It is often possible to circumvent this seemingly tight restriction by adding an observable child node which factorizes more.

The shape of the mask is broadcasted to plates using standard NumPy broadcasting rules. So, if the variable has plates (5,1,10), the mask could have a shape (),(1,),(1,1),(1,1,1),(10,),(1,10),(1,10),(5,1,1) or (5,1,10). In order to speed up the inference, missing values are automatically integrated out if they are not needed as latent variables to child nodes. This leads to faster convergence and more accurate approximations.

### 2.4.2 Choosing the inference method

Inference methods can be found in bayespy.inference package. Currently, only variational Bayesian approximation is implemented (bayespy.inference.VB). The inference engine is constructed by giving the stochastic nodes of the model.

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, C, X, alpha, tau)
```

There is no need to give any deterministic nodes. Currently, the inference engine does not automatically search for stochastic parents and children, thus it is important that all stochastic nodes of the model are given. This should be made more robust in future versions.

A node of the model can be obtained by using the name of the node as a key:

```
>>> Q['X'] <br/>
<
```

Note that the returned object is the same as the node object itself:

```
>>> Q['X'] is X True
```

Thus, one may use the object X when it is available. However, if the model and the inference engine are constructed in another function or module, the node object may not be available directly and this feature becomes useful.

### 2.4.3 Initializing the posterior approximation

The inference engines give some initialization to the stochastic nodes by default. However, the inference algorithms can be sensitive to the initialization, thus it is sometimes necessary to have better control over the initialization. For VB, the following initialization methods are available:

- initialize\_from\_prior: Use the current states of the parent nodes to update the node. This is the default initialization.
- initialize\_from\_parameters: Use the given parameter values for the distribution.
- initialize\_from\_value: Use the given value for the variable.
- initialize\_from\_random: Draw a random value for the variable. The random sample is drawn from the current state of the node's distribution.

Note that initialize\_from\_value and initialize\_from\_random initialize the distribution with a value of the variable instead of parameters of the distribution. Thus, the distribution is actually a delta distribution with a peak on the value after the initialization. This state of the distribution does not have proper natural parameter values nor normalization, thus the VB lower bound terms are np.nan for this initial state.

These initialization methods can be used to perform even a bit more complex initializations. For instance, a Gaussian distribution could be initialized with a random mean and variance 0.1. In our PCA model, this can be obtained by

```
>>> X.initialize_from_parameters(np.random.randn(1, 100, D), 10)
```

Note that the shape of the random mean is the sum of the plates (1, 100) and the variable shape (D,). In addition, instead of variance, GaussianARD uses precision as the second parameter, thus we initialized the variance to  $\frac{1}{10}$ . This random initialization is important in our PCA model because the default initialization gives C and X zero mean. If the mean of the other variable was zero when the other is updated, the other variable gets zero mean too. This would lead to an update algorithm where both means remain zeros and effectively no latent space is found. Thus, it is important to give non-zero random initialization for X if C is updated before X the first time. It is typical that at least some nodes need be initialized with some randomness.

By default, nodes are initialized with the method initialize\_from\_prior. The method is not very time consuming but if for any reason you want to avoid that default initialization computation, you can provide initialize=False when creating the stochastic node. However, the node does not have a proper state in that case, which leads to errors in VB learning unless the distribution is initialized using the above methods.

### 2.4.4 Running the inference algorithm

The approximation methods are based on iterative algorithms, which can be run using update method. By default, it takes one iteration step updating all nodes once:

```
>>> Q.update()
Iteration 1: loglike=-9.305259e+02 (... seconds)
```

The loglike tells the VB lower bound. The order in which the nodes are updated is the same as the order in which the nodes were given when creating Q. If you want to change the order or update only some of the nodes, you can give as arguments the nodes you want to update and they are updated in the given order:

```
>>> Q.update(C, X)
Iteration 2: loglike=-8.818976e+02 (... seconds)
```

It is also possible to give the same node several times:

```
>>> Q.update(C, X, C, tau)
Iteration 3: loglike=-8.071222e+02 (... seconds)
```

Note that each call to update is counted as one iteration step although not variables are necessarily updated. Instead of doing one iteration step, repeat keyword argument can be used to perform several iteration steps:

```
>>> Q.update(repeat=10)

Iteration 4: loglike=-7.167588e+02 (... seconds)

Iteration 5: loglike=-6.827873e+02 (... seconds)

Iteration 6: loglike=-6.259477e+02 (... seconds)

Iteration 7: loglike=-4.725400e+02 (... seconds)

Iteration 8: loglike=-3.270816e+02 (... seconds)

Iteration 9: loglike=-2.208865e+02 (... seconds)

Iteration 10: loglike=-1.658761e+02 (... seconds)

Iteration 11: loglike=-1.469468e+02 (... seconds)

Iteration 12: loglike=-1.420311e+02 (... seconds)

Iteration 13: loglike=-1.405139e+02 (... seconds)
```

The VB algorithm stops automatically if it converges, that is, the relative change in the lower bound is below some threshold:

```
>>> Q.update(repeat=1000)
Iteration 14: loglike=-1.396481e+02 (... seconds)
...
Iteration 488: loglike=-1.224106e+02 (... seconds)
Converged at iteration 488.
```

Now the algorithm stopped before taking 1000 iteration steps because it converged. The relative tolerance can be adjusted by providing tol keyword argument to the update method:

```
>>> Q.update(repeat=10000, tol=1e-6)
Iteration 489: loglike=-1.224094e+02 (... seconds)
...
Iteration 847: loglike=-1.222506e+02 (... seconds)
Converged at iteration 847.
```

Making the tolerance smaller, may improve the result but it may also significantly increase the iteration steps until convergence.

Instead of using update method of the inference engine VB, it is possible to use the update methods of the nodes directly as

```
>>> C.update()
or
>>> Q['C'].update()
```

However, this is not recommended, because the update method of the inference engine VB is a wrapper which, in addition to calling the nodes' update methods, checks for convergence and does a few other useful minor things. But if for any reason these direct update methods are needed, they can be used.

#### **Parameter expansion**

Sometimes the VB algorithm converges very slowly. This may happen when the variables are strongly coupled in the true posterior but factorized in the approximate posterior. This coupling leads to zigzagging of the variational parameters which progresses slowly. One solution to this problem is to use parameter expansion. The idea is to add an auxiliary variable which parameterizes the posterior approximation of several variables. Then optimizing this auxiliary variable actually optimizes several posterior approximations jointly leading to faster convergence.

The parameter expansion is model specific. Currently in BayesPy, only state-space models have built-in parameter expansions available. These state-space models contain a variable which is a dot product of two variables (plus some noise):

$$y = \mathbf{c}^T \mathbf{x} + \text{noise}$$

The parameter expansion can be motivated by noticing that we can add an auxiliary variable which rotates the variables  $\mathbf{c}$  and  $\mathbf{x}$  so that the dot product is unaffected:

$$y = \mathbf{c}^T \mathbf{x} + \text{noise} = \mathbf{c}^T \mathbf{R} \mathbf{R}^{-1} \mathbf{x} + \text{noise} = (\mathbf{R}^T \mathbf{c})^T (\mathbf{R}^{-1} \mathbf{x}) + \text{noise}$$

Now, applying this rotation to the posterior approximations  $q(\mathbf{c})$  and  $q(\mathbf{x})$ , and optimizing the VB lower bound with respect to the rotation leads to parameterized joint optimization of  $\mathbf{c}$  and  $\mathbf{x}$ .

The available parameter expansion methods are in module transformations:

```
>>> from bayespy.inference.vmp import transformations
```

First, you create the rotation transformations for the two variables:

```
>>> rotX = transformations.RotateGaussianARD(X)
>>> rotC = transformations.RotateGaussianARD(C, alpha)
```

Here, the rotation for C provides the ARD parameters alpha so they are updated simultaneously. In addition to RotateGaussianARD, there are a few other built-in rotations defined, for instance, RotateGaussian and RotateGaussianMarkovChain. It is extremely important that the model satisfies the assumptions made by

the rotation class and the user is mostly responsible for this. The optimizer for the rotations is constructed by giving the two rotations and the dimensionality of the rotated space:

```
>>> R = transformations.RotationOptimizer(rotC, rotX, D)
```

Now, calling rotate method will find optimal rotation and update the relevant nodes (X, C and alpha) accordingly:

```
>>> R.rotate()
```

Let us see how our iteration would have gone if we had used this parameter expansion. First, let us re-initialize our nodes and VB algorithm:

```
>>> alpha.initialize_from_prior()
>>> C.initialize_from_prior()
>>> X.initialize_from_parameters(np.random.randn(1, 100, D), 10)
>>> tau.initialize_from_prior()
>>> Q = VB(Y, C, X, alpha, tau)
```

Then, the rotation is set to run after each iteration step:

```
>>> Q.callback = R.rotate
```

Now the iteration converges to the relative tolerance  $10^{-6}$  much faster:

```
>>> Q.update(repeat=1000, tol=1e-6)
Iteration 1: loglike=-9.363500e+02 (... seconds)
...
Iteration 18: loglike=-1.221354e+02 (... seconds)
Converged at iteration 18.
```

The convergence took 18 iterations with rotations and 488 or 847 iterations without the parameter expansion. In addition, the lower bound is improved slightly. One can compare the number of iteration steps in this case because the cost per iteration step with or without parameter expansion is approximately the same. Sometimes the parameter expansion can have the drawback that it converges to a bad local optimum. Usually, this can be solved by updating the nodes near the observations a few times before starting to update the hyperparameters and to use parameter expansion. In any case, the parameter expansion is practically necessary when using state-space models in order to converge to a proper solution in a reasonable time.

# 2.5 Examining the results

After the results have been obtained, it is important to be able to examine the results easily. The results can be examined either numerically by inspecting numerical arrays or visually by plotting distributions of the nodes. In addition, the posterior distributions can be visualized during the learning algorithm and the results can saved into a file.

### 2.5.1 Plotting the results

The module plot offers some plotting basic functionality:

```
>>> import bayespy.plot as bpplt
```

The module contains matplotlib.pyplot module if the user needs that. For instance, interactive plotting can be enabled as:

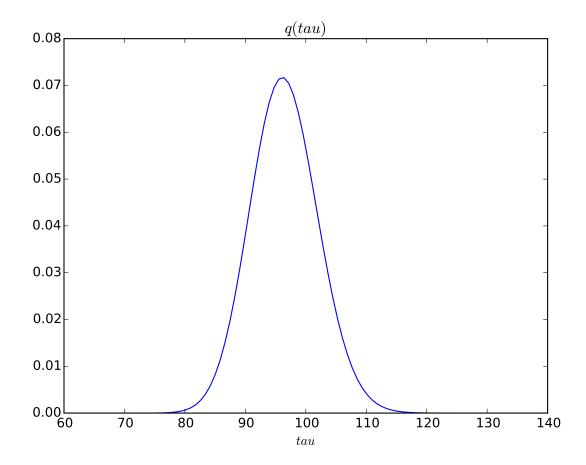
```
>>> bpplt.pyplot.ion()
```

The plot module contains some functions but it is not a very comprehensive collection, thus the user may need to write some problem- or model-specific plotting functions. The current collection is:

- pdf (): show probability density function of a scalar
- contour(): show probability density function of two-element vector
- hinton(): show the Hinton diagram
- plot (): show value as a function

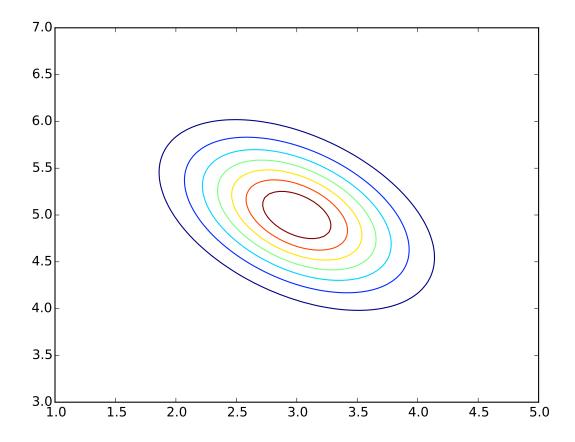
The probability density function of a scalar random variable can be plotted using the function pdf():

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pdf(Q['tau'], np.linspace(60, 140, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```



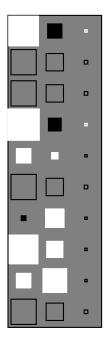
The variable tau models the inverse variance of the noise, for which the true value is  $0.1^{-2} = 100$ . Thus, the posterior captures the true value quite accurately. Similarly, the function contour () can be used to plot the probability density function of a 2-dimensional variable, for instance:

```
>>> V = Gaussian([3, 5], [[4, 2], [2, 5]])
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.contour(V, np.linspace(1, 5, num=100), np.linspace(3, 7, num=100))
<matplotlib.contour.QuadContourSet object at 0x...>
```



Both pdf() and contour() require that the user provides the grid on which the probability density function is computed. They also support several keyword arguments for modifying the output, similarly as plot and contour in matplotlib.pyplot. These functions can be used only for stochastic nodes. A few other plot types are also available as built-in functions. A Hinton diagram can be plotted as:

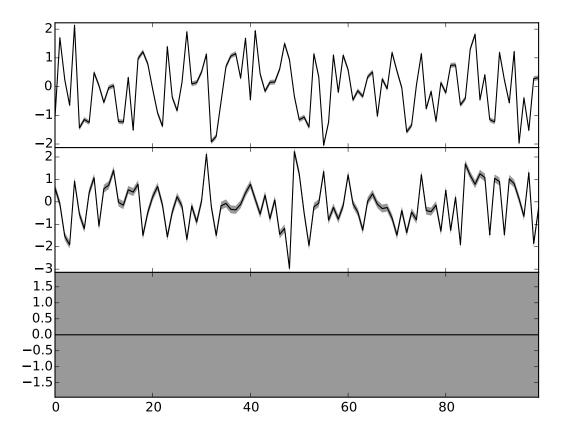
```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(C)
```



The diagram shows the elements of the matrix C. The size of the filled rectangle corresponds to the absolute value of the element mean, and white and black correspond to positive and negative values, respectively. The non-filled rectangle shows standard deviation. From this diagram it is clear that the third column of C has been pruned out and the rows that were missing in the data have zero mean and column-specific variance. The function  $\min()$  is a simple wrapper for node-specific Hinton diagram plotters, such as  $\texttt{gaussian\_hinton}()$  and  $\texttt{dirichlet\_hinton}()$ . Thus, the keyword arguments depend on the node which is plotted.

Another plotting function is plot (), which just plots the values of the node over one axis as a function:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(X, axis=-2)
```



Now, the axis is the second last axis which corresponds to  $n=0,\ldots,N-1$ . As D=3, there are three subplots. For Gaussian variables, the function shows the mean and two standard deviations. The plot shows that the third component has been pruned out, thus the method has been able to recover the true dimensionality of the latent space. It also has similar keyword arguments to plot function in matplotlib.pyplot. Again, plot () is a simple wrapper over node-specific plotting functions, thus it supports only some node classes.

### 2.5.2 Monitoring during the inference algorithm

It is possible to plot the distribution of the nodes during the learning algorithm. This is useful when the user is interested to see how the distributions evolve during learning and what is happening to the distributions. In order to utilize monitoring, the user must set plotters for the nodes that he or she wishes to monitor. This can be done either when creating the node or later at any time.

The plotters are set by creating a plotter object and providing this object to the node. The plotter is a wrapper of one of the plotting functions mentioned above: PDFPlotter, ContourPlotter, HintonPlotter or FunctionPlotter. Thus, our example model could use the following plotters:

```
>>> tau.set_plotter(bpplt.PDFPlotter(np.linspace(60, 140, num=100)))
>>> C.set_plotter(bpplt.HintonPlotter())
>>> X.set_plotter(bpplt.FunctionPlotter(axis=-2))
```

These could have been given at node creation as a keyword argument plotter:

```
>>> V = Gaussian([3, 5], [[4, 2], [2, 5]],
... plotter=bpplt.ContourPlotter(np.linspace(1, 5, num=100),
... np.linspace(3, 7, num=100)))
```

When the plotter is set, one can use the plot method of the node to perform plotting:

```
>>> V.plot()
<matplotlib.contour.QuadContourSet object at 0x...>
```

Nodes can also be plotted using the plot method of the inference engine:

```
>>> Q.plot('C')
```

This method remembers the figure in which a node has been plotted and uses that every time it plots the same node. In order to monitor the nodes during learning, it is possible to use the keyword argument plot:

```
>>> Q.update(repeat=5, plot=True, tol=np.nan)
Iteration 19: loglike=-1.221354e+02 (... seconds)
Iteration 20: loglike=-1.221354e+02 (... seconds)
Iteration 21: loglike=-1.221354e+02 (... seconds)
Iteration 22: loglike=-1.221354e+02 (... seconds)
Iteration 23: loglike=-1.221354e+02 (... seconds)
```

Each node which has a plotter set will be plotted after it is updated. Note that this may slow down the inference significantly if the plotting operation is time consuming.

### 2.5.3 Posterior parameters and moments

If the built-in plotting functions are not sufficient, it is possible to use matplotlib.pyplot for custom plotting. Each node has get\_moments method which returns the moments and they can be used for plotting. Stochastic exponential family nodes have natural parameter vectors which can also be used. In addition to plotting, it is also possible to just print the moments or parameters in the console.

### 2.5.4 Saving and loading results

The results of the inference engine can be easily saved and loaded using VB.save() and VB.load() methods:

```
>>> Q.save(filename='tmp.hdf5')
>>> Q.load(filename='tmp.hdf5')
```

The results are stored in a HDF5 file. The user may set an autosave file in which the results are automatically saved regularly. Autosave filename can be set at creation time by autosave\_filename keyword argument or later using VB.set\_autosave() method. If autosave file has been set, the VB.save() and VB.load() methods use that file by default. In order for the saving to work, all stochastic nodes must have been given (unique) names.

However, note that these methods do *not* save nor load the node definitions. It means that the user must create the nodes and the inference engine and then use VB.load() to set the state of the nodes and the inference engine. If there are any differences in the model that was saved and the one which is tried to update using loading, then loading does not work. Thus, the user should keep the model construction unmodified in a Python file in order to be able to load the results later. Or if the user wishes to share the results, he or she must share the model construction Python file with the HDF5 results file.

# 2.6 Advanced topics

This section contains brief information on how to implement some advanced methods in BayesPy. These methods include Riemannian conjugate gradient methods, pattern search, simulated annealing, collapsed variational inference, stochastic variational inference and black box variational inference. In order to use these methods properly, the user should understand them to some extent. They are also considered experimental, thus you may encounter bugs or unimplemented features. In any case, these methods may provide huge performance improvements easily compared to the standard VB-EM algorithm.

### 2.6.1 Gradient-based optimization

Variational Bayesian learning basically means that the parameters of the approximate posterior distributions are optimized to maximize the lower bound of the marginal log likelihood [3]. This optimization can be done by using gradient-based optimization methods. In order to improve the gradient-based methods, it is recommended to take into account the information geometry by using the Riemannian (a.k.a. natural) gradient. In fact, the standard VB-EM algorithm is equivalent to a gradient ascent method which uses the Riemannian gradient and step length 1. Thus, it is natural to try to improve this method by using non-linear conjugate gradient methods instead of gradient ascent. These optimization methods are especially useful when the VB-EM update equations are not available but one has to use fixed form approximation. But it is possible that the Riemannian conjugate gradient method improve performance even when the VB-EM update equations are available.

The optimization algorithm in VB.optimize() has a simple interface. Instead of using the default Riemannian geometry, one can use the Euclidean geometry by giving riemannian=False. It is also possible to choose the optimization method from gradient ascent (method='gradient') or conjugate gradient methods (only method='fletcher-reeves' implemented at the moment). For instance, we could optimize nodes C and X jointly using Euclidean gradient ascent as:

```
>>> Q = VB(Y, C, X, alpha, tau)
>>> Q.optimize(C, X, riemannian=False, method='gradient', maxiter=5)
Iteration ...
```

Note that this is very inefficient way of updating those nodes (bad geometry and not using conjugate gradients). Thus, one should understand the idea of these optimization methods, otherwise one may do something extremely inefficient. Most likely this method can be found useful in combination with the advanced tricks in the following sections.

**Note:** The Euclidean gradient has not been implemented for all nodes yet. The Euclidean gradient is required by the Euclidean geometry based optimization but also by the conjugate gradient methods in the Riemannian geometry. Thus, the Riemannian conjugate gradient may not yet work for all models.

It is possible to construct custom optimization algorithms with the tools provided by VB. For instance, VB.get\_parameters() and VB.set\_parameters() can be used to handle the parameters of nodes. VB.get\_gradients() is used for computing the gradients of nodes. The parameter and gradient objects are not numerical arrays but more complex nested lists not meant to be accessed by the user. Thus, for simple arithmetics with the parameter and gradient objects, use functions VB.add() and VB.dot(). Finally, VB.compute\_lowerbound() and VB.has\_converged() can be used to monitor the lower bound.

#### 2.6.2 Collapsed inference

The optimization method can be used efficiently in such a way that some of the variables are collapsed, that is, marginalized out [1]. The collapsed variables must be conditionally independent given the observations and all other variables. Probably, one also wants that the size of the marginalized variables is large and the size of the optimized variables is small. For instance, in our PCA example, we could optimize as follows:

```
>>> Q.optimize(C, tau, maxiter=10, collapsed=[X, alpha])
Iteration ...
```

The collapsed variables are given as a list. This optimization does basically the following: It first computes the gradients for C and tau and takes an update step using the desired optimization method. Then, it updates the collapsed variables by using the standard VB-EM update equations. These two steps are taken in turns. Effectively, this corresponds to collapsing the variables X and alpha in a particular way. The point of this method is that the number of parameters in the optimization reduces significantly and the collapsed variables are updated optimally. For more details, see [1].

It is possible to use this method in such a way, that the collapsed variables are not conditionally independent given the observations and all other variables. However, in that case, the method does not anymore correspond to collapsing the variables but just using VB-EM updates after gradient-based updates. The method does not check for conditional independence, so the user is free to do this.

**Note:** Although the Riemannian conjugate gradient method has not yet been implemented for all nodes, it may be possible to collapse those nodes and optimize the other nodes for which the Euclidean gradient is already implemented.

#### 2.6.3 Pattern search

The pattern search method estimates the direction in which the approximate posterior distributions are updating and performs a line search in that direction [4]. The search direction is based on the difference in the VB parameters on successive updates (or several updates). The idea is that the VB-EM algorithm may be slow because it just zigzags and this can be fixed by moving to the direction in which the VB-EM is slowly moving.

BayesPy offers a simple built-in pattern search method VB.pattern\_search(). The method updates the nodes twice, measures the difference in the parameters and performs a line search with a small number of function evaluations:

```
>>> Q.pattern_search(C, X)
Iteration ...
```

Similarly to the collapsed optimization, it is possible to collapse some of the variables in the pattern search. The same rules of conditional independence apply as above. The collapsed variables are given as list:

```
>>> Q.pattern_search(C, tau, collapsed=[X, alpha])
Iteration ...
```

Also, a maximum number of iterations can be set by using maxiter keyword argument. It is not always obvious whether a pattern search will improve the rate of convergence or not but if it seems that the convergence is slow because of zigzagging, it may be worth a try. Note that the computational cost of the pattern search is quite high, thus it is not recommended to perform it after every VB-EM update but every now and then, for instance, after every 10 iterations. In addition, it is possible to write a more customized VB learning algorithm which uses pattern searches by using the different methods of VB discussed above.

### 2.6.4 Deterministic annealing

The standard VB-EM algorithm converges to a local optimum which can often be inferior to the global optimum and many other local optima. Deterministic annealing aims at finding a better local optimum, hopefully even the global optimum [5]. It does this by increasing the weight on the entropy of the posterior approximation in the VB lower bound. Effectively, the annealed lower bound becomes closer to a uniform function instead of the original multimodal lower bound. The weight on the entropy is recovered slowly and the optimization is much more robust to initialization.

In BayesPy, the annealing can be set by using  $VB.set\_annealing()$ . The given annealing should be in range (0,1] but this is not validated in case the user wants to do something experimental. If annealing is set to 1, the original

VB lower bound is recovered. Annealing with 0 would lead to an improper uniform distribution, thus it will lead to errors. The entropy term is weighted by the inverse of this annealing term. An alternative view is that the model probability density functions are raised to the power of the annealing term.

Typically, the annealing is used in such a way that the annealing is small at the beginning and increased after every convergence of the VB algorithm until value 1 is reached. After the annealing value is increased, the algorithm continues from where it had just converged. The annealing can be used for instance as:

```
>>> beta = 0.1
>>> while beta < 1.0:
...     beta = min(beta*1.5, 1.0)
...     Q.set_annealing(beta)
...     Q.update(repeat=100, tol=1e-4)
Iteration ...</pre>
```

Here, the tol keyword argument is used to adjust the threshold for convergence. In this case, it is a bit larger than by default so the algorithm does not need to converge perfectly but a rougher convergence is sufficient for the next iteration with a new annealing value.

#### 2.6.5 Stochastic variational inference

In stochastic variational inference [2], the idea is to use mini-batches of large datasets to compute noisy gradients and learn the VB distributions by using stochastic gradient ascent. In order for it to be useful, the model must be such that it can be divided into "intermediate" and "global" variables. The number of intermediate variables increases with the data but the number of global variables remains fixed. The global variables are learnt in the stochastic optimization.

By denoting the data as  $Y = [Y_1, \dots, Y_N]$ , the intermediate variables as  $Z = [Z_1, \dots, Z_N]$  and the global variables as  $\theta$ , the model needs to have the following structure:

$$p(Y, Z, \theta) = p(\theta) \prod_{n=1}^{N} p(Y_n | Z_n, \theta) p(Z_n | \theta)$$

The algorithm consists of three steps which are iterated: 1) a random mini-batch of the data is selected, 2) the corresponding intermediate variables are updated by using normal VB update equations, and 3) the global variables are updated with (stochastic) gradient ascent as if there was as many replications of the mini-batch as needed to recover the original dataset size.

The learning rate for the gradient ascent must satisfy:

$$\sum_{i=1}^{\infty} \alpha_i = \infty \quad \text{and} \quad \sum_{i=1}^{\infty} \alpha^2 < \infty,$$

where i is the iteration number. An example of a valid learning parameter is  $\alpha_i = (\delta + i)^{-\gamma}$ , where  $\delta \ge 0$  is a delay and  $\gamma \in (0.5, 1]$  is a forgetting rate.

Stochastic variational inference is relatively easy to use in BayesPy. The idea is that the user creates a model for the size of a mini-batch and specifies a multiplier for those plate axes that are replicated. For the PCA example, the mini-batch model can be costructed as follows. We decide to use X as an intermediate variable and the other variables are global. The global variables alpha, C and tau are constructed identically as before. The intermediate variable X is constructed as:

Note that the plates are (1, 5) whereas they are (1, 100) in the full model. Thus, we need to provide a plates multiplier (1, 20) to define how the plates are replicated to get the full dataset. These multipliers do not need to be integers, in this case the latter plate axis is multiplied by 100/5 = 20. The remaining variables are defined as before:

```
>>> F = Dot(C, X)
>>> Y = GaussianARD(F, tau, name='Y')
```

Note that the plates of Y and F also correspond to the size of the mini-batch and they also deduce the plate multipliers from their parents, thus we do not need to specify the multiplier here explicitly (although it is ok to do so).

Let us construct the inference engine for the new mini-batch model:

```
>>> Q = VB(Y, C, X, alpha, tau)
```

Use random initialization for C to break the symmetry in C and X:

```
>>> C.initialize_from_random()
```

Then, stochastic variational inference algorithm could look as follows:

First, we ignore the bound checks because they are noisy. Then, the loop consists of three parts: 1) Draw a random mini-batch of the data (5 samples from 100). 2) Update the intermediate variable X. 3) Update global variables with gradient ascent using a proper learning rate.

#### 2.6.6 Black-box variational inference

TODO

**CHAPTER** 

**THREE** 

### **EXAMPLES**

# 3.1 Linear regression

#### 3.1.1 Data

The true parameters of the linear regression:

```
>>> k = 2 # slope
>>> c = 5 # bias
>>> s = 2 # noise standard deviation
```

#### Generate data:

```
>>> import numpy as np
>>> x = np.arange(10)
>>> y = k*x + c + s*np.random.randn(10)
```

### 3.1.2 **Model**

The regressors, that is, the input data:

```
>>> X = np.vstack([x, np.ones(len(x))]).T
```

Note that we added a column of ones to the regressor matrix for the bias term. We model the slope and the bias term in the same node so we do not factorize between them:

```
>>> from bayespy.nodes import GaussianARD
>>> B = GaussianARD(0, 1e-6, shape=(2,))
```

The first element is the slope which multiplies x and the second element is the bias term which multiplies the constant ones. Now we compute the dot product of X and B:

```
>>> from bayespy.nodes import SumMultiply
>>> F = SumMultiply('i,i', B, X)
```

The noise parameter:

```
>>> from bayespy.nodes import Gamma
>>> tau = Gamma(1e-3, 1e-3)
```

The noisy observations:

```
>>> Y = GaussianARD(F, tau)
```

### 3.1.3 Inference

Observe the data:

```
>>> Y.observe(y)
```

Construct the variational Bayesian (VB) inference engine by giving all stochastic nodes:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, B, tau)
```

Iterate until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-4.595948e+01 (... seconds)
...
Iteration 5: loglike=-4.495017e+01 (... seconds)
Converged at iteration 5.
```

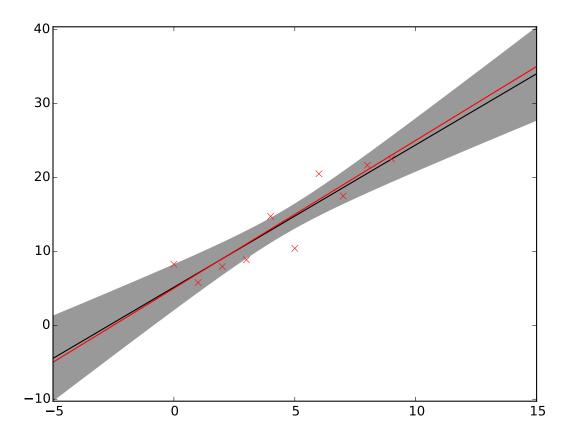
#### 3.1.4 Results

Create a simple predictive model for new inputs:

```
>>> xh = np.linspace(-5, 15, 100)
>>> Xh = np.vstack([xh, np.ones(len(xh))]).T
>>> Fh = SumMultiply('i,i', B, Xh)
```

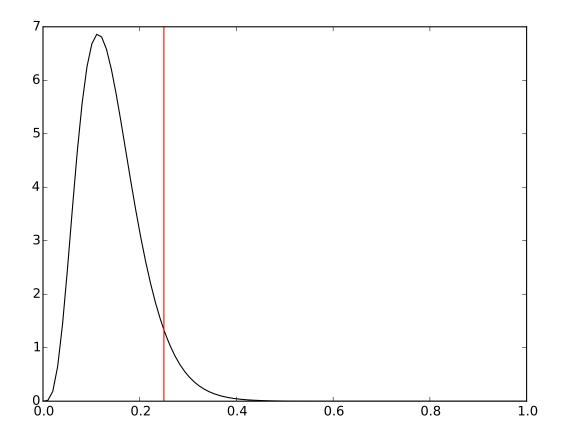
Note that we use the learned node B but create a new regressor array for predictions. Plot the predictive distribution of noiseless function values:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(Fh, x=xh, scale=2)
>>> bpplt.plot(y, x=x, color='r', marker='x', linestyle='None')
>>> bpplt.plot(k*xh+c, x=xh, color='r');
```

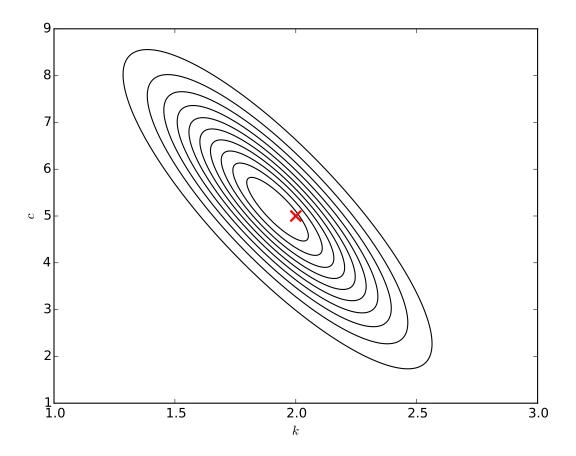


Note that the above plot shows two standard deviation of the posterior of the noiseless function, thus the data points may lie well outside this range. The red line shows the true linear function. Next, plot the distribution of the noise parameter and the true value,  $2^{-2} = 0.25$ :

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pdf(tau, np.linspace(1e-6,1,100), color='k')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.axvline(s**(-2), color='r')
<matplotlib.lines.Line2D object at 0x...>
```



The noise level is captured quite well, although the posterior has more mass on larger noise levels (smaller precision parameter values). Finally, plot the distribution of the regression parameters and mark the true value:



In this case, the true parameters are captured well by the posterior distribution.

### 3.1.5 Improving accuracy

The model can be improved by not factorizing between B and tau but learning their joint posterior distribution. This requires a slight modification to the model by using GaussianGammaISO node:

```
>>> from bayespy.nodes import GaussianGammaISO
>>> B_tau = GaussianGammaISO(np.zeros(2), le-6*np.identity(2), le-3, le-3)
```

This node contains both the regression parameter vector and the noise parameter. We compute the dot product similarly as before:

```
>>> F_tau = SumMultiply('i,i', B_tau, X)
```

However, Y is constructed as follows:

```
>>> Y = GaussianARD (F_tau, 1)
```

Because the noise parameter is already in F\_tau we can give a constant one as the second argument. The total noise parameter for Y is the product of the noise parameter in F\_tau and one. Now, inference is run similarly as before:

```
>>> Y.observe(y)
>>> Q = VB(Y, B_tau)
>>> Q.update(repeat=1000)
```

```
Iteration 1: loglike=-4.678478e+01 (... seconds)
Iteration 2: loglike=-4.678478e+01 (... seconds)
Converged at iteration 2.
```

Note that the method converges immediately. This happens because there is only one unobserved stochastic node so there is no need for iteration and the result is actually the exact true posterior distribution, not an approximation. Currently, the main drawback of using this approach is that BayesPy does not yet contain any plotting utilities for nodes that contain both Gaussian and gamma variables jointly.

### 3.1.6 Further extensions

The approach discussed in this example can easily be extended to non-linear regression and multivariate regression. For non-linear regression, the inputs are first transformed by some known non-linear functions and then linear regression is applied to this transformed data. For multivariate regression, X and B are concatenated appropriately: If there are more regressors, add more columns to both X and B. If there are more output dimensions, add plates to B.

### 3.2 Gaussian mixture model

This example demonstrates the use of Gaussian mixture model for flexible density estimation, clustering or classification.

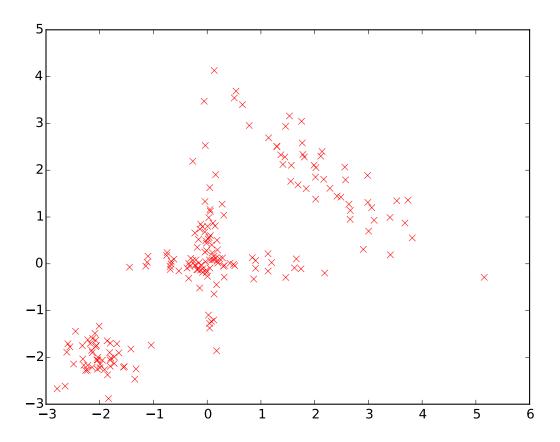
### 3.2.1 Data

First, let us generate some artificial data for the analysis. The data are two-dimensional vectors from one of the four different Gaussian distributions:

```
>>> import numpy as np
>>> y0 = np.random.multivariate_normal([0, 0], [[2, 0], [0, 0.1]], size=50)
>>> y1 = np.random.multivariate_normal([0, 0], [[0.1, 0], [0, 2]], size=50)
>>> y2 = np.random.multivariate_normal([2, 2], [[2, -1.5], [-1.5, 2]], size=50)
>>> y3 = np.random.multivariate_normal([-2, -2], [[0.5, 0], [0, 0.5]], size=50)
>>> y = np.vstack([y0, y1, y2, y3])
```

Thus, there are 200 data vectors in total. The data looks as follows:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'rx')
[<matplotlib.lines.Line2D object at 0x...>]
```



### 3.2.2 **Model**

For clarity, let us denote the number of the data vectors with  $\ensuremath{\mathbb{N}}$ 

```
>>> N = 200
```

and the dimensionality of the data vectors with D:

```
>>> D = 2
```

We will use a "large enough" number of Gaussian clusters in our model:

```
>>> K = 10
```

Cluster assignments Z and the prior for the cluster assignment probabilities alpha:

The mean vectors and the precision matrices of the clusters:

If either the mean or precision should be shared between clusters, then that node should not have plates, that is, plates=(). The data vectors are from a Gaussian mixture with cluster assignments Z and Gaussian component parameters mu and Lambda:

```
>>> from bayespy.nodes import Mixture
>>> Y = Mixture(Z, Gaussian, mu, Lambda,
... name='Y')
>>> Z.initialize_from_random()
>>> from bayespy.inference import VB
>>> Q = VB(Y, mu, Lambda, Z, alpha)
```

### 3.2.3 Inference

Before running the inference algorithm, we provide the data:

```
>>> Y.observe(y)
```

Then, run VB iteration until convergence:

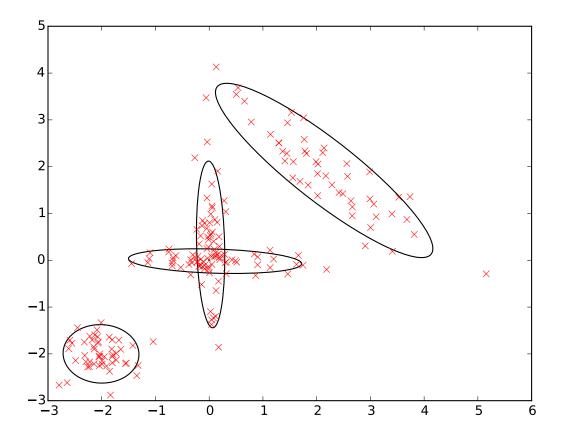
```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-1.401968e+03 (... seconds)
...
Iteration 48: loglike=-1.017893e+03 (... seconds)
Converged at iteration 48.
```

The algorithm converges very quickly. Note that the default update order of the nodes was such that mu and Lambda were updated before Z, which is what we wanted because Z was initialized randomly.

### 3.2.4 Results

For two-dimensional Gaussian mixtures, the mixture components can be plotted using gaussian\_mixture():

>>> bpplt.gaussian\_mixture(Y, scale=2)



The function is called with scale=2 which means that each ellipse shows two standard deviations. From the ten cluster components, the model uses effectively the correct number of clusters (4). These clusters capture the true density accurately.

In addition to clustering and density estimation, this model could also be used for classification by setting the known class assignments as observed.

### 3.2.5 Advanced next steps

### Joint node for mean and precision

The next step for improving the results could be to use GaussianWishart node for modelling the mean vectors mu and precision matrices Lambda jointly without factorization. This should improve the accuracy of the posterior approximation and the speed of the VB estimation. However, the implementation is a bit more complex.

### Fast collapsed inference

### 3.3 Bernoulli mixture model

This example considers data generated from a Bernoulli mixture model. One simple example process could be a questionnaire for election candidates. We observe a set of binary vectors, where each vector represents a candidate in the election and each element in these vectors correspond to a candidate's answer to a yes-or-no question. The goal is to find groups of similar candidates and analyze the answer patterns of these groups.

### 3.3.1 Data

First, we generate artificial data to analyze. Let us assume that the questionnaire contains ten yes-or-no questions. We assume that there are three groups with similar opinions. These groups could represent parties. These groups have the following answering patterns, which are represented by vectors with probabilities of a candidate answering yes to the questions:

```
>>> p0 = [0.1, 0.9, 0.1, 0.9, 0.1, 0.9, 0.1, 0.9, 0.1, 0.9]
>>> p1 = [0.1, 0.1, 0.1, 0.1, 0.1, 0.9, 0.9, 0.9, 0.9, 0.9]
>>> p2 = [0.9, 0.9, 0.9, 0.9, 0.9, 0.1, 0.1, 0.1, 0.1, 0.1]
```

Thus, the candidates in the first group are likely to answer no to questions 1, 3, 5, 7 and 9, and yes to questions 2, 4, 6, 8, 10. The candidates in the second group are likely to answer yes to the last five questions, whereas the candidates in the third group are likely to answer yes to the first five questions. For convenience, we form a NumPy array of these vectors:

```
>>> import numpy as np
>>> p = np.array([p0, p1, p2])
```

Next, we generate a hundred candidates. First, we randomly select the group for each candidate:

```
>>> from bayespy.utils import random
>>> z = random.categorical([1/3, 1/3, 1/3], size=100)
```

Using the group patterns, we generate yes-or-no answers for the candidates:

```
>>> x = random.bernoulli(p[z])
```

This is our simulated data to be analyzed.

### 3.3.2 Model

Now, we construct a model for learning the structure in the data. We have a dataset of hundred 10-dimensional binary vectors:

```
>>> N = 100
>>> D = 10
```

We will create a Bernoulli mixture model. We assume that the true number of groups is unknown to us, so we use a large enough number of clusters:

```
>>> K = 10
```

We use the categorical distribution for the group assignments and give the group assignment probabilities an uninformative Dirichlet prior:

```
>>> from bayespy.nodes import Categorical, Dirichlet
>>> R = Dirichlet(K*[1e-5],
... name='R')
>>> Z = Categorical(R,
... plates=(N,1),
... name='Z')
```

Each group has a probability of a yes answer for each question. These probabilities are given beta priors:

```
>>> from bayespy.nodes import Beta
>>> P = Beta([0.5, 0.5],
... plates=(D,K),
... name='P')
```

The answers of the candidates are modelled with the Bernoulli distribution:

```
>>> from bayespy.nodes import Mixture, Bernoulli
>>> X = Mixture(Z, Bernoulli, P)
```

Here, Z defines the group assignments and P the answering probability patterns for each group. Note how the plates of the nodes are matched: Z has plates (N, 1) and P has plates (D, K), but in the mixture node the last plate axis of P is discarded and thus the node broadcasts plates (N, 1) and (D, 1) resulting in plates (N, D) for X.

#### 3.3.3 Inference

In order to infer the variables in our model, we construct a variational Bayesian inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Z, R, X, P)
```

This also gives the default update order of the nodes. In order to find different groups, they must be initialized differently, thus we use random initialization for the group probability patterns:

```
>>> P.initialize_from_random()
```

We provide our simulated data:

```
>>> X.observe(x)
```

Now, we can run inference:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-6.872145e+02 (... seconds)
...
Iteration 17: loglike=-5.236921e+02 (... seconds)
Converged at iteration 17.
```

The algorithm converges in 17 iterations.

#### 3.3.4 Results

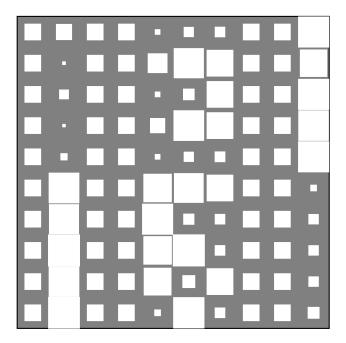
Now we can examine the approximate posterior distribution. First, let us plot the group assignment probabilities:

```
>>> import bayespy.plot as bpplt
>>> bpplt.hinton(R)
```



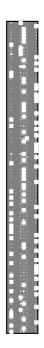
This plot shows that there are three dominant groups, which is equal to the true number of groups used to generate the data. However, there are still two smaller groups as the data does not give enough evidence to prune them out. The yes-or-no answer probability patterns for the groups can be plotted as:

```
>>> bpplt.hinton(P)
```



The three dominant groups have found the true patterns accurately. The patterns of the two minor groups some kind of mixtures of the three groups and they exist because the generated data happened to contain a few samples giving evidence for these groups. Finally, we can plot the group assignment probabilities for the candidates:

>>> bpplt.hinton(Z)



This plot shows the clustering of the candidates. It is possible to use  ${\tt HintonPlotter}$  to enable monitoring during the VB iteration by providing  ${\tt plotter=HintonPlotter}$  () for Z, P and R when creating the nodes.

### 3.4 Hidden Markov model

In this example, we will demonstrate the use of hidden Markov model in the case of known and unknown parameters. We will also use two different emission distributions to demonstrate the flexibility of the model construction.

### 3.4.1 Known parameters

This example follows the one presented in Wikipedia.

#### Model

Each day, the state of the weather is either 'rainy' or 'sunny'. The weather follows a first-order discrete Markov process. It has the following initial state probabilities

```
>>> a0 = [0.6, 0.4] # p(rainy)=0.6, p(sunny)=0.4
```

and state transition probabilities:

```
>>> A = [[0.7, 0.3], # p(rainy->rainy)=0.7, p(rainy->sunny)=0.3
... [0.4, 0.6]] # p(sunny->rainy)=0.4, p(sunny->sunny)=0.6
```

We will be observing one hundred samples:

```
>>> N = 100
```

The discrete first-order Markov chain is constructed as:

```
>>> from bayespy.nodes import CategoricalMarkovChain
>>> Z = CategoricalMarkovChain(a0, A, states=N)
```

However, instead of observing this process directly, we observe whether Bob is 'walking', 'shopping' or 'cleaning'. The probability of each activity depends on the current weather as follows:

```
>>> P = [[0.1, 0.4, 0.5], ... [0.6, 0.3, 0.1]]
```

where the first row contains activity probabilities on a rainy weather and the second row contains activity probabilities on a sunny weather. Using these emission probabilities, the observed process is constructed as:

```
>>> from bayespy.nodes import Categorical, Mixture
>>> Y = Mixture(Z, Categorical, P)
```

#### Data

In order to test our method, we'll generate artificial data from the model itself. First, draw realization of the weather process:

```
>>> weather = Z.random()
```

Then, using this weather, draw realizations of the activities:

```
>>> activity = Mixture(weather, Categorical, P).random()
```

#### Inference

Now, using this data, we set our variable Y to be observed:

```
>>> Y.observe(activity)
```

In order to run inference, we construct variational Bayesian inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, Z)
```

Note that we need to give all random variables to VB. In this case, the only random variables were Y and Z. Next we run the inference, that is, compute our posterior distribution:

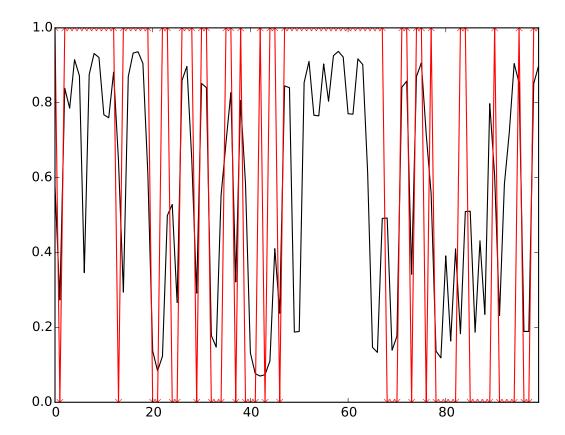
```
>>> Q.update()
Iteration 1: loglike=-1.095883e+02 (... seconds)
```

In this case, because there is only one unobserved random variable, we recover the exact posterior distribution and there is no need to iterate more than one step.

#### **Results**

One way to plot a 2-class categorical timeseries is to use the basic plot () function:

```
>>> import bayespy.plot as bpplt
>>> bpplt.plot(Z)
>>> bpplt.plot(1-weather, color='r', marker='x')
```



The black line shows the posterior probability of rain and the red line and crosses show the true state. Clearly, the method is not able to infer the weather very accurately in this case because the activies do not give that much information about the weather.

### 3.4.2 Unknown parameters

In this example, we consider unknown parameters for the Markov process and different emission distribution.

#### **Data**

We generate data from three 2-dimensional Gaussian distributions with different mean vectors and common standard deviation:

```
>>> import numpy as np
>>> mu = np.array([ [0,0], [3,4], [6,0] ])
>>> std = 2.0
```

Thus, the number of clusters is three:

```
>>> K = 3
```

And the number of samples is 200:

```
>>> N = 200
```

Each initial state is equally probable:

```
>>> p0 = np.ones(K) / K
```

State transition matrix is such that with probability 0.9 the process stays in the same state. The probability to move one of the other two states is 0.05 for both of those states.

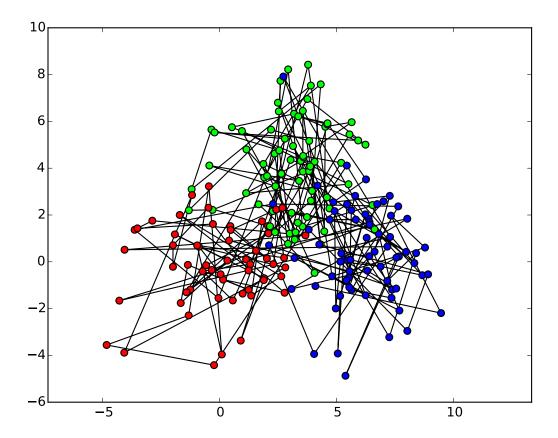
```
>>> q = 0.9
>>> r = (1-q) / (K-1)
>>> P = q*np.identity(K) + r*(np.ones((3,3))-np.identity(3))
```

Simulate the data:

```
>>> y = np.zeros((N,2))
>>> z = np.zeros(N)
>>> for n in range(N):
... z[n] = state
... y[n,:] = std*np.random.randn(2) + mu[state]
... state = np.random.choice(K, p=P[state])
```

Then, let us visualize the data:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pyplot.axis('equal')
(...)
>>> colors = [ [[1,0,0], [0,1,0], [0,0,1]][int(state)] for state in z ]
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'k-', zorder=-10)
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.scatter(y[:,0], y[:,1], c=colors, s=40)
<matplotlib.collections.PathCollection object at 0x...>
```



Consecutive states are connected by a solid black line and the dot color shows the true class.

### Model

Now, assume that we do not know the parameters of the process (initial state probability and state transition probabilities). We give these parameters quite non-informative priors, but it is possible to provide more informative priors if such information is available:

```
>>> from bayespy.nodes import Dirichlet
>>> a0 = Dirichlet(1e-3*np.ones(K))
>>> A = Dirichlet(1e-3*np.ones((K,K)))
```

The discrete Markov chain is constructed as:

```
>>> Z = CategoricalMarkovChain(a0, A, states=N)
```

Now, instead of using categorical emission distribution as before, we'll use Gaussian distribution. For simplicity, we use the true parameters of the Gaussian distributions instead of giving priors and estimating them. The known standard deviation can be converted to a precision matrix as:

```
>>> Lambda = std**(-2) * np.identity(2)
```

Thus, the observed process is a Gaussian mixture with cluster assignments from the hidden Markov process 2:

```
>>> from bayespy.nodes import Gaussian
>>> Y = Mixture(Z, Gaussian, mu, Lambda)
```

Note that Lambda does not have cluster plate axis because it is shared between the clusters.

#### Inference

Let us use the simulated data:

```
>>> Y.observe(y)
```

Because VB takes all the random variables, we need to provide A and a0 also:

```
>>> Q = VB(Y, Z, A, a0)
```

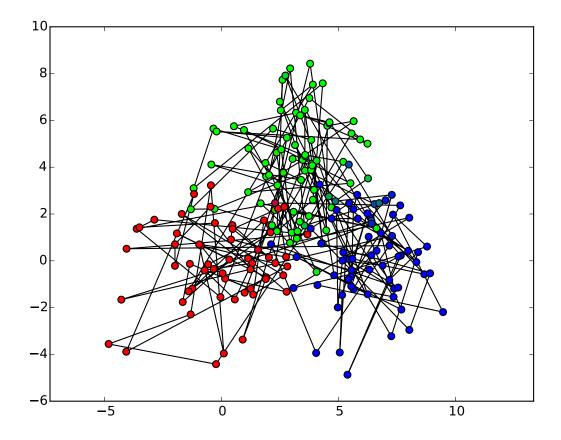
Then, run VB iteration until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-9.963054e+02 (... seconds)
...
Iteration 8: loglike=-9.235053e+02 (... seconds)
Converged at iteration 8.
```

#### Results

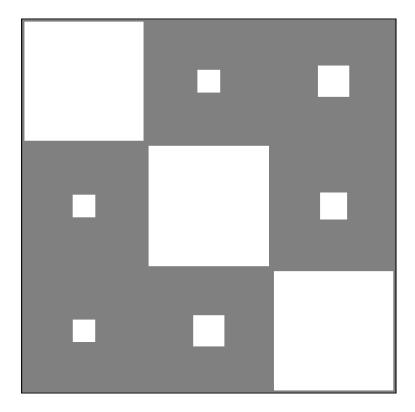
Plot the classification of the data similarly as the data:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pyplot.axis('equal')
(...)
>>> colors = Y.parents[0].get_moments()[0]
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'k-', zorder=-10)
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.scatter(y[:,0], y[:,1], c=colors, s=40)
<matplotlib.collections.PathCollection object at 0x...>
```



The data has been classified quite correctly. Even samples that are more in the region of another cluster are classified correctly if the previous and next sample provide enough evidence for the correct class. We can also plot the state transition matrix:

>>> bpplt.hinton(A)



Clearly, the learned state transition matrix is close to the true matrix. The models described above could also be used for classification by providing the known class assignments as observed data to  $\mathbb Z$  and the unknown class assignments as missing data.

# 3.5 Principal component analysis

This example uses a simple principal component analysis to find a two-dimensional latent subspace in a higher dimensional dataset.

### 3.5.1 Data

Let us create a Gaussian dataset with latent space dimensionality two and some observation noise:

```
>>> M = 20
>>> N = 100

>>> import numpy as np
>>> x = np.random.randn(N, 2)
>>> w = np.random.randn(M, 2)
>>> f = np.einsum('ik, jk->ij', w, x)
>>> y = f + 0.1*np.random.randn(M, N)
```

### 3.5.2 Model

We will use 10-dimensional latent space in our model and let it learn the true dimensionality:

```
>>> D = 10
```

Import relevant nodes:

```
>>> from bayespy.nodes import GaussianARD, Gamma, SumMultiply
```

The latent states:

```
>>> X = GaussianARD(0, 1, plates=(1,N), shape=(D,))
```

The loading matrix with automatic relevance determination (ARD) prior:

```
>>> alpha = Gamma(1e-5, 1e-5, plates=(D,))
>>> C = GaussianARD(0, alpha, plates=(M,1), shape=(D,))
```

Compute the dot product of the latent states and the loading matrix:

```
>>> F = SumMultiply('d, d->', X, C)
```

The observation noise:

```
>>> tau = Gamma(1e-5, 1e-5)
```

The observed variable:

```
>>> Y = GaussianARD(F, tau)
```

### 3.5.3 Inference

Observe the data:

```
>>> Y.observe(y)
```

We do not have missing data now, but they could be easily handled with mask keyword argument. Construct variational Bayesian (VB) inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, X, C, alpha, tau)
```

Initialize the latent subspace randomly, otherwise both X and C would converge to zero:

```
>>> C.initialize_from_random()
```

Now we could use VB.update() to run the inference. However, let us first create a parameter expansion to speed up the inference. The expansion is based on rotating the latent subspace optimally. This is optional but will usually improve the speed of the inference significantly, especially in high-dimensional problems:

```
>>> from bayespy.inference.vmp.transformations import RotateGaussianARD
>>> rot_X = RotateGaussianARD(X)
>>> rot_C = RotateGaussianARD(C, alpha)
```

By giving alpha for rot\_C, the rotation will also optimize alpha jointly with C. Now that we have defined the rotations for our variables, we need to construct an optimizer:

```
>>> from bayespy.inference.vmp.transformations import RotationOptimizer
>>> R = RotationOptimizer(rot_X, rot_C, D)
```

In order to use the rotations automatically, we need to set it as a callback function:

```
>>> Q.set_callback(R.rotate)
```

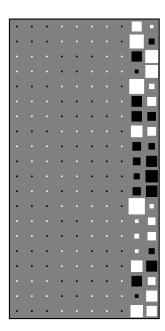
For more information about the rotation parameter expansion, see [7] and [6]. Now we can run the actual inference until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-2.339710e+03 (... seconds)
...
Iteration 23: loglike=6.500706e+02 (... seconds)
Converged at iteration 23.
```

### 3.5.4 Results

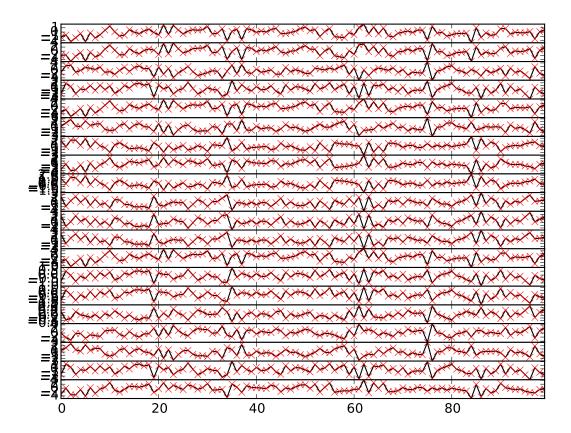
The results can be visualized, for instance, by plotting the Hinton diagram of the loading matrix:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(C)
```



The method has been able to prune out unnecessary latent dimensions and keep two components, which is the true number of components.

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(F)
>>> bpplt.plot(f, color='r', marker='x', linestyle='None')
```



The reconstruction of the noiseless function values are practically perfect in this simple example. Larger noise variance, more latent space dimensions and missing values would make this problem more difficult. The model construction could also be improved by having, for instance, C and tau in the same node without factorizing between them in the posterior approximation. This can be achieved by using GaussianGammaISO node.

## 3.6 Linear state-space model

### 3.6.1 Model

In linear state-space models a sequence of M-dimensional observations  $\mathbf{Y} = (\mathbf{y}_1, \dots, \mathbf{y}_N)$  is assumed to be generated from latent D-dimensional states  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_N)$  which follow a first-order Markov process:

$$\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-1} + \text{noise},$$
  
 $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \text{noise},$ 

where the noise is Gaussian,  $\bf A$  is the  $D \times D$  state dynamics matrix and  $\bf C$  is the  $M \times D$  loading matrix. Usually, the latent space dimensionality D is assumed to be much smaller than the observation space dimensionality M in order to model the dependencies of high-dimensional observations efficiently.

In order to construct the model in BayesPy, first import relevant nodes:

```
>>> from bayespy.nodes import GaussianARD, GaussianMarkovChain, Gamma, Dot
```

The data vectors will be 30-dimensional:

```
>>> M = 30
```

There will be 400 data vectors:

```
>>> N = 400
```

Let us use 10-dimensional latent space:

```
>>> D = 10
```

The state dynamics matrix **A** has ARD prior:

Note that **A** is a  $D \times D$ -dimensional matrix. However, in BayesPy it is modelled as a collection (plates=(D,)) of D-dimensional vectors (shape=(D,)) because this is how the variables factorize in the posterior approximation of the state dynamics matrix in GaussianMarkovChain. The latent states are constructed as

where the first two arguments are the mean and precision matrix of the initial state, the third argument is the state dynamics matrix and the fourth argument is the diagonal elements of the precision matrix of the innovation noise. The node also needs the length of the chain given as the keyword argument  $n=\mathbb{N}$ . Thus, the shape of this node is  $(\mathbb{N}, \mathbb{D})$ .

The linear mapping from the latent space to the observation space is modelled with the loading matrix which has ARD prior:

```
>>> gamma = Gamma(1e-5,
... 1e-5,
... plates=(D,),
... name='gamma')
>>> C = GaussianARD(0,
... gamma,
... shape=(D,),
... plates=(M,1),
... name='C')
```

Note that the plates for  $\mathbb{C}$  are (M, 1), thus the full shape of the node is  $(M, 1, \mathbb{D})$ . The unit plate axis is added so that  $\mathbb{C}$  broadcasts with  $\mathbb{X}$  when computing the dot product:

```
>>> F = Dot(C,
... X,
... name='F')
```

This dot product is computed over the D-dimensional latent space, thus the result is a  $M \times N$ -dimensional matrix which is now represented with plates (M, N) in BayesPy:

```
>>> F.plates (30, 400)
```

We also need to use random initialization either for C or X in order to find non-zero latent space because by default both C and X are initialized to zero because of their prior distributions. We use random initialization for C and then we must update X the first time before updating C:

```
>>> C.initialize_from_random()
```

The precision of the observation noise is given gamma prior:

```
>>> tau = Gamma(1e-5,
... 1e-5,
... name='tau')
```

The observations are noisy versions of the dot products:

```
>>> Y = GaussianARD(F, tau, name='Y')
```

The variational Bayesian inference engine is then construced as:

```
>>> from bayespy.inference import VB
>>> Q = VB(X, C, gamma, A, alpha, tau, Y)
```

Note that X is given before C, thus X is updated before C by default.

#### 3.6.2 Data

Now, let us generate some toy data for our model. Our true latent space is four dimensional with two noisy oscillator components, one random walk component and one white noise component.

The true linear mapping is just random:

```
>>> c = np.random.randn(M, 4)
```

Then, generate the latent states and the observations using the model equations:

We want to simulate missing values, thus we create a mask which randomly removes 80% of the data:

```
>>> from bayespy.utils import random
>>> mask = random.mask(M, N, p=0.2)
>>> Y.observe(y, mask=mask)
```

#### 3.6.3 Inference

As we did not define plotters for our nodes when creating the model, it is done now for some of the nodes:

```
>>> import bayespy.plot as bpplt
>>> X.set_plotter(bpplt.FunctionPlotter(center=True, axis=-2))
>>> A.set_plotter(bpplt.HintonPlotter())
>>> C.set_plotter(bpplt.HintonPlotter())
>>> tau.set_plotter(bpplt.PDFPlotter(np.linspace(0.02, 0.5, num=1000)))
```

This enables plotting of the approximate posterior distributions during VB learning. The inference engine can be run using VB.update() method:

```
>>> Q.update(repeat=10)
Iteration 1: loglike=-1.439704e+05 (... seconds)
...
Iteration 10: loglike=-1.051441e+04 (... seconds)
```

The iteration progresses a bit slowly, thus we'll consider parameter expansion to speed it up.

#### Parameter expansion

Section *Parameter expansion* discusses parameter expansion for state-space models to speed up inference. It is based on a rotating the latent space such that the posterior in the observation space is not affected:

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n = (\mathbf{C}\mathbf{R}^{-1})(\mathbf{R}\mathbf{x}_n)$$
.

Thus, the transformation is  $C \to CR^{-1}$  and  $X \to RX$ . In order to keep the dynamics of the latent states unaffected by the transformation, the state dynamics matrix **A** must be transformed accordingly:

$$\mathbf{R}\mathbf{x}_n = \mathbf{R}\mathbf{A}\mathbf{R}^{-1}\mathbf{R}\mathbf{x}_{n-1}$$
,

resulting in a transformation  $A \to RAR^{-1}$ . For more details, refer to [6] and [7]. In BayesPy, the transformations are available in bayespy.inference.vmp.transformations:

```
>>> from bayespy.inference.vmp import transformations
```

The rotation of the loading matrix along with the ARD parameters is defined as:

```
>>> rotC = transformations.RotateGaussianARD(C, gamma)
```

For rotating X, we first need to define the rotation of the state dynamics matrix:

```
>>> rotA = transformations.RotateGaussianARD(A, alpha)
```

Now we can define the rotation of the latent states:

```
>>> rotX = transformations.RotateGaussianMarkovChain(X, rotA)
```

The optimal rotation for all these variables is found using rotation optimizer:

```
>>> R = transformations.RotationOptimizer(rotX, rotC, D)
```

Set the parameter expansion to be applied after each iteration:

```
>>> Q.callback = R.rotate
```

Now, run iterations until convergence:

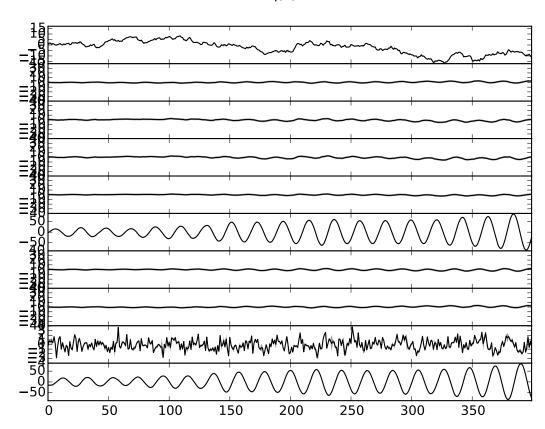
```
>>> Q.update(repeat=1000)
Iteration 11: loglike=-1.010806e+04 (... seconds)
...
Iteration 59: loglike=-8.906325e+03 (... seconds)
Converged at iteration 59.
```

### 3.6.4 Results

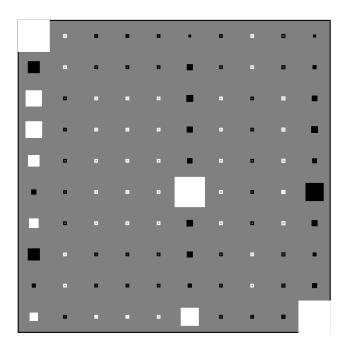
Because we have set the plotters, we can plot those nodes as:

```
>>> Q.plot(X, A, C, tau)
```

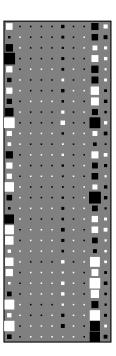


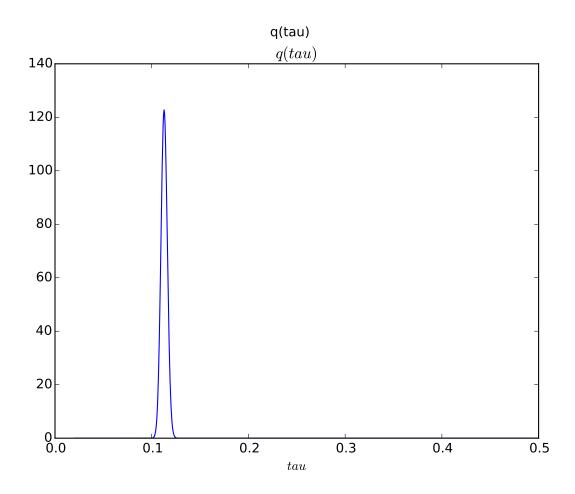


q(A)



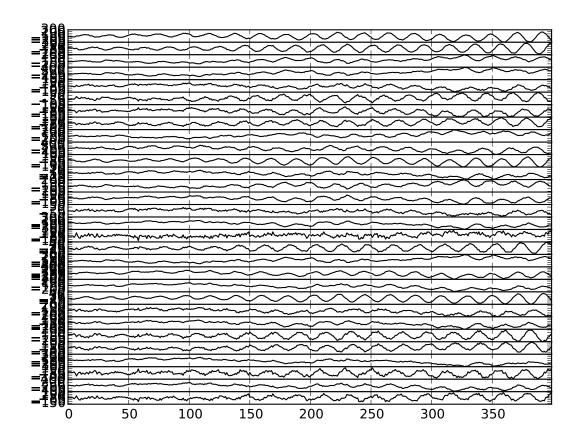
# q(C)





There are clearly four effective components in X: random walk (component number 1), random oscillation (7 and 10), and white noise (9). These dynamics are also visible in the state dynamics matrix Hinton diagram. Note that the white noise component does not have any dynamics. Also C shows only four effective components. The posterior of tau captures the true value  $3^{-2} \approx 0.111$  accurately. We can also plot predictions in the observation space:

>>> bpplt.plot(F, center=True)



We can also measure the performance numerically by computing root-mean-square error (RMSE) of the missing values:

```
>>> from bayespy.utils import misc
>>> misc.rmse(y[~mask], F.get_moments()[0][~mask])
5.18...
```

This is relatively close to the standard deviation of the noise (3), so the predictions are quite good considering that only 20% of the data was used.

### **DEVELOPER GUIDE**

This chapter provides basic information for developers about contributing, the theoretical background and the core structure. It is assumed that the reader has read and is familiar with *User guide*.

### 4.1 Workflow

The main forum for BayesPy development is GitHub. Bugs and other issues can be reported at https://github.com/bayespy/bayespy/issues. Contributions to the code and documentation are welcome and should be given as pull requests at https://github.com/bayespy/bayespy/pulls. In order to create pull requests, it is recommended to fork the git repository, make local changes and submit these changes as a pull request. The style guide for writing docstrings follows the style guide of NumPy, available at https://github.com/numpy/numpy/blob/master/doc/HOWTO\_DOCUMENT.rst.txt. Detailed instructions on development workflow can be read from NumPy guide, available at http://docs.scipy.org/doc/numpy/dev/gitwash/development\_workflow.html. BayesPy uses the following acronyms to start the commit message:

- API: an (incompatible) API change
- BLD: change related to building numpy
- BUG: bug fix
- DEMO: modification in demo code
- DEP: deprecate something, or remove a deprecated object
- DEV: development tool or utility
- DOC: documentation
- · ENH: enhancement
- MAINT: maintenance commit (refactoring, typos, etc.)
- REV: revert an earlier commit
- STY: style fix (whitespace, PEP8)
- TST: addition or modification of tests
- REL: related to releasing numpy

## 4.2 Variational message passing

This section briefly describes the variational message passing (VMP) framework, which is currently the only implemented inference engine in BayesPy. The variational Bayesian (VB) inference engine in BayesPy assumes that the posterior approximation factorizes with respect to nodes and plates. VMP is based on updating one node at a time (the plates in one node can be updated simultaneously) and iteratively updating all nodes in turns until convergence.

### 4.2.1 Standard update equation

The general update equation for the factorized approximation of node  $\theta$  is the following:

$$\log q(\boldsymbol{\theta}) = \langle \log p(\boldsymbol{\theta}|\operatorname{pa}(\boldsymbol{\theta})) \rangle + \sum_{\mathbf{x} \in \operatorname{ch}(\boldsymbol{\theta})} \langle \log p(\mathbf{x}|\operatorname{pa}(\mathbf{x})) \rangle + \operatorname{const}, \tag{4.1}$$

where  $pa(\theta)$  and  $ch(\theta)$  are the set of parents and children of  $\theta$ , respectively. Thus, the posterior approximation of a node is updated by taking a sum of the expectations of all log densities in which the node variable appears. The expectations are over the approximate distribution of all other variables than  $\theta$ . Actually, not all the variables are needed, because the non-constant part depends only on the Markov blanket of  $\theta$ . This leads to a local optimization scheme, which uses messages from neighbouring nodes.

The messages are simple for conjugate exponential family models. An exponential family distribution has the following log probability density function:

$$\log p(\mathbf{x}|\mathbf{\Theta}) = \mathbf{u}_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}} \boldsymbol{\phi}_{\mathbf{x}}(\mathbf{\Theta}) + g_{\mathbf{x}}(\mathbf{\Theta}) + f_{\mathbf{x}}(\mathbf{x}), \tag{4.2}$$

where  $\Theta = \{\theta_j\}$  is the set of parents,  $\mathbf{u}$  is the sufficient statistic vector,  $\phi$  is the natural parameter vector, g is the negative log normalizer, and f is the log base function. Note that the log density is linear with respect to the terms that are functions of  $\mathbf{x}$ :  $\mathbf{u}$  and f. If a parent has a conjugate prior, (4.2) is also linear with respect to the parent's sufficient statistic vector. Thus, (4.2) can be re-organized with respect to a parent  $\theta_j$  as

$$\log p(\mathbf{x}|\mathbf{\Theta}) = \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}_j}(\mathbf{x}, \{\boldsymbol{\theta}_k\}_{k \neq j}) + \text{const},$$

where  $\mathbf{u}_{\theta_j}$  is the sufficient statistic vector of  $\boldsymbol{\theta}_j$  and the constant part is constant with respect to  $\boldsymbol{\theta}_j$ . Thus, the update equation (4.1) for  $\boldsymbol{\theta}_j$  can be written as

$$\begin{split} \log q(\boldsymbol{\theta}_j) &= \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \langle \boldsymbol{\phi}_{\boldsymbol{\theta}_j} \rangle + f_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j) + \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \sum_{\mathbf{x} \in \mathrm{ch}(\boldsymbol{\theta}_j)} \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}_j} \rangle + \mathrm{const}, \\ &= \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \left( \langle \boldsymbol{\phi}_{\boldsymbol{\theta}_j} \rangle + \sum_{\mathbf{x} \in \mathrm{ch}(\boldsymbol{\theta}_j)} \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}_j} \rangle \right) + f_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j) + \mathrm{const}, \end{split}$$

where the summation is over all the child nodes of  $\theta_j$ . Because of the conjugacy,  $\langle \phi_{\theta_j} \rangle$  depends (multi)linearly on the parents' sufficient statistic vector. Similarly,  $\langle \phi_{\mathbf{x} \to \theta_j} \rangle$  depends (multi)linearly on the expectations of the children's and co-parents' sufficient statistics. This gives the following update equation for the natural parameter vector of the posterior approximation  $q(\phi_j)$ :

$$\tilde{\phi}_j = \langle \phi_{\theta_j} \rangle + \sum_{\mathbf{x} \in \text{ch}(\theta_j)} \langle \phi_{\mathbf{x} \to \theta_j} \rangle. \tag{4.3}$$

### 4.2.2 Variational messages

The update equation (4.3) leads to a message passing scheme: the term  $\langle \phi_{\theta_j} \rangle$  is a function of the parents' sufficient statistic vector and the term  $\langle \phi_{\mathbf{x} \to \theta_j} \rangle$  can be interpreted as a message from the child node  $\mathbf{x}$ . Thus, the message from the child node  $\mathbf{x}$  to the parent node  $\boldsymbol{\theta}$  is

$$\mathbf{m}_{\mathbf{x} \to \boldsymbol{\theta}} \equiv \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}} \rangle,$$

which can be computed as a function of the sufficient statistic vector of the co-parent nodes of  $\theta$  and the sufficient statistic vector of the child node  $\mathbf{x}$ . The message from the parent node  $\theta$  to the child node  $\mathbf{x}$  is simply the expectation of the sufficient statistic vector:

$$\mathbf{m}_{\boldsymbol{\theta} \to \mathbf{x}} \equiv \langle \mathbf{u}_{\boldsymbol{\theta}} \rangle.$$

In order to compute the expectation of the sufficient statistic vector we need to write  $q(\theta)$  as

$$\log q(\boldsymbol{\theta}) = \mathbf{u}(\boldsymbol{\theta})^{\mathrm{T}} \tilde{\boldsymbol{\phi}} + \tilde{q}(\tilde{\boldsymbol{\phi}}) + f(\boldsymbol{\theta}),$$

where  $\tilde{\phi}$  is the natural parameter vector of  $q(\theta)$ . Now, the expectation of the sufficient statistic vector is defined as

$$\langle \mathbf{u}_{\boldsymbol{\theta}} \rangle = -\frac{\partial \tilde{g}}{\partial \tilde{\phi}_{\boldsymbol{\theta}}} (\tilde{\phi}_{\boldsymbol{\theta}}). \tag{4.4}$$

We call this expectation of the sufficient statistic vector as the moments vector.

#### 4.2.3 Lower bound

Computing the VB lower bound is not necessary in order to find the posterior approximation, although it is extremely useful in monitoring convergence and possible bugs. The VB lower bound can be written as

$$\mathcal{L} = \langle \log p(\mathbf{Y}, \mathbf{X}) \rangle - \langle \log q(\mathbf{X}) \rangle,$$

where Y is the set of all observed variables and X is the set of all latent variables. It can also be written as

$$\mathcal{L} = \sum_{\mathbf{y} \in \mathbf{Y}} \langle \log p(\mathbf{y} | \text{pa}(\mathbf{y})) \rangle + \sum_{\mathbf{x} \in \mathbf{X}} \left[ \langle \log p(\mathbf{x} | \text{pa}(\mathbf{x})) \rangle - \langle \log q(\mathbf{x}) \right],$$

which shows that observed and latent variables contribute differently to the lower bound. These contributions have simple forms for exponential family nodes. Observed exponential family nodes contribute to the lower bound as follows:

$$\langle \log p(\mathbf{y}|\operatorname{pa}(\mathbf{y})) \rangle = \mathbf{u}(\mathbf{y})^T \langle \phi \rangle + \langle g \rangle + f(\mathbf{x}),$$

where y is the observed data. On the other hand, latent exponential family nodes contribute to the lower bound as follows:

$$\langle \log p(\mathbf{x}|\boldsymbol{\theta}) \rangle - \langle \log q(\mathbf{x}) \rangle = \langle \mathbf{u} \rangle^T (\langle \boldsymbol{\phi} \rangle - \tilde{\boldsymbol{\phi}}) + \langle g \rangle - \tilde{g}.$$

If a node is partially observed and partially unobserved, these formulas are applied plate-wise appropriately.

#### 4.2.4 Terms

To summarize, implementing VMP requires one to write for each stochastic exponential family node:

 $\langle \phi \rangle$ : the expectation of the prior natural parameter vector

Computed as a function of the messages from parents.

 $\phi$ : natural parameter vector of the posterior approximation

Computed as a sum of  $\langle \phi \rangle$  and the messages from children.

 $\langle \mathbf{u} \rangle$ : the posterior moments vector

Computed as a function of  $\tilde{\phi}$  as defined in (4.4).

 $\mathbf{u}(\mathbf{x})$ : the moments vector for given data

Computed as a function of of the observed data x.

 $\langle g \rangle$ : the expectation of the negative log normalizer of the prior

Computed as a function of parent moments.

 $\tilde{\boldsymbol{g}}$  : the negative log normalizer of the posterior approximation

Computed as a function of  $\tilde{\phi}$ .

 $f(\mathbf{x})$ : the log base measure for given data

Computed as a function of the observed data x.

 $\langle \phi_{{f x} 
ightarrow {m heta}} 
angle$  : the message to parent  ${m heta}$ 

Computed as a function of the moments of this node and the other parents.

Deterministic nodes require only the following terms:

 $\langle \mathbf{u} \rangle$ : the posterior moments vector

Computed as a function of the messages from the parents.

m: the message to a parent

Computed as a function of the messages from the other parents and all children.

# 4.3 Implementing inference engines

Currently, only variational Bayesian inference engine is implemented. This implementation is not very modular, that is, the inference engine is not well separated from the model construction. Thus, it is not straightforward to implement other inference engines at the moment. Improving the modularity of the inference engine and model construction is future work with high priority. In any case, BayesPy aims to be an efficient, simple and modular Bayesian package for variational inference at least.

# 4.4 Implementing nodes

The main goal of BayesPy is to provide a package which enables easy and flexible construction of simple and complex models with efficient inference. However, users may sometimes be unable to construct their models because the built-in nodes do not implement some specific features. Thus, one may need to implement new nodes in order to construct the model. BayesPy aims to make the implementation of new nodes both simple and fast. Probably, a large complex model can be constructed almost completely with the built-in nodes and the user needs to implement only a few nodes.

#### 4.4.1 Moments

In order to implement nodes, it is important to understand the messaging framework of the nodes. A node is a unit of calculation which communicates to its parent and child nodes using messages. These messages have types that need to match between nodes, that is, the child node needs to understand the messages its parents are sending and vice versa. Thus, a node defines which message type it requires from each of its parents, and only nodes that have that type of output message (i.e., the message to a child node) are valid parent nodes for that node.

The message type is defined by the moments of the parent node. The moments are a collection of expectations:  $\{\langle f_1(X)\rangle,\ldots,\langle f_N(X)\rangle\}$ . The functions  $f_1,\ldots,f_N$  (and the number of the functions) define the message type and they are the sufficient statistic as discussed in the previous section. Different message types are represented by Moments class hierarchy. For instance, GaussianMoments represents a message type with parent moments  $\{\langle \mathbf{x} \rangle, \langle \mathbf{x} \mathbf{x}^T \rangle\}$  and WishartMoments a message type with parent moments  $\{\langle \mathbf{\Lambda} \rangle, \langle \log |\mathbf{\Lambda}| \rangle\}$ .

Let us give an example: Gaussian node outputs GaussianMoments messages and Wishart node outputs WishartMoments messages. Gaussian node requires that it receives GaussianMoments messages from the mean parent node and WishartMoments messages from the precision parent node. Thus, Gaussian and Wishart are valid node classes as the mean and precision parent nodes of Gaussian node.

Note that several nodes may have the same output message type and some message types can be transformed to other message types using deterministic converter nodes. For instance, Gaussian and GaussianARD nodes both output GaussianMoments messages, deterministic SumMultiply also outputs GaussianMoments messages, and deterministic converter MarkovChainToGaussian converts GaussianMarkovChainMoments to GaussianMoments.

Each node specifies the message type requirements of its parents by Node.\_parent\_moments attribute which is a list of Moments sub-class instances. These moments objects have a few purpose when creating the node: 1) check that parents are sending proper messages; 2) if parents use different message type, try to add a converter which converts the messages to the correct type if possible; 3) if given parents are not nodes but numeric arrays, convert them to constant nodes with correct output message type.

When implementing a new node, it is not always necessary to implement a new moments class. If another node has the same sufficient statistic vector, thus the same moments, that moments class can be used. Otherwise, one must implement a simple moments class which has the following methods:

• Moments.compute\_fixed\_moments()

Computes the moments for a known value. This is used to compute the moments of constant numeric arrays and wrap them into constant nodes.

• Moments.compute\_dims\_from\_values()

Given a known value of the variable, return the shape of the variable dimensions in the moments. This is used to solve the shape of the moments array for constant nodes.

#### 4.4.2 Distributions

In order to implement a stochastic exponential family node, one must first write down the log probability density function of the node and derive the terms discussed in section *Terms*. These terms are implemented and collected as a class which is a subclass of <code>Distribution</code>. The main reason to implement these methods in another class instead of the node class itself is that these methods can be used without creating a node, for instance, in <code>Mixture</code> class.

For exponential family distributions, the distribution class is a subclass of ExponentialFamilyDistribution, and the relation between the terms in section *Terms* and the methods is as follows:

• ExponentialFamilyDistribution.compute\_phi\_from\_parents()

Computes the expectation of the natural parameters  $\langle \phi \rangle$  in the prior distribution given the moments of the parents.

• ExponentialFamilyDistribution.compute\_cqf\_from\_parents()

Computes the expectation of the negative log normalizer  $\langle g \rangle$  of the prior distribution given the moments of the parents.

• ExponentialFamilyDistribution.compute\_moments\_and\_cgf()

Computes the moments  $\langle \mathbf{u} \rangle$  and the negative log normalizer  $\tilde{g}$  of the posterior distribution given the natural parameters  $\tilde{\phi}$ .

• ExponentialFamilyDistribution.compute\_message\_to\_parent()

Computes the message  $\langle \phi_{\mathbf{x} \to \boldsymbol{\theta}} \rangle$  from the node  $\mathbf{x}$  to its parent node  $\boldsymbol{\theta}$  given the moments of the node and the other parents.

• ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f()

Computes  $\mathbf{u}(\mathbf{x})$  and  $f(\mathbf{x})$  for given observed value  $\mathbf{x}$ . Without this method, variables from this distribution cannot be observed.

For each stochastic exponential family node, one must write a distribution class which implements these methods. After that, the node class is basically a simple wrapper and it also stores the moments and the natural parameters of the current posterior approximation. Note that the distribution classes do not store node-specific information, they are more like static collections of methods. However, sometimes the implementations depend on some information, such as the dimensionality of the variable, and this information must be provided, if needed, when constructing the distribution object.

In addition to the methods listed above, it is necessary to implement a few more methods in some cases. This happens when the plates of the parent do not map to the plates directly as discussed in section *Irregular plates*. Then, one must write methods that implement this plate mapping and apply the same mapping to the mask array:

• ExponentialFamilyDistribution.plates\_from\_parent()

Given the plates of the parent, return the resulting plates of the child.

• ExponentialFamilyDistribution.plates\_to\_parent()

Given the plates of the child, return the plates of the parent that would have resulted them.

• ExponentialFamilyDistribution.compute\_mask\_to\_parent()

Given the mask array of the child, apply the plate mapping.

It is important to understand when one must implement these methods, because the default implementations in the base class will lead to errors or weird results.

### 4.4.3 Stochastic exponential family nodes

After implementing the distribution class, the next task is to implement the node class. First, we need to explain a few important attributes before we can explain how to implement a node class.

Stochastic exponential family nodes have two attributes that store the state of the posterior distribution:

• phi

The natural parameter vector  $\tilde{\phi}$  of the posterior approximation.

• u

The moments  $\langle \mathbf{u} \rangle$  of the posterior approximation.

Instead of storing these two variables as vectors (as in the mathematical formulas), they are stored as lists of arrays with convenient shapes. For instance, Gaussian node stores the moments as a list consisting of a vector  $\langle \mathbf{x} \rangle$  and a matrix  $\langle \mathbf{x} \mathbf{x}^T \rangle$  instead of reshaping and concatenating these into a single vector. The same applies for the natural parameters phi because it has the same shape as u.

The shapes of the arrays in the lists u and phi consist of the shape caused by the plates and the shape caused by the variable itself. For instance, the moments of Gaussian node have shape (D,) and (D, D), where D is the dimensionality of the Gaussian vector. In addition, if the node has plates, they are added to these shapes. Thus, for instance, if the Gaussian node has plates (3, 7) and D is 5, the shape of u[0] and phi[0] would be (3, 7, 5) and the shape of u[1] and phi[1] would be (3, 7, 5, 5). This shape information is stored in the following attributes:

```
• plates: a tuple
```

The plates of the node. In our example, (3, 7).

• dims: a list of tuples

The shape of each of the moments arrays (or natural parameter arrays) without plates. In our example, [(5, 5)].

Finally, three attributes define VMP for the node:

• \_moments: Moments sub-class instance

An object defining the moments of the node.

• \_parent\_moments: list of Moments sub-class instances

A list defining the moments requirements for each parent.

• \_distribution: Distribution sub-class instance

An object implementing the VMP formulas.

Basically, a node class is a collection of the above attributes. When a node is created, these attributes are defined. The base class for exponential family nodes, <code>ExponentialFamily</code>, provides a simple default constructor which does not need to be overwritten if <code>dims, \_moments</code>, <code>\_parent\_moments</code> and <code>\_distribution</code> can be provided as static class attributes. For instance, <code>Gamma</code> node defines these attributes statically. However, usually at least one of these attributes cannot be defined statically in the class. In that case, one must implement a class method which overloads <code>ExponentialFamily.\_constructor()</code>. The purpose of this method is to define all the attributes given the parent nodes. These are defined using a class method instead of <code>\_\_init\_</code> method in order to be able to use the class constructors statically, for instance, in <code>Mixture</code> class. This construction allows users to create mixtures of any exponential family distribution with simple syntax.

The parents of a node must be converted so that they have a correct message type, because the user may have provided numeric arrays or nodes with incorrect message type. Numeric arrays should be converted to constant nodes with correct message type. Incorrect message type nodes should be converted to correct message type nodes if possible. Thus, the constructor should use Node.\_ensure\_moments method to make sure the parent is a node with correct message type. Instead of calling this method for each parent node in the constructor, one can use ensureparents decorator to do this automatically. However, the decorator requires that \_parent\_moments attribute has already been defined statically. If this is not possible, the parent nodes must be converted manually in the constructor, because one should never assume that the parent nodes given to the constructor are nodes with correct message type or even nodes at all.

### 4.4.4 Deterministic nodes

Deterministic nodes are nodes that do not correspond to any probability distribution but rather a deterministic function. It does not have any moments or natural parameters to store. A deterministic node is implemented as a subclass of Deterministic base class. The new node class must implement the following methods:

• Deterministic.\_compute\_moments()

Computes the moments given the moments of the parents.

• Deterministic.\_compute\_message\_to\_parent()

given Computes the parent node the from the chilmessage to а message dren and the moments the other parents. In some cases, want Deterministic.\_compute\_message\_and\_mask\_to\_parent() implement or Deterministic.\_message\_to\_parent() instead in order to gain more control over efficient computa-

Similarly as in Distribution class, if the node handles plates irregularly, it is important to implement the following methods:

• Deterministic.\_plates\_from\_parent()

Given the plates of the parent, return the resulting plates of the child.

- Deterministic.\_plates\_to\_parent()

  Given the plates of the child, return the plates of the parent that would have resulted them.
- Deterministic.\_compute\_mask\_to\_parent()
   Given the mask array, convert it to a plate mask of the parent.

#### **Converter nodes**

Sometimes a node has incorrect message type but the message can be converted into a correct type. For instance, GaussianMarkovChain has GaussianMarkovChainMoments message type, which means moments  $\{\langle \mathbf{x}_n \rangle, \langle \mathbf{x}_n \mathbf{x}_n^T \rangle, \langle \mathbf{x}_n \mathbf{x}_{n-1}^T \rangle\}_{n=1}^N$ . These moments can be converted to GaussianMoments by ignoring the third element and considering the time axis as a plate axis. Thus, if a node requires GaussianMoments message from its parent, GaussianMarkovChain is a valid parent if its messages are modified properly. This conversion is implemented in MarkovChainToGaussian converter class. Converter nodes are simple deterministic nodes that have one parent node and they convert the messages to another message type.

For the user, it is not convenient if the exact message type has to be known and an explicit converter node needs to be created. Thus, the conversions are done automatically and the user will be unaware of them. In order to enable this automatization, when writing a converter node, one should register the converter to the moments class using Moments.add\_converter(). For instance, a class X which converts moments A to moments B is registered as A.add\_conveter(B, X). After that, Node.\_ensure\_moments() and Node.\_convert() methods are used to perform the conversions automatically. The conversion can consist of several consecutive converter nodes, and the least number of conversions is used.

**CHAPTER** 

**FIVE** 

# **USER API**

bayespy.nodes bayespy.inference bayespy.plot

# 5.1 bayespy.nodes

Package for nodes used to construct the model.

### 5.1.1 Stochastic nodes

Nodes for Gaussian variables:

Gaussian(mu, Lambda, **kwargs)	Node for Gaussian variables.
<pre>GaussianARD(mu, alpha[, ndim, shape])</pre>	Node for Gaussian variables with ARD prior.

### bayespy.nodes.Gaussian

class bayespy.nodes.Gaussian (mu, Lambda, \*\*kwargs)

Node for Gaussian variables.

The node represents a *D*-dimensional vector from the Gaussian distribution:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}),$$

where  $\mu$  is the mean vector and  $\Lambda$  is the precision matrix (i.e., inverse of the covariance matrix).

$$\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D, \quad \boldsymbol{\Lambda} \in \mathbb{R}^{D \times D}, \quad \boldsymbol{\Lambda}$$
 symmetric positive definite

**Parameters mu**: Gaussian-like node or GaussianGammaISO-like node or GaussianWishart-like node or array

Mean vector

Lambda: Wishart-like node or array

Precision matrix

#### See also:

Wishart, GaussianARD, GaussianWishart, GaussianGammaARD, GaussianGammaISO

\_\_init\_\_ (mu, Lambda, \*\*kwargs)
Create Gaussian node

#### **Methods**

```
__init__(mu, Lambda, **kwargs)
                                                  Create Gaussian node
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                                  Delete this node and the children
get_gradient(rg)
                                                  Computes gradient with respect to the natural parameters.
get_mask()
get_moments()
get_parameters()
                                                  Return parameters of the VB distribution.
get_riemannian_gradient()
                                                  Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                                  Return True if the node has a plotter
initialize_from_parameters(mu, Lambda)
initialize_from_prior()
initialize_from_random()
                                                  Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                                  Load the state of the node from a HDF5 file.
                                                  Compute the log probability density function Q(X) of this node.
logpdf(X[, mask])
                                                  Compute E[log p(X|parents) - log q(X)]
lower_bound_contribution([gradient, ...])
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                  Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                  Compute the probability density function of this node.
plot([fig])
                                                  Plot the node distribution using the plotter of the node
                                                  Draw a random sample from the distribution.
random()
rotate(R[, inv, logdet, Q])
rotate_matrix(R1, R2[, inv1, logdet1, inv2, ...])
                                                  The vector is reshaped into a matrix by stacking the row vectors.
save(group)
                                                  Save the state of the node into a HDF5 file.
set_parameters(x)
                                                  Set the parameters of the VB distribution.
set_plotter(plotter)
show()
unobserve()
update([annealing])
```

#### bayespy.nodes.Gaussian.\_\_init\_\_

```
Gaussian.__init__(mu, Lambda, **kwargs)
Create Gaussian node
```

### bayespy.nodes.Gaussian.add\_plate\_axis

Gaussian.add\_plate\_axis(to\_plate)

#### bayespy.nodes.Gaussian.broadcasting\_multiplier

Gaussian.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.Gaussian.delete

```
Gaussian.delete()
```

Delete this node and the children

### bayespy.nodes.Gaussian.get\_gradient

```
Gaussian.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

### bayespy.nodes.Gaussian.get\_mask

```
Gaussian.get_mask()
```

#### bayespy.nodes.Gaussian.get\_moments

```
Gaussian.get_moments()
```

#### bayespy.nodes.Gaussian.get\_parameters

```
Gaussian.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.Gaussian.get\_riemannian\_gradient

```
Gaussian.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

### bayespy.nodes.Gaussian.get\_shape

```
Gaussian.get_shape(ind)
```

### bayespy.nodes.Gaussian.has\_plotter

```
Gaussian.has_plotter()
```

Return True if the node has a plotter

# $bayes py. nodes. Gaussian. initialize\_from\_parameters$

 ${\tt Gaussian.initialize\_from\_parameters}~(\textit{mu}, \textit{Lambda})$ 

bayespy.nodes.Gaussian.observe

Gaussian.**observe**(*x*, \**args*, *mask=True*)

Fix moments, compute f and propagate mask.

```
bayespy.nodes.Gaussian.initialize_from_prior
Gaussian.initialize_from_prior()
bayespy.nodes.Gaussian.initialize_from_random
Gaussian.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Gaussian.initialize_from_value
Gaussian.initialize_from_value(x, *args)
bayespy.nodes.Gaussian.load
Gaussian.load(group)
    Load the state of the node from a HDF5 file.
bayespy.nodes.Gaussian.logpdf
Gaussian.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Gaussian.lower_bound_contribution
Gaussian.lower_bound_contribution (gradient=False, ignore_masked=True)
    Compute E[ \log p(X|parents) - \log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Gaussian.lowerbound
Gaussian.lowerbound()
bayespy.nodes.Gaussian.move_plates
Gaussian.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Gaussian.pdf

```
Gaussian.pdf (X, mask=True)
```

Compute the probability density function of this node.

### bayespy.nodes.Gaussian.plot

```
Gaussian.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Gaussian.random

```
Gaussian.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Gaussian.rotate

```
Gaussian.rotate(R, inv=None, logdet=None, Q=None)
```

#### bayespy.nodes.Gaussian.rotate\_matrix

```
Gaussian.rotate_matrix(R1, R2, inv1=None, logdet1=None, inv2=None, logdet2=None, Q=None)
```

The vector is reshaped into a matrix by stacking the row vectors.

Computes R1\*X\*R2', which is identical to kron(R1,R2)\*x (??)

Note that this is slightly different from the standard Kronecker product definition because Numpy stacks row vectors instead of column vectors.

### Parameters R1: ndarray

A matrix from the left

R2: ndarray

A matrix from the right

#### bayespy.nodes.Gaussian.save

```
Gaussian.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

#### bayespy.nodes.Gaussian.set\_parameters

```
Gaussian.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gaussian.set\_plotter

```
Gaussian.set_plotter(plotter)
```

### bayespy.nodes.Gaussian.show

```
Gaussian.show()
```

# bayespy.nodes.Gaussian.unobserve

```
Gaussian.unobserve()
```

#### bayespy.nodes.Gaussian.update

```
Gaussian.update(annealing=1.0)
```

### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.Gaussian.dims

```
Gaussian.dims = None
```

#### bayespy.nodes.Gaussian.plates

```
Gaussian.plates = None
```

#### bayespy.nodes.Gaussian.plates\_multiplier

```
Gaussian.plates_multiplier
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.GaussianARD

class bayespy.nodes.GaussianARD (mu, alpha, ndim=None, shape=None, \*\*kwargs)
 Node for Gaussian variables with ARD prior.

The node represents a D-dimensional vector from the Gaussian distribution:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \operatorname{diag}(\boldsymbol{\alpha})),$$

where  $\mu$  is the mean vector and diag $(\alpha)$  is the diagonal precision matrix (i.e., inverse of the covariance matrix).

$$\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D$$
,  $\alpha_d > 0$  for  $d = 0, \dots, D-1$ 

*Note:* The form of the posterior approximation is a Gaussian distribution with full covariance matrix instead of a diagonal matrix.

**Parameters mu**: Gaussian-like node or GaussianGammaISO-like node or GaussianGammaARD-like node or array

Mean vector

alpha: gamma-like node or array

Diagonal elements of the precision matrix

#### See also:

Gamma, Gaussian, GaussianGammaARD, GaussianGammaISO, GaussianWishart

\_\_init\_\_ (mu, alpha, ndim=None, shape=None, \*\*kwargs)
Create GaussianARD node.

#### **Methods**

init(mu, alpha[, ndim, shape])	Create GaussianARD node.
add_plate_axis(to_plate)	Create Gaussian IVD node.
broadcasting_multiplier(plates, *args)	
	Delete this node and the children
delete()	
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
<pre>get_shape(ind)</pre>	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_mean_and_covariance(mu, Cov)</pre>	
initialize_from_parameters(mu, alpha)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
	Continued on next page

### Table 5.5 – continued from previous page

```
pdf(X[, mask])
                                                            Compute the probability density function of this node.
plot([fig])
                                                            Plot the node distribution using the plotter of the node
random()
                                                            Draw a random sample from the distribution.
rotate(R[, inv, logdet, axis, Q, subset])
rotate_plates(Q[, plate_axis])
                                                            Approximate rotation of a plate axis.
save(group)
                                                            Save the state of the node into a HDF5 file.
                                                            Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
show()
unobserve()
update([annealing])
```

### bayespy.nodes.GaussianARD.\_\_init\_\_

```
GaussianARD.__init__ (mu, alpha, ndim=None, shape=None, **kwargs)
Create GaussianARD node.
```

# $bayespy.nodes. Gaussian ARD. add\_plate\_axis$

```
GaussianARD.add_plate_axis(to_plate)
```

#### bayespy.nodes.GaussianARD.broadcasting\_multiplier

```
GaussianARD.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.GaussianARD.delete

```
GaussianARD.delete()

Delete this node and the children
```

#### bayespy.nodes.GaussianARD.get\_gradient

```
GaussianARD.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.GaussianARD.get\_mask

```
GaussianARD.get_mask()
```

#### bayespy.nodes.GaussianARD.get\_moments

```
GaussianARD.get_moments()
```

#### bayespy.nodes.GaussianARD.get\_parameters

```
GaussianARD.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianARD.get\_riemannian\_gradient

```
GaussianARD.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

### bayespy.nodes.GaussianARD.get\_shape

```
GaussianARD.get_shape (ind)
```

### bayespy.nodes.GaussianARD.has\_plotter

```
GaussianARD.has_plotter()
Return True if the node has a plotter
```

### bayespy.nodes.GaussianARD.initialize\_from\_mean\_and\_covariance

```
GaussianARD.initialize_from_mean_and_covariance (mu, Cov)
```

### bayespy.nodes.GaussianARD.initialize\_from\_parameters

```
GaussianARD.initialize_from_parameters (mu, alpha)
```

### bayespy.nodes.GaussianARD.initialize\_from\_prior

```
GaussianARD.initialize_from_prior()
```

### bayespy.nodes.GaussianARD.initialize\_from\_random

```
GaussianARD.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

### bayespy.nodes.GaussianARD.initialize\_from\_value

```
GaussianARD.initialize_from_value(x, *args)
```

#### bayespy.nodes.GaussianARD.load

```
GaussianARD.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.GaussianARD.logpdf

```
GaussianARD.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.GaussianARD.lower\_bound\_contribution

```
GaussianARD.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.GaussianARD.lowerbound

```
GaussianARD.lowerbound()
```

#### bayespy.nodes.GaussianARD.move\_plates

```
GaussianARD.move_plates (from_plate, to_plate)
```

# bayespy.nodes.GaussianARD.observe

```
GaussianARD.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

### bayespy.nodes.GaussianARD.pdf

```
GaussianARD.pdf (X, mask=True)
```

Compute the probability density function of this node.

### bayespy.nodes.GaussianARD.plot

```
GaussianARD.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianARD.random

```
GaussianARD.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.GaussianARD.rotate

```
GaussianARD.rotate(R, inv=None, logdet=None, axis=-1, Q=None, subset=None)
```

#### bayespy.nodes.GaussianARD.rotate\_plates

```
GaussianARD.rotate_plates(Q, plate_axis=-1)
```

Approximate rotation of a plate axis.

Mean is rotated exactly but covariance/precision matrix is rotated approximately.

### bayespy.nodes.GaussianARD.save

```
GaussianARD.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

#### bayespy.nodes.GaussianARD.set\_parameters

```
GaussianARD.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianARD.set\_plotter

```
GaussianARD.set_plotter(plotter)
```

### bayespy.nodes.GaussianARD.show

```
GaussianARD.show()
```

#### bayespy.nodes.GaussianARD.unobserve

```
GaussianARD.unobserve()
```

#### bayespy.nodes.GaussianARD.update

```
GaussianARD.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.GaussianARD.dims

GaussianARD.dims = None

## bayespy.nodes.GaussianARD.plates

GaussianARD.plates = None

### bayespy.nodes.GaussianARD.plates\_multiplier

GaussianARD.plates\_multiplier

Plate multiplier is applied to messages to parents

Nodes for precision and scale variables:

Gamma(a, b, **kwargs)	Node for gamma random variables.
Wishart(n, V, **kwargs)	Node for Wishart random variables.
<pre>Exponential(l, **kwargs)</pre>	Node for exponential random variables.

### bayespy.nodes.Gamma

**class** bayespy.nodes.**Gamma** (*a*, *b*, \*\*kwargs)

Node for gamma random variables.

**Parameters a**: scalar or array

Shape parameter

**b** : gamma-like node or scalar or array

Rate parameter

\_\_init\_\_ (*a*, *b*, \*\*kwargs)

Create gamma random variable node

# Methods

init(a, b, **kwargs)	Create gamma random variable node
add_plate_axis(to_plate)	
as_diagonal_wishart()	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
	Continued on next page

Table 5.8 – continued from previous page

```
get_moments()
                                                Return parameters of the VB distribution.
get_parameters()
get_riemannian_gradient()
                                                Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                                Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                                Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
                                                Load the state of the node from a HDF5 file.
load(group)
                                                Compute the log probability density function Q(X) of this node.
logpdf(X[, mask])
lower_bound_contribution([gradient, ...])
                                                Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                Compute the probability density function of this node.
plot([fig])
                                                Plot the node distribution using the plotter of the node
random()
                                                Draw a random sample from the distribution.
                                                Save the state of the node into a HDF5 file.
save(group)
set_parameters(x)
                                                Set the parameters of the VB distribution.
set_plotter(plotter)
show()
                                                Print the distribution using standard parameterization.
unobserve()
update([annealing])
```

# bayespy.nodes.Gamma.\_\_init\_\_

```
Gamma . __init__ (a, b, **kwargs)

Create gamma random variable node
```

#### bayespy.nodes.Gamma.add\_plate\_axis

```
Gamma.add_plate_axis (to_plate)
```

# $bayespy.nodes. Gamma. as\_diagonal\_wish art$

```
Gamma.as_diagonal_wishart()
```

#### bayespy.nodes.Gamma.broadcasting\_multiplier

Gamma.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.Gamma.delete

```
Gamma.delete()
```

Delete this node and the children

#### bayespy.nodes.Gamma.get\_gradient

```
Gamma.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Gamma.get\_mask

```
Gamma.get_mask()
```

#### bayespy.nodes.Gamma.get\_moments

```
Gamma.get_moments()
```

### bayespy.nodes.Gamma.get\_parameters

```
Gamma.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gamma.get\_riemannian\_gradient

```
Gamma.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Gamma.get\_shape

```
Gamma.get_shape (ind)
```

#### bayespy.nodes.Gamma.has\_plotter

```
Gamma.has_plotter()
```

Return True if the node has a plotter

#### bayespy.nodes.Gamma.initialize\_from\_parameters

```
Gamma.initialize_from_parameters(*args)
```

#### bayespy.nodes.Gamma.initialize\_from\_prior

```
Gamma.initialize_from_prior()
```

#### bayespy.nodes.Gamma.initialize\_from\_random

```
Gamma.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Gamma.initialize\_from\_value

```
Gamma.initialize_from_value(x, *args)
```

#### bayespy.nodes.Gamma.load

```
Gamma.load(group)
```

Load the state of the node from a HDF5 file.

### bayespy.nodes.Gamma.logpdf

```
Gamma.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Gamma.lower\_bound\_contribution

```
Gamma.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X)]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.Gamma.lowerbound

```
Gamma.lowerbound()
```

### bayespy.nodes.Gamma.move\_plates

```
Gamma.move_plates (from_plate, to_plate)
```

### bayespy.nodes.Gamma.observe

```
Gamma.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Gamma.pdf

```
Gamma.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Gamma.plot

```
Gamma.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Gamma.random

```
Gamma.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Gamma.save

```
Gamma.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

#### bayespy.nodes.Gamma.set\_parameters

```
Gamma.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gamma.set\_plotter

```
Gamma.set_plotter(plotter)
```

#### bayespy.nodes.Gamma.show

```
Gamma.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Gamma.unobserve

```
Gamma.unobserve()
```

## bayespy.nodes.Gamma.update

```
Gamma.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

## bayespy.nodes.Gamma.dims

```
Gamma.dims = ((), ())
```

### bayespy.nodes.Gamma.plates

Gamma.plates = None

#### bayespy.nodes.Gamma.plates\_multiplier

#### Gamma.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.Wishart

class bayespy.nodes.Wishart (n, V, \*\*kwargs)

Node for Wishart random variables.

The random variable  $\Lambda$  is a  $D \times D$  positive-definite symmetric matrix.

$$p(\mathbf{\Lambda}) = \text{Wishart}(\mathbf{\Lambda}|N, \mathbf{V})$$

N, degrees of freedom, N > D - 1.

**V**: Wishart-like node or (...,D,D)-array

V, scale matrix.

\_\_init\_\_ (n, V, \*\*kwargs)
Create Wishart node.

### Methods

init(n, V, **kwargs)	Create Wishart node.
add_plate_axis(to_plate)	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
	Continued on next page

Table 5.10 – continued from previous page

```
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                               Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                               Load the state of the node from a HDF5 file.
logpdf(X[, mask])
                                               Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                               Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                               Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                               Compute the probability density function of this node.
plot([fig])
                                               Plot the node distribution using the plotter of the node
random()
                                               Draw a random sample from the distribution.
save(group)
                                               Save the state of the node into a HDF5 file.
set_parameters(x)
                                               Set the parameters of the VB distribution.
set_plotter(plotter)
show()
unobserve()
update([annealing])
```

### bayespy.nodes.Wishart.\_\_init\_\_

```
Wishart.__init__ (n, V, **kwargs)
Create Wishart node.
```

#### bayespy.nodes.Wishart.add\_plate\_axis

```
Wishart.add_plate_axis (to_plate)
```

#### bayespy.nodes.Wishart.broadcasting\_multiplier

```
Wishart.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Wishart.delete

```
Wishart.delete()
```

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Delete this node and the children

#### bayespy.nodes.Wishart.get\_gradient

```
Wishart.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

```
bayespy.nodes.Wishart.get_mask
Wishart.get_mask()
bayespy.nodes.Wishart.get_moments
Wishart.get_moments()
bayespy.nodes.Wishart.get_parameters
Wishart.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.Wishart.get_riemannian_gradient
Wishart.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.Wishart.get_shape
Wishart.get_shape(ind)
bayespy.nodes.Wishart.has_plotter
Wishart.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Wishart.initialize_from_parameters
Wishart.initialize_from_parameters(*args)
bayespy.nodes.Wishart.initialize_from_prior
```

bayespy.nodes.Wishart.initialize\_from\_value

Wishart.initialize\_from\_random()

Wishart.initialize\_from\_prior()

bayespy.nodes.Wishart.initialize\_from\_random

Wishart.initialize\_from\_value (x, \*args)

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Wishart.load

```
Wishart.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Wishart.logpdf

```
Wishart.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Wishart.lower\_bound\_contribution

```
Wishart.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

### bayespy.nodes.Wishart.lowerbound

```
Wishart.lowerbound()
```

#### bayespy.nodes.Wishart.move\_plates

```
Wishart.move_plates (from_plate, to_plate)
```

### bayespy.nodes.Wishart.observe

```
Wishart.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

### bayespy.nodes.Wishart.pdf

```
Wishart.pdf(X, mask=True)
```

Compute the probability density function of this node.

### bayespy.nodes.Wishart.plot

```
Wishart.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Wishart.random

```
Wishart.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Wishart.save

```
Wishart.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

### bayespy.nodes.Wishart.set\_parameters

```
Wishart.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Wishart.set\_plotter

```
Wishart.set_plotter(plotter)
```

### bayespy.nodes.Wishart.show

```
Wishart.show()
```

### bayespy.nodes.Wishart.unobserve

```
Wishart.unobserve()
```

### bayespy.nodes.Wishart.update

```
Wishart.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.Wishart.dims

```
Wishart.dims = None
```

#### bayespy.nodes.Wishart.plates

```
Wishart.plates = None
```

#### bayespy.nodes.Wishart.plates\_multiplier

```
Wishart.plates_multiplier
```

Plate multiplier is applied to messages to parents

### bayespy.nodes.Exponential

```
{f class} bayespy.nodes.Exponential (l,**kwargs)
```

Node for exponential random variables.

**Warning:** Use Gamma instead of this. Exponential(l) is equivalent to Gamma(1, l).

Parameters 1: gamma-like node or scalar or array

Rate parameter

#### See also:

Gamma, Poisson

#### **Notes**

For simplicity, this is just a gamma node with the first parent fixed to one. Note that this is a bit inconsistent with the BayesPy philosophy which states that the node does not only define the form of the prior distribution but more importantly the form of the posterior approximation. Thus, one might expect that this node would have exponential posterior distribution approximation. However, it has a gamma distribution. Also, the moments are gamma moments although only E[x] would be the moment of a exponential random variable. All this was done because: a) gamma was already implemented, so there was no need to implement anything, and b) people might easily use Exponential node as a prior definition and expect to get gamma posterior (which is what happens now). Maybe some day a pure Exponential node is implemented and the users are advised to use Gamma(1,b) if they want to use an exponential prior distribution but gamma posterior approximation.

```
__init__ (l, **kwargs)
```

#### **Methods**

```
--init__(l, **kwargs)

add_plate_axis(to_plate)

as_diagonal_wishart()

broadcasting_multiplier(plates, *args)

delete()

get_gradient(rg)

get_mask()

get_moments()

get_parameters()

get_riemannian_gradient()

get_shape(ind)

Delete this node and the children

Computes gradient with respect to the natural parameters.

Return parameters of the VB distribution.

Computes the Riemannian/natural gradient.
```

Continued on next page

Table 5.12 – continued from previous page

has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
$set_parameters(x)$	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
<pre>update([annealing])</pre>	

### bayespy.nodes.Exponential.\_\_init\_\_

```
Exponential.__init__(l, **kwargs)
```

### bayespy.nodes.Exponential.add\_plate\_axis

```
Exponential.add_plate_axis(to_plate)
```

### bayespy.nodes.Exponential.as\_diagonal\_wishart

```
Exponential.as_diagonal_wishart()
```

### bayespy.nodes.Exponential.broadcasting\_multiplier

Exponential.broadcasting\_multiplier(plates, \*args)

### bayespy.nodes.Exponential.delete

```
Exponential.delete()
```

Delete this node and the children

### bayespy.nodes.Exponential.get\_gradient

```
Exponential.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

```
bayespy.nodes.Exponential.get_mask
```

```
Exponential.get_mask()
```

#### bayespy.nodes.Exponential.get\_moments

```
Exponential.get_moments()
```

#### bayespy.nodes.Exponential.get\_parameters

```
Exponential.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Exponential.get\_riemannian\_gradient

```
Exponential.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

# bayespy.nodes.Exponential.get\_shape

```
Exponential.get_shape (ind)
```

#### bayespy.nodes.Exponential.has\_plotter

```
Exponential.has_plotter()
```

Return True if the node has a plotter

### bayespy.nodes.Exponential.initialize\_from\_parameters

```
Exponential.initialize_from_parameters(*args)
```

# $bayespy.nodes. Exponential. initialize\_from\_prior$

```
Exponential.initialize_from_prior()
```

# $bayespy.nodes. Exponential. initialize\_from\_random$

```
Exponential.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Exponential.initialize\_from\_value

```
Exponential.initialize_from_value(x, *args)
```

#### bayespy.nodes.Exponential.load

```
Exponential.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Exponential.logpdf

```
Exponential.logpdf (X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Exponential.lower\_bound\_contribution

```
Exponential.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Exponential.lowerbound

```
Exponential.lowerbound()
```

# bayespy.nodes.Exponential.move\_plates

```
Exponential.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Exponential.observe

```
Exponential.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

#### bayespy.nodes.Exponential.pdf

```
Exponential.pdf (X, mask=True)
```

Compute the probability density function of this node.

### bayespy.nodes.Exponential.plot

```
Exponential.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.nodes.Exponential.random

```
Exponential.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Exponential.save

```
Exponential.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

#### bayespy.nodes.Exponential.set\_parameters

```
Exponential.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Exponential.set\_plotter

```
Exponential.set_plotter(plotter)
```

### bayespy.nodes.Exponential.show

```
Exponential.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Exponential.unobserve

```
Exponential.unobserve()
```

# bayespy.nodes.Exponential.update

```
Exponential.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.Exponential.dims

```
Exponential.dims = ((), ())
```

### bayespy.nodes.Exponential.plates

```
Exponential.plates = None
```

#### bayespy.nodes.Exponential.plates\_multiplier

```
Exponential.plates_multiplier
```

Plate multiplier is applied to messages to parents

Nodes for modelling Gaussian and precision variables jointly (useful as prior for Gaussian nodes):

GaussianGammaISO(*args, **kwargs)	Node for Gaussian-gamma (isotropic) random variables.
GaussianGammaARD(mu, alpha, a, b, **kwargs)	Node for Gaussian and gamma random variables with ARD form.
<pre>GaussianWishart(*args, **kwargs)</pre>	Node for Gaussian-Wishart random variables.

### bayespy.nodes.GaussianGammalSO

```
class bayespy.nodes.GaussianGammaISO(*args, **kwargs)
```

Node for Gaussian-gamma (isotropic) random variables.

The prior:

$$p(x, \alpha | \mu, \Lambda, a, b)$$

$$p(x | \alpha, \mu, \Lambda) = \mathcal{N}(x | \mu, \alpha Lambda)$$

$$p(\alpha | a, b) = \mathcal{G}(\alpha | a, b)$$

The posterior approximation  $q(x,\alpha)$  has the same Gaussian-gamma form.

Currently, supports only vector variables.

#### **Methods**

```
__init__(*args, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
                                             Delete this node and the children
delete()
get_gaussian_mean_and_variance()
                                             Return the mean and variance of the distribution
                                             Computes gradient with respect to the natural parameters.
get_gradient(rg)
get_marginal_logpdf([gaussian, gamma])
                                             Get the (marginal) log pdf of a subset of the variables
get_mask()
get_moments()
                                             Return parameters of the VB distribution.
get_parameters()
get_riemannian_gradient()
                                             Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                             Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                             Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
                                                                               Continued on next page
```

### Table 5.15 – continued from previous page

	on made non provided page
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
plotmatrix()	Creates a matrix of marginal plots.
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

### bayespy.nodes.GaussianGammalSO.\_\_init\_\_

```
GaussianGammaISO.__init__(*args, **kwargs)
```

#### bayespy.nodes.GaussianGammalSO.add\_plate\_axis

GaussianGammaISO.add\_plate\_axis (to\_plate)

### $bayes py. nodes. Gaussian Gammal SO. broadcasting\_multiplier$

GaussianGammaISO.broadcasting\_multiplier(plates, \*args)

### bayespy.nodes.GaussianGammalSO.delete

```
GaussianGammaISO.delete()

Delete this node and the children
```

### bayespy.nodes.GaussianGammalSO.get\_gaussian\_mean\_and\_variance

```
GaussianGammaISO.get_gaussian_mean_and_variance()
```

Return the mean and variance of the distribution

### bayespy.nodes.GaussianGammalSO.get\_gradient

```
{\tt GaussianGammaISO.get\_gradient}\ (\textit{rg})
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.GaussianGammalSO.get\_marginal\_logpdf

```
GaussianGammaISO.get_marginal_logpdf(gaussian=None, gamma=None)
    Get the (marginal) log pdf of a subset of the variables
        Parameters gaussian: list or None
              Indices of the Gaussian variables to keep or None
            gamma: bool or None
              True if keep the gamma variable, otherwise False or None
        Returns function
              A function which computes log-pdf
bayespy.nodes.GaussianGammalSO.get_mask
GaussianGammaISO.get_mask()
bayespy.nodes.GaussianGammalSO.get_moments
GaussianGammaISO.get_moments()
bayespy.nodes.GaussianGammalSO.get_parameters
GaussianGammaISO.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.GaussianGammalSO.get_riemannian_gradient
GaussianGammaISO.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.GaussianGammalSO.get_shape
GaussianGammaISO.get_shape (ind)
bayespy.nodes.GaussianGammalSO.has_plotter
GaussianGammaISO.has_plotter()
```

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Return True if the node has a plotter

bayespy.nodes.GaussianGammalSO.initialize\_from\_parameters

GaussianGammaISO.initialize\_from\_parameters(\*args)

```
bayespy.nodes.GaussianGammalSO.initialize_from_prior
GaussianGammaISO.initialize_from_prior()
bayespy.nodes.GaussianGammalSO.initialize_from_random
GaussianGammaISO.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.GaussianGammalSO.initialize_from_value
GaussianGammaISO.initialize_from_value(x, *args)
bayespy.nodes.GaussianGammalSO.load
GaussianGammaISO.load(group)
    Load the state of the node from a HDF5 file.
bayespy.nodes.GaussianGammalSO.logpdf
GaussianGammaISO.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.GaussianGammalSO.lower_bound_contribution
GaussianGammaISO.lower_bound_contribution (gradient=False, ignore_masked=True)
    Compute E[ \log p(X|parents) - \log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.GaussianGammalSO.lowerbound
GaussianGammaISO.lowerbound()
bayespy.nodes.GaussianGammalSO.move_plates
GaussianGammaISO.move_plates (from_plate, to_plate)
bayespy.nodes.GaussianGammalSO.observe
GaussianGammaISO.observe(x, *args, mask=True)
    Fix moments, compute f and propagate mask.
```

#### bayespy.nodes.GaussianGammalSO.pdf

```
GaussianGammaISO.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.GaussianGammalSO.plot

```
GaussianGammaISO.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianGammalSO.plotmatrix

```
GaussianGammaISO.plotmatrix()
```

Creates a matrix of marginal plots.

On diagonal, are marginal plots of each variable. Off-diagonal plot (i,j) shows the joint marginal density of  $x_i$  and  $x_j$ .

### bayespy.nodes.GaussianGammalSO.random

```
GaussianGammaISO.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.GaussianGammalSO.save

```
GaussianGammaISO.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

#### bayespy.nodes.GaussianGammalSO.set\_parameters

```
GaussianGammaISO.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammalSO.set\_plotter

```
GaussianGammaISO.set_plotter(plotter)
```

#### bayespy.nodes.GaussianGammalSO.show

```
GaussianGammaISO.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.GaussianGammalSO.unobserve

GaussianGammaISO.unobserve()

#### bayespy.nodes.GaussianGammalSO.update

GaussianGammaISO.update (annealing=1.0)

#### **Attributes**

```
dims plates
```

plates\_multiplier Plate multiplier is applied to messages to parents

### bayespy.nodes.GaussianGammalSO.dims

GaussianGammaISO.dims = None

### bayespy.nodes.GaussianGammalSO.plates

GaussianGammaISO.plates = None

### bayespy.nodes.GaussianGammalSO.plates\_multiplier

```
GaussianGammaISO.plates_multiplier
```

Plate multiplier is applied to messages to parents

### bayespy.nodes.GaussianGammaARD

class bayespy.nodes.GaussianGammaARD (mu, alpha, a, b, \*\*kwargs)

Node for Gaussian and gamma random variables with ARD form.

The prior:

$$p(x, \tau | \mu, \alpha, a, b) = p(x | \tau, \mu, \alpha) p(\tau | a, b)$$
$$p(x | \alpha, \mu, \alpha) = \mathcal{N}(x | \mu, \text{diag}(\alpha \tau))$$
$$p(\tau | a, b) = \mathcal{G}(\tau | a, b)$$

The posterior approximation  $q(x,\tau)$  has the same Gaussian-gamma form.

Warning: Not yet implemented.

#### See also:

```
Gaussian, GaussianARD, Gamma, GaussianGammaISO, GaussianWishart
__init__(mu, alpha, a, b, **kwargs)
```

#### **Methods**

```
__init__(mu, alpha, a, b, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
                                               Delete this node and the children
delete()
                                               Computes gradient with respect to the natural parameters.
get_gradient(rg)
get_mask()
get_moments()
get_parameters()
                                               Return parameters of the VB distribution.
get_riemannian_gradient()
                                               Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                               Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                               Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                               Load the state of the node from a HDF5 file.
logpdf(X[, mask])
                                               Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                               Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                               Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                               Compute the probability density function of this node.
                                               Plot the node distribution using the plotter of the node
plot([fig])
                                               Draw a random sample from the distribution.
random()
                                               Save the state of the node into a HDF5 file.
save(group)
                                               Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
unobserve()
update([annealing])
```

#### bayespy.nodes.GaussianGammaARD.\_\_init\_\_

```
GaussianGammaARD.__init__ (mu, alpha, a, b, **kwargs)
```

#### bayespy.nodes.GaussianGammaARD.add\_plate\_axis

GaussianGammaARD.add\_plate\_axis(to\_plate)

#### bayespy.nodes.GaussianGammaARD.broadcasting\_multiplier

GaussianGammaARD.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.GaussianGammaARD.delete

```
GaussianGammaARD.delete()

Delete this node and the children
```

#### bayespy.nodes.GaussianGammaARD.get\_gradient

```
GaussianGammaARD.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

### bayespy.nodes.GaussianGammaARD.get\_mask

```
GaussianGammaARD.get_mask()
```

### bayespy.nodes.GaussianGammaARD.get\_moments

```
GaussianGammaARD.get_moments()
```

#### bayespy.nodes.GaussianGammaARD.get\_parameters

```
GaussianGammaARD.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammaARD.get\_riemannian\_gradient

```
GaussianGammaARD.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

### bayespy.nodes.GaussianGammaARD.get\_shape

```
GaussianGammaARD.get_shape (ind)
```

#### bayespy.nodes.GaussianGammaARD.has\_plotter

```
GaussianGammaARD.has_plotter()
Return True if the node has a plotter
```

#### bayespy.nodes.GaussianGammaARD.initialize\_from\_parameters

```
GaussianGammaARD.initialize_from_parameters(*args)
```

#### bayespy.nodes.GaussianGammaARD.initialize\_from\_prior

```
GaussianGammaARD.initialize_from_prior()
```

## bayespy.nodes.GaussianGammaARD.initialize\_from\_random

```
GaussianGammaARD.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

## bayespy.nodes.GaussianGammaARD.initialize\_from\_value

```
GaussianGammaARD.initialize_from_value (x, *args)
```

#### bayespy.nodes.GaussianGammaARD.load

```
GaussianGammaARD.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.GaussianGammaARD.logpdf

```
GaussianGammaARD.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.GaussianGammaARD.lower\_bound\_contribution

```
GaussianGammaARD.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.GaussianGammaARD.lowerbound

```
GaussianGammaARD.lowerbound()
```

# bayespy.nodes.GaussianGammaARD.move\_plates

GaussianGammaARD.move\_plates (from\_plate, to\_plate)

# bayespy.nodes.GaussianGammaARD.observe

```
GaussianGammaARD.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

#### bayespy.nodes.GaussianGammaARD.pdf

```
GaussianGammaARD.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.GaussianGammaARD.plot

```
GaussianGammaARD.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianGammaARD.random

```
GaussianGammaARD.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.GaussianGammaARD.save

```
GaussianGammaARD.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

# bayespy.nodes.GaussianGammaARD.set\_parameters

```
GaussianGammaARD.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammaARD.set\_plotter

```
GaussianGammaARD.set_plotter(plotter)
```

## bayespy.nodes.GaussianGammaARD.unobserve

```
GaussianGammaARD.unobserve()
```

# bayespy.nodes.GaussianGammaARD.update

```
GaussianGammaARD.update (annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.GaussianGammaARD.dims

GaussianGammaARD.dims = None

## bayespy.nodes.GaussianGammaARD.plates

GaussianGammaARD.plates = None

#### bayespy.nodes.GaussianGammaARD.plates\_multiplier

GaussianGammaARD.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.GaussianWishart

class bayespy.nodes.GaussianWishart (\*args, \*\*kwargs)

Node for Gaussian-Wishart random variables.

The prior:

$$p(x, \Lambda | \mu, \alpha, V, n)$$
  

$$p(x | \Lambda, \mu, \alpha) = (N)(x | \mu, \alpha^{-1} Lambda^{-1})$$
  

$$p(\Lambda | V, n) = (W)(\Lambda | n, V)$$

The posterior approximation  $q(x, \Lambda)$  has the same Gaussian-Wishart form.

Currently, supports only vector variables.

```
__init__(*args, **kwargs)
```

#### **Methods**

```
__init__(*args, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                             Delete this node and the children
                                             Computes gradient with respect to the natural parameters.
get_gradient(rg)
get_mask()
get_moments()
get_parameters()
                                             Return parameters of the VB distribution.
get_riemannian_gradient()
                                             Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                             Return True if the node has a plotter
initialize_from_parameters(*args)
                                                                               Continued on next page
```

Table 5.19 – continued from previous page

```
initialize_from_prior()
                                                Set the variable to a random sample from the current distribution.
initialize_from_random()
initialize_from_value(x, *args)
                                                Load the state of the node from a HDF5 file.
load(group)
logpdf(X[, mask])
                                                Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                                Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                Compute the probability density function of this node.
plot([fig])
                                                Plot the node distribution using the plotter of the node
random()
                                                Draw a random sample from the distribution.
save(group)
                                                Save the state of the node into a HDF5 file.
                                                Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
show()
                                                Print the distribution using standard parameterization.
unobserve()
update([annealing])
```

## bayespy.nodes.GaussianWishart.\_\_init\_\_

```
GaussianWishart.__init__(*args, **kwargs)
```

## bayespy.nodes.GaussianWishart.add\_plate\_axis

GaussianWishart.add\_plate\_axis(to\_plate)

#### bayespy.nodes.GaussianWishart.broadcasting\_multiplier

GaussianWishart.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.GaussianWishart.delete

```
GaussianWishart.delete()

Delete this node and the children
```

## bayespy.nodes.GaussianWishart.get\_gradient

```
GaussianWishart.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.GaussianWishart.get\_mask

```
GaussianWishart.get_mask()
```

## bayespy.nodes.GaussianWishart.get\_moments

```
GaussianWishart.get_moments()
```

## bayespy.nodes.GaussianWishart.get\_parameters

```
GaussianWishart.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.GaussianWishart.get\_riemannian\_gradient

```
GaussianWishart.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

# bayespy.nodes.GaussianWishart.get\_shape

```
GaussianWishart.get_shape(ind)
```

## bayespy.nodes.GaussianWishart.has\_plotter

```
GaussianWishart.has_plotter()
```

Return True if the node has a plotter

# $bayes py. nodes. Gaussian Wishart. initialize\_from\_parameters$

```
GaussianWishart.initialize_from_parameters(*args)
```

# bayespy.nodes.GaussianWishart.initialize\_from\_prior

```
GaussianWishart.initialize_from_prior()
```

# bayespy.nodes.GaussianWishart.initialize\_from\_random

```
GaussianWishart.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

# bayespy.nodes.GaussianWishart.initialize\_from\_value

```
GaussianWishart.initialize_from_value(x, *args)
```

#### bayespy.nodes.GaussianWishart.load

```
GaussianWishart.load (group)

Load the state of the node from a HDF5 file.
```

## bayespy.nodes.GaussianWishart.logpdf

```
GaussianWishart.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

## bayespy.nodes.GaussianWishart.lower\_bound\_contribution

```
GaussianWishart.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.GaussianWishart.lowerbound

```
GaussianWishart.lowerbound()
```

#### bayespy.nodes.GaussianWishart.move\_plates

```
GaussianWishart.move_plates (from_plate, to_plate)
```

# bayespy.nodes.GaussianWishart.observe

```
GaussianWishart.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

## bayespy.nodes.GaussianWishart.pdf

```
GaussianWishart.pdf (X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.GaussianWishart.plot

```
GaussianWishart.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.GaussianWishart.random

```
GaussianWishart.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.GaussianWishart.save

```
GaussianWishart.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

# bayespy.nodes.GaussianWishart.set\_parameters

```
GaussianWishart.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.GaussianWishart.set\_plotter

```
GaussianWishart.set_plotter(plotter)
```

# bayespy.nodes.GaussianWishart.show

```
GaussianWishart.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.GaussianWishart.unobserve

```
GaussianWishart.unobserve()
```

# bayespy.nodes.GaussianWishart.update

```
GaussianWishart.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.GaussianWishart.dims

```
GaussianWishart.dims = None
```

## bayespy.nodes.GaussianWishart.plates

```
GaussianWishart.plates = None
```

# bayespy.nodes.GaussianWishart.plates\_multiplier

```
GaussianWishart.plates_multiplier
```

Plate multiplier is applied to messages to parents

Nodes for discrete count variables:

Bernoulli(p, **kwargs)	Node for Bernoulli random variables.
Binomial(n, p, **kwargs)	Node for binomial random variables.
Categorical(p, **kwargs)	Node for categorical random variables.
<pre>Multinomial(n, p, **kwargs)</pre>	Node for multinomial random variables.
Poisson(l, **kwargs)	Node for Poisson random variables.

# bayespy.nodes.Bernoulli

```
class bayespy.nodes.Bernoulli(p, **kwargs)
```

Node for Bernoulli random variables.

The node models a binary random variable  $z \in \{0,1\}$  with prior probability  $p \in [0,1]$  for value one:

$$z \sim \text{Bernoulli}(p)$$
.

# **Parameters p**: beta-like node

Probability of a successful trial

## **Examples**

# Methods

init(p, **kwargs)	Create Bernoulli node.
add_plate_axis(to_plate)	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
	Continued on next page

Table 5.22 - continued from previous page

```
get_mask()
get_moments()
get_parameters()
                                               Return parameters of the VB distribution.
get_riemannian_gradient()
                                               Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                               Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                               Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
                                               Load the state of the node from a HDF5 file.
load(group)
logpdf(X[, mask])
                                               Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                               Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                               Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                               Compute the probability density function of this node.
                                               Plot the node distribution using the plotter of the node
plot([fig])
random()
                                               Draw a random sample from the distribution.
save(group)
                                               Save the state of the node into a HDF5 file.
set_parameters(x)
                                               Set the parameters of the VB distribution.
set_plotter(plotter)
show()
                                               Print the distribution using standard parameterization.
unobserve()
update([annealing])
```

# bayespy.nodes.Bernoulli.\_\_init\_\_

```
Bernoulli.__init__(p, **kwargs)
Create Bernoulli node.
```

#### bayespy.nodes.Bernoulli.add\_plate\_axis

```
Bernoulli.add_plate_axis(to_plate)
```

## bayespy.nodes.Bernoulli.broadcasting\_multiplier

```
Bernoulli.broadcasting.multiplier (plates, *args)
```

#### bayespy.nodes.Bernoulli.delete

```
Bernoulli.delete()
```

Delete this node and the children

#### bayespy.nodes.Bernoulli.get\_gradient

```
Bernoulli.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Bernoulli.get\_mask

```
Bernoulli.get_mask()
```

#### bayespy.nodes.Bernoulli.get\_moments

```
Bernoulli.get_moments()
```

### bayespy.nodes.Bernoulli.get\_parameters

```
Bernoulli.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Bernoulli.get\_riemannian\_gradient

```
Bernoulli.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Bernoulli.get\_shape

```
Bernoulli.get_shape (ind)
```

## bayespy.nodes.Bernoulli.has\_plotter

```
Bernoulli.has_plotter()
```

Return True if the node has a plotter

# bayespy.nodes.Bernoulli.initialize\_from\_parameters

```
Bernoulli.initialize_from_parameters(*args)
```

### bayespy.nodes.Bernoulli.initialize\_from\_prior

```
Bernoulli.initialize_from_prior()
```

## bayespy.nodes.Bernoulli.initialize\_from\_random

```
Bernoulli.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Bernoulli.initialize\_from\_value

```
Bernoulli.initialize_from_value(x, *args)
```

## bayespy.nodes.Bernoulli.load

```
Bernoulli.load(group)
```

Load the state of the node from a HDF5 file.

## bayespy.nodes.Bernoulli.logpdf

```
Bernoulli.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Bernoulli.lower\_bound\_contribution

```
Bernoulli.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Bernoulli.lowerbound

```
Bernoulli.lowerbound()
```

# bayespy.nodes.Bernoulli.move\_plates

```
Bernoulli.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Bernoulli.observe

```
Bernoulli.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Bernoulli.pdf

```
Bernoulli.pdf(X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Bernoulli.plot

```
Bernoulli.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Bernoulli.random

```
Bernoulli.random()
```

Draw a random sample from the distribution.

# bayespy.nodes.Bernoulli.save

```
Bernoulli.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

# bayespy.nodes.Bernoulli.set\_parameters

```
Bernoulli.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Bernoulli.set\_plotter

```
Bernoulli.set_plotter(plotter)
```

# bayespy.nodes.Bernoulli.show

```
Bernoulli.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Bernoulli.unobserve

```
Bernoulli.unobserve()
```

# bayespy.nodes.Bernoulli.update

```
Bernoulli.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Bernoulli.dims

```
Bernoulli.dims = None
```

### bayespy.nodes.Bernoulli.plates

```
Bernoulli.plates = None
```

## bayespy.nodes.Bernoulli.plates\_multiplier

```
Bernoulli.plates_multiplier
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.Binomial

```
class bayespy.nodes.Binomial (n, p, **kwargs)
```

Node for binomial random variables.

The node models the number of successes  $x \in \{0, \dots, n\}$  in n trials with probability p for success:

$$x \sim \text{Binomial}(n, p)$$
.

**Parameters n**: scalar or array

Number of trials

**p**: beta-like node or scalar or array

Probability of a success in a trial

# See also:

Bernoulli, Multinomial, Beta

## **Examples**

#### **Methods**

# Table 5.24 – continued from previous page

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<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
$set_parameters(x)$	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

## bayespy.nodes.Binomial.\_\_init\_\_

```
Binomial.__init__(n, p, **kwargs)

Create binomial node
```

# bayespy.nodes.Binomial.add\_plate\_axis

Binomial.add\_plate\_axis(to\_plate)

# bayespy.nodes.Binomial.broadcasting\_multiplier

Binomial.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.Binomial.delete

```
Binomial.delete()
```

Delete this node and the children

# $bayes py. nodes. Binomial. get\_gradient$

## Binomial.get\_gradient(rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient

is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

```
bayespy.nodes.Binomial.get_mask
```

```
Binomial.get_mask()
```

# bayespy.nodes.Binomial.get\_moments

```
Binomial.get_moments()
```

## bayespy.nodes.Binomial.get\_parameters

```
Binomial.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Binomial.get\_riemannian\_gradient

```
Binomial.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

# bayespy.nodes.Binomial.get\_shape

```
Binomial.get_shape(ind)
```

# bayespy.nodes.Binomial.has\_plotter

```
Binomial.has_plotter()
```

Return True if the node has a plotter

# $bayespy.nodes. Binomial. initialize\_from\_parameters$

```
Binomial.initialize_from_parameters(*args)
```

# bayespy.nodes.Binomial.initialize\_from\_prior

```
Binomial.initialize_from_prior()
```

## bayespy.nodes.Binomial.initialize\_from\_random

```
Binomial.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Binomial.initialize\_from\_value

```
Binomial.initialize_from_value(x, *args)
```

# bayespy.nodes.Binomial.load

```
Binomial.load(group)
```

Load the state of the node from a HDF5 file.

## bayespy.nodes.Binomial.logpdf

```
Binomial.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Binomial.lower\_bound\_contribution

```
Binomial.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Binomial.lowerbound

```
Binomial.lowerbound()
```

# bayespy.nodes.Binomial.move\_plates

```
Binomial.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Binomial.observe

```
Binomial.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Binomial.pdf

```
Binomial.pdf (X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Binomial.plot

```
Binomial.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Binomial.random

```
Binomial.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Binomial.save

```
Binomial.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

# bayespy.nodes.Binomial.set\_parameters

```
Binomial.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Binomial.set\_plotter

```
Binomial.set_plotter(plotter)
```

# bayespy.nodes.Binomial.show

```
Binomial.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Binomial.unobserve

```
Binomial.unobserve()
```

# bayespy.nodes.Binomial.update

```
Binomial.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Binomial.dims

```
Binomial.dims = None
```

## bayespy.nodes.Binomial.plates

```
Binomial.plates = None
```

# bayespy.nodes.Binomial.plates\_multiplier

```
Binomial.plates_multiplier
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.Categorical

```
class bayespy.nodes.Categorical(p, **kwargs)
```

Node for categorical random variables.

The node models a categorical random variable  $x \in \{0, \dots, K-1\}$  with prior probabilities  $\{p_0, \dots, p_{K-1}\}$  for each category:

$$p(x = k) = p_k$$
 for  $k \in \{0, \dots, K - 1\}$ .

**Parameters p**: Dirichlet-like node or (...,K)-array

Probabilities for each category

## See also:

```
Bernoulli, Multinomial, Dirichlet
```

\_\_**init**\_\_ (p, \*\*kwargs)

Create Categorical node.

## Methods

init(p, **kwargs)	Create Categorical node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
	Continued on next page

# Table 5.26 – continued from previous page

	, , ,
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	·
update([annealing])	

## bayespy.nodes.Categorical.\_\_init\_\_

```
Categorical.__init__(p, **kwargs)
Create Categorical node.
```

# bayespy.nodes.Categorical.add\_plate\_axis

```
Categorical.add_plate_axis(to_plate)
```

## bayespy.nodes.Categorical.broadcasting\_multiplier

Categorical.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.Categorical.delete

```
Categorical.delete()

Delete this node and the children
```

## bayespy.nodes.Categorical.get\_gradient

```
Categorical.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# bayespy.nodes.Categorical.get\_mask

```
Categorical.get_mask()
```

# $bayes py. nodes. Categorical. get\_moments$

```
Categorical.get_moments()
```

# bayespy.nodes.Categorical.get\_parameters

```
Categorical.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.Categorical.get_riemannian_gradient
Categorical.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.Categorical.get_shape
Categorical.get_shape (ind)
bayespy.nodes.Categorical.has_plotter
Categorical.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Categorical.initialize_from_parameters
Categorical.initialize_from_parameters(*args)
bayespy.nodes.Categorical.initialize_from_prior
Categorical.initialize_from_prior()
bayespy.nodes.Categorical.initialize_from_random
Categorical.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Categorical.initialize_from_value
Categorical.initialize_from_value(x, *args)
```

# bayespy.nodes.Categorical.load

```
Categorical.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Categorical.logpdf

```
Categorical.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Categorical.lower\_bound\_contribution

```
Categorical.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.Categorical.lowerbound

```
Categorical.lowerbound()
```

## bayespy.nodes.Categorical.move\_plates

```
Categorical.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Categorical.observe

```
Categorical.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Categorical.pdf

```
Categorical.pdf (X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Categorical.plot

```
Categorical.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Categorical.random

```
Categorical.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Categorical.save

```
Categorical.save (group)

Save the state of the node into a HDF5 file.
```

group can be the root

## bayespy.nodes.Categorical.set\_parameters

```
Categorical.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Categorical.set\_plotter

```
Categorical.set_plotter(plotter)
```

#### bayespy.nodes.Categorical.show

```
Categorical.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.Categorical.unobserve

```
Categorical.unobserve()
```

# bayespy.nodes.Categorical.update

```
Categorical.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Categorical.dims

```
Categorical.dims = None
```

# bayespy.nodes.Categorical.plates

```
Categorical.plates = None
```

# bayespy.nodes.Categorical.plates\_multiplier

Categorical.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.Multinomial

class bayespy.nodes.Multinomial(n, p, \*\*kwargs)

Node for multinomial random variables.

Assume there are K categories and N trials each of which leads a success for exactly one of the categories. Given the probabilities  $p_0, \ldots, p_{K-1}$  for the categories, multinomial distribution is gives the probability of any combination of numbers of successes for the categories.

The node models the number of successes  $x_k \in \{0, ..., n\}$  in n trials with probability  $p_k$  for success in K categories.

$$\text{Multinomial}(\mathbf{x}|N,\mathbf{p}) = \frac{N!}{x_0! \cdots x_{K-1}!} p_0^{x_0} \cdots p_{K-1}^{x_{K-1}}$$

Parameters n: scalar or array

N, number of trials

**p**: Dirichlet-like node or (...,K)-array

p, probabilities of successes for the categories

#### See also:

Dirichlet, Binomial, Categorical
\_\_init\_\_(n, p, \*\*kwargs)

Create Multinomial node.

#### **Methods**

	C + M 10 1 1 1
init(n, p, **kwargs)	Create Multinomial node.
add_plate_axis(to_plate)	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
	Continued on next page

# Table 5.28 – continued from previous page

```
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                 Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                 Compute the probability density function of this node.
plot([fig])
                                                 Plot the node distribution using the plotter of the node
random()
                                                 Draw a random sample from the distribution.
                                                 Save the state of the node into a HDF5 file.
save(group)
                                                 Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
show()
                                                 Print the distribution using standard parameterization.
unobserve()
update([annealing])
```

# bayespy.nodes.Multinomial.\_\_init\_\_

```
Multinomial.__init__(n, p, **kwargs)

Create Multinomial node.
```

# bayespy.nodes.Multinomial.add\_plate\_axis

```
Multinomial.add_plate_axis(to_plate)
```

# bayespy.nodes.Multinomial.broadcasting\_multiplier

```
Multinomial.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Multinomial.delete

```
Multinomial.delete()

Delete this node and the children
```

# bayespy.nodes.Multinomial.get\_gradient

```
Multinomial.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# $bayespy.nodes. Multinomial.get\_mask$

```
Multinomial.get_mask()
```

## bayespy.nodes.Multinomial.get\_moments

```
Multinomial.get_moments()
```

#### bayespy.nodes.Multinomial.get\_parameters

```
Multinomial.get_parameters()
Return parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Multinomial.get\_riemannian\_gradient

```
Multinomial.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

# bayespy.nodes.Multinomial.get\_shape

```
Multinomial.get_shape(ind)
```

# bayespy.nodes.Multinomial.has\_plotter

```
Multinomial.has_plotter()

Return True if the node has a plotter
```

## bayespy.nodes.Multinomial.initialize\_from\_parameters

```
Multinomial.initialize_from_parameters(*args)
```

# bayespy.nodes.Multinomial.initialize\_from\_prior

```
Multinomial.initialize_from_prior()
```

# bayespy.nodes.Multinomial.initialize\_from\_random

```
Multinomial.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

# bayespy.nodes.Multinomial.initialize\_from\_value

```
Multinomial.initialize_from_value(x, *args)
```

# bayespy.nodes.Multinomial.load

```
Multinomial.load(group)
```

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Multinomial.logpdf

```
Multinomial.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Multinomial.lower\_bound\_contribution

```
Multinomial.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.Multinomial.lowerbound

```
Multinomial.lowerbound()
```

# bayespy.nodes.Multinomial.move\_plates

```
Multinomial.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Multinomial.observe

```
Multinomial.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Multinomial.pdf

```
Multinomial.pdf (X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Multinomial.plot

```
Multinomial.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Multinomial.random

```
Multinomial.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Multinomial.save

```
Multinomial.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

## bayespy.nodes.Multinomial.set\_parameters

```
Multinomial.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Multinomial.set\_plotter

```
Multinomial.set_plotter(plotter)
```

#### bayespy.nodes.Multinomial.show

```
Multinomial.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.Multinomial.unobserve

```
Multinomial.unobserve()
```

# bayespy.nodes.Multinomial.update

```
Multinomial.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Multinomial.dims

```
Multinomial.dims = None
```

# bayespy.nodes.Multinomial.plates

Multinomial.plates = None

# $bayes py. nodes. Multinomial. plates\_multiplier$

```
Multinomial.plates_multiplier
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.Poisson

 ${f class}$  bayespy.nodes.Poisson( ${\it l, **kwargs}$ )

Node for Poisson random variables.

The node uses Poisson distribution:

$$p(x) = Poisson(x|\lambda)$$

where  $\lambda$  is the rate parameter.

Parameters 1: gamma-like node or scalar or array

 $\lambda$ , rate parameter

# See also:

```
Gamma, Exponential
```

\_\_**init**\_\_(*l*, \*\*kwargs)

Create Poisson random variable node

## **Methods**

init(l, **kwargs)	Create Poisson random variable node
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
get_riemannian_gradient()	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
initialize_from_prior()	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
	Continued on next page

# Table 5.30 – continued from previous page

	1 1 5
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Poisson.\_\_init\_\_

```
Poisson.__init__(l, **kwargs)

Create Poisson random variable node
```

# bayespy.nodes.Poisson.add\_plate\_axis

Poisson.add\_plate\_axis(to\_plate)

# bayespy.nodes.Poisson.broadcasting\_multiplier

Poisson.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.Poisson.delete

```
Poisson.delete()
```

Delete this node and the children

# bayespy.nodes.Poisson.get\_gradient

```
Poisson.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Poisson.get\_mask

```
Poisson.get_mask()
```

#### bayespy.nodes.Poisson.get\_moments

```
Poisson.get_moments()
```

#### bayespy.nodes.Poisson.get\_parameters

```
Poisson.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.Poisson.get_riemannian_gradient
Poisson.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.Poisson.get_shape
Poisson.get_shape (ind)
bayespy.nodes.Poisson.has_plotter
Poisson.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Poisson.initialize_from_parameters
Poisson.initialize_from_parameters(*args)
bayespy.nodes.Poisson.initialize_from_prior
Poisson.initialize_from_prior()
bayespy.nodes.Poisson.initialize_from_random
Poisson.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Poisson.initialize_from_value
Poisson.initialize_from_value(x, *args)
bayespy.nodes.Poisson.load
```

Poisson.load(group)

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Poisson.logpdf

```
Poisson.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Poisson.lower\_bound\_contribution

```
Poisson.lower_bound_contribution (gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.Poisson.lowerbound

```
Poisson.lowerbound()
```

## bayespy.nodes.Poisson.move\_plates

```
Poisson.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Poisson.observe

```
Poisson.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Poisson.pdf

```
Poisson.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.Poisson.plot

```
Poisson.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Poisson.random

```
Poisson.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Poisson.save

```
Poisson.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

# bayespy.nodes.Poisson.set\_parameters

```
Poisson.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Poisson.set\_plotter

```
Poisson.set_plotter(plotter)
```

#### bayespy.nodes.Poisson.show

```
Poisson.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.Poisson.unobserve

```
Poisson.unobserve()
```

# bayespy.nodes.Poisson.update

```
Poisson.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Poisson.dims

```
Poisson.dims = ((),)
```

# bayespy.nodes.Poisson.plates

```
Poisson.plates = None
```

# bayespy.nodes.Poisson.plates\_multiplier

```
Poisson.plates_multiplier
```

Plate multiplier is applied to messages to parents

Nodes for probabilities:

Beta(alpha, **kwargs)	Node for beta random variables.
Dirichlet(*args, **kwargs)	Node for Dirichlet random variables.

# bayespy.nodes.Beta

```
class bayespy.nodes.Beta(alpha, **kwargs)
```

Node for beta random variables.

The node models a probability variable  $p \in [0, 1]$  as

$$p \sim \text{Beta}(a, b)$$

where a and b are prior counts for success and failure, respectively.

**Parameters alpha**: (...,2)-shaped array

Two-element vector containing  $\boldsymbol{a}$  and  $\boldsymbol{b}$ 

#### **Examples**

#### **Methods**

```
__init__(alpha, **kwargs)
                                              Create beta node
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                              Delete this node and the children
get_gradient(rg)
                                              Computes gradient with respect to the natural parameters.
get_mask()
get_moments()
get_parameters()
                                              Return parameters of the VB distribution.
                                              Computes the Riemannian/natural gradient.
get_riemannian_gradient()
get_shape(ind)
                                              Return True if the node has a plotter
has_plotter()
                                                                                Continued on next page
```

# Table 5.33 - continued from previous page

```
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                                Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                                Load the state of the node from a HDF5 file.
logpdf(X[, mask])
                                                Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                                Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                Compute the probability density function of this node.
plot([fig])
                                                Plot the node distribution using the plotter of the node
random()
                                                Draw a random sample from the distribution.
save(group)
                                                Save the state of the node into a HDF5 file.
set_parameters(x)
                                                Set the parameters of the VB distribution.
set_plotter(plotter)
show()
                                                Print the distribution using standard parameterization.
unobserve()
update([annealing])
```

# bayespy.nodes.Beta.\_\_init\_\_

```
Beta.__init__(alpha, **kwargs)
Create beta node
```

#### bayespy.nodes.Beta.add\_plate\_axis

```
Beta.add_plate_axis(to_plate)
```

#### bayespy.nodes.Beta.broadcasting\_multiplier

```
Beta.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Beta.delete

```
Beta.delete()
```

Delete this node and the children

### bayespy.nodes.Beta.get\_gradient

```
Beta.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

```
bayespy.nodes.Beta.get_mask
Beta.get_mask()
bayespy.nodes.Beta.get_moments
Beta.get_moments()
bayespy.nodes.Beta.get_parameters
Beta.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.Beta.get_riemannian_gradient
Beta.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.Beta.get_shape
Beta.get_shape(ind)
bayespy.nodes.Beta.has_plotter
Beta.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Beta.initialize_from_parameters
Beta.initialize_from_parameters(*args)
bayespy.nodes.Beta.initialize_from_prior
Beta.initialize_from_prior()
bayespy.nodes.Beta.initialize_from_random
Beta.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Beta.initialize_from_value
Beta.initialize_from_value(x, *args)
```

#### bayespy.nodes.Beta.load

```
Beta.load(group)
```

Load the state of the node from a HDF5 file.

## bayespy.nodes.Beta.logpdf

```
Beta.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

## bayespy.nodes.Beta.lower\_bound\_contribution

```
{\tt Beta.lower\_bound\_contribution} \ (\textit{gradient=False}, \textit{ignore\_masked=True})
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.Beta.lowerbound

```
Beta.lowerbound()
```

#### bayespy.nodes.Beta.move\_plates

```
Beta.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Beta.observe

```
Beta.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Beta.pdf

```
Beta.pdf(X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Beta.plot

```
Beta.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Beta.random

```
Beta.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Beta.save

```
Beta.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

## bayespy.nodes.Beta.set\_parameters

```
Beta.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Beta.set\_plotter

```
Beta.set_plotter(plotter)
```

## bayespy.nodes.Beta.show

```
Beta.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Beta.unobserve

```
Beta.unobserve()
```

## bayespy.nodes.Beta.update

```
Beta.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

## bayespy.nodes.Beta.dims

```
Beta.dims = None
```

## bayespy.nodes.Beta.plates

```
Beta.plates = None
```

## bayespy.nodes.Beta.plates\_multiplier

```
Beta.plates_multiplier
```

Plate multiplier is applied to messages to parents

## bayespy.nodes.Dirichlet

```
class bayespy.nodes.Dirichlet(*args, **kwargs)
```

Node for Dirichlet random variables.

The node models a set of probabilities  $\{\pi_0, \dots, \pi_{K-1}\}$  which satisfy  $\sum_{k=0}^{K-1} \pi_k = 1$  and  $\pi_k \in [0,1] \ \forall k = 0, \dots, K-1$ .

$$p(\pi_0, \dots, \pi_{K-1}) = Dirichlet(\alpha_0, \dots, \alpha_{K-1})$$

where  $\alpha_k$  are concentration parameters.

The posterior approximation has the same functional form but with different concentration parameters.

**Parameters alpha**: (...,K)-shaped array

Prior counts  $\alpha_k$ 

#### See also:

```
Beta, Categorical, Multinomial, CategoricalMarkovChain
__init__(*args, **kwargs)
```

## Methods

```
__init__(*args, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                              Delete this node and the children
get_gradient(rg)
                                              Computes gradient with respect to the natural parameters.
get_mask()
get_moments()
get_parameters()
                                              Return parameters of the VB distribution.
get_riemannian_gradient()
                                              Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                              Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                              Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                              Load the state of the node from a HDF5 file.
                                              Compute the log probability density function Q(X) of this node.
logpdf(X[, mask])
                                              Compute E[ \log p(X|parents) - \log q(X) ]
lower_bound_contribution([gradient, ...])
lowerbound()
```

Continued on next page

Table 5.35 - continued from previous page

```
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                 Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                 Compute the probability density function of this node.
plot([fig])
                                                 Plot the node distribution using the plotter of the node
random()
                                                 Draw a random sample from the distribution.
save(group)
                                                 Save the state of the node into a HDF5 file.
                                                 Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
                                                 Print the distribution using standard parameterization.
show()
unobserve()
update([annealing])
```

## bayespy.nodes.Dirichlet.\_\_init\_\_

```
Dirichlet.__init__(*args, **kwargs)
```

#### bayespy.nodes.Dirichlet.add\_plate\_axis

```
Dirichlet.add_plate_axis (to_plate)
```

### bayespy.nodes.Dirichlet.broadcasting\_multiplier

```
Dirichlet.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.Dirichlet.delete

```
Dirichlet.delete()
```

Delete this node and the children

## bayespy.nodes.Dirichlet.get\_gradient

```
Dirichlet.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Dirichlet.get\_mask

```
Dirichlet.get_mask()
```

### bayespy.nodes.Dirichlet.get\_moments

```
Dirichlet.get_moments()
```

#### bayespy.nodes.Dirichlet.get\_parameters

```
Dirichlet.get_parameters()
    Return parameters of the VB distribution.
    The parameters should be such that they can be used for optimization, that is, use log transformation for
    positive parameters.
bayespy.nodes.Dirichlet.get_riemannian_gradient
Dirichlet.get_riemannian_gradient()
    Computes the Riemannian/natural gradient.
bayespy.nodes.Dirichlet.get_shape
Dirichlet.get_shape(ind)
bayespy.nodes.Dirichlet.has_plotter
Dirichlet.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Dirichlet.initialize_from_parameters
Dirichlet.initialize_from_parameters(*args)
bayespy.nodes.Dirichlet.initialize_from_prior
Dirichlet.initialize_from_prior()
bayespy.nodes. Dirichlet. in itialize\_from\_random
Dirichlet.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Dirichlet.initialize_from_value
Dirichlet.initialize_from_value(x, *args)
bayespy.nodes.Dirichlet.load
```

Dirichlet.load(group)

Load the state of the node from a HDF5 file.

#### bayespy.nodes.Dirichlet.logpdf

```
Dirichlet.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Dirichlet.lower\_bound\_contribution

```
Dirichlet.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.Dirichlet.lowerbound

```
Dirichlet.lowerbound()
```

## bayespy.nodes.Dirichlet.move\_plates

```
Dirichlet.move_plates (from_plate, to_plate)
```

## bayespy.nodes.Dirichlet.observe

```
Dirichlet.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Dirichlet.pdf

```
Dirichlet.pdf(X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.Dirichlet.plot

```
Dirichlet.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Dirichlet.random

```
Dirichlet.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Dirichlet.save

```
Dirichlet.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

## bayespy.nodes.Dirichlet.set\_parameters

```
Dirichlet.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Dirichlet.set\_plotter

```
Dirichlet.set_plotter(plotter)
```

## bayespy.nodes.Dirichlet.show

```
Dirichlet.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.Dirichlet.unobserve

```
Dirichlet.unobserve()
```

## bayespy.nodes.Dirichlet.update

```
Dirichlet.update(annealing=1.0)
```

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.nodes.Dirichlet.dims

```
Dirichlet.dims = None
```

## bayespy.nodes.Dirichlet.plates

```
Dirichlet.plates = None
```

#### bayespy.nodes.Dirichlet.plates\_multiplier

## Dirichlet.plates\_multiplier

Plate multiplier is applied to messages to parents

Nodes for dynamic variables:

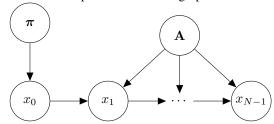
CategoricalMarkovChain(pi, A[, states])	Node for categorical Markov chain random variables.
<pre>GaussianMarkovChain(mu, Lambda, A, nu[, n,])</pre>	Node for Gaussian Markov chain random variables.
SwitchingGaussianMarkovChain(mu, Lambda, B,)	Node for Gaussian Markov chain random variables with switching d
VaryingGaussianMarkovChain(mu, Lambda, B, S, nu)	Node for Gaussian Markov chain random variables with time-varyin

## bayespy.nodes.CategoricalMarkovChain

class bayespy.nodes.CategoricalMarkovChain(pi, A, states=None, \*\*kwargs)

Node for categorical Markov chain random variables.

The node models a Markov chain which has a discrete set of K possible states and the next state depends only on the previous state and the state transition probabilities. The graphical model is shown below:



where  $\pi$  contains the probabilities for the initial state and A is the state transition probability matrix. It is possible to have A varying in time.

$$p(x_0, \dots, x_{N-1}) = p(x_0) \prod_{n=1}^{N-1} p(x_n | x_{n-1}),$$

where

$$p(x_0 = k) = \pi_k, \quad \text{for } k \in \{0, \dots, K - 1\},$$

$$p(x_n = j | x_{n-1} = i) = a_{ij}^{(n-1)} \quad \text{for } n = 1, \dots, N - 1, \ i \in \{1, \dots, K - 1\}, \ j \in \{1, \dots, K - 1\}$$

$$a_{ij}^{(n)} = [\mathbf{A}_n]_{ij}$$

This node can be used to construct hidden Markov models by using Mixture for the emission distribution.

Parameters pi : Dirichlet-like node or (...,K)-array

 $\pi$ , probabilities for the first state. K-dimensional Dirichlet.

A: Dirichlet-like node or (K,K)-array or (...,1,K,K)-array or (...,N-1,K,K)-array

A, probabilities for state transitions. K-dimensional Dirichlet with plates (K,) or (...,1,K) or (...,N-1,K).

states: int, optional

N, the length of the chain.

See also:

Categorical, Dirichlet, GaussianMarkovChain, Mixture, SwitchingGaussianMarkovChain

```
__init__ (pi, A, states=None, **kwargs)
Create categorical Markov chain
```

## **Methods**

init(pi, A[, states])	Create categorical Markov chain
add_plate_axis(to_plate)	č
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
get_moments()	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
$set_parameters(x)$	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

## bayespy.nodes.CategoricalMarkovChain.\_\_init\_\_

```
CategoricalMarkovChain.__init__(pi, A, states=None, **kwargs)
Create categorical Markov chain
```

# bayespy.nodes.CategoricalMarkovChain.add\_plate\_axis

CategoricalMarkovChain.add\_plate\_axis (to\_plate)

## bayespy.nodes.CategoricalMarkovChain.broadcasting\_multiplier

CategoricalMarkovChain.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.CategoricalMarkovChain.delete

```
CategoricalMarkovChain.delete()

Delete this node and the children
```

## bayespy.nodes.CategoricalMarkovChain.get\_gradient

```
CategoricalMarkovChain.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.CategoricalMarkovChain.get\_mask

```
CategoricalMarkovChain.get_mask()
```

## bayespy.nodes.CategoricalMarkovChain.get\_moments

```
CategoricalMarkovChain.get_moments()
```

#### bayespy.nodes.CategoricalMarkovChain.get\_parameters

```
CategoricalMarkovChain.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## $bayes py.nodes. Categorical Markov Chain. get\_riemannian\_gradient$

```
CategoricalMarkovChain.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

## bayespy.nodes.CategoricalMarkovChain.get\_shape

```
CategoricalMarkovChain.get_shape(ind)
```

### bayespy.nodes.CategoricalMarkovChain.has\_plotter

```
CategoricalMarkovChain.has_plotter()
Return True if the node has a plotter
```

## bayespy.nodes.CategoricalMarkovChain.initialize\_from\_parameters

```
{\tt Categorical Markov Chain. initialize\_from\_parameters}~(*args)
```

```
bayespy.nodes.CategoricalMarkovChain.initialize_from_prior
CategoricalMarkovChain.initialize_from_prior()
bayespy.nodes.CategoricalMarkovChain.initialize_from_random
CategoricalMarkovChain.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.CategoricalMarkovChain.initialize_from_value
CategoricalMarkovChain.initialize_from_value(x, *args)
bayespy.nodes.CategoricalMarkovChain.load
CategoricalMarkovChain.load(group)
    Load the state of the node from a HDF5 file.
bayespy.nodes.CategoricalMarkovChain.logpdf
CategoricalMarkovChain.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.CategoricalMarkovChain.lower_bound_contribution
CategoricalMarkovChain.lower_bound_contribution(gradient=False,
                                                                                       ig-
                                                            nore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.CategoricalMarkovChain.lowerbound
CategoricalMarkovChain.lowerbound()
bayespy.nodes.CategoricalMarkovChain.move_plates
CategoricalMarkovChain.move_plates (from_plate, to_plate)
bayespy.nodes.CategoricalMarkovChain.observe
```

CategoricalMarkovChain.observe(x, \*args, mask=True)

Fix moments, compute f and propagate mask.

#### bayespy.nodes.CategoricalMarkovChain.pdf

```
CategoricalMarkovChain.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.CategoricalMarkovChain.plot

```
CategoricalMarkovChain.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.nodes.CategoricalMarkovChain.random

```
CategoricalMarkovChain.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.CategoricalMarkovChain.save

```
CategoricalMarkovChain.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

### bayespy.nodes.CategoricalMarkovChain.set\_parameters

```
CategoricalMarkovChain.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.CategoricalMarkovChain.set\_plotter

```
CategoricalMarkovChain.set_plotter(plotter)
```

## bayespy.nodes.CategoricalMarkovChain.show

```
CategoricalMarkovChain.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.CategoricalMarkovChain.unobserve

```
CategoricalMarkovChain.unobserve()
```

#### bayespy.nodes.CategoricalMarkovChain.update

CategoricalMarkovChain.update(annealing=1.0)

#### **Attributes**

dims
plates
plates\_multiplier Plate multiplier is applied to messages to parents

## bayespy.nodes.CategoricalMarkovChain.dims

CategoricalMarkovChain.dims = None

## bayespy.nodes.CategoricalMarkovChain.plates

CategoricalMarkovChain.plates = None

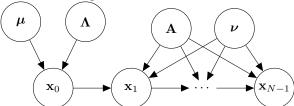
## bayespy.nodes.CategoricalMarkovChain.plates\_multiplier

CategoricalMarkovChain.plates\_multiplier
Plate multiplier is applied to messages to parents

## bayespy.nodes.GaussianMarkovChain

**class** bayespy.nodes.**GaussianMarkovChain** (*mu*, *Lambda*, *A*, *nu*, *n=None*, *inputs=None*, \*\*kwargs) Node for Gaussian Markov chain random variables.

In a simple case, the graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state, A is the state dynamics matrix and  $\nu$  is the precision of the innovation noise. It is possible that A and/or  $\nu$  are different for each transition instead of being constant.

The probability distribution is

$$p(\mathbf{x}_0, \dots, \mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n | \mathbf{x}_{n-1})$$

where

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda})$$
$$p(\mathbf{x}_n | \mathbf{x}_{n-1}) = \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu}_{n-1})).$$

```
Parameters \mathbf{mu}: Gaussian-like node or (...,D)-array \boldsymbol{\mu}, mean of x_0, D-dimensional with plates (...)

Lambda: Wishart-like node or (...,D,D)-array

\boldsymbol{\Lambda}, precision matrix of x_0, D \times D-dimensional with plates (...)

\boldsymbol{\Lambda}: Gaussian-like node or (D,D)-array or (...,1,D,D)-array or (...,N-1,D,D)-array

\boldsymbol{\Lambda}, state dynamics matrix, D-dimensional with plates (D,) or (...,1,D) or (...,N-1,D)

\boldsymbol{nu}: gamma-like node or (D,)-array or (...,1,D)-array or (...,N-1,D)-array

\boldsymbol{\nu}, diagonal elements of the precision of the innovation process, plates (D,) or (...,1,D) or (...,N-1,D)

\boldsymbol{n}: int, optional

\boldsymbol{N}, the length of the chain. Must be given if \boldsymbol{\Lambda} and \boldsymbol{\nu} are constant over time.
```

## See also:

```
Gaussian, GaussianARD, Wishart, Gamma, SwitchingGaussianMarkovChain, VaryingGaussianMarkovChain, CategoricalMarkovChain

__init__ (mu, Lambda, A, nu, n=None, inputs=None, **kwargs)

Create GaussianMarkovChain node.
```

#### **Methods**

init(mu, Lambda, A, nu[, n, inputs])	Create GaussianMarkovChain node.
add_plate_axis(to_plate)	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
get_riemannian_gradient()	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
move_plates(from_plate, to_plate)	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
<pre>rotate(R[, inv, logdet])</pre>	
save(group)	Save the state of the node into a HDF5 file.
set_parameters(x)	Set the parameters of the VB distribution.
	Continued on next page

## Table 5.40 – continued from previous page

```
set_plotter(plotter)
show()
unobserve()
update([annealing])
```

## bayespy.nodes.GaussianMarkovChain.\_\_init\_\_

```
GaussianMarkovChain.__init__ (mu, Lambda, A, nu, n=None, inputs=None, **kwargs)
Create GaussianMarkovChain node.
```

### bayespy.nodes.GaussianMarkovChain.add\_plate\_axis

```
GaussianMarkovChain.add_plate_axis(to_plate)
```

### bayespy.nodes.GaussianMarkovChain.broadcasting\_multiplier

```
GaussianMarkovChain.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.GaussianMarkovChain.delete

```
GaussianMarkovChain.delete()

Delete this node and the children
```

## bayespy.nodes.GaussianMarkovChain.get\_gradient

```
GaussianMarkovChain.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.GaussianMarkovChain.get\_mask

```
GaussianMarkovChain.get_mask()
```

#### bayespy.nodes.GaussianMarkovChain.get\_moments

```
GaussianMarkovChain.get_moments()
```

#### bayespy.nodes.GaussianMarkovChain.get\_parameters

```
GaussianMarkovChain.get_parameters()
Return parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

```
bayespy.nodes.GaussianMarkovChain.get_riemannian_gradient
```

```
GaussianMarkovChain.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

#### bayespy.nodes.GaussianMarkovChain.get\_shape

```
GaussianMarkovChain.get_shape(ind)
```

#### bayespy.nodes.GaussianMarkovChain.has\_plotter

```
GaussianMarkovChain.has_plotter()
Return True if the node has a plotter
```

## $bayes py. nodes. Gaussian Markov Chain. initialize\_from\_parameters$

```
GaussianMarkovChain.initialize_from_parameters(*args)
```

# $bayes py. nodes. Gaussian Markov Chain. in itialize\_from\_prior$

```
GaussianMarkovChain.initialize_from_prior()
```

## bayespy.nodes.GaussianMarkovChain.initialize\_from\_random

```
GaussianMarkovChain.initialize_from_random()

Set the variable to a random sample from the current distribution.
```

## bayespy.nodes.GaussianMarkovChain.initialize\_from\_value

```
GaussianMarkovChain.initialize_from_value(x, *args)
```

# bayespy.nodes.GaussianMarkovChain.load

```
GaussianMarkovChain.load(group)

Load the state of the node from a HDF5 file.
```

## bayespy.nodes.GaussianMarkovChain.logpdf

```
GaussianMarkovChain.logpdf (X, mask=True)

Compute the log probability density function Q(X) of this node.
```

#### bayespy.nodes.GaussianMarkovChain.lower\_bound\_contribution

```
GaussianMarkovChain.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.GaussianMarkovChain.lowerbound

GaussianMarkovChain.lowerbound()

#### bayespy.nodes.GaussianMarkovChain.move\_plates

GaussianMarkovChain.move\_plates (from\_plate, to\_plate)

## bayespy.nodes.GaussianMarkovChain.observe

```
GaussianMarkovChain.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

## bayespy.nodes.GaussianMarkovChain.pdf

```
GaussianMarkovChain.pdf (X, mask=True)

Compute the probability density function of this node.
```

## bayespy.nodes.GaussianMarkovChain.plot

```
GaussianMarkovChain.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianMarkovChain.random

```
GaussianMarkovChain.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.GaussianMarkovChain.rotate

```
GaussianMarkovChain.rotate(R, inv=None, logdet=None)
```

## bayespy.nodes.GaussianMarkovChain.save

```
GaussianMarkovChain.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

## bayespy.nodes.GaussianMarkovChain.set\_parameters

```
{\tt Gaussian Markov Chain.set\_parameters}\ (x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.GaussianMarkovChain.set\_plotter

```
GaussianMarkovChain.set_plotter(plotter)
```

#### bayespy.nodes.GaussianMarkovChain.show

GaussianMarkovChain.show()

## bayespy.nodes.GaussianMarkovChain.unobserve

GaussianMarkovChain.unobserve()

# bayes py. nodes. Gaussian Markov Chain. update

GaussianMarkovChain.update(annealing=1.0)

#### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

## bayespy.nodes.GaussianMarkovChain.dims

GaussianMarkovChain.dims = None

## bayespy.nodes.GaussianMarkovChain.plates

GaussianMarkovChain.plates = None

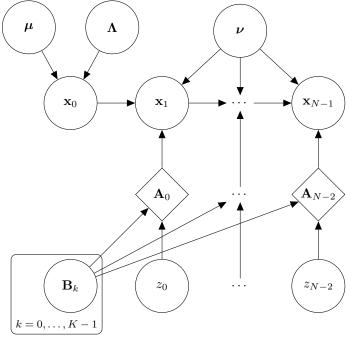
#### bayespy.nodes.GaussianMarkovChain.plates\_multiplier

GaussianMarkovChain.plates\_multiplier
Plate multiplier is applied to messages to parents

#### bayespy.nodes.SwitchingGaussianMarkovChain

Node for Gaussian Markov chain random variables with switching dynamics.

The node models a sequence of Gaussian variables :math:  $mathbf\{x\}_0$ , ldots, mathbf $\{x\}_k$ . With linear Markovian dynamics. The dynamics may change in time, which is obtained by having a set of matrices and at each time selecting one of them as the state dynamics matrix. The graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state,  $\nu$  is the precision of the innovation noise, and  $\mathbf{A}_n$  are the state dynamics matrix obtained by selecting one of the matrices  $\{\mathbf{B}_k\}_{k=0}^{K-1}$  at each time. The selections are provided by  $z_n \in \{0, \dots, K-1\}$ . The probability distribution is

$$p(\mathbf{x}_0, \dots, \mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n | \mathbf{x}_{n-1})$$

where

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$p(\mathbf{x}_n | \mathbf{x}_{n-1}) = \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu})), \quad \text{for } n = 1, \dots, N-1,$$

$$\mathbf{A}_n = \mathbf{B}_{z_n}, \quad \text{for } n = 0, \dots, N-2.$$

Parameters mu: Gaussian-like node or (...,D)-array

 $\mu$ , mean of  $x_0$ , D-dimensional with plates (...)

**Lambda**: Wishart-like node or (...,D,D)-array

 $\Lambda$ , precision matrix of  $x_0$ ,  $D \times D$  -dimensional with plates (...)

```
B : Gaussian-like node or (...,D,D,K)-array \{\mathbf{B}_k\}_{k=0}^{K-1}, \text{ a set of state dynamics matrix, } D\times K\text{-dimensional with plates (...,D)}
```

 ${f Z}$  : categorical-like node or (...,N-1)-array

 $\{z_0, \dots, z_{N-2}\}$ , time-dependent selection, K-categorical with plates (...,N-1)

nu: gamma-like node or (...,D)-array

 $\nu$ , diagonal elements of the precision of the innovation process, plates (...,D)

n: int, optional

N, the length of the chain. Must be given if **Z** does not have plates over the time domain (which would not make sense).

#### See also:

```
Gaussian, GaussianARD, Wishart, Gamma, GaussianMarkovChain, VaryingGaussianMarkovChain, Categorical, CategoricalMarkovChain
```

#### **Notes**

Equivalent model block can be constructed with GaussianMarkovChain by explicitly using Gate to select the state dynamics matrix. However, that approach is not very efficient for large datasets because it does not utilize the structure of  $\mathbf{A}_n$ , thus it explicitly computes huge moment arrays.

```
__init__ (mu, Lambda, B, Z, nu, n=None, **kwargs)
Create SwitchingGaussianMarkovChain node.
```

## Methods

init(mu, Lambda, B, Z, nu[, n])	Create SwitchingGaussianMarkovChain node.
	Create Switching Gaussianiviar Rovenani node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	D1 (4) 1 14 171
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
get_moments()	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
-	Continued on next page

## Table 5.42 – continued from previous page

random()	Draw a random sample from the distribution.	
<pre>rotate(R[, inv, logdet])</pre>		
save(group)	Save the state of the node into a HDF5 file.	
set_parameters(x)	Set the parameters of the VB distribution.	
set_plotter(plotter)	•	
show()		
unobserve()		
update([annealing])		

### bayespy.nodes.SwitchingGaussianMarkovChain.\_\_init\_\_

SwitchingGaussianMarkovChain.\_\_init\_\_ (mu, Lambda, B, Z, nu, n=None, \*\*kwargs)
Create SwitchingGaussianMarkovChain node.

#### bayespy.nodes.SwitchingGaussianMarkovChain.add\_plate\_axis

SwitchingGaussianMarkovChain.add\_plate\_axis(to\_plate)

## bayespy.nodes.SwitchingGaussianMarkovChain.broadcasting\_multiplier

SwitchingGaussianMarkovChain.broadcasting multiplier (plates, \*args)

## bayespy.nodes.SwitchingGaussianMarkovChain.delete

SwitchingGaussianMarkovChain.delete()

Delete this node and the children

# $bayes py. nodes. Switching Gaussian Markov Chain. get\_gradient$

SwitchingGaussianMarkovChain.get\_gradient (rg) Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## $bayes py. nodes. Switching Gaussian Markov Chain. get\_mask$

SwitchingGaussianMarkovChain.get\_mask()

## bayespy.nodes.SwitchingGaussianMarkovChain.get\_moments

SwitchingGaussianMarkovChain.get\_moments()

#### bayespy.nodes.SwitchingGaussianMarkovChain.get\_parameters

```
SwitchingGaussianMarkovChain.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.SwitchingGaussianMarkovChain.get\_riemannian\_gradient

```
SwitchingGaussianMarkovChain.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

## bayespy.nodes.SwitchingGaussianMarkovChain.get\_shape

SwitchingGaussianMarkovChain.get\_shape(ind)

## bayespy.nodes.SwitchingGaussianMarkovChain.has\_plotter

```
SwitchingGaussianMarkovChain.has_plotter()
Return True if the node has a plotter
```

## bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_parameters

SwitchingGaussianMarkovChain.initialize\_from\_parameters(\*args)

## bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_prior

```
SwitchingGaussianMarkovChain.initialize_from_prior()
```

## $bayes py. nodes. Switching Gaussian Markov Chain. initialize\_from\_random$

```
SwitchingGaussianMarkovChain.initialize_from_random()
Set the variable to a random sample from the current distribution.
```

### bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_value

```
SwitchingGaussianMarkovChain.initialize_from_value (x, *args)
```

## bayespy.nodes.SwitchingGaussianMarkovChain.load

```
SwitchingGaussianMarkovChain.load(group)
Load the state of the node from a HDF5 file.
```

#### bayespy.nodes.SwitchingGaussianMarkovChain.logpdf

```
SwitchingGaussianMarkovChain.logpdf (X, mask=True) Compute the log probability density function Q(X) of this node.
```

#### bayespy.nodes.SwitchingGaussianMarkovChain.lower\_bound\_contribution

```
SwitchingGaussianMarkovChain.lower_bound_contribution(gradient=False, ig-nore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.SwitchingGaussianMarkovChain.lowerbound

```
SwitchingGaussianMarkovChain.lowerbound()
```

## bayespy.nodes.SwitchingGaussianMarkovChain.move\_plates

SwitchingGaussianMarkovChain.move\_plates(from\_plate, to\_plate)

## bayespy.nodes.SwitchingGaussianMarkovChain.observe

```
SwitchingGaussianMarkovChain.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

### bayespy.nodes.SwitchingGaussianMarkovChain.pdf

```
SwitchingGaussianMarkovChain.pdf (X, mask=True)

Compute the probability density function of this node.
```

### bayespy.nodes.SwitchingGaussianMarkovChain.plot

```
SwitchingGaussianMarkovChain.plot(fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.SwitchingGaussianMarkovChain.random

```
{\tt Switching Gaussian Markov Chain.} \textbf{random()}
```

Draw a random sample from the distribution.

#### bayespy.nodes.SwitchingGaussianMarkovChain.rotate

SwitchingGaussianMarkovChain.rotate(R, inv=None, logdet=None)

## bayespy.nodes.SwitchingGaussianMarkovChain.save

```
SwitchingGaussianMarkovChain. save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

## bayespy.nodes.SwitchingGaussianMarkovChain.set\_parameters

```
SwitchingGaussianMarkovChain.set_parameters (x)
Set the parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.SwitchingGaussianMarkovChain.set\_plotter

SwitchingGaussianMarkovChain.set\_plotter(plotter)

## bayespy.nodes.SwitchingGaussianMarkovChain.show

SwitchingGaussianMarkovChain.show()

# bayes py. nodes. Switching Gaussian Markov Chain. unobserve

SwitchingGaussianMarkovChain.unobserve()

## bayespy.nodes.SwitchingGaussianMarkovChain.update

SwitchingGaussianMarkovChain.update(annealing=1.0)

## **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

#### bayespy.nodes.SwitchingGaussianMarkovChain.dims

SwitchingGaussianMarkovChain.dims = None

#### bayespy.nodes.SwitchingGaussianMarkovChain.plates

SwitchingGaussianMarkovChain.plates = None

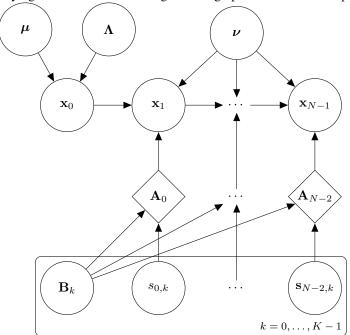
## bayespy.nodes.SwitchingGaussianMarkovChain.plates\_multiplier

SwitchingGaussianMarkovChain.plates\_multiplier Plate multiplier is applied to messages to parents

## bayespy.nodes.VaryingGaussianMarkovChain

**class** bayespy.nodes.**VaryingGaussianMarkovChain** (*mu*, *Lambda*, *B*, *S*, *nu*, *n=None*, \*\*kwargs) Node for Gaussian Markov chain random variables with time-varying dynamics.

The node models a sequence of Gaussian variables  $\mathbf{x}_0, \dots, \mathbf{x}_{N-1}$  with linear Markovian dynamics. The time variability of the dynamics is obtained by modelling the state dynamics matrix as a linear combination of a set of matrices with time-varying linear combination weights. The graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state,  $\nu$  is the precision of the innovation noise, and  $A_n$  are the state dynamics matrix obtained by mixing matrices  $B_k$  with weights  $s_{n,k}$ .

The probability distribution is

$$p(\mathbf{x}_0, \dots, \mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n | \mathbf{x}_{n-1})$$

where

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$p(\mathbf{x}_n | \mathbf{x}_{n-1}) = \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu})), \quad \text{for } n = 1, \dots, N-1,$$

$$\mathbf{A}_n = \sum_{k=0}^{K-1} s_{n,k} \mathbf{B}_k, \quad \text{for } n = 0, \dots, N-2.$$

```
Parameters mu: Gaussian-like node or (...,D)-array
```

 $\mu$ , mean of  $x_0$ , D-dimensional with plates (...)

Lambda: Wishart-like node or (...,D,D)-array

 $\Lambda$ , precision matrix of  $x_0$ ,  $D \times D$  -dimensional with plates (...)

**B**: Gaussian-like node or (...,D,D,K)-array

 $\{\mathbf{B}_k\}_{k=0}^{K-1}$ , a set of state dynamics matrix,  $D \times K$ -dimensional with plates (...,D)

S: Gaussian-like node or (...,N-1,K)-array

 $\{s_0, \dots, s_{N-2}\}$ , time-varying weights of the linear combination, K-dimensional with plates (...,N-1)

**nu**: gamma-like node or (...,D)-array

 $\nu$ , diagonal elements of the precision of the innovation process, plates (...,D)

**n**: int, optional

N, the length of the chain. Must be given if **S** does not have plates over the time domain (which would not make sense).

#### See also:

```
Gaussian, GaussianARD, Wishart, Gamma, GaussianMarkovChain, SwitchingGaussianMarkovChain
```

#### **Notes**

Equivalent model block can be constructed with GaussianMarkovChain by explicitly using SumMultiply to compute the linear combination. However, that approach is not very efficient for large datasets because it does not utilize the structure of  $A_n$ , thus it explicitly computes huge moment arrays.

#### References

# [8]

```
__init__ (mu, Lambda, B, S, nu, n=None, **kwargs)
Create VaryingGaussianMarkovChain node.
```

## Methods

init(mu, Lambda, B, S, nu[, n])	Create VaryingGaussianMarkovChain node.
add_plate_axis(to_plate)	
<pre>broadcasting_multiplier(plates, *args)</pre>	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
	Continued on next page

## Table 5.44 – continued from previous page

```
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                                Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
load(group)
                                               Load the state of the node from a HDF5 file.
logpdf(X[, mask])
                                               Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                               Compute E[ \log p(X|parents) - \log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                               Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                               Compute the probability density function of this node.
plot([fig])
                                               Plot the node distribution using the plotter of the node
random()
                                               Draw a random sample from the distribution.
rotate(R[, inv, logdet])
save(group)
                                               Save the state of the node into a HDF5 file.
                                               Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
show()
unobserve()
update([annealing])
```

#### bayespy.nodes.VaryingGaussianMarkovChain.\_\_init\_\_

```
VaryingGaussianMarkovChain.__init__ (mu, Lambda, B, S, nu, n=None, **kwargs)
Create VaryingGaussianMarkovChain node.
```

### bayespy.nodes.VaryingGaussianMarkovChain.add\_plate\_axis

VaryingGaussianMarkovChain.add\_plate\_axis(to\_plate)

## bayespy.nodes.VaryingGaussianMarkovChain.broadcasting\_multiplier

VaryingGaussianMarkovChain.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.VaryingGaussianMarkovChain.delete

```
VaryingGaussianMarkovChain.delete()

Delete this node and the children
```

## bayespy.nodes.VaryingGaussianMarkovChain.get\_gradient

```
VaryingGaussianMarkovChain.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.VaryingGaussianMarkovChain.get\_mask

VaryingGaussianMarkovChain.get\_mask()

## bayespy.nodes.VaryingGaussianMarkovChain.get\_moments

VaryingGaussianMarkovChain.get\_moments()

## bayespy.nodes.VaryingGaussianMarkovChain.get\_parameters

VaryingGaussianMarkovChain.get\_parameters()

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## $bayespy. nodes. Varying Gaussian Markov Chain. get\_riemannian\_gradient$

VaryingGaussianMarkovChain.get\_riemannian\_gradient()
Computes the Riemannian/natural gradient.

## bayespy.nodes.VaryingGaussianMarkovChain.get\_shape

VaryingGaussianMarkovChain.get\_shape(ind)

## bayespy.nodes.VaryingGaussianMarkovChain.has\_plotter

VaryingGaussianMarkovChain.has\_plotter()
Return True if the node has a plotter

## $bayes py. nodes. Varying Gaussian Markov Chain. initialize\_from\_parameters$

VaryingGaussianMarkovChain.initialize\_from\_parameters(\*args)

# bayespy.nodes.VaryingGaussianMarkovChain.initialize\_from\_prior

VaryingGaussianMarkovChain.initialize\_from\_prior()

## $bayes py. nodes. Varying Gaussian Markov Chain. initialize\_from\_random$

VaryingGaussianMarkovChain.initialize\_from\_random()
Set the variable to a random sample from the current distribution.

## bayespy.nodes.VaryingGaussianMarkovChain.initialize\_from\_value

VaryingGaussianMarkovChain.initialize\_from\_value(x, \*args)

#### bayespy.nodes.VaryingGaussianMarkovChain.load

VaryingGaussianMarkovChain.load (group)

Load the state of the node from a HDF5 file.

## bayespy.nodes.VaryingGaussianMarkovChain.logpdf

VaryingGaussianMarkovChain.logpdf (*X*, *mask=True*)

Compute the log probability density function Q(*X*) of this node.

## bayespy.nodes.VaryingGaussianMarkovChain.lower\_bound\_contribution

VaryingGaussianMarkovChain.lower\_bound\_contribution(gradient=False, ig-nore\_masked=True)

Compute E[  $\log p(X|parents) - \log q(X)$  ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.VaryingGaussianMarkovChain.lowerbound

VaryingGaussianMarkovChain.lowerbound()

## bayespy.nodes.VaryingGaussianMarkovChain.move\_plates

VaryingGaussianMarkovChain.move\_plates (from\_plate, to\_plate)

### bayespy.nodes.VaryingGaussianMarkovChain.observe

VaryingGaussianMarkovChain.**observe**(*x*, \**args*, *mask=True*) Fix moments, compute f and propagate mask.

## bayespy.nodes.VaryingGaussianMarkovChain.pdf

VaryingGaussianMarkovChain.**pdf** (*X*, *mask=True*) Compute the probability density function of this node.

## bayespy.nodes.VaryingGaussianMarkovChain.plot

VaryingGaussianMarkovChain.plot(fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.VaryingGaussianMarkovChain.random

```
VaryingGaussianMarkovChain.random()

Draw a random sample from the distribution.
```

## bayespy.nodes.VaryingGaussianMarkovChain.rotate

VaryingGaussianMarkovChain.rotate(R, inv=None, logdet=None)

## bayespy.nodes.VaryingGaussianMarkovChain.save

```
VaryingGaussianMarkovChain.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

## bayespy.nodes.VaryingGaussianMarkovChain.set\_parameters

```
VaryingGaussianMarkovChain.set_parameters (x)
Set the parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.VaryingGaussianMarkovChain.set\_plotter

VaryingGaussianMarkovChain.set\_plotter(plotter)

## bayespy.nodes.VaryingGaussianMarkovChain.show

VaryingGaussianMarkovChain.show()

## bayes py. nodes. Varying Gaussian Markov Chain. unobserve

VaryingGaussianMarkovChain.unobserve()

# bayes py. nodes. Varying Gaussian Markov Chain. update

VaryingGaussianMarkovChain.update(annealing=1.0)

# **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

#### bayespy.nodes.VaryingGaussianMarkovChain.dims

VaryingGaussianMarkovChain.dims = None

## bayespy.nodes.VaryingGaussianMarkovChain.plates

VaryingGaussianMarkovChain.plates = None

### bayespy.nodes.VaryingGaussianMarkovChain.plates\_multiplier

VaryingGaussianMarkovChain.plates\_multiplier

Plate multiplier is applied to messages to parents

Other stochastic nodes:

Mixture(z, node\_class, \*params[, cluster\_plate]) Node for exponential family mixture variables.

## bayespy.nodes.Mixture

class bayespy.nodes.Mixture(z, node\_class, \*params, cluster\_plate=-1, \*\*kwargs)
 Node for exponential family mixture variables.

The node represents a random variable which is sampled from a mixture distribution. It is possible to mix any exponential family distribution. The probability density function is

$$p(x|z=k,\boldsymbol{\theta}_0,\ldots,\boldsymbol{\theta}_{K-1})=\phi(x|\boldsymbol{\theta}_k),$$

where  $\phi$  is the probability density function of the mixed exponential family distribution and  $\theta_0, \dots, \theta_{K-1}$  are the parameters of each cluster. For instance,  $\phi$  could be the Gaussian probability density function  $\mathcal N$  and  $\theta_k = \{\mu_k, \Lambda_k\}$  where  $\mu_k$  and  $\Lambda_k$  are the mean vector and precision matrix for cluster k.

Parameters z : categorical-like node or array

z, cluster assignment

**node\_class**: stochastic exponential family node class

Mixed distribution

params: types specified by the mixed distribution

Parameters of the mixed distribution. If some parameters should vary between clusters, those parameters' plate axis *cluster\_plate* should have a size which equals the number of clusters. For parameters with shared values, that plate axis should have length 1. At least one parameter should vary between clusters.

cluster\_plate : int, optional

Negative integer defining which plate axis is used for the clusters in the parameters. That plate axis is ignored from the parameters when considering the plates for this node. By default, mix over the last plate axis.

### See also:

Categorical, CategoricalMarkovChain

## **Examples**

A simple 2-dimensional Gaussian mixture model with three clusters for 100 samples can be constructed, for instance, as:

#### Methods

```
__init__(z, node_class, *params[, cluster_plate])
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                                  Delete this node and the children
get_gradient(rg)
                                                  Computes gradient with respect to the natural parameters.
get_mask()
get_moments()
                                                  Return parameters of the VB distribution.
get_parameters()
get_riemannian_gradient()
                                                  Computes the Riemannian/natural gradient.
get_shape(ind)
has_plotter()
                                                  Return True if the node has a plotter
initialize_from_parameters(*args)
initialize_from_prior()
initialize_from_random()
                                                  Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)
integrated_logpdf_from_parents(x, index)
                                                  Approximates the posterior predictive pdf int p(x|parents) q(parents) dparents
                                                  Load the state of the node from a HDF5 file.
load(group)
logpdf(X[, mask])
                                                  Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient, ...])
                                                  Compute E[ log p(X|parents) - log q(X) ]
lowerbound()
move_plates(from_plate, to_plate)
observe(x, *args[, mask])
                                                  Fix moments, compute f and propagate mask.
pdf(X[, mask])
                                                  Compute the probability density function of this node.
plot([fig])
                                                  Plot the node distribution using the plotter of the node
                                                  Draw a random sample from the distribution.
random()
save(group)
                                                  Save the state of the node into a HDF5 file.
                                                  Set the parameters of the VB distribution.
set_parameters(x)
set_plotter(plotter)
unobserve()
```

## bayespy.nodes.Mixture.\_\_init\_\_

update([annealing])

```
Mixture.__init__(z, node_class, *params, cluster_plate=-1, **kwargs)
```

#### bayespy.nodes.Mixture.add\_plate\_axis

```
Mixture.add_plate_axis (to_plate)
```

## bayespy.nodes.Mixture.broadcasting\_multiplier

```
Mixture.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.Mixture.delete

```
Mixture.delete()
```

Delete this node and the children

## bayespy.nodes.Mixture.get\_gradient

```
Mixture.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Mixture.get\_mask

```
Mixture.get_mask()
```

## bayespy.nodes.Mixture.get\_moments

```
Mixture.get_moments()
```

## bayespy.nodes.Mixture.get\_parameters

```
Mixture.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Mixture.get\_riemannian\_gradient

```
Mixture.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Mixture.get\_shape

```
Mixture.get_shape(ind)
```

#### bayespy.nodes.Mixture.has\_plotter

```
Mixture.has_plotter()
```

Return True if the node has a plotter

#### bayespy.nodes.Mixture.initialize\_from\_parameters

```
Mixture.initialize_from_parameters(*args)
```

## bayespy.nodes.Mixture.initialize\_from\_prior

```
Mixture.initialize_from_prior()
```

### bayespy.nodes.Mixture.initialize\_from\_random

```
Mixture.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

### bayespy.nodes.Mixture.initialize\_from\_value

```
Mixture.initialize_from_value(x, *args)
```

## bayespy.nodes.Mixture.integrated\_logpdf\_from\_parents

```
Mixture.integrated_logpdf_from_parents(x, index)
```

Approximates the posterior predictive pdf int p(x|parents) q(parents) dparents in log-scale as int  $q(parents_i)$  exp( int  $q(parents_i)$  log  $p(x|parents_i)$  dparents\_i) dparents\_i.

## bayespy.nodes.Mixture.load

```
Mixture.load(group)
```

Load the state of the node from a HDF5 file.

## bayespy.nodes.Mixture.logpdf

```
Mixture.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

## bayespy.nodes.Mixture.lower\_bound\_contribution

```
Mixture.lower_bound_contribution (gradient=False, ignore\_masked=True)
Compute E[ log p(X|parents) - log q(X)]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Mixture.lowerbound

```
Mixture.lowerbound()
```

## bayespy.nodes.Mixture.move\_plates

```
Mixture.move_plates (from_plate, to_plate)
```

## bayespy.nodes.Mixture.observe

```
Mixture.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Mixture.pdf

```
Mixture.pdf (X, mask=True)
```

Compute the probability density function of this node.

### bayespy.nodes.Mixture.plot

```
Mixture.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.Mixture.random

```
Mixture.random()
```

Draw a random sample from the distribution.

# bayespy.nodes.Mixture.save

```
Mixture.save(group)
```

Save the state of the node into a HDF5 file.

group can be the root

## bayespy.nodes.Mixture.set\_parameters

```
Mixture.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Mixture.set\_plotter

Mixture.set\_plotter(plotter)

## bayespy.nodes.Mixture.unobserve

Mixture.unobserve()

## bayespy.nodes.Mixture.update

Mixture.update (annealing=1.0)

## **Attributes**

dims plates

plates\_multiplier Plate multiplier is applied to messages to parents

## bayespy.nodes.Mixture.dims

Mixture.dims = None

## bayespy.nodes.Mixture.plates

Mixture.plates = None

## bayespy.nodes.Mixture.plates\_multiplier

Mixture.plates\_multiplier

Plate multiplier is applied to messages to parents

## 5.1.2 Deterministic nodes

Dot(*args, **kwargs)	Node for computing inner product of several Gaussian vectors.
<pre>SumMultiply(*args[, iterator_axis])</pre>	Node for computing general products and sums of Gaussian nodes.
$Gate(Z, X[, gated\_plate, moments])$	Deterministic gating of one node.

# bayespy.nodes.Dot

bayespy.nodes.Dot(\*args, \*\*kwargs)

Node for computing inner product of several Gaussian vectors.

This is a simple wrapper of the much more general SumMultiply. For now, it is here for backward compatibility.

## bavespy.nodes.SumMultiply

class bayespy.nodes.SumMultiply(\*args, iterator\_axis=None, \*\*kwargs)

Node for computing general products and sums of Gaussian nodes.

The node is similar to *numpy.einsum*, which is a very general function for computing dot products, sums, products and other sums of products of arrays.

For instance, the equivalent of

```
np.einsum('abc,bd,ca->da', X, Y, Z)
would be given as
SumMultiply('abc,bd,ca->da', X, Y, Z)
or
SumMultiply(X, [0,1,2], Y, [1,3], Z, [2,0], [3,0])
```

which is similar to the other syntax of numpy.einsum.

This node operates similarly as numpy.einsum. However, you must use all the elements of each node, that is, an operation like np.einsum('ii->i',X) is not allowed. Thus, for each node, each axis must be given unique id. The id identifies which axes correspond to which axes between the different nodes. Also, Ellipsis ('...') is not yet supported for simplicity. It would also have some problems with constant inputs (because how to determine ndim), so let us just forget it for now.

Each output axis must appear in the input mappings.

The keys must refer to variable dimension axes only, not plate axes.

The input nodes may be Gaussian-gamma (isotropic) nodes.

The output message is Gaussian-gamma (isotropic) if any of the input nodes is Gaussian-gamma.

### **Notes**

This operation can be extremely slow if not used wisely. For large and complex operations, it is sometimes more efficient to split the operation into multiple nodes. For instance, the example above could probably be computed faster by

```
XZ = SumMultiply(X, [0,1,2], Z, [2,0], [0,1])

F = SumMultiply(XZ, [0,1], Y, [1,2], [2,0])
```

because the third axis ('c') could be summed out already in the first operation. This same effect applies also to numpy.einsum in general.

## **Examples**

Sum over the rows: 'ij->j'

Inner product of three vectors: 'i,i,i'

Matrix-vector product: 'ij,j->i' Matrix-matrix product: 'ik,kj->ij'

Outer product: 'i,j->ij'

Vector-matrix-vector product: 'i,ij,j'

```
__init__(Node1, map1, Node2, map2, ..., NodeN, mapN[, map_out])
```

### **Methods**

```
__init__(Node1, map1, Node2, map2, ..., ...)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                             Delete this node and the children
get_mask()
get_moments()
get_parameters()
get_shape(ind)
has_plotter()
                                             Return True if the node has a plotter
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
                                             Plot the node distribution using the plotter of the node
plot([fig])
set_plotter(plotter)
```

### bayespy.nodes.SumMultiply.\_\_init\_\_

```
SumMultiply.__init__(Node1, map1, Node2, map2, ..., NodeN, mapN[, map_out])
```

### bayespy.nodes.SumMultiply.add\_plate\_axis

```
SumMultiply.add_plate_axis (to_plate)
```

# bayespy.nodes.SumMultiply.broadcasting\_multiplier

```
SumMultiply.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.SumMultiply.delete

```
SumMultiply.delete()

Delete this node and the children
```

### bayespy.nodes.SumMultiply.get\_mask

```
SumMultiply.get_mask()
```

### bayespy.nodes.SumMultiply.get\_moments

```
SumMultiply.get_moments()
```

# bayespy.nodes.SumMultiply.get\_parameters

```
SumMultiply.get_parameters()
```

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### bayespy.nodes.SumMultiply.get\_shape

```
SumMultiply.get_shape (ind)
```

### bayespy.nodes.SumMultiply.has\_plotter

```
SumMultiply.has_plotter()

Return True if the node has a plotter
```

# bayespy.nodes.SumMultiply.lower\_bound\_contribution

```
SumMultiply.lower_bound_contribution(gradient=False, **kwargs)
```

# bayespy.nodes.SumMultiply.move\_plates

```
SumMultiply.move_plates (from_plate, to_plate)
```

### bayespy.nodes.SumMultiply.plot

```
SumMultiply.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.SumMultiply.set\_plotter

```
SumMultiply.set_plotter(plotter)
```

### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.SumMultiply.plates

```
SumMultiply.plates = None
```

### bayespy.nodes.SumMultiply.plates\_multiplier

```
SumMultiply.plates_multiplier
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.Gate

```
class bayespy.nodes.Gate (Z, X, gated\_plate=-1, moments=None, **kwargs) Deterministic gating of one node.
```

Gating is performed over one plate axis.

Note: You should not use gating for several variables which parents of a same node if the gates use the same gate assignments. In such case, the results will be wrong. The reason is a general one: A stochastic node may not be a parent of another node via several paths unless at most one path has no other stochastic nodes between them.

```
__init__(Z, X, gated_plate=-1, moments=None, **kwargs)
```

#### Methods

```
__init__(Z, X[, gated_plate, moments])
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
set_plotter(plotter)

Plot the node distribution using the plotter of the node
set_plotter(plotter)
```

### bayespy.nodes.Gate.\_\_init\_\_

```
Gate.__init__(Z, X, gated_plate=-1, moments=None, **kwargs)
```

# bayespy.nodes.Gate.add\_plate\_axis

```
Gate.add_plate_axis (to_plate)
```

#### bayespy.nodes.Gate.broadcasting\_multiplier

```
Gate.broadcasting_multiplier(plates, *args)
```

### bayespy.nodes.Gate.delete

```
Gate.delete()
```

Delete this node and the children

# $bayespy.nodes. Gate. get\_mask$

```
Gate.get_mask()
```

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Gate.plates = None

```
bayespy.nodes.Gate.get_moments
Gate.get_moments()
bayespy.nodes.Gate.get_shape
Gate.get_shape(ind)
bayespy.nodes.Gate.has_plotter
Gate.has_plotter()
     Return True if the node has a plotter
bayespy.nodes.Gate.lower_bound_contribution
Gate.lower_bound_contribution(gradient=False, **kwargs)
bayespy.nodes.Gate.move_plates
Gate.move_plates (from_plate, to_plate)
bayespy.nodes.Gate.plot
Gate.plot (fig=None, **kwargs)
    Plot the node distribution using the plotter of the node
     Because the distributions are in general very difficult to plot, the user must specify some functions which
     performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is,
     functions that perform plotting for a node.
bayespy.nodes.Gate.set_plotter
Gate.set_plotter(plotter)
Attributes
            plates
            plates_multiplier
                                      Plate multiplier is applied to messages to parents
bayespy.nodes.Gate.plates
```

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#### bayespy.nodes.Gate.plates\_multiplier

Gate.plates\_multiplier

Plate multiplier is applied to messages to parents

# 5.2 bayespy.inference

Package for Bayesian inference engines

# 5.2.1 Inference engines

VB(\*nodes[, tol, autosave\_filename, ...]) Variational Bayesian (VB) inference engine

# bayespy.inference.VB

class bayespy.inference.VB (\*nodes, tol=1e-05, autosave\_filename=None, autosave\_iterations=0, callback=None)

Variational Bayesian (VB) inference engine

Parameters nodes: nodes

Nodes that form the model. Must include all at least all stochastic nodes of the model.

tol: double, optional

Convergence criterion. Tolerance for the relative change in the VB lower bound.

autosave\_filename : string, optional
 Filename for automatic saving
autosave\_iterations : int, optional

Iteration interval between each automatic saving

callback: callable, optional

Function which is called after each update iteration step

**\_\_init\_** (\*nodes, tol=1e-05, autosave\_filename=None, autosave\_iterations=0, callback=None)

### Methods

```
__init__(*nodes[, tol, autosave_filename, ...])
add(x1, x2[, scale])
                                                 Add two vectors (in parameter format)
compute_lowerbound([ignore_masked])
compute_lowerbound_terms(*nodes)
dot(x1, x2)
                                                 Computes dot products of given vectors (in parameter format)
get_gradients(*nodes[, euclidian])
                                                 Computes gradients (both Riemannian and normal)
get_iteration_by_nodes()
get_parameters(*nodes)
                                                 Get parameters of the nodes
gradient_step(*nodes[, scale])
                                                 Update nodes by taking a gradient ascent step
has_converged([tol])
                                                                                 Continued on next page
```

# Table 5.55 – continued from previous page

```
load(*nodes[, filename])
loglikelihood_lowerbound()
optimize(*nodes[, maxiter, verbose, method, ...])
                                                   Optimize nodes using Riemannian conjugate gradient
pattern_search(*nodes[, collapsed, maxiter])
                                                   Perform simple pattern search [4].
plot(*nodes)
                                                   Plot the distribution of the given nodes (or all nodes)
plot_iteration_by_nodes([axes, diff])
                                                   Plot the cost function per node during the iteration.
save(*nodes[, filename])
                                                   Set deterministic annealing from range (0, 1].
set_annealing(annealing)
set_autosave(filename[, iterations])
set_callback(callback)
set_parameters(x, *nodes)
                                                   Set parameters of the nodes
update(*nodes[, repeat, plot, tol, verbose])
```

#### bayespy.inference.VB.\_\_init\_\_

```
VB.__init__ (*nodes, tol=1e-05, autosave_filename=None, autosave_iterations=0, callback=None)
```

# bayespy.inference.VB.add

```
VB. add (x1, x2, scale=1)
Add two vectors (in parameter format)
```

### bayespy.inference.VB.compute\_lowerbound

```
VB.compute_lowerbound(ignore_masked=True)
```

#### bayespy.inference.VB.compute\_lowerbound\_terms

```
VB.compute_lowerbound_terms (*nodes)
```

### bayespy.inference.VB.dot

```
VB. dot (x1, x2)
```

Computes dot products of given vectors (in parameter format)

### bayespy.inference.VB.get\_gradients

```
VB.get_gradients (*nodes, euclidian=False)
Computes gradients (both Riemannian and normal)
```

# bayespy.inference.VB.get\_iteration\_by\_nodes

```
VB.get_iteration_by_nodes()
```

# bayespy.inference.VB.get\_parameters VB.get\_parameters (\*nodes) Get parameters of the nodes bayespy.inference.VB.gradient\_step VB.gradient\_step(\*nodes, scale=1.0) Update nodes by taking a gradient ascent step bayespy.inference.VB.has\_converged VB.has\_converged(tol=None) bayespy.inference.VB.load VB.load(\*nodes, filename=None) bayespy.inference.VB.loglikelihood\_lowerbound VB.loglikelihood\_lowerbound() bayespy.inference.VB.optimize VB.optimize (\*nodes, maxiter=10, verbose=True, method='fletcher-reeves', riemannian=True, collapsed=None, tol=None) Optimize nodes using Riemannian conjugate gradient bayespy.inference.VB.pattern\_search VB.pattern\_search (\*nodes, collapsed=None, maxiter=3) Perform simple pattern search [4]. Some of the variables can be collapsed. bayespy.inference.VB.plot VB.plot(\*nodes) Plot the distribution of the given nodes (or all nodes) bayespy.inference.VB.plot\_iteration\_by\_nodes VB.plot\_iteration\_by\_nodes (axes=None, diff=False) Plot the cost function per node during the iteration. Handy tool for debugging.

### bayespy.inference.VB.save

```
VB.save(*nodes, filename=None)
```

# bayespy.inference.VB.set\_annealing

### VB.set\_annealing(annealing)

Set deterministic annealing from range (0, 1].

With 1, no annealing, standard updates.

With smaller values, entropy has more weight and model probability equations less. With 0, one would obtain improper uniform distributions.

### bayespy.inference.VB.set\_autosave

```
VB.set_autosave (filename, iterations=None)
```

# bayespy.inference.VB.set\_callback

VB.set\_callback(callback)

# bayespy.inference.VB.set\_parameters

```
VB.set_parameters (x, *nodes) Set parameters of the nodes
```

# bayespy.inference.VB.update

```
VB.update(*nodes, repeat=1, plot=False, tol=None, verbose=True)
```

### **Attributes**

ignore\_bound\_checks

### bayespy.inference.VB.ignore\_bound\_checks

VB.ignore\_bound\_checks

# 5.2.2 Parameter expansions

vmp.transformations.RotationOptimizer()	Optimizer for rotation parameter expansion in state-spa
$ extsf{vmp.transformations.RotateGaussian}(X)$	Rotation parameter expansion for bayespy.nodes.
$ extsf{vmp.transformations.RotateGaussianARD}(X, *alpha)$	Rotation parameter expansion for bayespy.nodes.
$\verb vmp.transformations.RotateGaussianMarkovChain (X,) $	Rotation parameter expansion for bayespy.nodes.

# Table 5.57 – continued from previous page

vmp.transformations.RotateSwitchingMarkovChain(X, ...)
vmp.transformations.RotateVaryingMarkovChain(X, ...)
vmp.transformations.RotateMultiple(\*rotators)

Rotation for bayespy.nodes.VaryingGaussian
Rotation for bayespy.nodes.SwitchingGaussian
Identical parameter expansion for several nodes simultations.

# bayespy.inference.vmp.transformations.RotationOptimizer

class bayespy.inference.vmp.transformations.RotationOptimizer (block1, block2, D) Optimizer for rotation parameter expansion in state-space models

Rotates one model block with  $\mathbf{R}$  and one model block with  $\mathbf{R}^{-1}$ .

Parameters block1: rotator object

The first rotation parameter expansion object

block2: rotator object

The second rotation parameter expansion object

D: int

Dimensionality of the latent space

#### References

```
[7], [6]
__init__(block1, block2, D)
```

### Methods

```
__init__(block1, block2, D)
rotate([maxiter, check_gradient, verbose, ...]) Optimize the rotation of two separate model blocks jointly.
```

### bayespy.inference.vmp.transformations.RotationOptimizer.\_\_init\_\_

```
RotationOptimizer.__init__(block1, block2, D)
```

### bayespy.inference.vmp.transformations.RotationOptimizer.rotate

```
RotationOptimizer.rotate(maxiter=10, check_gradient=False, verbose=False, check_bound=False)
```

Optimize the rotation of two separate model blocks jointly.

If some variable is the dot product of two Gaussians, rotating the two Gaussians optimally can make the inference algorithm orders of magnitude faster.

First block is rotated with  $\mathbf{R}$  and the second with  $\mathbf{R}^{-T}$ .

Blocks must have methods: bound(U,s,V) and rotate(R).

# bayespy.inference.vmp.transformations.RotateGaussian

```
class bayespy.inference.vmp.transformations.RotateGaussian (X) Rotation parameter expansion for bayespy.nodes.Gaussian

__init__(X)
```

#### **Methods**

# bayespy.inference.vmp.transformations.RotateGaussian.\_\_init\_\_

```
RotateGaussian.__init__(X)
```

# bayespy.inference.vmp.transformations.RotateGaussian.bound

RotateGaussian.bound(*R*, *logdet=None*, *inv=None*)

### bayespy.inference.vmp.transformations.RotateGaussian.get\_bound\_terms

RotateGaussian.get\_bound\_terms(R, logdet=None, inv=None)

### bayespy.inference.vmp.transformations.RotateGaussian.nodes

```
RotateGaussian.nodes()
```

### bayespy.inference.vmp.transformations.RotateGaussian.rotate

RotateGaussian.rotate(R, inv=None, logdet=None)

# bayespy.inference.vmp.transformations.RotateGaussian.setup

```
RotateGaussian.setup()
```

This method should be called just before optimization.

### bayespy.inference.vmp.transformations.RotateGaussianARD

```
 \begin{array}{ll} \textbf{class} \ \texttt{bayespy.inference.vmp.transformations.RotateGaussianARD} \ (X, \ *alpha, \ axis=-1, \\ precompute=False, \\ subset=None) \\ \textbf{Rotation parameter expansion for bayespy.nodes.GaussianARD} \end{array}
```

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The model:

```
alpha ~ N(a, b) X ~ N(mu, alpha)
```

X can be an array (e.g., GaussianARD).

Transform q(X) and q(alpha) by rotating X.

Requirements: \* X and alpha do not contain any observed values

```
__init__(X, *alpha, axis=-1, precompute=False, subset=None)
```

Precompute tells whether to compute some moments once in the setup function instead of every time in the bound function. However, they are computed a bit differently in the bound function so it can be useful too. Precomputation is probably beneficial only when there are large axes that are not rotated (by R nor Q) and they are not contained in the plates of alpha, and the dimensions for R and Q are quite small.

### **Methods**

```
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
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precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
precompute tells whether to compute some moments once in the setup function instruction.
```

### bayespy.inference.vmp.transformations.RotateGaussianARD.\_init\_

```
RotateGaussianARD.__init__(X, *alpha, axis=-1, precompute=False, subset=None)
```

Precompute tells whether to compute some moments once in the setup function instead of every time in the bound function. However, they are computed a bit differently in the bound function so it can be useful too. Precomputation is probably beneficial only when there are large axes that are not rotated (by R nor Q) and they are not contained in the plates of alpha, and the dimensions for R and Q are quite small.

#### bayespy.inference.vmp.transformations.RotateGaussianARD.bound

RotateGaussianARD.bound(R, logdet=None, inv=None, Q=None)

### bayespy.inference.vmp.transformations.RotateGaussianARD.get\_bound\_terms

RotateGaussianARD.get\_bound\_terms (R, logdet=None, inv=None, Q=None)

### bayespy.inference.vmp.transformations.RotateGaussianARD.nodes

RotateGaussianARD.nodes()

### bayespy.inference.vmp.transformations.RotateGaussianARD.rotate

RotateGaussianARD.rotate(R, inv=None, logdet=None, Q=None)

### bayespy.inference.vmp.transformations.RotateGaussianARD.setup

```
RotateGaussianARD.setup(plate_axis=None)
```

This method should be called just before optimization.

For efficiency, sum over axes that are not in mu, alpha nor rotation.

If using Q, set rotate\_plates to True.

# bayespy.inference.vmp.transformations.RotateGaussianMarkovChain

```
class bayespy.inference.vmp.transformations.RotateGaussianMarkovChain(X,
```

\*args)

Rotation parameter expansion for bayespy.nodes.GaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise. Unit variance only?

Maybe: Assume innovation noise with unit variance? Would it help make this function more general with respect to A.

TODO: Allow constant A or not rotating A.

A may vary in time.

Shape of A: (N,D,D) Shape of AA: (N,D,D,D)

No plates for X.

```
__init__(X, *args)
```

#### **Methods**

```
Linit_(X, *args)
bound(R[, logdet, inv])
get_bound_terms(R[, logdet, inv])
nodes()
rotate(R[, inv, logdet])
setup()
This method should be called just before optimization.
```

### bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.\_\_init\_\_

```
RotateGaussianMarkovChain.__init__(X, *args)
```

# bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.bound

RotateGaussianMarkovChain.bound(R, logdet=None, inv=None)

#### bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.get\_bound\_terms

RotateGaussianMarkovChain.get\_bound\_terms(R, logdet=None, inv=None)

#### bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.nodes

RotateGaussianMarkovChain.nodes()

# bayes py. in ference. vmp. transformations. Rotate Gaussian Markov Chain. rotate

RotateGaussianMarkovChain.rotate(*R*, *inv=None*, *logdet=None*)

### bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.setup

```
RotateGaussianMarkovChain.setup()
```

This method should be called just before optimization.

# bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain

 ${f class}$  bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain( ${\it X}, {\it B}, {\it Z},$ 

 $B_{rotator}$ 

Rotation for bayespy.nodes.VaryingGaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise.

```
A_n = B_{z_n}
```

Gaussian B: (..., K, D) x (D) Categorical Z: (..., N-1) x (K) GaussianMarkovChain X: (...) x (N,D)

No plates for X.

```
__init__ (X, B, Z, B_rotator)
```

# Methods

```
\begin{tabular}{ll} -- init_-(X, B, Z, B\_rotator) \\ bound(R[, logdet, inv]) \\ get\_bound\_terms(R[, logdet, inv]) \\ nodes() \\ rotate(R[, inv, logdet]) \\ setup() & This method should be called just before optimization. \\ \end{tabular}
```

# bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.\_\_init\_\_

RotateSwitchingMarkovChain.\_\_init\_\_(X, B, Z, B\_rotator)

# bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.bound

RotateSwitchingMarkovChain.bound(R, logdet=None, inv=None)

### bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.get\_bound\_terms

RotateSwitchingMarkovChain.get\_bound\_terms(R, logdet=None, inv=None)

### bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.nodes

RotateSwitchingMarkovChain.nodes()

### bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.rotate

RotateSwitchingMarkovChain.rotate(R, inv=None, logdet=None)

### bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.setup

```
RotateSwitchingMarkovChain.setup()
```

This method should be called just before optimization.

# bayespy.inference.vmp.transformations.RotateVaryingMarkovChain

class bayespy.inference.vmp.transformations.RotateVaryingMarkovChain(X, B, S, B\_rotator)

Rotation for bayespy.nodes.SwitchingGaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise.

$$A_n = \sum_k B_k s_{kn}$$

Gaussian B: (1,D) x (D,K) Gaussian S: (N,1) x (K) MC X: () x (N+1,D)

No plates for X.

\_\_init\_\_ (*X*, *B*, *S*, *B\_rotator*)

### Methods

# bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.\_\_init\_\_

RotateVaryingMarkovChain.\_\_init\_\_(X, B, S, B\_rotator)

#### bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.bound

RotateVaryingMarkovChain.bound(*R*, *logdet=None*, *inv=None*)

# $bayes py. inference. vmp. transformations. Rotate Varying Markov Chain. get\_bound\_terms$

RotateVaryingMarkovChain.get\_bound\_terms (R, logdet=None, inv=None)

### bayes py. inference. vmp. transformations. Rotate Varying Markov Chain. nodes

RotateVaryingMarkovChain.nodes()

#### bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.rotate

RotateVaryingMarkovChain.rotate(*R*, *inv=None*, *logdet=None*)

### bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.setup

```
RotateVaryingMarkovChain.setup()
```

This method should be called just before optimization.

### bayespy.inference.vmp.transformations.RotateMultiple

```
class bayespy.inference.vmp.transformations.RotateMultiple(*rotators)
    Identical parameter expansion for several nodes simultaneously
```

Performs the same rotation for multiple nodes and combines the cost effect.

```
__init__(*rotators)
```

# Methods

```
..init_.(*rotators)
bound(R[, logdet, inv])
get_bound_terms(R[, logdet, inv])
nodes()
rotate(R[, inv, logdet])
setup()
```

# bayespy.inference.vmp.transformations.RotateMultiple.\_\_init\_\_

RotateMultiple.\_\_init\_\_(\*rotators)

### bayespy.inference.vmp.transformations.RotateMultiple.bound

RotateMultiple.bound(R, logdet=None, inv=None)

# bayespy.inference.vmp.transformations.RotateMultiple.get\_bound\_terms

RotateMultiple.get\_bound\_terms (R, logdet=None, inv=None)

# bayespy.inference.vmp.transformations.RotateMultiple.nodes

```
RotateMultiple.nodes()
```

### bayespy.inference.vmp.transformations.RotateMultiple.rotate

```
RotateMultiple.rotate(R, inv=None, logdet=None)
```

#### bayespy.inference.vmp.transformations.RotateMultiple.setup

```
RotateMultiple.setup()
```

# 5.3 bayespy.plot

Functions for plotting nodes.

# 5.3.1 Functions

pdf(Z, x, *args[, name, axes, fig])	Plot probability density function of a scalar variable.
contour(Z, x, y[, n, axes, fig])	Plot 2-D probability density function of a 2-D variable.
plot(Y[, axis, scale, center])	Plot a variable or an array as 1-D function with errorbars
hinton(X, **kwargs)	Plot the Hinton diagram of a node
	Plot Gaussian mixture as ellipses in 2-D

# bayespy.plot.pdf

```
bayespy.plot.pdf (Z, x, *args, name=None, axes=None, fig=None, **kwargs) Plot probability density function of a scalar variable.
```

### Parameters **Z**: node or function

Stochastic node or log pdf function

x: array

Grid points

# bayespy.plot.contour

```
bayespy.plot.contour (Z, x, y, n=None, axes=None, fig=None, **kwargs) Plot 2-D probability density function of a 2-D variable.
```

Parameters Z: node or function

Stochastic node or log pdf function

x: array

Grid points on x axis

y: array

Grid points on y axis

# bayespy.plot.plot

```
bayespy.plot.plot (Y, axis=-1, scale=2, center=False, **kwargs)
Plot a variable or an array as 1-D function with errorbars
```

# bayespy.plot.hinton

```
bayespy.plot.hinton(X, **kwargs)

Plot the Hinton diagram of a node
```

The keyword arguments depend on the node type. For some node types, the diagram also shows uncertainty with non-filled rectangles. Currently, beta-like, Gaussian-like and Dirichlet-like nodes are supported.

Parameters X: node

# bayespy.plot.gaussian\_mixture

```
bayespy.plot.gaussian_mixture (X, scale=1, fill=False, axes=None, **kwargs)
Plot Gaussian mixture as ellipses in 2-D
```

# 5.3.2 Plotters

Plotter(plotter, *args, **kwargs)	Wrapper for plotting functions and base class for node plotters
PDFPlotter(x_grid, **kwargs)	Plotter of probability density function of a scalar node
ContourPlotter(x1_grid, x2_grid, **kwargs)	Plotter of probability density function of a two-dimensional node
HintonPlotter(**kwargs)	Plotter of the Hinton diagram of a node
FunctionPlotter(**kwargs)	Plotter of a node as a 1-dimensional function
<pre>GaussianTimeseriesPlotter(**kwargs)</pre>	Plotter of a Gaussian node as a timeseries
CategoricalMarkovChainPlotter(**kwargs)	Plotter of a Categorical timeseries

# bayespy.plot.Plotter

```
class bayespy.plot.Plotter (plotter, *args, **kwargs)
Wrapper for plotting functions and base class for node plotters
```

The purpose of this class is to collect all the parameters needed by a plotting function and provide a callable interface which needs only the node as the input.

Plotter instances are callable objects that plot a given node using a specified plotting function.

Parameters plotter: function

Plotting function to use

args: defined by the plotting function

Additional inputs needed by the plotting function

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**kwargs**: defined by the plotting function

Additional keyword arguments supported by the plotting function

# **Examples**

First, create a gamma variable:

```
>>> import numpy as np
>>> from bayespy.nodes import Gamma
>>> x = Gamma(4, 5)
```

The probability density function can be plotted as:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pdf(x, np.linspace(0.1, 10, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```

However, this can be problematic when one needs to provide a plotting function for the inference engine as the inference engine gives only the node as input. Thus, we need to create a simple plotter wrapper:

```
>>> p = bpplt.Plotter(bpplt.pdf, np.linspace(0.1, 10, num=100))
```

Now, this callable object p needs only the node as the input:

```
>>> p(x)
[<matplotlib.lines.Line2D object at 0x...>]
```

Thus, it can be given to the inference engine to use as a plotting function:

```
>>> x = Gamma(4, 5, plotter=p)
>>> x.plot()
[<matplotlib.lines.Line2D object at 0x...>]
__init__(plotter, *args, **kwargs)
```

# Methods

```
__init__(plotter, *args, **kwargs)
```

bayespy.plot.Plotter.\_\_init\_\_

```
Plotter.__init__(plotter, *args, **kwargs)
```

# bayespy.plot.PDFPlotter

```
class bayespy.plot.PDFPlotter(x_grid, **kwargs)
```

Plotter of probability density function of a scalar node

```
Parameters x_grid : array
```

Numerical grid on which the density function is computed and plotted

See also:

pdf

```
__init__ (x_grid, **kwargs)
     Methods
                               __init__(x_grid, **kwargs)
     bayespy.plot.PDFPlotter.__init__
     PDFPlotter.__init__(x_grid, **kwargs)
bayespy.plot.ContourPlotter
class bayespy.plot.ContourPlotter (x1-grid, x2-grid, **kwargs)
     Plotter of probability density function of a two-dimensional node
          Parameters x1_grid : array
                  Grid for the first dimension
              x2_grid: array
                  Grid for the second dimension
     See also:
     contour
     __init__ (x1_grid, x2_grid, **kwargs)
     Methods
                                 _init__(x1_grid, x2_grid, **kwargs)
     bayespy.plot.ContourPlotter.__init__
     ContourPlotter.__init__(x1_grid, x2_grid, **kwargs)
bayespy.plot.HintonPlotter
class bayespy.plot.HintonPlotter(**kwargs)
     Plotter of the Hinton diagram of a node
     See also:
     hinton
     __init__(**kwargs)
     Methods
```

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```
init_(**kwargs)
     bayespy.plot.HintonPlotter.__init__
     HintonPlotter.__init__(**kwargs)
bayespy.plot.FunctionPlotter
class bayespy.plot.FunctionPlotter(**kwargs)
     Plotter of a node as a 1-dimensional function
     See also:
     plot
     __init__(**kwargs)
     Methods
                             __init__(**kwargs)
     bayespy.plot.FunctionPlotter.__init__
     FunctionPlotter.__init__(**kwargs)
bayespy.plot.GaussianTimeseriesPlotter
class bayespy.plot.GaussianTimeseriesPlotter(**kwargs)
     Plotter of a Gaussian node as a timeseries
     __init__(**kwargs)
     Methods
                              __init__(**kwargs)
     bayespy.plot.GaussianTimeseriesPlotter.__init__
     GaussianTimeseriesPlotter.__init__(**kwargs)
bayespy.plot.CategoricalMarkovChainPlotter
class bayespy.plot.CategoricalMarkovChainPlotter(**kwargs)
     Plotter of a Categorical timeseries
     __init__(**kwargs)
```

# Methods

bayespy.plot.CategoricalMarkovChainPlotter.\_\_init\_

CategoricalMarkovChainPlotter.\_\_init\_\_(\*\*kwargs)

5.3. bayespy.plot

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**CHAPTER** 

SIX

# **DEVELOPER API**

This chapter contains API specifications which are relevant to BayesPy developers and contributors.

# 6.1 Developer nodes

The following base classes are useful if writing new nodes:

```
node.Node(*parents, **kwargs)
stochastic.Stochastic(*args[, initialize, dims])
expfamily.ExponentialFamily(*args, **kwargs)
deterministic.Deterministic(*args, **kwargs)

Base class for all nodes.

Base class for nodes that are stochastic.

A base class for nodes using natural parameterization phi.

Base class for deterministic nodes.
```

# 6.1.1 bayespy.inference.vmp.nodes.node.Node

```
class bayespy.inference.vmp.nodes.node.Node(*parents, **kwargs)
    Base class for all nodes.

mask dims plates parents children name
Sub-classes must implement: 1. For computing the message to children:
    get_moments(self):

2.For computing the message to parents: _get_message_and_mask_to_parent(self, index)
Sub-classes may need to re-implement: 1. If they manipulate plates:
    _compute_mask_to_parent(index, mask) _plates_to_parent(self, index) _plates_from_parent(self, index)
__init__(*parents, **kwargs)
```

### **Methods**

# Table 6.2 – continued from previous page

get\_moments()
get\_shape(ind)
has\_plotter()
move\_plates(from\_plate, to\_plate)
plot([fig])
set\_plotter(plotter)
Return True if the node has a plotter
Plot the node distribution using the plotter of the node

# bayespy.inference.vmp.nodes.node.Node.\_\_init\_\_

Node.\_\_init\_\_(\*parents, \*\*kwargs)

# bayespy.inference.vmp.nodes.node.Node.add\_plate\_axis

Node.add\_plate\_axis(to\_plate)

# bayespy.inference.vmp.nodes.node.Node.broadcasting\_multiplier

 $static \ Node.broadcasting\_multiplier (plates, *args)$ 

# bayespy.inference.vmp.nodes.node.Node.delete

Node.delete()

Delete this node and the children

# bayespy.inference.vmp.nodes.node.Node.get\_mask

Node.get\_mask()

### bayespy.inference.vmp.nodes.node.Node.get\_moments

Node.get\_moments()

### bayespy.inference.vmp.nodes.node.Node.get\_shape

Node.get\_shape(ind)

# bayespy.inference.vmp.nodes.node.Node.has\_plotter

Node.has\_plotter()

Return True if the node has a plotter

# bayespy.inference.vmp.nodes.node.Node.move\_plates

Node.move\_plates (from\_plate, to\_plate)

# bayespy.inference.vmp.nodes.node.Node.plot

```
Node.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.inference.vmp.nodes.node.Node.set\_plotter

```
Node.set_plotter(plotter)
```

### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.inference.vmp.nodes.node.Node.plates

```
Node.plates = None
```

# bayespy.inference.vmp.nodes.node.Node.plates\_multiplier

```
Node.plates_multiplier
```

Plate multiplier is applied to messages to parents

# 6.1.2 bayespy.inference.vmp.nodes.stochastic.Stochastic

Base class for nodes that are stochastic.

u observed

**Sub-classes must implement:** \_compute\_message\_to\_parent(parent, index, u\_self, \*u\_parents) \_update\_distribution\_and\_lowerbound(self, m, \*u) lowerbound(self) \_compute\_dims initialize\_from\_prior()

If you want to be able to observe the variable: \_compute\_fixed\_moments\_and\_f

Sub-classes may need to re-implement: 1. If they manipulate plates:

```
_compute_mask_to_parent(index, mask) _compute_plates_to_parent(self, index, plates) _compute_plates_from_parent(self, index, plates)
```

```
__init__ (*args, initialize=True, dims=None, **kwargs)
```

# Methods

```
__init__(*args[, initialize, dims])

Continued on next page
```

# Table 6.4 – continued from previous page

```
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
                                              Delete this node and the children
delete()
get_mask()
get_moments()
get_shape(ind)
                                              Return True if the node has a plotter
has_plotter()
                                              Load the state of the node from a HDF5 file.
load(group)
lowerbound()
move_plates(from_plate, to_plate)
observe(x[, mask])
                                              Fix moments, compute f and propagate mask.
plot([fig])
                                              Plot the node distribution using the plotter of the node
                                              Draw a random sample from the distribution.
random()
                                              Save the state of the node into a HDF5 file.
save(group)
set_plotter(plotter)
unobserve()
update([annealing])
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.\_\_init\_\_

```
Stochastic.__init__(*args, initialize=True, dims=None, **kwargs)
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.add\_plate\_axis

```
Stochastic.add_plate_axis(to_plate)
```

### bayespy.inference.vmp.nodes.stochastic.Stochastic.broadcasting\_multiplier

```
Stochastic.broadcasting_multiplier(plates, *args)
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.delete

```
Stochastic.delete()

Delete this node and the children
```

### bayespy.inference.vmp.nodes.stochastic.Stochastic.get\_mask

```
Stochastic.get_mask()
```

### bayespy.inference.vmp.nodes.stochastic.Stochastic.get\_moments

```
Stochastic.get_moments()
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.get\_shape

```
Stochastic.get_shape(ind)
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.has\_plotter

```
Stochastic.has_plotter()

Return True if the node has a plotter
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.load

```
Stochastic.load (group)

Load the state of the node from a HDF5 file.
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.lowerbound

```
Stochastic.lowerbound()
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.move\_plates

```
Stochastic.move_plates (from_plate, to_plate)
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.observe

```
Stochastic.observe (x, mask=True)
Fix moments, compute f and propagate mask.
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.plot

```
Stochastic.plot (fig=None, **kwargs)

Plot the node distribution using the plotter of the node
```

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.inference.vmp.nodes.stochastic.Stochastic.random

```
Stochastic.random()
```

Draw a random sample from the distribution.

# bayespy.inference.vmp.nodes.stochastic.Stochastic.save

```
Stochastic.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.set\_plotter

```
Stochastic.set_plotter(plotter)
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.unobserve

```
Stochastic.unobserve()
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.update

```
Stochastic.update (annealing=1.0)
```

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.plates

```
Stochastic.plates = None
```

# bayespy.inference.vmp.nodes.stochastic.Stochastic.plates\_multiplier

```
Stochastic.plates_multiplier

Plate multiplier is applied to messages to parents
```

# 6.1.3 bayespy.inference.vmp.nodes.expfamily.ExponentialFamily

```
A base class for nodes using natural parameterization phi.

phi

Sub-classes must implement the following static methods: _compute_message_to_parent(index, __u_self, __*u_parents) _compute_phi_from_parents(*u_parents, __mask) _compute_moments_and_cgf(phi, __mask) _compute_fixed_moments_and_f(x, __mask=True)

Sub-classes may need to re-implement: 1. If they manipulate plates:

_compute_mask_to_parent(index, __mask) __compute_plates_to_parent(self, __index, __plates) __compute_plates_from_parent(self, index, plates)

_init__(*args, **kwargs)
```

class bayespy.inference.vmp.nodes.expfamily.ExponentialFamily(\*args, \*\*kwargs)

#### **Methods**

Continued on next page

# Table 6.6 – continued from previous page

<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	<u>-</u>
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	-
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	•
load(group)	Load the state of the node from a HDF5 file.
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(group)	Save the state of the node into a HDF5 file.
$set_parameters(x)$	Set the parameters of the VB distribution.
set_plotter(plotter)	
unobserve()	
update([annealing])	

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.\_\_init\_\_

```
ExponentialFamily.__init__(*args, **kwargs)
```

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.add\_plate\_axis

ExponentialFamily.add\_plate\_axis (to\_plate)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.broadcasting\_multiplier

ExponentialFamily.broadcasting\_multiplier(plates, \*args)

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.delete

```
ExponentialFamily.delete()

Delete this node and the children
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_gradient

```
ExponentialFamily.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_mask

```
ExponentialFamily.get_mask()
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_moments

```
ExponentialFamily.get_moments()
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_parameters

```
ExponentialFamily.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_riemannian\_gradient

```
ExponentialFamily.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_shape

```
ExponentialFamily.get_shape (ind)
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.has\_plotter

```
ExponentialFamily.has_plotter()
```

Return True if the node has a plotter

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_parameters

```
ExponentialFamily.initialize_from_parameters(*args)
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_prior

```
ExponentialFamily.initialize_from_prior()
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_random

```
ExponentialFamily.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_value

```
ExponentialFamily.initialize_from_value (x, *args)
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.load

```
ExponentialFamily.load(group)

Load the state of the node from a HDF5 file.
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.logpdf

```
ExponentialFamily.logpdf (X, mask=True)
Compute the log probability density function Q(X) of this node.
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.lower\_bound\_contribution

```
ExponentialFamily.lower_bound_contribution (gradient=False, ignore\_masked=True)

Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.lowerbound

```
ExponentialFamily.lowerbound()
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.move\_plates

```
ExponentialFamily.move_plates (from_plate, to_plate)
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.observe

```
ExponentialFamily.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.pdf

```
ExponentialFamily.pdf (X, mask=True)

Compute the probability density function of this node.
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plot

```
ExponentialFamily.plot (fig=None, **kwargs)

Plot the node distribution using the plotter of the node
```

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.random

```
ExponentialFamily.random()
```

Draw a random sample from the distribution.

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.save

```
ExponentialFamily.save (group)
Save the state of the node into a HDF5 file.
group can be the root
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.set\_parameters

```
ExponentialFamily.set_parameters (x) Set the parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.set\_plotter

```
ExponentialFamily.set_plotter(plotter)
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.unobserve

ExponentialFamily.unobserve()

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.update

ExponentialFamily.update(annealing=1.0)

### **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

# bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.dims

ExponentialFamily.dims = None

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plates

ExponentialFamily.plates = None

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plates\_multiplier

```
ExponentialFamily.plates_multiplier

Plate multiplier is applied to messages to parents
```

# 6.1.4 bayespy.inference.vmp.nodes.deterministic.Deterministic

```
class bayespy.inference.vmp.nodes.deterministic.Deterministic(*args, **kwargs)
     Base class for deterministic nodes.
     Sub-classes must implement: 1. For implementing the deterministic function:
           _compute_moments(self, *u)
         2.One of the following options: a) Simple methods:
               _compute_message_to_parent(self, index, m, *u) not? _compute_mask_to_parent(self, index,
               mask)
            (a)More control with: _compute_message_and_mask_to_parent(self, index, m, *u)
     Sub-classes may need to re-implement: 1. If they manipulate plates:
           _compute_mask_to_parent(index,
                                             mask)
                                                      _compute_plates_to_parent(self,
                                                                                       index,
                                                                                                 plates)
           _compute_plates_from_parent(self, index, plates)
     __init__ (*args, **kwargs)
```

### **Methods**

```
-_init__(*args, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
set_plotter(plotter)

Plot the node distribution using the plotter of the node
set_plotter(plotter)
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.\_\_init\_\_

```
Deterministic.__init__(*args, **kwargs)
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.add\_plate\_axis

Deterministic.add\_plate\_axis(to\_plate)

# bayespy.inference.vmp.nodes.deterministic.Deterministic.broadcasting\_multiplier

Deterministic.broadcasting\_multiplier(plates, \*args)

# bayespy.inference.vmp.nodes.deterministic.Deterministic.delete

```
Deterministic.delete()

Delete this node and the children
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_mask

```
Deterministic.get_mask()
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_moments

```
Deterministic.get_moments()
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_shape

```
Deterministic.get_shape (ind)
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.has\_plotter

```
Deterministic.has_plotter()

Return True if the node has a plotter
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.lower\_bound\_contribution

```
Deterministic.lower_bound_contribution(gradient=False, **kwargs)
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.move\_plates

```
Deterministic.move_plates (from_plate, to_plate)
```

### bayespy.inference.vmp.nodes.deterministic.Deterministic.plot

```
Deterministic.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.deterministic.Deterministic.set\_plotter

```
Deterministic.set_plotter(plotter)
```

# **Attributes**

```
plates plates multiplier Plate multiplier is applied to messages to parents
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.plates

```
Deterministic.plates = None
```

# bayespy.inference.vmp.nodes.deterministic.Deterministic.plates\_multiplier

```
Deterministic.plates_multiplier
```

Plate multiplier is applied to messages to parents

The following nodes are examples of special nodes that remain hidden for the user although they are often implicitly used:

```
constant.Constant(moments, x, **kwargs)

gaussian.GaussianToGaussianGammaISO(X, **kwargs)

gaussian.GaussianGammaISOToGaussianGammaARD(X, ...)

gaussian.WrapToGaussianGammaISO(*parents, ...)

gaussian.WrapToGaussianGammaARD(mu_alpha, ...)

gaussian.WrapToGaussianWishart(X, Lambda, ...)

Wraps Gaussian and Wishart nodes into a Gaussian-Wishart
```

# 6.1.5 bayespy.inference.vmp.nodes.constant.Constant

```
class bayespy.inference.vmp.nodes.constant.Constant(moments, x, **kwargs)
    Node for presenting constant values.
```

The node wraps arrays into proper node type.

```
__init__ (moments, x, **kwargs)
```

### **Methods**

```
__init__(moments, x, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
                                             Delete this node and the children
get_mask()
get_moments()
get_shape(ind)
has_plotter()
                                             Return True if the node has a plotter
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
                                             Plot the node distribution using the plotter of the node
set_plotter(plotter)
set_value(x)
```

### bayespy.inference.vmp.nodes.constant.Constant.\_init\_

```
Constant.__init__(moments, x, **kwargs)
```

# bayespy.inference.vmp.nodes.constant.Constant.add\_plate\_axis

```
Constant.add_plate_axis(to_plate)
```

# bayespy.inference.vmp.nodes.constant.Constant.broadcasting\_multiplier

```
Constant.broadcasting_multiplier(plates, *args)
```

# bayespy.inference.vmp.nodes.constant.Constant.delete

```
Constant.delete()
```

Delete this node and the children

# bayespy.inference.vmp.nodes.constant.Constant.get\_mask

```
Constant.get_mask()
```

# bayespy.inference.vmp.nodes.constant.Constant.get\_moments

```
Constant.get_moments()
```

### bayespy.inference.vmp.nodes.constant.Constant.get\_shape

```
Constant.get_shape(ind)
```

# bayespy.inference.vmp.nodes.constant.Constant.has\_plotter

```
Constant.has_plotter()
```

Return True if the node has a plotter

### bayespy.inference.vmp.nodes.constant.Constant.lower\_bound\_contribution

```
Constant.lower_bound_contribution(gradient=False, **kwargs)
```

# bayespy.inference.vmp.nodes.constant.Constant.move\_plates

```
Constant.move_plates (from_plate, to_plate)
```

# bayespy.inference.vmp.nodes.constant.Constant.plot

```
Constant.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.inference.vmp.nodes.constant.Constant.set\_plotter

```
Constant.set_plotter(plotter)
```

#### bayespy.inference.vmp.nodes.constant.Constant.set\_value

```
Constant.set_value(x)
```

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.constant.Constant.plates

```
Constant.plates = None
```

### bayespy.inference.vmp.nodes.constant.Constant.plates\_multiplier

```
Constant.plates_multiplier
```

Plate multiplier is applied to messages to parents

# 6.1.6 bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO

```
class bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammaISO(X, **kwargs)
```

Converter for Gaussian moments to Gaussian-gamma isotropic moments

Combines the Gaussian moments with gamma moments for a fixed value 1.

```
__init__ (X, **kwargs)
```

```
-_init__(X,**kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
set_plotter(plotter)

Plot the node distribution using the plotter of the node
set_plotter(plotter)
```

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.\_\_init\_\_

GaussianToGaussianGammaISO.\_\_init\_\_(X, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.add\_plate\_axis

GaussianToGaussianGammaISO.add\_plate\_axis (to\_plate)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.broadcasting\_multiplier

GaussianToGaussianGammaISO.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.delete

GaussianToGaussianGammaISO.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_mask

GaussianToGaussianGammaISO.get\_mask()

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_moments

GaussianToGaussianGammaISO.get\_moments()

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_shape

GaussianToGaussianGammaISO.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.has\_plotter

GaussianToGaussianGammaISO.has\_plotter()
Return True if the node has a plotter

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.lower\_bound\_contribution

GaussianToGaussianGammaISO.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.move\_plates

GaussianToGaussianGammaISO.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plot

GaussianToGaussianGammaISO.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.set\_plotter

GaussianToGaussianGammaISO.set\_plotter(plotter)

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plates

GaussianToGaussianGammaISO.plates = None

## bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plates\_multiplier

GaussianToGaussianGammaISO.plates\_multiplier
Plate multiplier is applied to messages to parents

# 6.1.7 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOToGaussianGammaARD(X,

\*\*kwargs)

Converter for Gaussian-gamma ISO moments to Gaussian-gamma ARD moments

```
__init__(X, **kwargs)
```

```
__init__(X, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])

Plot the node distribution using the plotter of the node

Continued on next page
```

### Table 6.15 – continued from previous page

set\_plotter(plotter)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.\_\_init\_\_

GaussianGammaISOToGaussianGammaARD.\_\_init\_\_(X, \*\*kwargs)

bayespy.inference.vmp.nodes.qaussian.GaussianGammaISOToGaussianGammaARD.add\_plate\_axis

GaussianGammaISOToGaussianGammaARD.add\_plate\_axis(to\_plate)

 $bayes py. inference. vmp. nodes. gaussian. Gaussian Gammal SOTo Gaussian Gamma ARD. broadcasting \_multiplier and the state of the sta$ 

GaussianGammaISOToGaussianGammaARD.broadcasting multiplier (plates, \*args)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.delete

 ${\tt GaussianGammaISOToGaussianGammaARD.} \textbf{delete()}$ 

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.get\_mask

GaussianGammaISOToGaussianGammaARD.get\_mask()

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.get\_moments

 ${\tt GaussianGammaISOToGaussianGammaARD.} \textbf{get\_moments} \ ()$ 

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.get\_shape

GaussianGammaISOToGaussianGammaARD.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.has\_plotter

GaussianGammaISOToGaussianGammaARD.has\_plotter()

Return True if the node has a plotter

 $bayes py. in ference. vmp. nodes. gaussian. Gaussian Gammal SOTo Gaussian Gamma ARD. lower\_bound\_contribution for the contribution of the contri$ 

GaussianGammaISOToGaussianGammaARD.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.move\_plates

GaussianGammaISOToGaussianGammaARD.move\_plates(from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plot

GaussianGammaISOToGaussianGammaARD.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.set\_plotter

 ${\tt GaussianGammaISOToGaussianGammaARD.} \textbf{set\_plotter} \ (plotter)$ 

#### **Attributes**

plates
plates\_multiplier Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plates

GaussianGammaISOToGaussianGammaARD.plates = None

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plates\_multiplier

GaussianGammaISOToGaussianGammaARD.plates\_multiplier
Plate multiplier is applied to messages to parents

## 6.1.8 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart

```
__init__(X_alpha, **kwargs)
```

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.\_\_init\_\_

GaussianGammaARDToGaussianWishart.\_\_init\_\_(X\_alpha, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.add\_plate\_axis

GaussianGammaARDToGaussianWishart.add\_plate\_axis(to\_plate)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.broadcasting\_multiplier

GaussianGammaARDToGaussianWishart.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.delete

GaussianGammaARDToGaussianWishart.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_mask

GaussianGammaARDToGaussianWishart.get\_mask()

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_moments

GaussianGammaARDToGaussianWishart.get\_moments()

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_shape

GaussianGammaARDToGaussianWishart.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.has\_plotter

GaussianGammaARDToGaussianWishart.has\_plotter()
Return True if the node has a plotter

 $bayes py. inference. vmp. nodes. gaussian. Gaussian Gamma ARD To Gaussian Wishart. lower\_bound\_contribution$ 

GaussianGammaARDToGaussianWishart.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.move\_plates

GaussianGammaARDToGaussianWishart.move\_plates(from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plot

GaussianGammaARDToGaussianWishart.**plot** (*fig=None*, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.set\_plotter

GaussianGammaARDToGaussianWishart.set\_plotter(plotter)

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plates

GaussianGammaARDToGaussianWishart.plates = None

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plates\_multiplier

GaussianGammaARDToGaussianWishart.plates\_multiplier
Plate multiplier is applied to messages to parents

# 6.1.9 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO

```
{\bf class} \ {\bf bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaISO} \ (*parents, **kwargs)
```

```
__init__ (*parents, **kwargs)
```

```
-_init__(*parents, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
set_plotter(plotter)

Plot the node distribution using the plotter of the node
set_plotter(plotter)
```

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.\_\_init\_\_

WrapToGaussianGammaISO.\_\_init\_\_(\*parents, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.add\_plate\_axis

WrapToGaussianGammaISO.add\_plate\_axis (to\_plate)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.broadcasting\_multiplier

WrapToGaussianGammaISO.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.delete

WrapToGaussianGammaISO.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_mask

WrapToGaussianGammaISO.get\_mask()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_moments

WrapToGaussianGammaISO.get\_moments()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_shape

WrapToGaussianGammaISO.get\_shape (ind)

 $bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gammal SO. has\_plotter$ 

WrapToGaussianGammaISO.has\_plotter()
Return True if the node has a plotter

 $bayes py. in ference. vmp. nodes. gaussian. Wrap To Gaussian Gammal SO. lower\_bound\_contribution$ 

WrapToGaussianGammaISO.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.move\_plates

WrapToGaussianGammaISO.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.plot

```
WrapToGaussianGammaISO.plot (fig=None, **kwargs)
Plot the node distribution using the plotter of the node
```

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.set\_plotter

WrapToGaussianGammaISO.set\_plotter(plotter)

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.plates

WrapToGaussianGammaISO.plates = None

# $bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gammal SO. plates\_multiplier$

```
WrapToGaussianGammaISO.plates_multiplier
Plate multiplier is applied to messages to parents
```

# 6.1.10 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD

```
__init__ (mu_alpha, tau, **kwargs)
```

```
-_init__(mu_alpha, tau, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()
get_moments()
get_shape(ind)
has_plotter()
lower_bound_contribution([gradient])
move_plates(from_plate, to_plate)
plot([fig])
set_plotter(plotter)

Plot the node distribution using the plotter of the node
set_plotter(plotter)
```

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD...init\_

WrapToGaussianGammaARD.\_\_init\_\_(mu\_alpha, tau, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.add\_plate\_axis

WrapToGaussianGammaARD.add\_plate\_axis(to\_plate)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.broadcasting\_multiplier

WrapToGaussianGammaARD.broadcasting\_multiplier(plates, \*args)

bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gamma ARD. delete

WrapToGaussianGammaARD.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_mask

WrapToGaussianGammaARD.get\_mask()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_moments

WrapToGaussianGammaARD.get\_moments()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_shape

WrapToGaussianGammaARD.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.has\_plotter

WrapToGaussianGammaARD.has\_plotter()
Return True if the node has a plotter

 $bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gamma ARD. lower\_bound\_contribution$ 

WrapToGaussianGammaARD.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.move\_plates

WrapToGaussianGammaARD.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plot

```
{\tt WrapToGaussianGammaARD.plot}~(\textit{fig=None},~**kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.set\_plotter

```
WrapToGaussianGammaARD.set_plotter(plotter)
```

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plates

WrapToGaussianGammaARD.plates = None

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plates\_multiplier

```
WrapToGaussianGammaARD.plates_multiplier
Plate multiplier is applied to messages to parents
```

# 6.1.11 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart

Wraps Gaussian and Wishart nodes into a Gaussian-Wishart node.

#### The following node combinations can be wrapped:

- · Gaussian and Wishart
- · Gaussian-gamma and Wishart
- · Gaussian-Wishart and gamma

```
__init__ (X, Lambda, **kwargs)
```

```
__init__(X, Lambda, **kwargs)
add_plate_axis(to_plate)
broadcasting_multiplier(plates, *args)
delete()
get_mask()

Continued on next page
```

### Table 6.23 - continued from previous page

get\_moments()
get\_shape(ind)
has\_plotter()
lower\_bound\_contribution([gradient])
move\_plates(from\_plate, to\_plate)
plot([fig])
set\_plotter(plotter)
Plot the node distribution using the plotter of the node

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.\_\_init\_\_

WrapToGaussianWishart.\_\_init\_\_(X, Lambda, \*\*kwargs)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.add\_plate\_axis

WrapToGaussianWishart.add\_plate\_axis(to\_plate)

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.broadcasting\_multiplier

WrapToGaussianWishart.broadcasting\_multiplier(plates, \*args)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.delete

WrapToGaussianWishart.delete()

Delete this node and the children

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_mask

WrapToGaussianWishart.get\_mask()

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_moments

WrapToGaussianWishart.get\_moments()

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_shape

WrapToGaussianWishart.get\_shape (ind)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.has\_plotter

WrapToGaussianWishart.has\_plotter()
Return True if the node has a plotter

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.lower\_bound\_contribution

WrapToGaussianWishart.lower\_bound\_contribution (gradient=False, \*\*kwargs)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.move\_plates

WrapToGaussianWishart.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plot

```
WrapToGaussianWishart.plot (fig=None, **kwargs)
Plot the node distribution using the plotter of the node
```

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.set\_plotter

WrapToGaussianWishart.set\_plotter(plotter)

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plates

WrapToGaussianWishart.plates = None

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plates\_multiplier

```
WrapToGaussianWishart.plates_multiplier
Plate multiplier is applied to messages to parents
```

## 6.2 Moments

node.Moments	Base class for defining the expectation of the su
gaussian.GaussianMoments(ndim)	Class for the moments of Gaussian variables.
gaussian_markov_chain.GaussianMarkovChainMoments	
gaussian.GaussianGammaISOMoments(ndim)	Class for the moments of Gaussian-gamma-ISC
gaussian.GaussianGammaARDMoments(ndim)	Class for the moments of Gaussian-gamma-AR
gaussian.GaussianWishartMoments	Class for the moments of Gaussian-Wishart var
gamma.GammaMoments	Class for the moments of gamma variables.
wishart.WishartMoments	
beta.BetaMoments	Class for the moments of beta variables.
dirichlet.DirichletMoments	Class for the moments of Dirichlet variables.
bernoulli.BernoulliMoments()	Class for the moments of Bernoulli variables.
$\verb binomial.BinomialMoments (N) $	Class for the moments of binomial variables
categorical.CategoricalMoments(categories)	Class for the moments of categorical variables.
<pre>categorical_markov_chain.CategoricalMarkovChainMoments()</pre>	Class for the moments of categorical Markov ch

Continue

### Table 6.25 – continued from previous page

multinomial.MultinomialMoments poisson.PoissonMoments

Class for the moments of multinomial variables Class for the moments of Poisson variables

## 6.2.1 bayespy.inference.vmp.nodes.node.Moments

class bayespy.inference.vmp.nodes.node.Moments

Base class for defining the expectation of the sufficient statistics.

The benefits:

- •Write statistic-specific features in one place only. For instance, covariance from Gaussian message.
- •Different nodes may have identically defined statistic so you need to implement related features only once. For instance, Gaussian and GaussianARD differ on the prior but the moments are the same.
- •General processing nodes which do not change the type of the moments may "inherit" the features from the parent node. For instance, slicing operator.
- •Conversions can be done easily in both of the above cases if the message conversion is defined in the moments class. For instance, GaussianMarkovChain to Gaussian and VaryingGaussianMarkovChain to Gaussian.

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

```
add_converter(moments_to, converter)
compute_dims_from_values(x)
compute_fixed_moments(x)
get_converter(moments_to)
Finds conversion to another moments type if possible.
```

## bayespy.inference.vmp.nodes.node.Moments.add\_converter

classmethod Moments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.node.Moments.compute\_dims\_from\_values

Moments.compute\_dims\_from\_values(x)

#### bayespy.inference.vmp.nodes.node.Moments.compute\_fixed\_moments

Moments.compute\_fixed\_moments(x)

#### bayespy.inference.vmp.nodes.node.Moments.get\_converter

Moments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.2 bayespy.inference.vmp.nodes.gaussian.GaussianMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianMoments (ndim)
 Class for the moments of Gaussian variables.

```
__init__(ndim)
```

#### **Methods**

init(ndim)	
<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.\_\_init\_\_

```
GaussianMoments.__init__(ndim)
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.add\_converter

GaussianMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.compute\_dims\_from\_values

```
GaussianMoments.compute_dims_from_values (x)
Return the shape of the moments for a fixed value.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.compute\_fixed\_moments

```
GaussianMoments.compute_fixed_moments (x)

Compute the moments for a fixed value
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.get\_converter

```
GaussianMoments.get_converter (moments_to)
Finds conversion to another moments type if possible.
```

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.3 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments

class bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### Methods

```
add_converter(moments_to, converter)
compute_dims_from_values(x)
compute_fixed_moments(x)
get_converter(moments_to)
Finds conversion to another moments type if possible.
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.add\_converter

GaussianMarkovChainMoments.add\_converter (moments\_to, converter)

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.compute\_dims\_from\_v

GaussianMarkovChainMoments.compute\_dims\_from\_values(x)

 $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Moments. compute\_fixed\_moments. com$ 

GaussianMarkovChainMoments.compute\_fixed\_moments(x)

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.get\_converter

GaussianMarkovChainMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.4 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOMoments(ndim)
 Class for the moments of Gaussian-gamma-ISO variables.

```
__init__(ndim)
```

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

#### Methods

init(ndim)	Create moments object for Gaussian-gamma isotropic variables
<pre>add_converter(moments_to, converter)</pre>	
<pre>compute_dims_from_values(x, alpha)</pre>	Return the shape of the moments for a fixed value.
<pre>compute_fixed_moments(x, alpha)</pre>	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.\_init\_

```
GaussianGammaISOMoments.__init__(ndim)
```

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.add\_converter

GaussianGammaISOMoments.add\_converter (moments\_to, converter)

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.compute\_dims\_from\_values

 ${\tt GaussianGammaISOMoments.compute\_dims\_from\_values}\ (x, alpha)$ 

Return the shape of the moments for a fixed value.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.compute\_fixed\_moments

GaussianGammaISOMoments.compute\_fixed\_moments(x, alpha)

Compute the moments for a fixed value

x is a mean vector. alpha is a precision scale

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.get\_converter

GaussianGammaISOMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.5 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments(ndim)
 Class for the moments of Gaussian-gamma-ARD variables.

\_\_init\_\_(ndim)

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

#### **Methods**

init(ndim)	Create moments object for Gaussian-gamma isotropic variables
<pre>add_converter(moments_to, converter)</pre>	
<pre>compute_dims_from_values(x, alpha)</pre>	Return the shape of the moments for a fixed value.
compute_fixed_moments(x, alpha)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.\_\_init\_\_

```
GaussianGammaARDMoments...init...(ndim)
Create moments object for Gaussian-gamma isotropic variables
ndim=0: scalar ndim=1: vector ndim=2: matrix ...
```

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.add\_converter

GaussianGammaARDMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.compute\_dims\_from\_values

GaussianGammaARDMoments.compute\_dims\_from\_values (x, alpha)
Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.compute\_fixed\_moments

```
GaussianGammaARDMoments.compute_fixed_moments (x, alpha)
Compute the moments for a fixed value

x is a mean vector. alpha is a precision scale
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.get\_converter

GaussianGammaARDMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their

converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.6 bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments
 Class for the moments of Gaussian-Wishart variables.

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### Methods

add_converter(moments_to, converter)	
<pre>compute_dims_from_values(x, Lambda)</pre>	Return the shape of the moments for a fixed value.
<pre>compute_fixed_moments(x, Lambda)</pre>	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.add\_converter

GaussianWishartMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.compute\_dims\_from\_values

```
GaussianWishartMoments.compute_dims_from_values (x, Lambda)
Return the shape of the moments for a fixed value.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.compute\_fixed\_moments

```
GaussianWishartMoments.compute_fixed_moments (x, Lambda)
Compute the moments for a fixed value

x is a vector. Lambda is a precision matrix
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.get\_converter

```
{\tt GaussianWishartMoments.get\_converter}~(\textit{moments\_to})
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.7 bayespy.inference.vmp.nodes.gamma.GammaMoments

```
class bayespy.inference.vmp.nodes.gamma.GammaMoments
    Class for the moments of gamma variables.
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### Methods

add_converter(moments_to, converter)	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

## bayespy.inference.vmp.nodes.gamma.GammaMoments.add\_converter

GammaMoments.add\_converter (moments\_to, converter)

## bayespy.inference.vmp.nodes.gamma.GammaMoments.compute\_dims\_from\_values

```
GammaMoments.compute_dims_from_values (x)
Return the shape of the moments for a fixed value.
```

#### bayespy.inference.vmp.nodes.gamma.GammaMoments.compute\_fixed\_moments

```
GammaMoments.compute_fixed_moments (x)
Compute the moments for a fixed value
```

## bayespy.inference.vmp.nodes.gamma.GammaMoments.get\_converter

```
GammaMoments.get_converter (moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.8 bayespy.inference.vmp.nodes.wishart.WishartMoments

#### **Methods**

add_converter(moments_to, converter)	
$compute\_dims\_from\_values(x)$	Compute the dimensions of phi and u.
compute_fixed_moments(Lambda)	Compute moments for fixed x.
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.add\_converter

WishartMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.compute\_dims\_from\_values

```
WishartMoments.compute_dims_from_values (x)
Compute the dimensions of phi and u.
```

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.compute\_fixed\_moments

```
WishartMoments.compute_fixed_moments (Lambda)
Compute moments for fixed x.
```

## bayespy.inference.vmp.nodes.wishart.WishartMoments.get\_converter

```
WishartMoments.get_converter(moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.9 bayespy.inference.vmp.nodes.beta.BetaMoments

Initialize self. See help(type(self)) for accurate signature.

```
class bayespy.inference.vmp.nodes.beta.BetaMoments
    Class for the moments of beta variables.
__init__()
```

#### Methods

add_converter(moments_to, converter)	
<pre>compute_dims_from_values(p)</pre>	Return the shape of the moments for a fixed value.
compute_fixed_moments(p)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.beta.BetaMoments.add\_converter

BetaMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.beta.BetaMoments.compute\_dims\_from\_values

```
{\tt BetaMoments.compute\_dims\_from\_values}\ (p)
```

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.beta.BetaMoments.compute\_fixed\_moments

```
BetaMoments.compute_fixed_moments(p)
```

Compute the moments for a fixed value

#### bayespy.inference.vmp.nodes.beta.BetaMoments.get\_converter

```
BetaMoments.get_converter (moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.10 bayespy.inference.vmp.nodes.dirichlet.DirichletMoments

class bayespy.inference.vmp.nodes.dirichlet.DirichletMoments
 Class for the moments of Dirichlet variables.

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### Methods

add_converter(moments_to, converter)	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
compute_fixed_moments(p)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.add\_converter

DirichletMoments.add\_converter (moments\_to, converter)

### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.compute\_dims\_from\_values

```
DirichletMoments.compute_dims_from_values (x)
Return the shape of the moments for a fixed value.
```

## bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.compute\_fixed\_moments

```
DirichletMoments.compute_fixed_moments(p)

Compute the moments for a fixed value
```

## bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.get\_converter

```
DirichletMoments.get_converter (moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.11 bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments

class bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments
 Class for the moments of Bernoulli variables.

```
__init__()
```

#### Methods

init()	
<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.\_init\_

```
BernoulliMoments.__init__()
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.add\_converter

BernoulliMoments.add\_converter (moments\_to, converter)

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.compute\_dims\_from\_values

```
BernoulliMoments.compute_dims_from_values(x)
```

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.compute\_fixed\_moments

```
{\tt BernoulliMoments.compute\_fixed\_moments}\,(x)
```

Compute the moments for a fixed value

## bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.get\_converter

```
BernoulliMoments.get_converter(moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.12 bayespy.inference.vmp.nodes.binomial.BinomialMoments

```
class bayespy.inference.vmp.nodes.binomial.BinomialMoments (N)
```

Class for the moments of binomial variables

```
__init__(N)
```

#### **Methods**

init(N)	
<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.binomial.BinomialMoments.\_\_init\_\_

```
BinomialMoments.__init__(N)
```

#### bayespy.inference.vmp.nodes.binomial.BinomialMoments.add\_converter

BinomialMoments.add\_converter (moments\_to, converter)

### bayespy.inference.vmp.nodes.binomial.BinomialMoments.compute\_dims\_from\_values

```
BinomialMoments.compute_dims_from_values(x)
```

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

### bayespy.inference.vmp.nodes.binomial.BinomialMoments.compute\_fixed\_moments

```
BinomialMoments.compute_fixed_moments(x)
```

Compute the moments for a fixed value

### bayespy.inference.vmp.nodes.binomial.BinomialMoments.get\_converter

```
BinomialMoments.get_converter(moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.13 bayespy.inference.vmp.nodes.categorical.CategoricalMoments

```
__init__(categories)
```

Create moments object for categorical variables

#### Methods

init(categories)	Create moments object for categorical variables
<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.\_\_init\_\_

```
CategoricalMoments...init...(categories)

Create moments object for categorical variables
```

## bayespy.inference.vmp.nodes.categorical.CategoricalMoments.add\_converter

CategoricalMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.compute\_dims\_from\_values

CategoricalMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

The observations are scalar.

### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.compute\_fixed\_moments

CategoricalMoments.compute\_fixed\_moments(x)

Compute the moments for a fixed value

### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.get\_converter

CategoricalMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.14 bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMomer

class bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments(categorical Markov chain variables.

\_\_init\_\_(categories)

Create moments object for categorical Markov chain variables.

#### **Methods**

init(categories)	Create moments object for categorical Markov chain variables.
<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.\_\_init\_\_

CategoricalMarkovChainMoments...init..(categories)

Create moments object for categorical Markov chain variables.

#### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.add\_converter

CategoricalMarkovChainMoments.add\_converter (moments\_to, converter)

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.compute\_dims\_fro

```
CategoricalMarkovChainMoments.compute_dims_from_values (x) Return the shape of the moments for a fixed value.
```

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.compute\_fixed\_mo

```
CategoricalMarkovChainMoments.compute_fixed_moments(x)
Compute the moments for a fixed value
```

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.get\_converter

```
CategoricalMarkovChainMoments.get_converter(moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.15 bayespy.inference.vmp.nodes.multinomial.MultinomialMoments

class bayespy.inference.vmp.nodes.multinomial.MultinomialMoments
 Class for the moments of multinomial variables.

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.add\_converter

MultinomialMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.compute\_dims\_from\_values

```
MultinomialMoments.compute_dims_from_values (x)
Return the shape of the moments for a fixed value.
```

#### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.compute\_fixed\_moments

```
MultinomialMoments.compute_fixed_moments(x)
Compute the moments for a fixed value

x must be a vector of counts.
```

### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.get\_converter

```
MultinomialMoments.get_converter(moments.to)
Finds conversion to another moments type if possible.
```

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

## 6.2.16 bayespy.inference.vmp.nodes.poisson.PoissonMoments

#### **Methods**

add_converter(moments_to, converter)	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
get_converter(moments_to)	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.add\_converter

PoissonMoments.add\_converter (moments\_to, converter)

### bayespy.inference.vmp.nodes.poisson.PoissonMoments.compute\_dims\_from\_values

```
PoissonMoments.compute_dims_from_values(x)
```

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.compute\_fixed\_moments

```
PoissonMoments.compute_fixed_moments (x)
Compute the moments for a fixed value
```

### bayespy.inference.vmp.nodes.poisson.PoissonMoments.get\_converter

PoissonMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.3 Distributions

stochastic.Distribution	A base class for the VMP for
expfamily.ExponentialFamilyDistribution	Sub-classes implement distrib
gaussian.GaussianDistribution	Class for the VMP formulas
gaussian.GaussianARDDistribution(shape, ndim_mu)	
gaussian.GaussianGammaISODistribution	Class for the VMP formulas
gaussian.GaussianGammaARDDistribution()	
gaussian.GaussianWishartDistribution	Class for the VMP formulas
${ t gaussian\_markov\_chain.GaussianMarkovChainDistribution}(N,D)$	Sub-classes implement distrib
$\verb"gaussian_markov_chain.SwitchingGaussianMarkovChainDistribution" (N, D, K)$	Sub-classes implement distrib
${ t gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution} (N,D)$	Sub-classes implement distrib
gamma.GammaDistribution	Class for the VMP formulas
wishart.WishartDistribution	Sub-classes implement distrib
beta.BetaDistribution	Class for the VMP formulas
dirichlet.DirichletDistribution	Class for the VMP formulas
bernoulli.BernoulliDistribution()	Class for the VMP formulas
binomial.BinomialDistribution( $N$ )	Class for the VMP formulas
categorical.CategoricalDistribution(categories)	Class for the VMP formulas
categorical_markov_chain.CategoricalMarkovChainDistribution()	Class for the VMP formulas
multinomial.MultinomialDistribution(trials)	Class for the VMP formulas
poisson.PoissonDistribution	Class for the VMP formulas

## 6.3.1 bayespy.inference.vmp.nodes.stochastic.Distribution

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#### **Methods**

compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.stochastic.Distribution.compute\_mask\_to\_parent

```
Distribution.compute_mask_to_parent (index, mask)

Maps the mask to the plates of a parent.
```

#### bayespy.inference.vmp.nodes.stochastic.Distribution.compute\_message\_to\_parent

```
Distribution.compute_message_to_parent (parent, index, u_self, *u_parents)

Compute the message to a parent node.
```

### bayespy.inference.vmp.nodes.stochastic.Distribution.plates\_from\_parent

```
Distribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.stochastic.Distribution.plates\_to\_parent

```
Distribution.plates_to_parent (index, plates)
```

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.stochastic.Distribution.random

```
Distribution.random(*params, plates=None)
Draw a random sample from the distribution.
```

# 6.3.2 bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution

```
class bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution
    Sub-classes implement distribution specific computations.
```

```
__init__()
Initialize self. See help(type(self)) for accurate signature.
```

```
compute_cgf_from_parents(*u_parents)
compute_fixed_moments_and_f(x[, mask])
                                                   Compute the standard gradient with respect to the natural parameters.
compute_gradient(g, u, phi)
compute_logpdf(u, phi, g, f, ndims)
                                                   Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f].
                                                   Maps the mask to the plates of a parent.
compute_mask_to_parent(index, mask)
compute_message_to_parent(parent, index, ...)
compute_moments_and_cgf(phi[, mask])
compute_phi_from_parents(*u_parents[, mask])
                                                   Resolve the plate mapping from a parent.
plates_from_parent(index, plates)
plates_to_parent(index, plates)
                                                   Resolves the plate mapping to a parent.
random(*params[, plates])
                                                   Draw a random sample from the distribution.
```

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_cqf\_from\_parents

ExponentialFamilyDistribution.compute\_cqf\_from\_parents(\*u\_parents)

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f

ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f(x, mask=True)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_gradient

ExponentialFamilyDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_logpdf

ExponentialFamilyDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_mask\_to\_parent

ExponentialFamilyDistribution.compute\_mask\_to\_parent(index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_message\_to\_parent

ExponentialFamilyDistribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_moments\_and\_cgf

ExponentialFamilyDistribution.compute\_moments\_and\_cgf(phi, mask=True)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_phi\_from\_parents

ExponentialFamilyDistribution.compute\_phi\_from\_parents(\*u\_parents, mask=True)

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### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.plates\_from\_parent

ExponentialFamilyDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.plates\_to\_parent

ExponentialFamilyDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.random

ExponentialFamilyDistribution.random(\*params, plates=None)

Draw a random sample from the distribution.

## 6.3.3 bayespy.inference.vmp.nodes.gaussian.GaussianDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianDistribution
 Class for the VMP formulas of Gaussian variables.

Currently, supports only vector variables.

#### **Notes**

Message passing equations:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}),$$

 $\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D$ ,  $\boldsymbol{\Lambda} \in \mathbb{R}^{D \times D}$ ,  $\boldsymbol{\Lambda}$  symmetric positive definite

$$\log \mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Lambda}) = -\frac{1}{2}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Lambda}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{\mu} - \frac{1}{2}\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{\mu} + \frac{1}{2}\log|\boldsymbol{\Lambda}| - \frac{D}{2}\log(2\pi)$$

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

compute_cgf_from_parents(u_mu_Lambda)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
	Continued on next page

### Table 6.45 – continued from previous page

compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_mu_Lambda[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_cgf\_from\_parents

$$g(\boldsymbol{\mu}, \boldsymbol{\Lambda}) = -\frac{1}{2}\operatorname{tr}(\boldsymbol{\mu}\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Lambda}) + \frac{1}{2}\log|\boldsymbol{\Lambda}|$$

## bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_fixed\_moments\_and\_f

GaussianDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \begin{bmatrix} \mathbf{x} \\ \mathbf{x} \mathbf{x}^T \end{bmatrix} \\ f(\mathbf{x}) &= -\frac{D}{2} \log(2\pi) \end{aligned}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_gradient

GaussianDistribution.compute\_gradient (g, u, phi)

Compute the standard gradient with respect to the natural parameters.

Gradient of the moments:

$$\begin{split} \mathrm{d}\overline{\mathbf{u}} &= \begin{bmatrix} \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1 - \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_1 \\ -\frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} - \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} \end{bmatrix} \\ &= \begin{bmatrix} 2(\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_2\overline{u}_1 + (\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_1 \\ u_2d\phi_2u_2 - 2u_1u_1^\mathrm{T}d\phi_2u_1u_1^\mathrm{T} + 2(u_2 - u_1u_1^\mathrm{T})d\phi_1u_1^\mathrm{T} \end{bmatrix} \end{split}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} (\overline{u}_2 - \overline{u}_1 \overline{u}_1^T) \tilde{\nabla}_1 + 2(u_2 - u_1 u_1^T) \tilde{\nabla}_2 u_1 \\ (u_2 - u_1 u_1^T) \tilde{\nabla}_1 u_1^T + u_1 \tilde{\nabla}_1^T (u_2 - u_1 u_1^T) + 2u_2 \tilde{\nabla}_2 u_2 - 2u_1 u_1^T \tilde{\nabla}_2 u_1 u_1^T \end{bmatrix}$$

# $bayes py. inference. vmp. nodes. gaussian. Gaussian Distribution. compute\_log pdf$

GaussianDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_mask\_to\_parent

GaussianDistribution.compute\_mask\_to\_parent(index, mask)

Maps the mask to the plates of a parent.

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#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_message\_to\_parent

GaussianDistribution.compute\_message\_to\_parent (parent, index, u, u\_mu\_Lambda)

Compute the message to a parent node.

$$\begin{split} \phi_{\boldsymbol{\mu}}(\mathbf{x}, \boldsymbol{\Lambda}) &= \begin{bmatrix} \boldsymbol{\Lambda} \mathbf{x} \\ -\frac{1}{2} \boldsymbol{\Lambda} \end{bmatrix} \\ \phi_{\boldsymbol{\Lambda}}(\mathbf{x}, \boldsymbol{\mu}) &= \begin{bmatrix} -\frac{1}{2} \mathbf{x} \mathbf{x}^{\mathrm{T}} + \frac{1}{2} \mathbf{x} \boldsymbol{\mu}^{\mathrm{T}} + \frac{1}{2} \boldsymbol{\mu} \mathbf{x}^{\mathrm{T}} - \frac{1}{2} \boldsymbol{\mu} \boldsymbol{\mu}^{\mathrm{T}} \\ \frac{1}{2} \end{bmatrix} \end{split}$$

# $bayes py. inference. vmp. nodes. gaussian. Gaussian Distribution. compute\_moments\_and\_cgf$

GaussianDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $q(\phi)$ .

$$\begin{aligned} \overline{\mathbf{u}}(\phi) &= \begin{bmatrix} -\frac{1}{2}\phi_2^{-1}\phi_1 \\ \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^{\mathrm{T}}\phi_2^{-1} - \frac{1}{2}\phi_2^{-1} \end{bmatrix} \\ g_{\phi}(\phi) &= \frac{1}{4}\phi_1^{\mathrm{T}}\phi_2^{-1}\phi_1 + \frac{1}{2}\log|-2\phi_2| \end{aligned}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_phi\_from\_parents

GaussianDistribution.compute\_phi\_from\_parents (u\_mu\_Lambda, mask=True)

Compute the natural parameter vector given parent moments.

$$\phi(oldsymbol{\mu},oldsymbol{\Lambda}) = egin{bmatrix} oldsymbol{\Lambda}oldsymbol{\mu} \ -rac{1}{2}oldsymbol{\Lambda} \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.plates\_from\_parent

GaussianDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.plates\_to\_parent

GaussianDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.random

GaussianDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

## 6.3.4 bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution

 $\begin{array}{c} \textbf{class} \ \texttt{bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution} \ (\textit{shape}, \\ & \textit{...} \\ & \textit{ndim\_mu}) \end{array}$ 

Log probability density function:

$$\log p(x|\mu,\alpha) = -\frac{1}{2}x^T \operatorname{diag}(\alpha)x + x^T \operatorname{diag}(\alpha)\mu - \frac{1}{2}\mu^T \operatorname{diag}(\alpha)\mu + \frac{1}{2}\sum_i \log \alpha_i - \frac{D}{2}\log(2\pi)$$

Parent has moments:

$$\begin{bmatrix} \alpha \circ \mu \\ \alpha \circ \mu \circ \mu \\ \alpha \\ \log(\alpha) \end{bmatrix}$$

\_\_init\_\_ (shape, ndim\_mu)

#### Methods

init(shape, ndim_mu)	
compute_cgf_from_parents(u_mu_alpha)	Compute the value of the cumulant generating function.
$compute_fixed_moments_and_f(x[, mask])$	Compute $u(x)$ and $f(x)$ for given $x$ .
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u,)</pre>	
compute_moments_and_cgf(phi[, mask])	
<pre>compute_phi_from_parents(u_mu_alpha[, mask])</pre>	
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the Gaussian distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.\_\_init\_\_

GaussianARDDistribution.\_\_init\_\_(shape, ndim\_mu)

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_cqf\_from\_parents

GaussianARDDistribution.compute\_cgf\_from\_parents(u\_mu\_alpha)
Compute the value of the cumulant generating function.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_fixed\_moments\_and\_f

 $\label{lem:compute_fixed_moments_and_f} \begin{subarray}{c} Gaussian ARD Distribution. {\bf compute\_fixed\_moments\_and\_f} \end{subarray} (x, mask=True) \\ Compute u(x) and f(x) for given x. \end{subarray}$ 

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### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_gradient

GaussianARDDistribution.compute\_gradient (g, u, phi)

Compute the standard gradient with respect to the natural parameters.

Gradient of the moments:

$$\begin{split} \mathrm{d}\overline{\mathbf{u}} &= \begin{bmatrix} \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1 - \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_1 \\ -\frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} - \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\phi_1\mathrm{d}\phi_1^\mathrm{T}\phi_2^{-1} \end{bmatrix} \\ &= \begin{bmatrix} 2(\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_2\overline{u}_1 + (\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_1 \\ u_2d\phi_2u_2 - 2u_1u_1^\mathrm{T}d\phi_2u_1u_1^\mathrm{T} + 2(u_2 - u_1u_1^\mathrm{T})d\phi_1u_1^\mathrm{T} \end{bmatrix} \end{split}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} (\overline{u}_2 - \overline{u}_1 \overline{u}_1^T) \tilde{\nabla}_1 + 2(u_2 - u_1 u_1^T) \tilde{\nabla}_2 u_1 \\ (u_2 - u_1 u_1^T) \tilde{\nabla}_1 u_1^T + u_1 \tilde{\nabla}_1^T (u_2 - u_1 u_1^T) + 2u_2 \tilde{\nabla}_2 u_2 - 2u_1 u_1^T \tilde{\nabla}_2 u_1 u_1^T \end{bmatrix}$$

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_logpdf

GaussianARDDistribution.compute\_logpdf (u, phi, g, f, ndims)

Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_mask\_to\_parent

GaussianARDDistribution.compute\_mask\_to\_parent (index, mask)
Maps the mask to the plates of a parent.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_message\_to\_parent

GaussianARDDistribution.compute\_message\_to\_parent(parent, index, u, u\_mu\_alpha)

...

$$m = \begin{bmatrix} x \\ \left[-\frac{1}{2}, \dots, -\frac{1}{2}\right] \\ -\frac{1}{2} \operatorname{diag}(xx^T) \\ \left[\frac{1}{2}, \dots, \frac{1}{2}\right] \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_moments\_and\_cgf

GaussianARDDistribution.compute\_moments\_and\_cqf (phi, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_phi\_from\_parents

GaussianARDDistribution.compute\_phi\_from\_parents(u\_mu\_alpha, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.plates\_from\_parent

GaussianARDDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.plates\_to\_parent

GaussianARDDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.random

 ${\tt Gaussian ARD Distribution.random} \, (*phi, plates = None)$ 

Draw a random sample from the Gaussian distribution.

### 6.3.5 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODistribution
 Class for the VMP formulas of Gaussian-Gamma-ISO variables.

Currently, supports only vector variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_mu_Lambda, u_a, u_b)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x, alpha[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_mu_Lambda, u_a, u_b)	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
<pre>random(*params[, plates])</pre>	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_cgf\_from\_parents

```
GaussianGammaISODistribution.compute_cgf_from_parents (u\_mu\_Lambda, u\_a, u\_b)
Compute \mathrm{E}_{q(p)}[g(p)]
```

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_fixed\_moments\_and\_f

```
GaussianGammaISODistribution.compute_fixed_moments_and_f (x, alpha, mask=True) Compute the moments and f(x) for a fixed value.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_gradient

GaussianGammaISODistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_logpdf

GaussianGammaISODistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_mask\_to\_parent

GaussianGammaISODistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_message\_to\_parent

```
GaussianGammaISODistribution.compute_message_to_parent (parent, index, u, u\_mu\_Lambda, u\_a, u\_b)
```

Compute the message to a parent node.

### $bayes py. inference. vmp. nodes. gaussian. Gaussian Gammal SOD is tribution. compute\_moments\_and\_cgf$

GaussianGammaISODistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $q(\phi)$ .

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_phi\_from\_parents

```
GaussianGammaISODistribution.compute_phi_from_parents(u_mu_Lambda, u_a, u_b, mask=True)

Compute the natural parameter vector given parent moments.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.plates\_from\_parent

```
GaussianGammaISODistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.plates\_to\_parent

```
GaussianGammaISODistribution.plates_to_parent (index, plates)
Resolves the plate mapping to a parent.
```

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.random

GaussianGammaISODistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

### 6.3.6 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution

\_\_init\_\_()

#### Methods

```
__init__()
compute_cgf_from_parents(*u_parents)
compute_fixed_moments_and_f(x[, mask])
                                                   Compute the standard gradient with respect to the natural parameters.
compute_gradient(g, u, phi)
compute_logpdf(u, phi, g, f, ndims)
                                                   Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)
                                                   Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, ...)
compute_moments_and_cgf(phi[, mask])
compute_phi_from_parents(*u_parents[, mask])
plates_from_parent(index, plates)
                                                   Resolve the plate mapping from a parent.
plates_to_parent(index, plates)
                                                   Resolves the plate mapping to a parent.
random(*params[, plates])
                                                   Draw a random sample from the distribution.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.\_\_init\_\_

GaussianGammaARDDistribution.\_\_init\_\_()

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_cgf\_from\_parents

 ${\tt GaussianGammaARDDistribution.} \textbf{compute\_cgf\_from\_parents} \ (*u\_parents)$ 

#### bayespy.inference.vmp.nodes.qaussian.GaussianGammaARDDistribution.compute\_fixed\_moments\_and\_f

 ${\tt GaussianGammaARDDistribution.} \textbf{compute\_fixed\_moments\_and\_f} \ (x, mask=True)$ 

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_gradient

GaussianGammaARDDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_logpdf

GaussianGammaARDDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_mask\_to\_parent

GaussianGammaARDDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_message\_to\_parent

GaussianGammaARDDistribution.compute\_message\_to\_parent(parent, index, u\_self, \*u\_parents)

### $bayes py. inference. vmp. nodes. gaussian. Gaussian Gamma ARD Distribution. compute\_moments\_and\_cgf$

GaussianGammaARDDistribution.compute\_moments\_and\_cgf(phi, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_phi\_from\_parents

GaussianGammaARDDistribution.compute\_phi\_from\_parents(\*u\_parents, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.plates\_from\_parent

 ${\tt GaussianGammaARDDistribution.plates\_from\_parent}~(index, plates)$ 

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.plates\_to\_parent

 ${\tt GaussianGammaARDDistribution.plates\_to\_parent}~(\textit{index}, \textit{plates})$ 

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.random

GaussianGammaARDDistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

### 6.3.7 bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution
 Class for the VMP formulas of Gaussian-Wishart variables.

Currently, supports only vector variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

Methods

```
compute_cgf_from_parents(u_mu_alpha, u_V, u_n)
                                                        Compute E_{q(p)}[g(p)]
                                                        Compute the moments and f(x) for a fixed value.
compute_fixed_moments_and_f(x, Lambda[, mask])
                                                        Compute the standard gradient with respect to the natural parameters.
compute_gradient(g, u, phi)
compute_logpdf(u, phi, g, f, ndims)
                                                        Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)
                                                        Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u, ...)
                                                       Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])
                                                        Compute the moments and g(\phi).
compute_phi_from_parents(u_mu_alpha, u_V, u_n)
                                                        Compute the natural parameter vector given parent moments.
plates_from_parent(index, plates)
                                                        Resolve the plate mapping from a parent.
plates_to_parent(index, plates)
                                                        Resolves the plate mapping to a parent.
random(*params[, plates])
                                                        Draw a random sample from the distribution.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_cqf\_from\_parents

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_fixed\_moments\_and\_f

```
GaussianWishartDistribution.compute_fixed_moments_and_f(x, tambda, tambda, tambda Compute the moments and tambda for a fixed value.
```

### $bayes py. inference. vmp. nodes. gaussian. Gaussian Wishart Distribution. compute\_gradient$

```
GaussianWishartDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_logpdf

```
GaussianWishartDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_mask\_to\_parent

```
GaussianWishartDistribution.compute_mask_to_parent (index, mask)

Maps the mask to the plates of a parent.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_message\_to\_parent

```
GaussianWishartDistribution.compute_message_to_parent (parent, index, u, u\_mu\_alpha, u\_V, u\_n)

Compute the message to a parent node.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_moments\_and\_cgf

```
GaussianWishartDistribution.compute_moments_and_cgf (phi, mask=True)

Compute the moments and g(\phi).
```

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_phi\_from\_parents

```
GaussianWishartDistribution.compute_phi_from_parents(u\_mu\_alpha, u\_V, u\_n, mask=True)
```

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.plates\_from\_parent

```
GaussianWishartDistribution.plates_from_parent (index, plates)
```

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.plates\_to\_parent

```
{\tt GaussianWishartDistribution.plates\_to\_parent} \ (index, plates)
```

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.random

```
GaussianWishartDistribution.random(*params, plates=None)
Draw a random sample from the distribution.
```

## $6.3.8\ bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Distribution$

```
 \textbf{class} \ \texttt{bayespy.inference.vmp.nodes.gaussian\_markov\_chain.} \textbf{GaussianMarkovChainDistribution} \ (N, \\ D)
```

Sub-classes implement distribution specific computations.

```
\_init\_(N, D)
```

#### **Methods**

```
__init__(N, D)
compute_cgf_from_parents(u_mu, u_Lambda, ...)
                                                     Compute CGF using the moments of the parents.
compute_fixed_moments_and_f(x[, mask])
                                                     Compute u(x) and f(x) for given x.
compute_gradient(g, u, phi)
                                                     Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)
                                                     Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)
compute_message_to_parent(parent, index, u, ...)
                                                     Compute a message to a parent.
compute_moments_and_cqf(phi[, mask])
                                                     Compute the moments and the cumulant-generating function.
compute_phi_from_parents(u_mu, u_Lambda, ...)
                                                     Compute the natural parameters using parents' moments.
plates_from_parent(index, plates)
                                                     Compute the plates using information of a parent node.
plates_to_parent(index, plates)
                                                     Computes the plates of this node with respect to a parent.
                                                     Draw a random sample from the distribution.
random(*params[, plates])
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.\_\_init\_\_

```
GaussianMarkovChainDistribution.__init_(N, D)
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_cgf\_from\_parketering

```
 \begin{tabular}{ll} {\tt Gaussian Markov Chain Distribution. \bf compute\_cgf\_from\_parents} (u\_mu, & u\_Lambda, \\ & u\_A, u\_v, *u\_inputs) \end{tabular}
```

Compute CGF using the moments of the parents.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_fixed\_mon

```
GaussianMarkovChainDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute u(x) and f(x) for given x.
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_gradient

```
GaussianMarkovChainDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_logpdf

```
GaussianMarkovChainDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

 $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Distribution. compute\_mask\_to\_particle for the comput$ 

```
GaussianMarkovChainDistribution.compute_mask_to_parent(index, mask)
```

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_message\_

```
GaussianMarkovChainDistribution.compute_message_to_parent(parent, index, u, u_mu, u_Lambda, u_A, u_v, *u_inputs)
```

Compute a message to a parent.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_moments

```
GaussianMarkovChainDistribution.compute_moments_and_cgf (phi, mask=True)
Compute the moments and the cumulant-generating function.
```

This basically performs the filtering and smoothing for the variable.

```
Parameters phi
```

Returns u

g

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_phi\_from\_page 1.00 to 1.00 t

```
GaussianMarkovChainDistribution.compute_phi_from_parents(u\_mu, u\_Lambda, u\_A, u\_v, *u\_inputs, mask=True)
```

Compute the natural parameters using parents' moments.

Parameters u\_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.

dims: tuple

Shape of the variable part of phi.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.plates\_from\_parent

```
GaussianMarkovChainDistribution.plates_from_parent(index, plates)
```

Compute the plates using information of a parent node.

If the plates of the parents are: mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D) N: ()

the resulting plates of this node are (...)

Parameters index: int

Index of the parent to use.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.plates\_to\_parent

```
GaussianMarkovChainDistribution.plates_to_parent (index, plates)
```

Computes the plates of this node with respect to a parent.

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

```
mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D)
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.random

```
GaussianMarkovChainDistribution.random(*params, plates=None)
Draw a random sample from the distribution.
```

## 6.3.9 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDi

class bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribut:

Sub-classes implement distribution specific computations.

```
\_init\_(N, D, K)
```

Methods

```
\_init\_(N, D, K)
compute_cgf_from_parents(u_mu, u_Lambda, ...)
                                                     Compute CGF using the moments of the parents.
compute_fixed_moments_and_f(x[, mask])
                                                     Compute u(x) and f(x) for given x.
compute_gradient(g, u, phi)
                                                     Compute the standard gradient with respect to the natural parameters.
                                                     Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f].
compute_logpdf(u, phi, g, f, ndims)
compute_mask_to_parent(index, mask)
compute_message_to_parent(parent, index, u, ...)
                                                     Compute a message to a parent.
compute_moments_and_cgf(phi[, mask])
                                                     Compute the moments and the cumulant-generating function.
compute_phi_from_parents(u_mu, u_Lambda, ...)
                                                     Compute the natural parameters using parents' moments.
plates_from_parent(index, plates)
                                                     Compute the plates using information of a parent node.
plates_to_parent(index, plates)
                                                     Computes the plates of this node with respect to a parent.
random(*params[, plates])
                                                     Draw a random sample from the distribution.
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.\_\_init\_\_

SwitchingGaussianMarkovChainDistribution.\_\_init\_(N, D, K)

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

SwitchingGaussianMarkovChainDistribution.compute\_cgf\_from\_parents ( $u\_mu$ ,  $u\_Lambda$ ,  $u\_B$ ,  $u\_Z$ ,  $u\_v$ )

Compute CGF using the moments of the parents.

## $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_chain. Switching Gaussian Markov Chain Distribution. Compute\_chain Distribution. Compute\_$

```
\label{lem:compute_fixed_moments_and_f} Switching \textit{GaussianMarkovChainDistribution.} \textbf{compute\_fixed\_moments\_and\_f} \ (x, \\ mask=True) \\ Compute \ u(x) \ and \ f(x) \ for \ given \ x.
```

## $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute a property of the computed proper$

```
SwitchingGaussianMarkovChainDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

## $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_chain. Switching Gaussian Markov Chain Distribution. Compute\_chain Distrib$

```
SwitchingGaussianMarkovChainDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

## $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute the property of the property o$

```
SwitchingGaussianMarkovChainDistribution.compute_mask_to_parent(index, mask)
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

```
SwitchingGaussianMarkovChainDistribution.compute_message_to_parent (parent, index, u, u_mu, u_lambda, u_B, u_Z, u_v)
```

Compute a message to a parent.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

```
\label{lem:compute_moments_and_cgf} Switching \textit{Gaussian} \textit{MarkovChainDistribution.} \textbf{compute_moments\_and\_cgf} \ (phi, \\ \textit{mask=True}) \\ Compute the moments and the cumulant-generating function.}
```

This basically performs the filtering and smoothing for the variable.

```
Parameters phi
Returns u
```

## $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_markov\_chain. Switching Gaussian Markov Chain Distribution. Switching Gaussian Marko$

```
SwitchingGaussianMarkovChainDistribution.compute_phi_from_parents (u\_mu, u\_Lambda, u\_B, u\_Z, u\_v, mask=True)
```

Compute the natural parameters using parents' moments.

```
Parameters u_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.
```

dims : tuple

Shape of the variable part of phi.

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.plates\_fro

```
SwitchingGaussianMarkovChainDistribution.plates_from_parent (index, plates)
Compute the plates using information of a parent node.

If the plates of the parents are: mu: (...) Lambda: (...) B: (...,D) S: (...,N-1) v: (...,N-1,D) N: ()
the resulting plates of this node are (...)

Parameters index: int
Index of the parent to use.
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.plates\_to\_

```
SwitchingGaussianMarkovChainDistribution.plates_to_parent (index, plates)

Computes the plates of this node with respect to a parent.
```

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

```
mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D)
```

### $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. random and the contraction of the contract$

```
SwitchingGaussianMarkovChainDistribution.random(*params, plates=None)

Draw a random sample from the distribution.
```

## 6.3.10 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDis

 ${\bf class} \ {\bf bayespy.inference.vmp.nodes.gaussian\_markov\_chain.Varying Gaussian Markov Chain Distribution Chain Chai$ 

Sub-classes implement distribution specific computations.

```
\_init\_(N, D)
```

#### **Methods**

Compute CGF using the moments of the parents.
Compute $u(x)$ and $f(x)$ for given $x$ .
Compute the standard gradient with respect to the natural parameters.
Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
Compute a message to a parent.
Compute the moments and the cumulant-generating function.
Compute the natural parameters using parents' moments.
Compute the plates using information of a parent node.
Computes the plates of this node with respect to a parent.
Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.\_\_init\_\_

```
VaryingGaussianMarkovChainDistribution.\_init_-(N, D)
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_cg

```
VaryingGaussianMarkovChainDistribution.compute_cgf_from_parents(u\_mu, u\_Lambda, u\_B, u\_S, u\_v)
```

Compute CGF using the moments of the parents.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_fix

```
\label{lem:compute_fixed_moments_and_f} \begin{tabular}{ll} $$Varying Gaussian Markov Chain Distribution. {\bf compute_fixed_moments_and_f}(x, \\ $mask=True)$ \\ Compute $u(x)$ and $f(x)$ for given $x$. \\ \end{tabular}
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_gr

```
VaryingGaussianMarkovChainDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_log

```
VaryingGaussianMarkovChainDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

### $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Varying Gaussian Markov Chain Distribution. compute\_markov\_chain. Varying Gaussian Markov Chain Distribution. Compute\_markov Ch$

VaryingGaussianMarkovChainDistribution.compute\_mask\_to\_parent(index, mask)

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_m

```
VaryingGaussianMarkovChainDistribution.compute_message_to_parent (parent, index, u, u_mu, u_Lambda, u_B, u_S, u_v)
```

Compute a message to a parent.

## $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Varying Gaussian Markov Chain Distribution. compute\_model and the property of the p$

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_ph

```
VaryingGaussianMarkovChainDistribution.compute_moments_and_cgf (phi, mask=True)

Compute the moments and the cumulant-generating function.
```

This basically performs the filtering and smoothing for the variable.

```
Parameters phi
Returns u
g
```

Parameters u\_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.

dims: tuple

Shape of the variable part of phi.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.plates\_from

VaryingGaussianMarkovChainDistribution.plates\_from\_parent (index, plates)

Compute the plates using information of a parent node.

If the plates of the parents are: mu: (...) Lambda: (...) B: (...,D) S: (...,N-1) v: (...,N-1,D) N: ()

the resulting plates of this node are (...)

Parameters index: int

Index of the parent to use.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.plates\_to\_pa

VaryingGaussianMarkovChainDistribution.plates\_to\_parent (index, plates)

Computes the plates of this node with respect to a parent.

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

```
mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D)
```

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.random

VaryingGaussianMarkovChainDistribution.random(\*params, plates=None)

Draw a random sample from the distribution.

### 6.3.11 bayespy.inference.vmp.nodes.gamma.GammaDistribution

class bayespy.inference.vmp.nodes.gamma.GammaDistribution

Class for the VMP formulas of gamma variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

compute_cgf_from_parents(*u_parents)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
	Continued on next page

### Table 6.53 – continued from previous page

compute_message_to_parent(parent, index,)	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(*u_parents[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_cgf\_from\_parents

```
\label{eq:compute_cgf_from_parents} \textbf{Compute} \ \textbf{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_fixed\_moments\_and\_f

GammaDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_gradient

GammaDistribution.compute\_gradient (g, u, phi)Compute the moments and  $q(\phi)$ .

$$d\overline{\mathbf{u}} = \begin{bmatrix} -\frac{d\phi_2}{phi_1} + \frac{\phi_2}{\phi_1^2} d\phi_1\\ \psi^{(1)}(\phi_2) d\phi_2 - \frac{1}{\phi_1} d\phi_1 \end{bmatrix}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} \nabla_1 \frac{\phi_2}{\phi_1^2} - \nabla_2 \frac{1}{\phi_1} \\ \nabla_2 \psi^{(1)}(\phi_2) - \nabla_1 \frac{1}{\phi_1} \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_logpdf

GammaDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_mask\_to\_parent

GammaDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_message\_to\_parent

GammaDistribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

Compute the message to a parent node.

### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_moments\_and\_cgf

GammaDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

$$\overline{\mathbf{u}}(\phi) = \begin{bmatrix} -\frac{\phi_2}{\phi_1} \\ \psi(\phi_2) - \log(-\phi_1) \end{bmatrix}$$
$$g_{\phi}(\phi) = TODO$$

### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_phi\_from\_parents

GammaDistribution.compute\_phi\_from\_parents (\*u\_parents, mask=True)

Compute the natural parameter vector given parent moments.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.plates\_from\_parent

GammaDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.gamma.GammaDistribution.plates\_to\_parent

GammaDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.random

GammaDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

## 6.3.12 bayespy.inference.vmp.nodes.wishart.WishartDistribution

class bayespy.inference.vmp.nodes.wishart.WishartDistribution
 Sub-classes implement distribution specific computations.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

```
compute_cgf_from_parents(*u_parents)
compute_fixed_moments_and_f(Lambda[, mask])
compute_gradient(g, u, phi)
compute_logpdf(u, phi, g, f, ndims)

Compute u(x) and f(x) for given x.
Compute the standard gradient with respect to the natural parameters.
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].

Continued on next page
```

### Table 6.54 – continued from previous page

compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	
compute_moments_and_cgf(phi[, mask])	
<pre>compute_phi_from_parents(*u_parents[, mask])</pre>	
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_cgf\_from\_parents

WishartDistribution.compute\_cgf\_from\_parents(\*u\_parents)

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_fixed\_moments\_and\_f

WishartDistribution.compute\_fixed\_moments\_and\_f (Lambda, mask=True)
Compute u(x) and f(x) for given x.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_gradient

WishartDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_logpdf

WishartDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_mask\_to\_parent

WishartDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_message\_to\_parent

WishartDistribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_moments\_and\_cqf

WishartDistribution.compute\_moments\_and\_cgf (phi, mask=True)

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_phi\_from\_parents

WishartDistribution.compute\_phi\_from\_parents(\*u\_parents, mask=True)

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.plates\_from\_parent

```
WishartDistribution.plates_from_parent (index, plates)
```

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.plates\_to\_parent

```
WishartDistribution.plates_to_parent (index, plates)
```

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.random

```
WishartDistribution.random(*params, plates=None)
```

Draw a random sample from the distribution.

### 6.3.13 bayespy.inference.vmp.nodes.beta.BetaDistribution

class bayespy.inference.vmp.nodes.beta.BetaDistribution

Class for the VMP formulas of beta variables.

Although the realizations are scalars (probability p), the moments is a two-dimensional vector: [log(p), log(1-p)].

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_alpha)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(p[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_alpha[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_cgf\_from\_parents

```
BetaDistribution.compute_cgf_from_parents (u_alpha) Compute \mathrm{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_fixed\_moments\_and\_f

BetaDistribution.compute\_fixed\_moments\_and\_f (p, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_gradient

BetaDistribution.compute\_gradient (g, u, phi)

Compute the moments and  $g(\phi)$ .

 $psi(phi_1) - psi(sum_d phi_{1,d})$ 

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \left[ (\psi^{(1)}(\phi_1) - \psi^{(1)}(\sum_d \phi_{1,d}) \nabla_1 \right]$$

### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_logpdf

BetaDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_mask\_to\_parent

BetaDistribution.compute\_mask\_to\_parent(index, mask)

Maps the mask to the plates of a parent.

### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_message\_to\_parent

BetaDistribution.compute\_message\_to\_parent (parent, index, u\_self, u\_alpha) Compute the message to a parent node.

### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_moments\_and\_cgf

BetaDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_phi\_from\_parents

BetaDistribution.compute\_phi\_from\_parents (*u\_alpha*, *mask=True*)

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.beta.BetaDistribution.plates\_from\_parent

BetaDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.beta.BetaDistribution.plates\_to\_parent

```
BetaDistribution.plates_to_parent (index, plates)
```

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.beta.BetaDistribution.random

```
{\tt BetaDistribution.random(*phi, plates=None)}
```

Draw a random sample from the distribution.

### 6.3.14 bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution

class bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution
 Class for the VMP formulas of Dirichlet variables.

```
__init__()
```

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_alpha)	Compute $E_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_alpha[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_cgf\_from\_parents

```
DirichletDistribution.compute_cgf_from_parents (u\_alpha) Compute \mathrm{E}_{q(p)}[g(p)]
```

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_fixed\_moments\_and\_f

```
DirichletDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_gradient

```
DirichletDistribution.compute_gradient (g, u, phi)
Compute the moments and g(\phi).
```

 $psi(phi_1) - psi(sum_d phi_{1,d})$ 

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \left[ (\psi^{(1)}(\phi_1) - \psi^{(1)}(\sum_d \phi_{1,d}) \nabla_1 \right]$$

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_logpdf

DirichletDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_mask\_to\_parent

DirichletDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_message\_to\_parent

DirichletDistribution.compute\_message\_to\_parent (parent, index, u\_self, u\_alpha)
Compute the message to a parent node.

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_moments\_and\_cgf

DirichletDistribution.compute\_moments\_and\_cgf (phi, mask=True) Compute the moments and  $g(\phi)$ .

$$\overline{\mathbf{u}}(\boldsymbol{\phi}) = \left[ \psi(\phi_1) - \psi(\sum_d \phi_{1,d}) \right]$$
$$g_{\boldsymbol{\phi}}(\boldsymbol{\phi}) = TODO$$

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_phi\_from\_parents

DirichletDistribution.compute\_phi\_from\_parents (u\_alpha, mask=True)

Compute the natural parameter vector given parent moments.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.plates\_from\_parent

 ${\tt DirichletDistribution.plates\_from\_parent} \ ({\it index}, {\it plates})$ 

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.plates\_to\_parent

DirichletDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.random

```
DirichletDistribution.random(*phi, plates=None)

Draw a random sample from the distribution.
```

### 6.3.15 bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution

class bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution
 Class for the VMP formulas of Bernoulli variables.

```
__init__()
```

#### **Methods**

init()	
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.\_\_init\_\_

```
BernoulliDistribution.__init__()
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_cgf\_from\_parents

```
BernoulliDistribution.compute_cgf_from_parents (u_p) Compute \mathrm{E}_{q(p)}[g(p)]
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_fixed\_moments\_and\_f

```
BernoulliDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_gradient

```
BernoulliDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_logpdf

```
BernoulliDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

### $bayes py. inference. vmp. nodes. bernoulli. Bernoulli Distribution. compute\_mask\_to\_parent$

```
BernoulliDistribution.compute_mask_to_parent (index, mask)

Maps the mask to the plates of a parent.
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_message\_to\_parent

```
BernoulliDistribution.compute_message_to_parent (parent, index, u\_self, u\_p)
Compute the message to a parent node.
```

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_moments\_and\_cgf

```
BernoulliDistribution.compute_moments_and_cgf (phi, mask=True)
Compute the moments and g(\phi).
```

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_phi\_from\_parents

```
BernoulliDistribution.compute_phi_from_parents (u_p, mask=True)
Compute the natural parameter vector given parent moments.
```

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.plates\_from\_parent

```
BernoulliDistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.plates\_to\_parent

```
BernoulliDistribution.plates_to_parent (index, plates)
Resolves the plate mapping to a parent.
```

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.random

```
BernoulliDistribution.random(*phi, plates=None)
Draw a random sample from the distribution.
```

### 6.3.16 bayespy.inference.vmp.nodes.binomial.BinomialDistribution

```
class bayespy.inference.vmp.nodes.binomial.BinomialDistribution (N) Class for the VMP formulas of binomial variables.
```

```
_{-}init_{-}(N)
```

#### **Methods**

init(N)	
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p[, mask])</pre>	Compute the natural parameter vector given parent moments.
plates_from_parent(index, plates)	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.\_\_init\_\_

```
BinomialDistribution.__init__(N)
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_cgf\_from\_parents

```
BinomialDistribution.compute_cgf_from_parents (u_p) Compute \mathbf{E}_{q(p)}[g(p)]
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_fixed\_moments\_and\_f

```
BinomialDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_gradient

```
BinomialDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_logpdf

```
BinomialDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_mask\_to\_parent

BinomialDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_message\_to\_parent

BinomialDistribution.compute\_message\_to\_parent (parent, index,  $u\_self$ ,  $u\_p$ )

Compute the message to a parent node.

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_moments\_and\_cgf

BinomialDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_phi\_from\_parents

BinomialDistribution.compute\_phi\_from\_parents (u\_p, mask=True)

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.plates\_from\_parent

BinomialDistribution.plates\_from\_parent (index, plates)
Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.plates\_to\_parent

BinomialDistribution.plates\_to\_parent (index, plates)
Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.random

BinomialDistribution.random(\*phi, plates=None)
Draw a random sample from the distribution.

## 6.3.17 bayespy.inference.vmp.nodes.categorical.CategoricalDistribution

\_\_init\_\_ (categories)

Create VMP formula node for a categorical variable

categories is the total number of categories.

#### Methods

init(categories)	Create VMP formula node for a categorical variable
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
<pre>compute_gradient(g, u, phi)</pre>	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u, u_p)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.\_\_init\_\_

```
CategoricalDistribution.__init__(categories)
Create VMP formula node for a categorical variable
categories is the total number of categories.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_cgf\_from\_parents

```
CategoricalDistribution.compute_cgf_from_parents (u_p) Compute \mathrm{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_fixed\_moments\_and\_f

```
CategoricalDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_gradient

```
CategoricalDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_logpdf

```
CategoricalDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_mask\_to\_parent

```
CategoricalDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_message\_to\_parent

CategoricalDistribution.compute\_message\_to\_parent (parent, index, u, u-p)

Compute the message to a parent node.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_moments\_and\_cgf

CategoricalDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_phi\_from\_parents

CategoricalDistribution.compute\_phi\_from\_parents (u.p, mask=True)

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.plates\_from\_parent

CategoricalDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.plates\_to\_parent

CategoricalDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.random

CategoricalDistribution.random(\*phi, plates=None)
Draw a random sample from the distribution.

## 6.3.18 bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution

class bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution

Class for the VMP formulas of categorical Markov chain variables.

\_\_init\_\_ (categories, states)

Create VMP formula node for a categorical variable

categories is the total number of categories. states is the length of the chain.

Continued on next page

### Table 6.60 – continued from previous page

#### **Methods**

init(categories, states)	Create VMP formula node for a categorical variable
compute_cgf_from_parents(u_p0, u_P)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u,)</pre>	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p0, u_P[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.\_\_init\_\_

```
CategoricalMarkovChainDistribution.__init__(categories, states)
Create VMP formula node for a categorical variable

categories is the total number of categories. states is the length of the chain.
```

## $bayes py. inference. vmp. nodes. categorical\_markov\_chain. Categorical Markov Chain Distribution. compute\_cgf\_free transference and the computer of the comp$

```
 \label{lem:compute_cgf_from_parents}  \text{CategoricalMarkovChainDistribution.}  \textbf{compute\_cgf\_from\_parents} \; (\textit{u\_p0}, \textit{u\_P}) \\  \text{Compute} \; \mathrm{E}_{q(p)}[g(p)]
```

### $bayes py. inference. vmp. nodes. categorical\_markov\_chain. Categorical Markov Chain Distribution. compute\_fixed\_markov\_chain. Categorical Markov Chain Distribution. C$

```
CategoricalMarkovChainDistribution.compute_fixed_moments_and_f(x, mask=True)

Compute the moments and f(x) for a fixed value.
```

## $bayes py. in ference. vmp. nodes. categorical\_markov\_chain. Categorical Markov Chain Distribution. compute\_gradient and the property of the$

```
CategoricalMarkovChainDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

## $bayes py. inference. vmp. nodes. categorical\_markov\_chain. Categorical Markov Chain Distribution. compute\_log pdote the computes and the com$

```
CategoricalMarkovChainDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

## $bayes py. inference. vmp. nodes. categorical\_markov\_chain. Categorical Markov Chain Distribution. compute\_maskappa and the compute\_maskappa and the compute\_maskappa and the compute\_maskappa. The compute\_maskappa and the compute\_maskappa and the compute\_maskappa and the compute\_maskappa. The compute\_maskappa and the compute\_maskappa and the compute\_maskappa and the compute\_maskappa. The compute\_maskappa and the compute\_maskappa and the compute\_maskappa and the compute\_maskappa and the compute\_maskappa. The compute\_maskappa and the compute\_$

```
CategoricalMarkovChainDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messa

CategoricalMarkovChainDistribution.compute\_message\_to\_parent (parent, index, u,  $u_p0$ ,  $u_pP$ )

Compute the message to a parent node.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_mome

CategoricalMarkovChainDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_phi\_free

```
CategoricalMarkovChainDistribution.compute_phi_from_parents(u_p0, u_P, mask=True)
```

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.plates\_from\_par

```
CategoricalMarkovChainDistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.plates\_to\_paren

```
CategoricalMarkovChainDistribution.plates_to_parent(index, plates)
```

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.random

```
CategoricalMarkovChainDistribution.random(*phi, plates=None)

Draw a random sample from the distribution.
```

### 6.3.19 bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution

```
__init__(trials)
```

Create VMP formula node for a multinomial variable

trials is the total number of trials.

### Methods

init(trials)	Create VMP formula node for a multinomial variable
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u, u_p)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi)	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.\_\_init\_\_

MultinomialDistribution.\_\_init\_\_(trials)

Create VMP formula node for a multinomial variable

trials is the total number of trials.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_cqf\_from\_parents

```
MultinomialDistribution.compute_cgf_from_parents (u\_p) Compute \mathrm{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_fixed\_moments\_and\_f

```
MultinomialDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_gradient

```
MultinomialDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_logpdf

```
MultinomialDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_mask\_to\_parent

```
MultinomialDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_message\_to\_parent

```
MultinomialDistribution.compute_message_to_parent (parent, index, u, u_p) Compute the message to a parent node.
```

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_moments\_and\_cgf

MultinomialDistribution.compute\_moments\_and\_cgf (phi, mask=True) Compute the moments and  $g(\phi)$ .

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_phi\_from\_parents

MultinomialDistribution.compute\_phi\_from\_parents (u.p, mask=True)

Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.plates\_from\_parent

MultinomialDistribution.plates\_from\_parent(index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.plates\_to\_parent

MultinomialDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.random

MultinomialDistribution.random(\*phi)

Draw a random sample from the distribution.

### 6.3.20 bayespy.inference.vmp.nodes.poisson.PoissonDistribution

class bayespy.inference.vmp.nodes.poisson.PoissonDistribution
 Class for the VMP formulas of Poisson variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

### Methods

compute_cgf_from_parents(u_lambda)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_lambda[, mask])</pre>	Compute the natural parameter vector given parent moments.
	Continued on next page

### Table 6.62 - continued from previous page

<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi)	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_cgf\_from\_parents

```
PoissonDistribution.compute_cgf_from_parents (u\_lambda) Compute \mathbf{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_fixed\_moments\_and\_f

```
PoissonDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_gradient

```
PoissonDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_logpdf

```
PoissonDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_mask\_to\_parent

```
PoissonDistribution.compute_mask_to_parent (index, mask)

Maps the mask to the plates of a parent.
```

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_message\_to\_parent

```
PoissonDistribution.compute_message_to_parent (parent, index, u, u_lambda) Compute the message to a parent node.
```

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_moments\_and\_cgf

```
PoissonDistribution.compute_moments_and_cgf (phi, mask=True)
Compute the moments and g(\phi).
```

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_phi\_from\_parents

```
PoissonDistribution.compute_phi_from_parents (u_lambda, mask=True)

Compute the natural parameter vector given parent moments.
```

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.plates\_from\_parent

PoissonDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.plates\_to\_parent

PoissonDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.random

PoissonDistribution.random(\*phi)

Draw a random sample from the distribution.

## 6.4 Utility functions

linalg random	Random variable generators.
optimize misc	-

### 6.4.1 bayespy.utils.linalg

General numerical functions and methods.

### **Functions**

block_banded_solve(A, B, y)	Invert symmetric, banded, positive-definite matrix.
chol(C)	
$chol_inv(U)$	
${ t chol_logdet}({ t U})$	
$chol_solve(U, b[, out, matrix])$	
dot(*arrays)	Compute matrix-matrix product.
inner(*args[, ndim])	Compute inner product.
inv(A[, ndim])	General array inversion.
logdet_chol(U)	
$logdet_cov(C)$	
logdet_tri( <b>R</b> )	Logarithm of the absolute value of the determinant of a triangular matrix.
$m_dot(A, b)$	
mmdot(A, B[, ndim])	Compute matrix-matrix product.
mvdot(A, b[, ndim])	Compute matrix-vector product.
	Continued on next page

### Table 6.64 – continued from previous page

outer(A, B[, ndim])	Computes outer product over the last axes of A and B.
<pre>solve_triangular(U, B, **kwargs)</pre>	
tracedot(A,B)	Computes trace(A*B).
transpose(X[, ndim])	Transpose the matrix.

### bayespy.utils.linalg.block\_banded\_solve

 $\verb|bayespy.utils.linalg.block_banded_solve| (A, B, y)$ 

Invert symmetric, banded, positive-definite matrix.

A contains the diagonal blocks.

B contains the superdiagonal blocks (their transposes are the subdiagonal blocks).

Shapes: A: (..., N, D, D) B: (..., N-1, D, D) y: (..., N, D)

The algorithm is basically LU decomposition.

Computes only the diagonal and super-diagonal blocks of the inverse. The true inverse is dense, in general.

Assume each block has the same size.

Return: \* inverse blocks \* solution to the system \* log-determinant

### bayespy.utils.linalg.chol

bayespy.utils.linalg.chol(C)

#### bayespy.utils.linalg.chol\_inv

bayespy.utils.linalg.chol\_inv(U)

### bayespy.utils.linalg.chol\_logdet

bayespy.utils.linalg.chol\_logdet(U)

### bayespy.utils.linalg.chol\_solve

bayespy.utils.linalg.chol\_solve(*U*, *b*, out=None, matrix=False)

### bayespy.utils.linalg.dot

bayespy.utils.linalg.dot(\*arrays)

Compute matrix-matrix product.

You can give multiple arrays, the dot product is computed from left to right: A1\*A2\*A3\*...\*AN. The dot product is computed over the last two axes of each arrays. All other axes must be broadcastable.

### bayespy.utils.linalg.inner

```
bayespy.utils.linalg.inner(*args, ndim=1)
    Compute inner product.
```

The number of arrays is arbitrary. The number of dimensions is arbitrary.

### bayespy.utils.linalg.inv

```
bayespy.utils.linalg.inv(A, ndim=1)
General array inversion.
```

Supports broadcasting and inversion of multidimensional arrays. For instance, an array with shape (4,3,2,3,2) could mean that there are four (3\*2) x (3\*2) matrices to be inverted. This can be done by inv(A, ndim=2). For inverting scalars, ndim=0. For inverting matrices, ndim=1.

### bayespy.utils.linalg.logdet\_chol

```
bayespy.utils.linalg.logdet_chol(U)
```

### bayespy.utils.linalg.logdet\_cov

```
bayespy.utils.linalg.logdet_cov(C)
```

### bayespy.utils.linalg.logdet\_tri

```
bayespy.utils.linalg.logdet_tri(R)
```

Logarithm of the absolute value of the determinant of a triangular matrix.

#### bayespy.utils.linalg.m\_dot

```
bayespy.utils.linalg.m_dot(A, b)
```

#### bayespy.utils.linalg.mmdot

```
bayespy.utils.linalg.mmdot(A, B, ndim=1)
Compute matrix-matrix product.
```

Applies broadcasting.

### bayespy.utils.linalg.mvdot

```
bayespy.utils.linalg.mvdot(A, b, ndim=1)
```

Compute matrix-vector product.

Applies broadcasting.

## bayespy.utils.linalg.outer

```
bayespy.utils.linalg.outer(A, B, ndim=1)
```

Computes outer product over the last axes of A and B.

The other axes are broadcasted. Thus, if A has shape (..., N) and B has shape (..., M), then the result has shape (..., N, M).

Using the argument *ndim* it is possible to change that how many axes trailing axes are used for the outer product. For instance, if ndim=3, A and B have shapes (...,N1,N2,N3) and (...,M1,M2,M3), the result has shape (...,N1,M1,N2,M2,N3,M3).

## bayespy.utils.linalg.solve\_triangular

```
bayespy.utils.linalg.solve_triangular(U, B, **kwargs)
```

# bayespy.utils.linalg.tracedot

```
bayespy.utils.linalg.tracedot (A, B)
Computes trace(A*B).
```

## bayespy.utils.linalg.transpose

```
bayespy.utils.linalg.transpose (X, ndim=1)
    Transpose the matrix.
```

# 6.4.2 bayespy.utils.random

General functions random sampling and distributions.

#### **Functions**

lpha_beta_recursion(logp0, logP)	Compute alpha-beta recursion for Markov chain	
pernoulli(p[, size])	Draw random samples from the Bernoulli distribution.	
categorical(p[, size])	Draw random samples from a categorical distribution.	
correlation(D)	Draw a random correlation matrix.	
covariance(D[, size])	Draw a random covariance matrix.	
dirichlet(alpha[, size])	Draw random samples from the Dirichlet distribution.	
gamma_entropy(a, log_b, gammaln_a, psi_a,)	Entropy of $\mathcal{G}(a,b)$ .	
gamma_logpdf(bx, logx, a_logx, a_logb, gammaln_a)	Log-density of $\mathcal{G}(x a,b)$ .	
gaussian_entropy(logdet_V, D)	Compute the entropy of a Gaussian distribution.	
gaussian_gamma_to_t(mu, Cov, a, b[, ndim])	Integrates gamma distribution to obtain parameters of t distribution	
gaussian_logpdf(yVy, yVmu, muVmu, logdet_V, D	) Log-density of a Gaussian distribution.	
ntervals(N, length[, amount, gap])	Return random non-overlapping parts of a sequence.	
nvwishart_rand(nu, V)		
.ogodds_to_probability(x)	Solves p from $log(p/(1-p))$	
nask(*shape[, p])	Return a boolean array of the given shape.	
$\operatorname{orth}(D)$	Draw random orthogonal matrix.	
phere([N])	Draw random points uniformly on a unit sphere.	
vd(s)	Draw a random matrix given its singular values.	

Continued on next page

## Table 6.65 – continued from previous page

```
t_logpdf(z2, logdet_cov, nu, D)
wishart_rand(nu, V)
```

Draw a random sample from the Wishart distribution.

## bayespy.utils.random.alpha\_beta\_recursion

```
bayespy.utils.random.alpha_beta_recursion(logp0, logP)
```

Compute alpha-beta recursion for Markov chain

Initial state log-probabilities are in p0 and state transition log-probabilities are in P. The probabilities do not need to be scaled to sum to one, but they are interpreted as below:

```
log p0 = log P(z_0) + log P(y_0|z_0) log P[...,n,:,:] = log P(z_{n+1}|z_n) + log P(y_{n+1}|z_n+1)
```

## bayespy.utils.random.bernoulli

```
bayespy.utils.random.bernoulli(p, size=None)
```

Draw random samples from the Bernoulli distribution.

## bayespy.utils.random.categorical

```
bayespy.utils.random.categorical(p, size=None)
```

Draw random samples from a categorical distribution.

## bayespy.utils.random.correlation

```
bayespy.utils.random.correlation(D)
```

Draw a random correlation matrix.

## bayespy.utils.random.covariance

```
bayespy.utils.random.covariance(D, size=())
```

Draw a random covariance matrix.

Draws from inverse-Wishart distribution. The distribution of each element is independent of the dimensionality of the matrix.

```
C ~ Inv-W(I, D)
```

#### Parameters D: int

Dimensionality of the covariance matrix.

### bayespy.utils.random.dirichlet

```
bayespy.utils.random.dirichlet(alpha, size=None)
```

Draw random samples from the Dirichlet distribution.

## bayespy.utils.random.gamma\_entropy

```
bayespy.utils.random.gamma_entropy (a, log\_b, gammaln\_a, psi\_a, a\_psi\_a) Entropy of \mathcal{G}(a,b).

If you want to get the gradient, just let each parameter be a gradient of that term.

Parameters \mathbf{a}: ndarray
\mathbf{a}
\mathbf{log\_b}: ndarray
\mathbf{log}(b)
\mathbf{gammaln\_a}: ndarray
\mathbf{log}\Gamma(a)
\mathbf{psi\_a}: ndarray
\psi(a)
\mathbf{a\_psi\_a}: ndarray
a\psi(a)
```

## bayespy.utils.random.gamma\_logpdf

```
bayespy.utils.random.gamma_logpdf(bx, logx, a\_logx, a\_logb, gammaln\_a) Log-density of \mathcal{G}(x|a,b).
```

If you want to get the gradient, just let each parameter be a gradient of that term.

```
Parameters bx : ndarray bx

logx : ndarray log(x)

a_logx : ndarray a log(x)

a_logb : ndarray a log(b)

gammaln_a : ndarray log \Gamma(a)
```

## bayespy.utils.random.gaussian\_entropy

```
bayespy.utils.random.gaussian_entropy(logdet_V, D)
```

Compute the entropy of a Gaussian distribution.

If you want to get the gradient, just let each parameter be a gradient of that term.

Parameters logdet\_V : ndarray or double

The log-determinant of the precision matrix.

D: int

The dimensionality of the distribution.

## bayespy.utils.random.gaussian\_gamma\_to\_t

```
bayespy.utils.random.gaussian_gamma_to_t (mu, Cov, a, b, ndim=1)
Integrates gamma distribution to obtain parameters of t distribution
```

## bayespy.utils.random.gaussian\_logpdf

```
bayespy.utils.random.gaussian_logpdf (yVy, yVmu, muVmu, logdet_{-}V, D)
   Log-density of a Gaussian distribution.
   \mathcal{G}(\mathbf{y}|\boldsymbol{\mu},\mathbf{V}^{-1})
Parameters \mathbf{y}V\mathbf{y}: ndarray or double
   \mathbf{y}^{T}\mathbf{V}\mathbf{y}
\mathbf{y}V\mathbf{m}\mathbf{u}: ndarray or double
   \mathbf{y}^{T}\mathbf{V}\boldsymbol{\mu}
\mathbf{m}\mathbf{u}V\mathbf{m}\mathbf{u}: ndarray or double
   \boldsymbol{\mu}^{T}\mathbf{V}\boldsymbol{\mu}
\mathbf{logdet}_{-}\mathbf{V}: ndarray or double
   \mathbf{Log}_{-}
\mathbf{d}
\mathbf{v}
```

## bayespy.utils.random.intervals

```
bayespy.utils.random.intervals(N, length, amount=1, gap=0)
```

Dimensionality of the distribution.

Return random non-overlapping parts of a sequence.

```
For instance, N=16, length=2 and amount=4: [0, |\mathbf{1}, \mathbf{2}|, 3, 4, 5, |\mathbf{6}, \mathbf{7}|, 8, 9, |\mathbf{10}, \mathbf{11}|, |\mathbf{12}, \mathbf{13}|, 14, 15] that is, [1,2,6,7,10,11,12,13]
```

However, the function returns only the indices of the beginning of the sequences, that is, in the example: [1,6,10,12]

# bayespy.utils.random.invwishart\_rand

```
bayespy.utils.random.invwishart_rand(nu, V)
```

#### bayespy.utils.random.logodds\_to\_probability

## bayespy.utils.random.mask

```
bayespy.utils.random.mask (*shape, p=0.5)
Return a boolean array of the given shape.
```

#### Parameters d0, d1, ..., dn: int

Shape of the output.

**p**: value in range [0,1]

A probability that the elements are *True*.

## bayespy.utils.random.orth

```
bayespy.utils.random.orth (D) Draw random orthogonal matrix.
```

## bayespy.utils.random.sphere

```
bayespy.utils.random.sphere (N=1)
```

Draw random points uniformly on a unit sphere.

Returns (latitude,longitude) in degrees.

## bayespy.utils.random.svd

```
bayespy.utils.random.svd(s)
```

Draw a random matrix given its singular values.

## bayespy.utils.random.t\_logpdf

```
bayespy.utils.random.t_logpdf(z2, logdet_cov, nu, D)
```

# $bayes py. utils. random. wishart\_rand$

```
bayespy.utils.random.wishart_rand (nu, V)
Draw a random sample from the Wishart distribution.
```

Parameters nu: int

# 6.4.3 bayespy.utils.optimize

#### **Functions**

<pre>check_gradient(f, x0[, verbose])</pre>	Simple wrapper for SciPy's gradient checker.
minimize(f, x0[, maxiter, verbose])	Simple wrapper for SciPy's optimize.

## bayespy.utils.optimize.check\_gradient

```
bayespy.utils.optimize.check_gradient (f, x0, verbose=True)
Simple wrapper for SciPy's gradient checker.

The given function must return a tuple: (value, gradient).

Returns relative
```

# bayespy.utils.optimize.minimize

```
bayespy.utils.optimize.minimize (f, x0, maxiter=None, verbose=False)
Simple wrapper for SciPy's optimize.
The given function must return a tuple: (value, gradient).
```

# 6.4.4 bayespy.utils.misc

General numerical functions and methods.

## **Functions**

T(X)	Transpose the matrix.
$add_axes(X[, num, axis])$	
add_leading_axes(x, n)	
add_trailing_axes(x, n)	
array_to_scalar(x)	
$atleast_nd(X, d)$	
<pre>axes_to_collapse(shape_x, shape_to)</pre>	
$block\_banded(D, B)$	Construct a symmetric block-banded matrix.
broadcasted_shape(*shapes)	Computes the resulting broadcasted shape for a given set of shapes.
broadcasted_shape_from_arrays(*arrays)	Computes the resulting broadcasted shape for a given set of arrays.
broadcasting_multiplier(plates, *args)	Compute the plate multiplier for given shapes.
ceildiv(a, b)	Compute a divided by b and rounded up.
$check\_gradient(x0, f, df, eps)$	
chol(C)	
${ t chol_{-inv}(U)}$	
${\tt chol\_logdet}({\tt U})$	
$chol_solve(U, b)$	
${ m cholesky}({ m K})$	
composite_function(function_list)	Construct a function composition from a list of functions.
diag(X[, ndim])	Create a diagonal array given the diagonal elements.
diagonal(A)	
dist_haversine(c1, c2[, radius])	
first(L)	
<pre>gaussian_logpdf(y_invcov_y, y_invcov_mu,)</pre>	
$get_diag(X[, ndim])$	Get the diagonal of an array.
grid(x1, x2)	Returns meshgrid as a (M*N,2)-shape array.
identity(*shape)	
is_callable(f)	
is_numeric(a)	
	Continued on next page

Table 6.67 – continued from previous page

```
is_shape_subset(sub_shape, full_shape)
is_string(s)
isinteger(x)
kalman_filter(y, U, A, V, mu0, Cov0[, out])
                                                    Perform Kalman filtering to obtain filtered mean and covariance.
logdet_chol(U)
logsumexp(X[, axis, keepdims])
                                                    Compute log(sum(exp(X))) in a numerically stable way
m_chol(C)
m_{chol_{inv}(U)}
m_chol_logdet(U)
m_{chol_solve}(U, B[, out])
m_digamma(a, d)
m_{dot}(A, b)
m_outer(A, B)
m_solve_triangular(U, B, **kwargs)
make_equal_length(*shapes)
                                                    Make tuples equal length.
                                                    Add trailing unit axes so that arrays have equal ndim
make_equal_ndim(*arrays)
mean(X[, axis, keepdims])
                                                    Compute the mean, ignoring NaNs.
moveaxis(A, axis_from, axis_to)
                                                    Move the axis axis_from to position axis_to.
multiply_shapes(*shapes)
                                                    Compute element-wise product of lists/tuples.
nans([size])
nested_iterator(max_inds)
remove_whitespace(s)
repeat_to_shape(A, s)
rmse(y1, y2[, axis])
rts_smoother(mu, Cov, A, V[, removethis])
                                                    Perform Rauch-Tung-Striebel smoothing to obtain the posterior.
safe_indices(inds, shape)
                                                    Makes sure that indices are valid for given shape.
                                                    Remove leading axes that have unit length.
squeeze(X)
squeeze_to_dim(X, dim)
sum_multiply(*args[, axis, sumaxis, keepdims])
sum_multiply_to_plates(*arrays[, to_plates, ...])
                                                    Compute the product of the arguments and sum to the target shape.
sum_product(*args[, axes_to_keep, ...])
sum_to_dim(A, dim)
                                                    Sum leading axes of A such that A has dim dimensions.
sum_to_shape(X, s)
                                                    Sum axes of the array such that the resulting shape is as given.
                                                    Make X symmetric.
symm(X)
tempfile([prefix, suffix])
trues(shape)
unique(1)
                                                    Remove duplicate items from a list while preserving order.
vb_optimize(x0, set_values, lowerbound[, ...])
vb_optimize_nodes(*nodes)
write_to_hdf5(group, data, name)
                                                    Writes the given array into the HDF5 file.
zipper_merge(*lists)
                                                    Combines lists by alternating elements from them.
```

# bayespy.utils.misc.T

```
bayespy.utils.misc.\mathbf{T}(X)
Transpose the matrix.
```

## bayespy.utils.misc.add\_axes

```
bayespy.utils.misc.add_axes (X, num=1, axis=0)
```

## bayespy.utils.misc.add\_leading\_axes

bayespy.utils.misc.add\_leading\_axes (x, n)

## bayespy.utils.misc.add\_trailing\_axes

bayespy.utils.misc.add\_trailing\_axes(x, n)

## bayespy.utils.misc.array\_to\_scalar

bayespy.utils.misc.array\_to\_scalar(x)

## bayespy.utils.misc.atleast\_nd

bayespy.utils.misc.atleast\_nd(X, d)

## bayespy.utils.misc.axes\_to\_collapse

bayespy.utils.misc.axes\_to\_collapse(shape\_x, shape\_to)

#### bayespy.utils.misc.block\_banded

bayespy.utils.misc.block\_banded(D, B)

Construct a symmetric block-banded matrix.

D contains square diagonal blocks. B contains super-diagonal blocks.

The resulting matrix is:

## bayespy.utils.misc.broadcasted\_shape

bayespy.utils.misc.broadcasted\_shape(\*shapes)

Computes the resulting broadcasted shape for a given set of shapes.

Uses the broadcasting rules of NumPy. Raises an exception if the shapes do not broadcast.

## bayespy.utils.misc.broadcasted\_shape\_from\_arrays

bayespy.utils.misc.broadcasted\_shape\_from\_arrays(\*arrays)

Computes the resulting broadcasted shape for a given set of arrays.

Raises an exception if the shapes do not broadcast.

## bayespy.utils.misc.broadcasting\_multiplier

bayespy.utils.misc.broadcasting\_multiplier(plates, \*args)

Compute the plate multiplier for given shapes.

The first shape is compared to all other shapes (using NumPy broadcasting rules). All the elements which are non-unit in the first shape but 1 in all other shapes are multiplied together.

This method is used, for instance, for computing a correction factor for messages to parents: If this node has non-unit plates that are unit plates in the parent, those plates are summed. However, if the message has unit axis for that plate, it should be first broadcasted to the plates of this node and then summed to the plates of the parent. In order to avoid this broadcasting and summing, it is more efficient to just multiply by the correct factor. This method computes that factor. The first argument is the full plate shape of this node (with respect to the parent). The other arguments are the shape of the message array and the plates of the parent (with respect to this node).

## bayespy.utils.misc.ceildiv

```
bayespy.utils.misc.ceildiv (a, b)
Compute a divided by b and rounded up.
```

## bayespy.utils.misc.check\_gradient

```
bayespy.utils.misc.check_gradient (x0, f, df, eps)
```

# bayespy.utils.misc.chol

```
bayespy.utils.misc.chol(C)
```

## bayespy.utils.misc.chol\_inv

```
bayespy.utils.misc.chol_inv(U)
```

## bayespy.utils.misc.chol\_logdet

```
bayespy.utils.misc.chol_logdet(U)
```

#### bayespy.utils.misc.chol\_solve

```
bayespy.utils.misc.chol_solve(U, b)
```

#### bayespy.utils.misc.cholesky

```
bayespy.utils.misc.cholesky(K)
```

## bayespy.utils.misc.composite\_function

```
bayespy.utils.misc.composite_function(function_list)
```

Construct a function composition from a list of functions.

Given a list of functions [f,g,h], constructs a function  $h \circ g \circ f$ . That is, returns a function z, for which z(x) = h(g(f(x))).

## bayespy.utils.misc.diag

```
bayespy.utils.misc.diag(X, ndim=1)
```

Create a diagonal array given the diagonal elements.

The diagonal array can be multi-dimensional. By default, the last axis is transformed to two axes (diagonal matrix) but this can be changed using ndim keyword. For instance, an array with shape (K,L,M,N) can be transformed to a set of diagonal 4-D tensors with shape (K,L,M,N,M,N) by giving ndim=2. If ndim=3, the result has shape (K,L,M,N,L,M,N), and so on.

Diagonality means that for the resulting array Y holds:  $Y[...,i_1,i_2,...,i_ndim,j_1,j_2,...,j_ndim]$  is zero if  $i_n!=j_n$  for any n.

## bayespy.utils.misc.diagonal

```
bayespy.utils.misc.diagonal(A)
```

## bayespy.utils.misc.dist\_haversine

```
bayespy.utils.misc.dist_haversine(c1, c2, radius=6372795)
```

## bayespy.utils.misc.first

```
bayespy.utils.misc.first(L)
```

## bayespy.utils.misc.gaussian\_logpdf

bayespy.utils.misc.gaussian\_logpdf(y\_invcov\_y, y\_invcov\_mu, mu\_invcov\_mu, logdetcov, D)

## bayespy.utils.misc.get\_diag

```
bayespy.utils.misc.get_diag(X, ndim=1)
```

Get the diagonal of an array.

If ndim>1, take the diagonal of the last 2\*ndim axes.

## bayespy.utils.misc.grid

```
bayespy.utils.misc.grid(x1, x2)
```

Returns meshgrid as a (M\*N,2)-shape array.

## bayespy.utils.misc.identity

bayespy.utils.misc.identity(\*shape)

## bayespy.utils.misc.is\_callable

bayespy.utils.misc.is\_callable (f)

## bayespy.utils.misc.is\_numeric

bayespy.utils.misc.is\_numeric(a)

## bayespy.utils.misc.is\_shape\_subset

bayespy.utils.misc.is\_shape\_subset (sub\_shape, full\_shape)

## bayespy.utils.misc.is\_string

bayespy.utils.misc.is\_string(s)

## bayespy.utils.misc.isinteger

bayespy.utils.misc.isinteger(x)

## bayespy.utils.misc.kalman\_filter

bayespy.utils.misc.kalman\_filter(y, U, A, V, mu0, Cov0, out=None)

Perform Kalman filtering to obtain filtered mean and covariance.

The parameters of the process may vary in time, thus they are given as iterators instead of fixed values.

## **Parameters** y : (N,D) array

"Normalized" noisy observations of the states, that is, the observations multiplied by the precision matrix U (and possibly other transformation matrices).

U: (N,D,D) array or N-list of (D,D) arrays

Precision matrix (i.e., inverse covariance matrix) of the observation noise for each time instance.

A: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Dynamic matrix for each time instance.

V: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Covariance matrix of the innovation noise for each time instance.

### Returns mu: array

Filtered mean of the states.

Cov: array

Filtered covariance of the states.

## See also:

rts\_smoother

## bayespy.utils.misc.logdet\_chol

bayespy.utils.misc.logdet\_chol(U)

## bayespy.utils.misc.logsumexp

bayespy.utils.misc.logsumexp(X, axis=None, keepdims=False) Compute log(sum(exp(X))) in a numerically stable way

## bayespy.utils.misc.m\_chol

bayespy.utils.misc.m\_chol(C)

## bayespy.utils.misc.m\_chol\_inv

bayespy.utils.misc.m\_chol\_inv(U)

## bayespy.utils.misc.m\_chol\_logdet

bayespy.utils.misc.m\_chol\_logdet(U)

### bayespy.utils.misc.m\_chol\_solve

bayespy.utils.misc.m\_chol\_solve(U, B, out=None)

## bayespy.utils.misc.m\_digamma

bayespy.utils.misc.m\_digamma (a, d)

# bayespy.utils.misc.m\_dot

bayespy.utils.misc.m\_dot(A, b)

## bayespy.utils.misc.m\_outer

bayespy.utils.misc.m\_outer(A, B)

#### bayespy.utils.misc.m\_solve\_triangular

bayespy.utils.misc.m\_solve\_triangular(U, B, \*\*kwargs)

## bayespy.utils.misc.make\_equal\_length

```
bayespy.utils.misc.make_equal_length(*shapes)
Make tuples equal length.
```

Add leading 1s to shorter tuples.

## bayespy.utils.misc.make\_equal\_ndim

```
bayespy.utils.misc.make_equal_ndim(*arrays)

Add trailing unit axes so that arrays have equal ndim
```

# bayespy.utils.misc.mean

```
bayespy.utils.misc.mean (X, axis=None, keepdims=False)
Compute the mean, ignoring NaNs.
```

## bayespy.utils.misc.moveaxis

```
bayespy.utils.misc.moveaxis(A, axis_from, axis_to)
Move the axis axis_from to position axis_to.
```

## bayespy.utils.misc.multiply\_shapes

```
bayespy.utils.misc.multiply_shapes(*shapes)
```

Compute element-wise product of lists/tuples.

Shorter lists are concatenated with leading 1s in order to get lists with the same length.

## bayespy.utils.misc.nans

```
bayespy.utils.misc.nans(size=())
```

#### bayespy.utils.misc.nested\_iterator

```
bayespy.utils.misc.nested_iterator(max_inds)
```

## bayespy.utils.misc.remove\_whitespace

```
bayespy.utils.misc.remove_whitespace(s)
```

## bayespy.utils.misc.repeat\_to\_shape

```
bayespy.utils.misc.repeat_to_shape (A, s)
```

### bayespy.utils.misc.rmse

```
bayespy.utils.misc.rmse(y1, y2, axis=None)
```

## bayespy.utils.misc.rts\_smoother

```
bayespy.utils.misc.rts_smoother(mu, Cov, A, V, removethis=None)
```

Perform Rauch-Tung-Striebel smoothing to obtain the posterior.

The function returns the posterior mean and covariance of each state. The parameters of the process may vary in time, thus they are given as iterators instead of fixed values.

Parameters mu: (N,D) array

Mean of the states from Kalman filter.

Cov: (N,D,D) array

Covariance of the states from Kalman filter.

A: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Dynamic matrix for each time instance.

V: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Covariance matrix of the innovation noise for each time instance.

Returns mu: array

Posterior mean of the states.

Cov: array

Posterior covariance of the states.

#### See also:

kalman\_filter

## bayespy.utils.misc.safe\_indices

```
bayespy.utils.misc.safe_indices(inds, shape)
```

Makes sure that indices are valid for given shape.

The shorter shape determines the length.

For instance,

```
>>> safe_indices( (3, 4, 5), (1, 6) )
(0, 5)
```

## bayespy.utils.misc.squeeze

```
bayespy.utils.misc.squeeze(X)
```

Remove leading axes that have unit length.

For instance, a shape (1,1,4,1,3) will be reshaped to (4,1,3).

## bayespy.utils.misc.squeeze\_to\_dim

```
bayespy.utils.misc.squeeze_to_dim(X, dim)
```

## bayespy.utils.misc.sum\_multiply

bayespy.utils.misc.sum.multiply(\*args, axis=None, sumaxis=True, keepdims=False)

## bayespy.utils.misc.sum\_multiply\_to\_plates

```
bayespy.utils.misc.sum_multiply_to_plates (*arrays, to_plates=(), from_plates=None, ndim=0)

Compute the product of the arguments and sum to the target shape.
```

## bayespy.utils.misc.sum\_product

```
bayespy.utils.misc.sum_product(*args, axes_to_keep=None, axes_to_sum=None, keep-dims=False)
```

## bayespy.utils.misc.sum\_to\_dim

```
bayespy.utils.misc.sum_to_dim(A, dim)
Sum leading axes of A such that A has dim dimensions.
```

## bayespy.utils.misc.sum\_to\_shape

```
bayespy.utils.misc.sum_to_shape (X,s)
Sum axes of the array such that the resulting shape is as given.
```

Thus, the shape of the result will be s or an error is raised.

### bayespy.utils.misc.symm

```
bayespy.utils.misc.symm(X)

Make X symmetric.
```

## bayespy.utils.misc.tempfile

```
bayespy.utils.misc.tempfile(prefix='', suffix='')
```

#### bayespy.utils.misc.trues

```
bayespy.utils.misc.trues(shape)
```

#### bayespy.utils.misc.unique

```
bayespy.utils.misc.unique (l) Remove duplicate items from a list while preserving order.
```

## bayespy.utils.misc.vb\_optimize

```
bayespy.utils.misc.vb_optimize(x0, set_values, lowerbound, gradient=None)
```

## bayespy.utils.misc.vb\_optimize\_nodes

```
bayespy.utils.misc.vb_optimize_nodes(*nodes)
```

## bayespy.utils.misc.write\_to\_hdf5

```
bayespy.utils.misc.write_to_hdf5 (group, data, name)
Writes the given array into the HDF5 file.
```

## bayespy.utils.misc.zipper\_merge

```
bayespy.utils.misc.zipper_merge(*lists)
```

Combines lists by alternating elements from them.

Combining lists [1,2,3], ['a','b','c'] and [42,666,99] results in [1,'a',42,2,'b',666,3,'c',99]

The lists should have equal length or they are assumed to have the length of the shortest list.

This is known as alternating merge or zipper merge.

#### **Classes**

```
\begin{tabular}{ll} CholeskyDense(K) \\ CholeskySparse(K) \\ TestCase([methodName]) & Simple base class for unit testing. \\ \end{tabular}
```

## bayespy.utils.misc.CholeskyDense

```
class bayespy.utils.misc.CholeskyDense(K)
```

```
__init__(K)
```

## Methods

```
..init_.(K)
logdet()
solve(b)
trace_solve_gradient(dK)
```

#### bayespy.utils.misc.CholeskyDense.\_\_init\_\_

```
CholeskyDense.__init__(K)
```

#### bayespy.utils.misc.CholeskyDense.logdet

```
CholeskyDense.logdet()
```

# bayespy.utils.misc.CholeskyDense.solve

```
CholeskyDense.solve(b)
```

## bayespy.utils.misc.CholeskyDense.trace\_solve\_gradient

```
CholeskyDense.trace_solve_gradient (dK)
```

# bayespy.utils.misc.CholeskySparse

```
class bayespy.utils.misc.CholeskySparse(K)
```

```
\_init\_(K)
```

#### **Methods**

```
__init__(K)
logdet()
solve(b)
trace_solve_gradient(dK)
```

#### bayespy.utils.misc.CholeskySparse.\_\_init\_\_

```
CholeskySparse.__init__(K)
```

#### bayespy.utils.misc.CholeskySparse.logdet

```
CholeskySparse.logdet()
```

## bayespy.utils.misc.CholeskySparse.solve

```
CholeskySparse.solve(b)
```

## bayespy.utils.misc.CholeskySparse.trace\_solve\_gradient

```
{\tt CholeskySparse.trace\_solve\_gradient}\ (dK)
```

## bayespy.utils.misc.TestCase

```
class bayespy.utils.misc.TestCase (methodName='runTest')
```

Simple base class for unit testing.

Adds NumPy's features to Python's unittest.

```
__init__ (methodName='runTest')
```

Create an instance of the class that will use the named test method when executed. Raises a ValueError if the instance does not have a method with the specified name.

#### **Methods**

```
__init__([methodName])
                                                         Create an instance of the class that will use the named test method when e
addCleanup(function, *args, **kwargs)
                                                         Add a function, with arguments, to be called when the test is completed.
addTypeEqualityFunc(typeobj, function)
                                                         Add a type specific assertEqual style function to compare a type.
assertAllClose(A, B[, msg, rtol, atol])
assertAlmostEqual(first, second[, places, ...])
                                                         Fail if the two objects are unequal as determined by their difference round
assertAlmostEquals(*args, **kwargs)
assertArrayEqual(A, B[, msg])
assertCountEqual(first, second[, msg])
                                                         An unordered sequence comparison asserting that the same elements, reg-
assertDictContainsSubset(subset, dictionary)
                                                         Checks whether dictionary is a superset of subset.
assertDictEqual(d1, d2[, msg])
assertEqual(first, second[, msg])
                                                         Fail if the two objects are unequal as determined by the '==' operator.
assertEquals(*args, **kwargs)
assertFalse(expr[, msg])
                                                         Check that the expression is false.
assertGreater(a, b[, msg])
                                                         Just like self.assertTrue(a > b), but with a nicer default message.
assertGreaterEqual(a, b[, msg])
                                                         Just like self.assertTrue(a \ge b), but with a nicer default message.
assertIn(member, container[, msg])
                                                         Just like self.assertTrue(a in b), but with a nicer default message.
                                                         Just like self.assertTrue(a is b), but with a nicer default message.
assertIs(expr1, expr2[, msg])
assertIsInstance(obj, cls[, msg])
                                                         Same as self.assertTrue(isinstance(obj, cls)), with a nicer default message
                                                         Same as self.assertTrue(obj is None), with a nicer default message.
assertIsNone(obj[, msg])
assertIsNot(expr1, expr2[, msg])
                                                         Just like self.assertTrue(a is not b), but with a nicer default message.
assertIsNotNone(obj[, msg])
                                                         Included for symmetry with assertIsNone.
                                                         Just like self.assertTrue(a < b), but with a nicer default message.
assertLess(a, b[, msg])
assertLessEqual(a, b[, msg])
                                                         Just like self.assertTrue(a \le b), but with a nicer default message.
assertListEqual(list1, list2[, msg])
                                                         A list-specific equality assertion.
assertLogs([logger, level])
                                                         Fail unless a log message of level level or higher is emitted on logger_name
assertMessage(M1, M2)
assertMessageToChild(X, u)
assertMultiLineEqual(first, second[, msg])
                                                         Assert that two multi-line strings are equal.
assertNotAlmostEqual(first, second[, ...])
                                                         Fail if the two objects are equal as determined by their difference rounded
assertNotAlmostEquals(*args, **kwargs)
assertNotEqual(first, second[, msg])
                                                         Fail if the two objects are equal as determined by the '!=' operator.
assertNotEquals(*args, **kwargs)
assertNotIn(member, container[, msg])
                                                         Just like self.assertTrue(a not in b), but with a nicer default message.
assertNotIsInstance(obj, cls[, msg])
                                                         Included for symmetry with assertIsInstance.
                                                         Fail the test if the text matches the regular expression.
assertNotRegex(text, unexpected_regex[, msg])
                                                         Fail unless an exception of class excClass is raised by callableObj when it
assertRaises(excClass[, callableObj])
assertRaisesRegex(expected_exception, ...[, ...])
                                                         Asserts that the message in a raised exception matches a regex.
assertRaisesRegexp(*args, **kwargs)
assertRegex(text, expected_regex[, msg])
                                                         Fail the test unless the text matches the regular expression.
assertRegexpMatches(*args, **kwargs)
assertSequenceEqual(seq1, seq2[, msg, seq_type])
                                                         An equality assertion for ordered sequences (like lists and tuples).
assertSetEqual(set1, set2[, msg])
                                                         A set-specific equality assertion.
assertTrue(expr[, msg])
                                                         Check that the expression is true.
                                                         A tuple-specific equality assertion.
assertTupleEqual(tuple1, tuple2[, msg])
assertWarns(expected_warning[, callable_obj])
                                                         Fail unless a warning of class warnClass is triggered by callable_obj when
assertWarnsRegex(expected_warning, ...[, ...])
                                                         Asserts that the message in a triggered warning matches a regexp.
```

Run the test without collecting errors in a TestResult

debug()

assert\_(\*args, \*\*kwargs)
countTestCases()

defaultTestResult()

```
doCleanups()
                                                        Execute all cleanup functions.
                                                        Fail immediately, with the given message.
fail([msg])
failIf(*args, **kwargs)
failIfAlmostEqual(*args, **kwargs)
failIfEqual(*args, **kwargs)
failUnless(*args, **kwargs)
failUnlessAlmostEqual(*args, **kwargs)
failUnlessEqual(*args, **kwargs)
failUnlessRaises(*args, **kwargs)
id()
run([result])
setUp()
                                                        Hook method for setting up the test fixture before exercising it.
setUpClass()
                                                        Hook method for setting up class fixture before running tests in the class.
                                                        Returns a one-line description of the test, or None if no description has be
shortDescription()
skipTest(reason)
                                                        Skip this test.
subTest([msg])
                                                        Return a context manager that will return the enclosed block of code in a
tearDown()
                                                        Hook method for deconstructing the test fixture after testing it.
tearDownClass()
                                                        Hook method for deconstructing the class fixture after running all tests in
```

#### bayespy.utils.misc.TestCase.\_\_init\_\_

```
TestCase.__init__(methodName='runTest')
```

Create an instance of the class that will use the named test method when executed. Raises a ValueError if the instance does not have a method with the specified name.

#### bayespy.utils.misc.TestCase.addCleanup

```
TestCase.addCleanup (function, *args, **kwargs)
```

Add a function, with arguments, to be called when the test is completed. Functions added are called on a LIFO basis and are called after tearDown on test failure or success.

Cleanup items are called even if setUp fails (unlike tearDown).

### bayespy.utils.misc.TestCase.addTypeEqualityFunc

```
TestCase.addTypeEqualityFunc(typeobj, function)
```

Add a type specific assertEqual style function to compare a type.

This method is for use by TestCase subclasses that need to register their own type equality functions to provide nicer error messages.

#### Args:

**typeobj:** The data type to call this function on when both values are of the same type in assertEqual().

**function:** The callable taking two arguments and an optional msg= argument that raises self.failureException with a useful error message when the two arguments are not equal.

#### bayespy.utils.misc.TestCase.assertAllClose

TestCase.assertAllClose (A, B, msg='Arrays not almost equal', rtol=0.0001, atol=0)

#### bayespy.utils.misc.TestCase.assertAlmostEqual

TestCase.assertAlmostEqual (first, second, places=None, msg=None, delta=None)

Fail if the two objects are unequal as determined by their difference rounded to the given number of decimal places (default 7) and comparing to zero, or by comparing that the between the two objects is more than the given delta.

Note that decimal places (from zero) are usually not the same as significant digits (measured from the most significant digit).

If the two objects compare equal then they will automatically compare almost equal.

#### bayespy.utils.misc.TestCase.assertAlmostEquals

```
TestCase.assertAlmostEquals(*args, **kwargs)
```

#### bayespy.utils.misc.TestCase.assertArrayEqual

```
TestCase.assertArrayEqual (A, B, msg='Arrays not equal')
```

#### bayespy.utils.misc.TestCase.assertCountEqual

```
TestCase.assertCountEqual (first, second, msg=None)
```

An unordered sequence comparison asserting that the same elements, regardless of order. If the same element occurs more than once, it verifies that the elements occur the same number of times.

**self.assertEqual(Counter(list(first)),** Counter(list(second)))

#### **Example:**

- [0, 1, 1] and [1, 0, 1] compare equal.
- [0, 0, 1] and [0, 1] compare unequal.

## bayespy.utils.misc.TestCase.assertDictContainsSubset

```
\texttt{TestCase.} \textbf{assertDictContainsSubset} \ (\textit{subset}, \textit{dictionary}, \textit{msg} = None)
```

Checks whether dictionary is a superset of subset.

#### bayespy.utils.misc.TestCase.assertDictEqual

```
TestCase.assertDictEqual(d1, d2, msg=None)
```

#### bayespy.utils.misc.TestCase.assertEqual

```
TestCase.assertEqual (first, second, msg=None)
```

Fail if the two objects are unequal as determined by the '==' operator.

#### bayespy.utils.misc.TestCase.assertEquals

```
TestCase.assertEquals(*args, **kwargs)
```

### bayespy.utils.misc.TestCase.assertFalse

```
TestCase.assertFalse(expr, msg=None)
```

Check that the expression is false.

#### bayespy.utils.misc.TestCase.assertGreater

```
TestCase.assertGreater(a, b, msg=None)
```

Just like self.assertTrue(a > b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertGreaterEqual

```
TestCase.assertGreaterEqual (a, b, msg=None)
```

Just like self.assertTrue( $a \ge b$ ), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertIn

```
TestCase.assertIn (member, container, msg=None)
```

Just like self.assertTrue(a in b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertls

```
TestCase.assertIs(expr1, expr2, msg=None)
```

Just like self.assertTrue(a is b), but with a nicer default message.

## bayespy.utils.misc.TestCase.assertIsInstance

```
TestCase.assertIsInstance(obj, cls, msg=None)
```

Same as self.assertTrue(isinstance(obj, cls)), with a nicer default message.

## bayespy.utils.misc.TestCase.assertIsNone

```
TestCase.assertIsNone(obj, msg=None)
```

Same as self.assertTrue(obj is None), with a nicer default message.

## bayespy.utils.misc.TestCase.assertIsNot

```
TestCase.assertIsNot (expr1, expr2, msg=None)
```

Just like self.assertTrue(a is not b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertlsNotNone

```
TestCase.assertIsNotNone (obj, msg=None)
Included for symmetry with assertIsNone.
```

#### bayespy.utils.misc.TestCase.assertLess

```
TestCase.assertLess(a, b, msg=None)
```

Just like self.assertTrue(a < b), but with a nicer default message.

## bayespy.utils.misc.TestCase.assertLessEqual

```
TestCase.assertLessEqual(a, b, msg=None)
```

Just like self.assertTrue( $a \le b$ ), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertListEqual

```
TestCase.assertListEqual (list1, list2, msg=None)
```

A list-specific equality assertion.

**Args:** list1: The first list to compare. list2: The second list to compare. msg: Optional message to use on failure instead of a list of

differences.

### bayespy.utils.misc.TestCase.assertLogs

```
TestCase.assertLogs(logger=None, level=None)
```

Fail unless a log message of level *level* or higher is emitted on *logger\_name* or its children. If omitted, *level* defaults to INFO and *logger* defaults to the root logger.

This method must be used as a context manager, and will yield a recording object with two attributes: *output* and *records*. At the end of the context manager, the *output* attribute will be a list of the matching formatted log messages and the *records* attribute will be a list of the corresponding LogRecord objects.

#### Example:

#### bayespy.utils.misc.TestCase.assertMessage

```
TestCase.assertMessage (M1, M2)
```

#### bayespy.utils.misc.TestCase.assertMessageToChild

```
TestCase.assertMessageToChild (X, u)
```

#### bayespy.utils.misc.TestCase.assertMultiLineEqual

TestCase.assertMultiLineEqual (first, second, msg=None)

Assert that two multi-line strings are equal.

#### bayespy.utils.misc.TestCase.assertNotAlmostEqual

TestCase.assertNotAlmostEqual (first, second, places=None, msg=None, delta=None)

Fail if the two objects are equal as determined by their difference rounded to the given number of decimal places (default 7) and comparing to zero, or by comparing that the between the two objects is less than the given delta.

Note that decimal places (from zero) are usually not the same as significant digits (measured from the most significant digit).

Objects that are equal automatically fail.

#### bayespy.utils.misc.TestCase.assertNotAlmostEquals

TestCase.assertNotAlmostEquals(\*args, \*\*kwargs)

#### bayespy.utils.misc.TestCase.assertNotEqual

TestCase.assertNotEqual (first, second, msg=None)

Fail if the two objects are equal as determined by the '!=' operator.

#### bayespy.utils.misc.TestCase.assertNotEquals

TestCase.assertNotEquals(\*args, \*\*kwargs)

#### bayespy.utils.misc.TestCase.assertNotIn

TestCase.assertNotIn (member, container, msg=None)

Just like self.assertTrue(a not in b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertNotIsInstance

TestCase.assertNotIsInstance(obj, cls, msg=None)

Included for symmetry with assertIsInstance.

#### bayespy.utils.misc.TestCase.assertNotRegex

TestCase.assertNotRegex (text, unexpected\_regex, msg=None)

Fail the test if the text matches the regular expression.

#### bayespy.utils.misc.TestCase.assertRaises

```
TestCase.assertRaises (excClass, callableObj=None, *args, **kwargs)
```

Fail unless an exception of class excClass is raised by callableObj when invoked with arguments args and keyword arguments kwargs. If a different type of exception is raised, it will not be caught, and the test case will be deemed to have suffered an error, exactly as for an unexpected exception.

If called with callableObj omitted or None, will return a context object used like this:

```
with self.assertRaises(SomeException):
    do_something()
```

An optional keyword argument 'msg' can be provided when assertRaises is used as a context object.

The context manager keeps a reference to the exception as the 'exception' attribute. This allows you to inspect the exception after the assertion:

```
with self.assertRaises(SomeException) as cm:
    do_something()
the_exception = cm.exception
self.assertEqual(the_exception.error_code, 3)
```

## bayespy.utils.misc.TestCase.assertRaisesRegex

```
TestCase.assertRaisesRegex (expected_exception, expected_regex, callable_obj=None, *args, **kwargs)
```

Asserts that the message in a raised exception matches a regex.

**Args:** expected\_exception: Exception class expected to be raised. expected\_regex: Regex (re pattern object or string) expected

to be found in error message.

callable\_obj: Function to be called. msg: Optional message used in case of failure. Can only be used when assertRaisesRegex is used as a context manager.

args: Extra args. kwargs: Extra kwargs.

#### bayespy.utils.misc.TestCase.assertRaisesRegexp

```
TestCase.assertRaisesRegexp(*args, **kwargs)
```

#### bayespy.utils.misc.TestCase.assertRegex

```
TestCase.assertRegex (text, expected_regex, msg=None)
Fail the test unless the text matches the regular expression.
```

#### bayespy.utils.misc.TestCase.assertRegexpMatches

```
TestCase.assertRegexpMatches(*args, **kwargs)
```

#### bayespy.utils.misc.TestCase.assertSequenceEqual

```
TestCase.assertSequenceEqual (seq1, seq2, msg=None, seq_type=None)
```

An equality assertion for ordered sequences (like lists and tuples).

For the purposes of this function, a valid ordered sequence type is one which can be indexed, has a length, and has an equality operator.

**Args:** seq1: The first sequence to compare. seq2: The second sequence to compare. seq\_type: The expected datatype of the sequences, or None if no

datatype should be enforced.

msg: Optional message to use on failure instead of a list of differences.

#### bayespy.utils.misc.TestCase.assertSetEqual

```
TestCase.assertSetEqual (set1, set2, msg=None)
```

A set-specific equality assertion.

**Args:** set1: The first set to compare. set2: The second set to compare. msg: Optional message to use on failure instead of a list of

differences.

assertSetEqual uses ducktyping to support different types of sets, and is optimized for sets specifically (parameters must support a difference method).

#### bayespy.utils.misc.TestCase.assertTrue

```
TestCase.assertTrue(expr, msg=None)
```

Check that the expression is true.

#### bayespy.utils.misc.TestCase.assertTupleEqual

```
TestCase.assertTupleEqual (tuple1, tuple2, msg=None)
```

A tuple-specific equality assertion.

**Args:** tuple1: The first tuple to compare. tuple2: The second tuple to compare. msg: Optional message to use on failure instead of a list of

differences.

#### bayespy.utils.misc.TestCase.assertWarns

```
TestCase.assertWarns(expected_warning, callable_obj=None, *args, **kwargs)
```

Fail unless a warning of class warnClass is triggered by callable\_obj when invoked with arguments args and keyword arguments kwargs. If a different type of warning is triggered, it will not be handled: depending on the other warning filtering rules in effect, it might be silenced, printed out, or raised as an exception.

If called with callable\_obj omitted or None, will return a context object used like this:

```
with self.assertWarns(SomeWarning):
    do_something()
```

An optional keyword argument 'msg' can be provided when assertWarns is used as a context object.

The context manager keeps a reference to the first matching warning as the 'warning' attribute; similarly, the 'filename' and 'lineno' attributes give you information about the line of Python code from which the warning was triggered. This allows you to inspect the warning after the assertion:

```
with self.assertWarns(SomeWarning) as cm:
    do_something()
the_warning = cm.warning
self.assertEqual(the_warning.some_attribute, 147)
```

#### bayespy.utils.misc.TestCase.assertWarnsRegex

Asserts that the message in a triggered warning matches a regexp. Basic functioning is similar to assertWarns() with the addition that only warnings whose messages also match the regular expression are considered successful matches.

**Args:** expected\_warning: Warning class expected to be triggered. expected\_regex: Regex (re pattern object or string) expected

to be found in error message.

callable\_obj: Function to be called. msg: Optional message used in case of failure. Can only be used when assertWarnsRegex is used as a context manager.

args: Extra args. kwargs: Extra kwargs.

## bayespy.utils.misc.TestCase.assert

```
TestCase.assert_(*args, **kwargs)
```

## bayespy.utils.misc.TestCase.countTestCases

```
TestCase.countTestCases()
```

### bayespy.utils.misc.TestCase.debug

```
TestCase.debug()
```

Run the test without collecting errors in a TestResult

## bayes py. utils. misc. Test Case. default Test Result

```
TestCase.defaultTestResult()
```

## bayespy.utils.misc.TestCase.doCleanups

```
TestCase.doCleanups()
```

Execute all cleanup functions. Normally called for you after tearDown.

```
bayespy.utils.misc.TestCase.fail
TestCase.fail(msg=None)
    Fail immediately, with the given message.
bayespy.utils.misc.TestCase.faillf
TestCase.failIf(*args, **kwargs)
bayespy.utils.misc.TestCase.failIfAlmostEqual
TestCase.failIfAlmostEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.faillfEqual
TestCase.failIfEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnless
TestCase.failUnless(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessAlmostEqual
TestCase.failUnlessAlmostEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessEqual
TestCase.failUnlessEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessRaises
TestCase.failUnlessRaises(*args, **kwargs)
bayespy.utils.misc.TestCase.id
TestCase.id()
bayespy.utils.misc.TestCase.run
TestCase.run (result=None)
bayespy.utils.misc.TestCase.setUp
TestCase.setUp()
    Hook method for setting up the test fixture before exercising it.
```

#### bayespy.utils.misc.TestCase.setUpClass

```
TestCase.setUpClass()
```

Hook method for setting up class fixture before running tests in the class.

#### bayespy.utils.misc.TestCase.shortDescription

```
TestCase.shortDescription()
```

Returns a one-line description of the test, or None if no description has been provided.

The default implementation of this method returns the first line of the specified test method's docstring.

#### bayespy.utils.misc.TestCase.skipTest

```
TestCase.skipTest (reason)
Skip this test.
```

### bayespy.utils.misc.TestCase.subTest

```
TestCase.subTest (msg=None, **params)
```

Return a context manager that will return the enclosed block of code in a subtest identified by the optional message and keyword parameters. A failure in the subtest marks the test case as failed but resumes execution at the end of the enclosed block, allowing further test code to be executed.

## bayespy.utils.misc.TestCase.tearDown

```
TestCase.tearDown()
```

Hook method for deconstructing the test fixture after testing it.

#### bayespy.utils.misc.TestCase.tearDownClass

```
{\tt TestCase.tearDownClass}\,(\,)
```

Hook method for deconstructing the class fixture after running all tests in the class.

#### **Attributes**

longMessage maxDiff

## bayespy.utils.misc.TestCase.longMessage

```
TestCase.longMessage = True
```

## bayespy.utils.misc.TestCase.maxDiff

TestCase.maxDiff = 640

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