waveform representation of a Brownian particle's wanderings was just such a waveform. To represent a nonlinear operator with a Brownian input, Wiener used a Stieltjes form of the Volterra functional which is equivalent to our familiar Riemann form of the Volterra functionals with the input, $x(t)$, being white Gaussian time function[1] . With these, he formed an orthogonal set of functionals for the analysis of nonlinear systems. He called the orthogonal set he formed, G-functionals. A time function, $x(t)$, is said to be white if its power density spectrum is a constant so that $\phi_{xx}(j\omega) = A$ for which its autocorrelation function is $\phi_{xx}(\tau) = \overline{x(t)x(t+\tau)} = A\delta(t)$ in which the overbar indicates the time average and $\delta(t)$is the unit impulse. A white time function is not a physical waveform since its total average power is infinite. However, it is an accurate idealisation of a physical waveform for which the power density spectrum is flat over a band of frequencies considerably wider than the bandwidth of the system to which is being applied as an input. This idealisation results in great analytical simplification.

2.6 The Wiener G-functionals

As I stated above, Wiener called his orthogonal set of functionals G-functionals since they are orthogonal with respect to a white Gaussian input. That is, he formed the G-functionals, $G_p[k_p;x(t)]$ in which k_p is the kernel and $x(t)$ is the input of the p^{th}-order G-functional, to satisfy the condition

$$\overline{G_p\left[k_p;x(t)\right] G_q\left[k_p;x(t)\right]} = 0 \qquad p \neq q \tag{2.4}$$

when $x(t)$ is from a white Gaussian process with the power density $\phi_{xx}(j\omega) = A$

A p^{th}-order G-functional, $G_p[k_p;x(t)]$, is the sum of Volterra functionals of orders less than or equal to p. The form of a p^{th}-degree G-functional, is

$$G_p\left[k_p;x(t)\right] = \sum_{n=0}^{p} K_{n(p)}\left[x(t)\right] \ . \tag{2.5}$$

All kernels of order less than p are determined by the leading kernel, k_p, to satisfy the orthogonality condition. A p^{th}-degree G-functional, $G_p\left[k_p;x(t)\right]$, thus is the sum of Volterra functionals $K_{n(p)}$ of orders less than or equal to p which are specified only by the leading Volterra kernel,k_p. The first few G-functionals are:

$$G_0\left[k_0;x(t)\right] = k_0 \ , \tag{2.6}$$

[1] Wiener expressed the Volterra operators as Stieltjes integrals with the input being a member of an ensemble of Brownian waveforms. Since Brownian motion is differentiable almost nowhere, the Riemann form of these integrals does not formally exist. However, the statistical properties of the Volterra operators expressed in the Riemann form with a white Gaussian input is identical with those of the Volterra operators expressed in Stieltjes form with the input being a Brownian waveform. For convenience, we use the Riemann form with a white Gaussian input.

$$G_1\left[k_1;x(t)\right] = \int_{-\infty}^{+\infty} k_1(\tau_1)\,x(t-\tau_1)\,d\tau_1 \;, \tag{2.7}$$

$$G_2\left[k_2;x(t)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} k_2(\tau_1,\tau_2)\,x(t-\tau_1)\,x(t-\tau_2)\,d\tau_1 d\tau_2 + k_{0(2)} \;, \tag{2.8}$$

in which

$$k_{0(2)} = -A\int_{-\infty}^{+\infty} k_2(\tau_1,\tau_1)\,d\tau_1 \;, \tag{2.9}$$

and

$$G_3\left[k_3;x(t)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} k_3(\tau_1,\tau_2,\tau_3)\,x(t-\tau_1)\,x(t-\tau_2)\,x(t-\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \int_{-\infty}^{+\infty} k_{1(3)}(\tau_1)\,x(t-\tau_1)\,d\tau_1 \;, \tag{2.10}$$

in which

$$k_{1(3)}(\tau_1) = -3A\int_{-\infty}^{+\infty} k_3(\tau_1,\tau_2,\tau_2)\,d\tau_2 \;. \tag{2.11}$$

2.7 System Modelling with the G-functionals

In terms of the G-functionals, the p^{th}-order model of a nonlinear system is

$$y_p(t) = \sum_{n=0}^{p} G_n\left[k_n;x(t)\right] \;. \tag{2.12}$$

The kernels, k_n, are called the Wiener kernels of the nonlinear system. A practical method was developed [2] by which the Wiener kernels can be determined by crosscorrelating the system response, y(t), with the white Gaussian input, x(t), as

$$k_n(\tau_1,\tau_2,\ldots,\tau_n) = \begin{cases} \frac{1}{n!A^n}\overline{y(t)x(t-\tau_1)\,x(t-\tau_2)\ldots x(t-\tau_n)} & t_i \geq 0, i = 1,2,\ldots,n\;,\\ & n = 0,1,2,\ldots,p\;,\\ 0 \text{ otherwise.} \end{cases} \tag{2.13}$$

In L_2 -norm, this representation results in the optimum p^{th}-order model of the given system. Note that it is optimum only for the white Gaussian input. Once the optimum p^{th}-order model of a system has been determined, the corresponding p^{th}-order Volterra series can be obtained simply by summing the Volterra kernels with the same order of each orthogonal functional.

Wiener originally showed that his set of G-functionals is complete in the class of nonexplosive nonlinear systems whose memory is not infinite which I call the Wiener class of systems [6]. By nonexplosive in this context I mean systems for which the output has a finite mean-square value when the input is from a white Gaussian process. Although infinite memory linear systems are not physical, there is