Importance Sampling

- Let our proposal be of the form: $q(z) = \tilde{q}(z)/\mathcal{Z}_q.$

$$\mathbb{E}[f] = \int f(z)p(z)dz \quad = \int f(z)\frac{p(z)}{q(z)}q(z)dz = \frac{Z_q}{Z_p}\int f(z)\frac{\tilde{p}(z)}{\tilde{q}(z)}q(z)dz$$

$$\approx \frac{\mathcal{Z}_q}{\mathcal{Z}_p}\frac{1}{N}\sum_n \frac{\tilde{p}(z^n)}{\tilde{q}(z^n)}f(z^n) = \frac{\mathcal{Z}_q}{\mathcal{Z}_p}\frac{1}{N}\sum_n w^n f(z^n),$$

- But we can use the same weights to approximate $\mathcal{Z}_q/\mathcal{Z}_p$:

$$\frac{\mathcal{Z}_p}{\mathcal{Z}_q} = \frac{1}{\mathcal{Z}_q}\int \tilde{p}(z)dz = \int \frac{\tilde{p}(z)}{\tilde{q}(z)}q(z)dz \approx \frac{1}{N}\sum_n \frac{\tilde{p}(z^n)}{\tilde{q}^n(z)} = \frac{1}{N}\sum_n w^n.$$

- Hence:

$$\mathbb{E}[f] \approx \sum_{n=1}^{N} \frac{w^n}{\sum_{m=1}^{N} w^m} f(z^n), \qquad z^n \sim q(z).$$

Consistent but biased.