Section 2: Learning

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- Beyond Linear Functions
- Backpropagation
- Nearest Neighbors
- Scikit-learn Tutorial

Beyond Linear Functions

Example: beyond linear functions

Regression: $x \in \mathbb{R}, y \in \mathbb{R}$

Linear functions:

$$\phi(x) = x$$

$$\mathcal{F}_1 = \{x \mapsto w_1 x + w_2 x^2 : w_1 \in \mathbb{R}, w_2 = 0\}$$

Quadratic functions:

$$\phi(x) = [x, x^2]$$

$$\mathcal{F}_2 = \{x \mapsto w_1 x + w_2 x^2 : w_1 \in \mathbb{R}, w_2 \in \mathbb{R}\}$$

[whiteboard]

Example: even more flexible functions

Regression: $x \in \mathbb{R}, y \in \mathbb{R}$

Piecewise constant functions:

$$\phi(x) = [\mathbf{1}[0 < x \leq 1], \mathbf{1}[1 < x \leq 2], \dots]$$

$$\mathcal{F}_3 = \{x \mapsto \textstyle\sum_{j=1}^{10} w_j \mathbf{1}[j-1 < x \leq j] : \mathbf{w} \in \mathbb{R}^{10}\}$$

[whiteboard]

Backpropagation

Backpropagation

$$\mathsf{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$

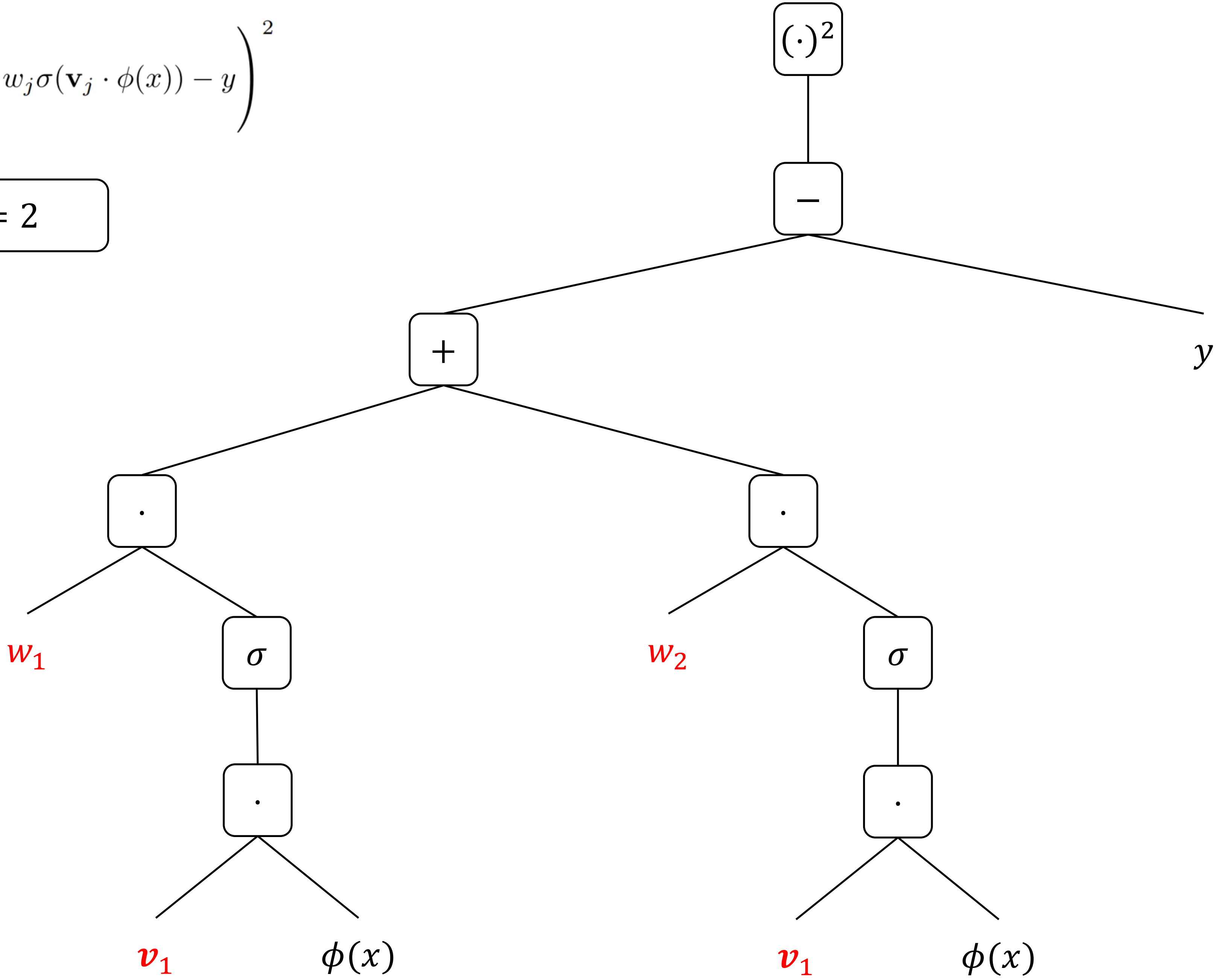
Backpropagation

Weights

$$\mathrm{Loss}(x, y, \mathbf{w}) = \left(\underbrace{\boxed{\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x))}}_{\text{Predicted}} - \underbrace{\boxed{y}}_{\text{Actual}} \right)^2$$

$$\text{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$

Consider $k = 2$



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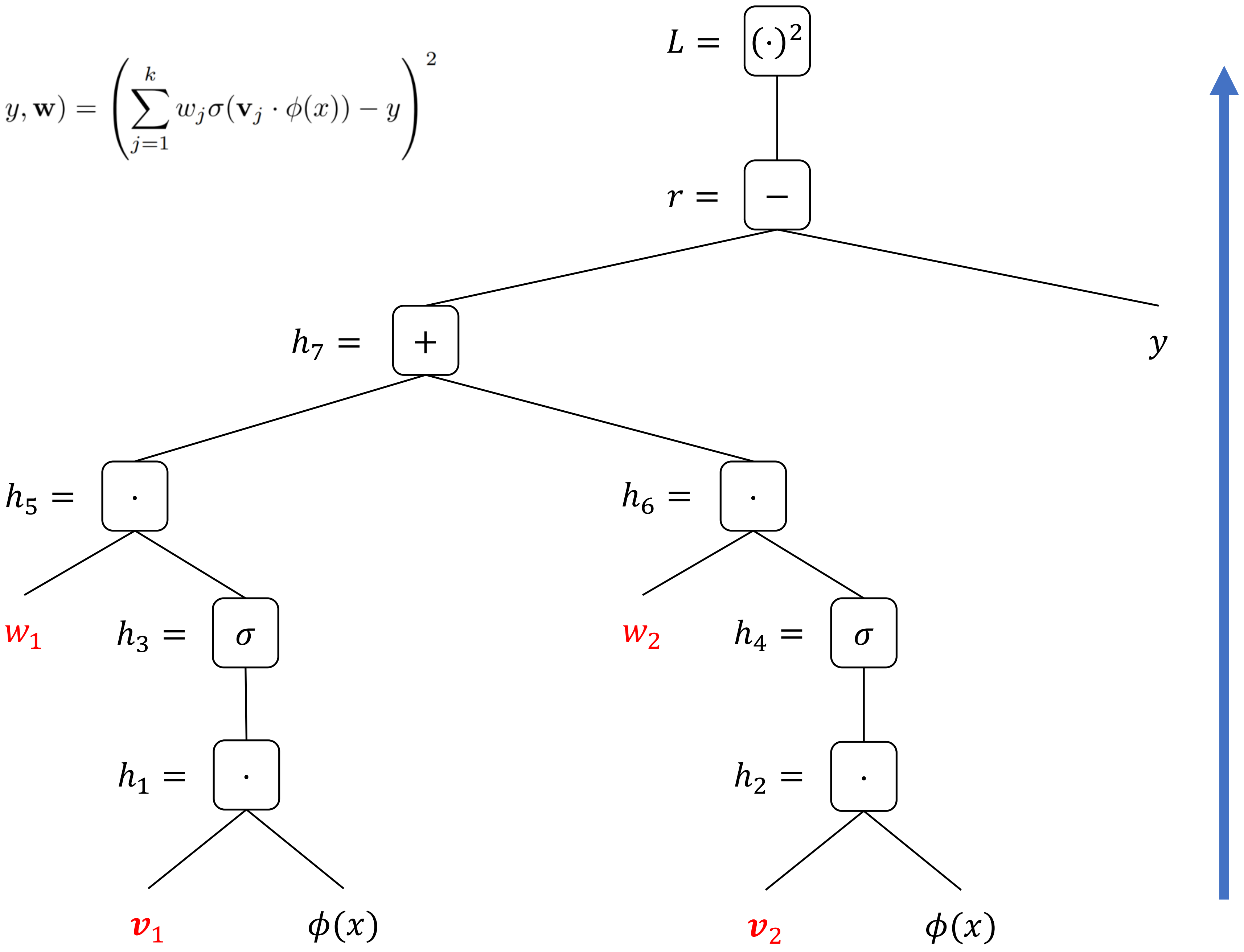
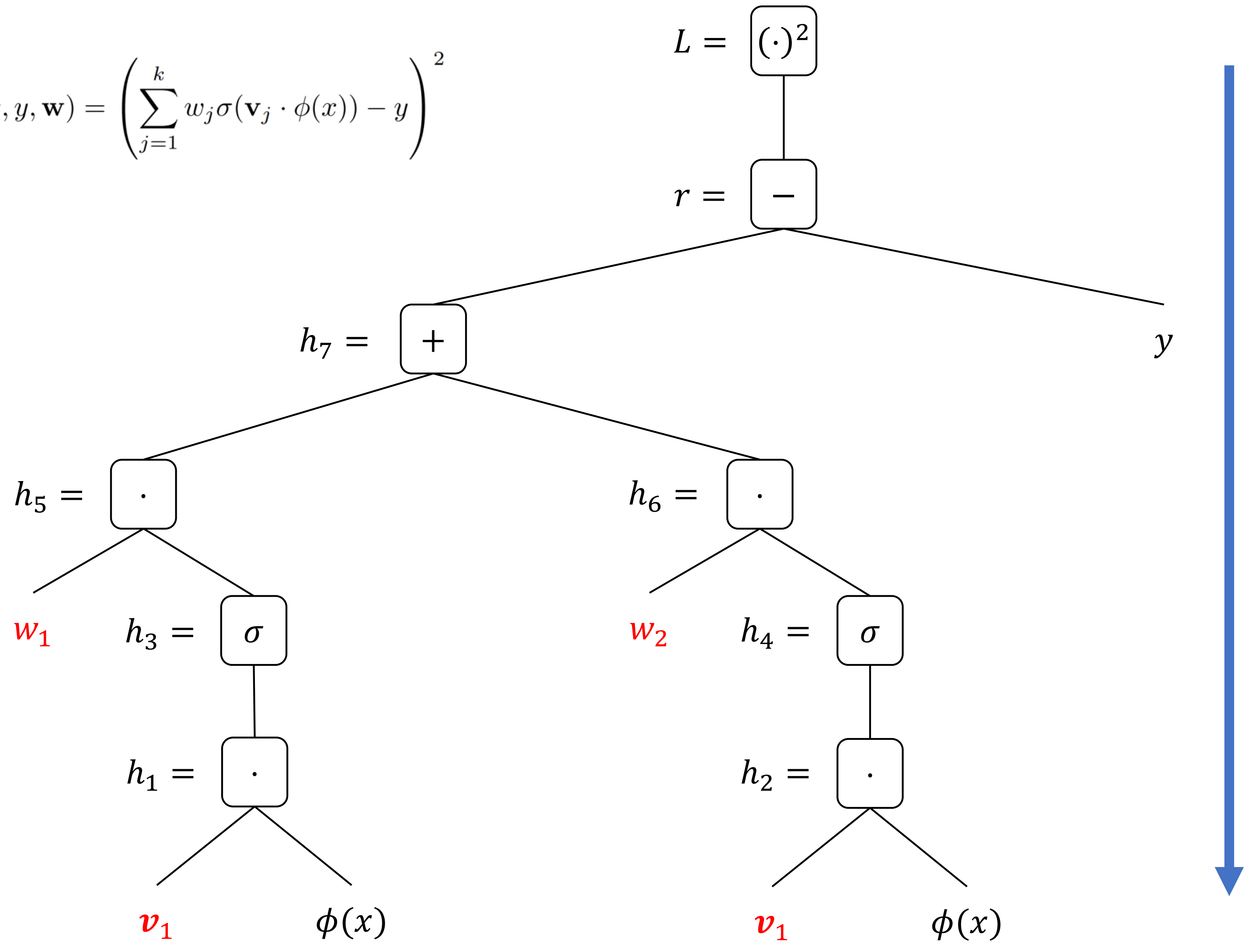


Forward Pass

Compute each node and store intermediate values

$$\text{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$



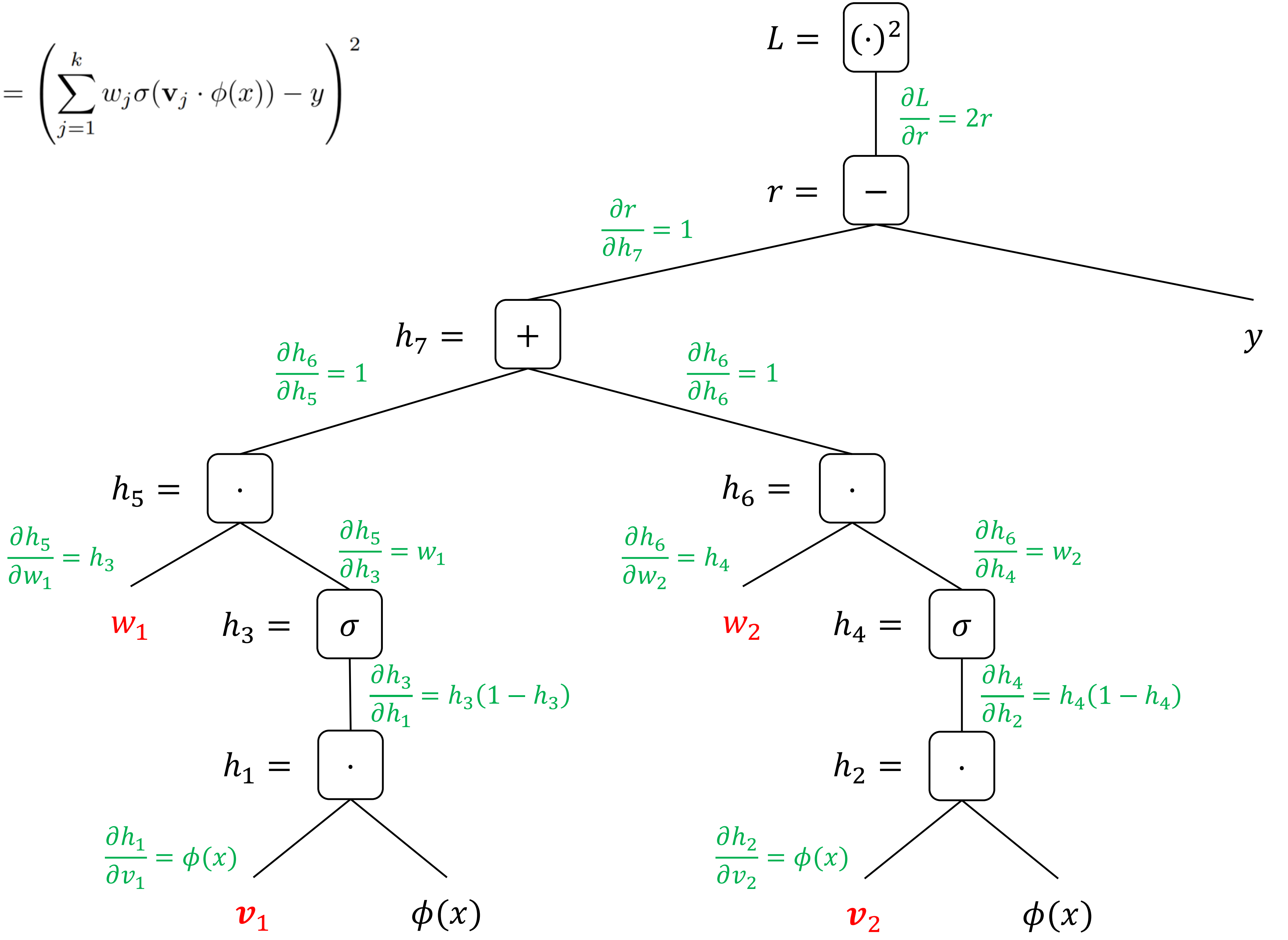
Backward Pass

Compute derivatives with respect to intermediate variables:

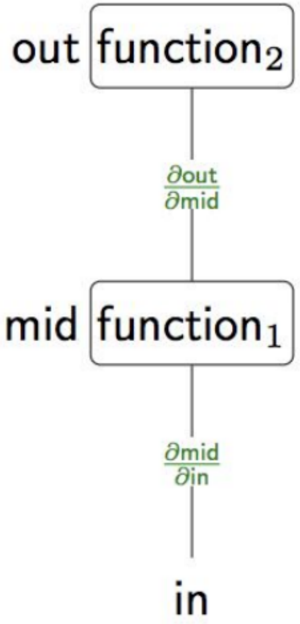
$$\frac{\partial L}{\partial r}, \frac{\partial r}{\partial h_6}, \frac{\partial h_6}{\partial h_4}, \frac{\partial h_6}{\partial h_4}, etc$$

[Whiteboard]

$$\mathrm{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$

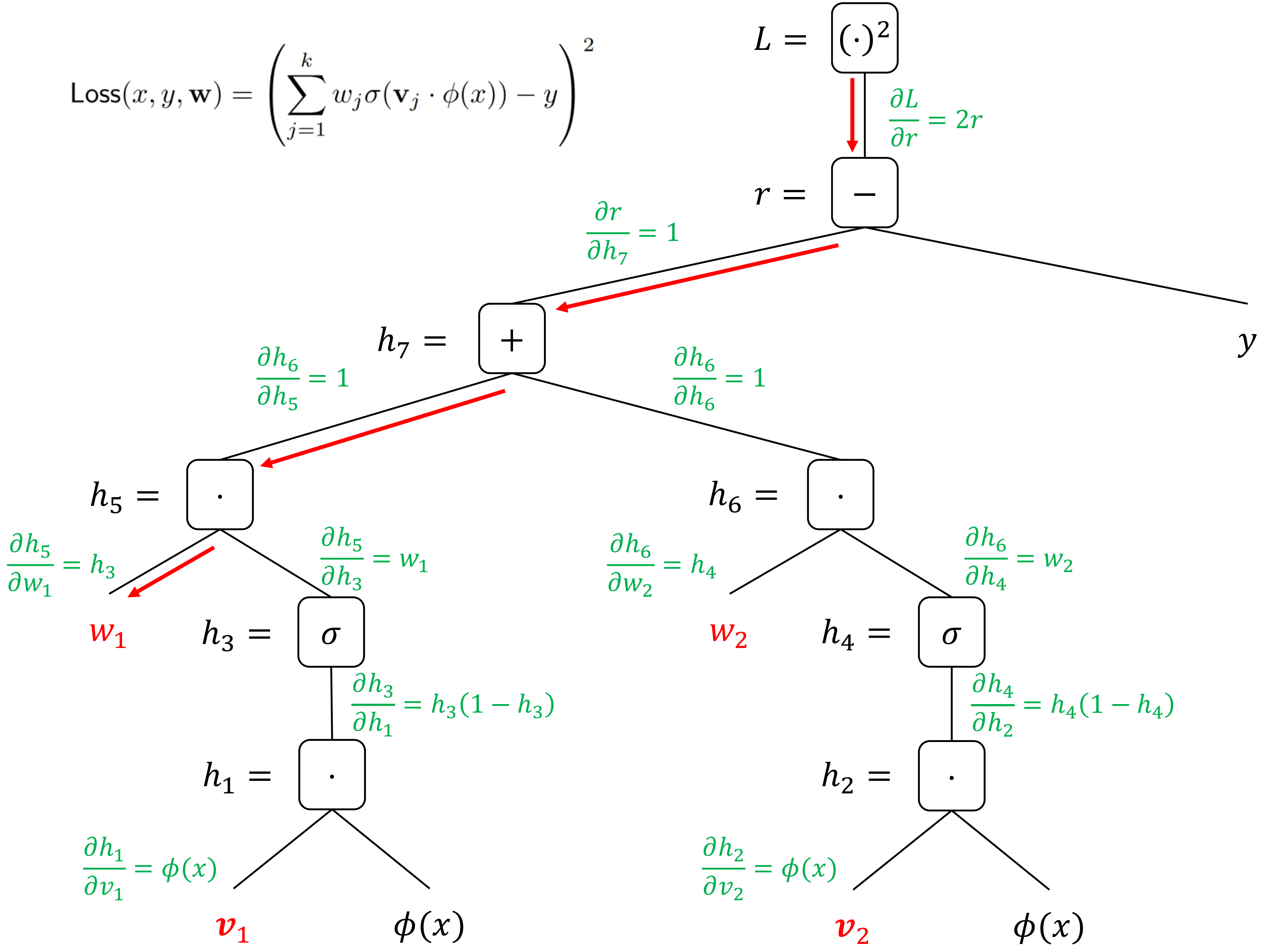


Backpropagation



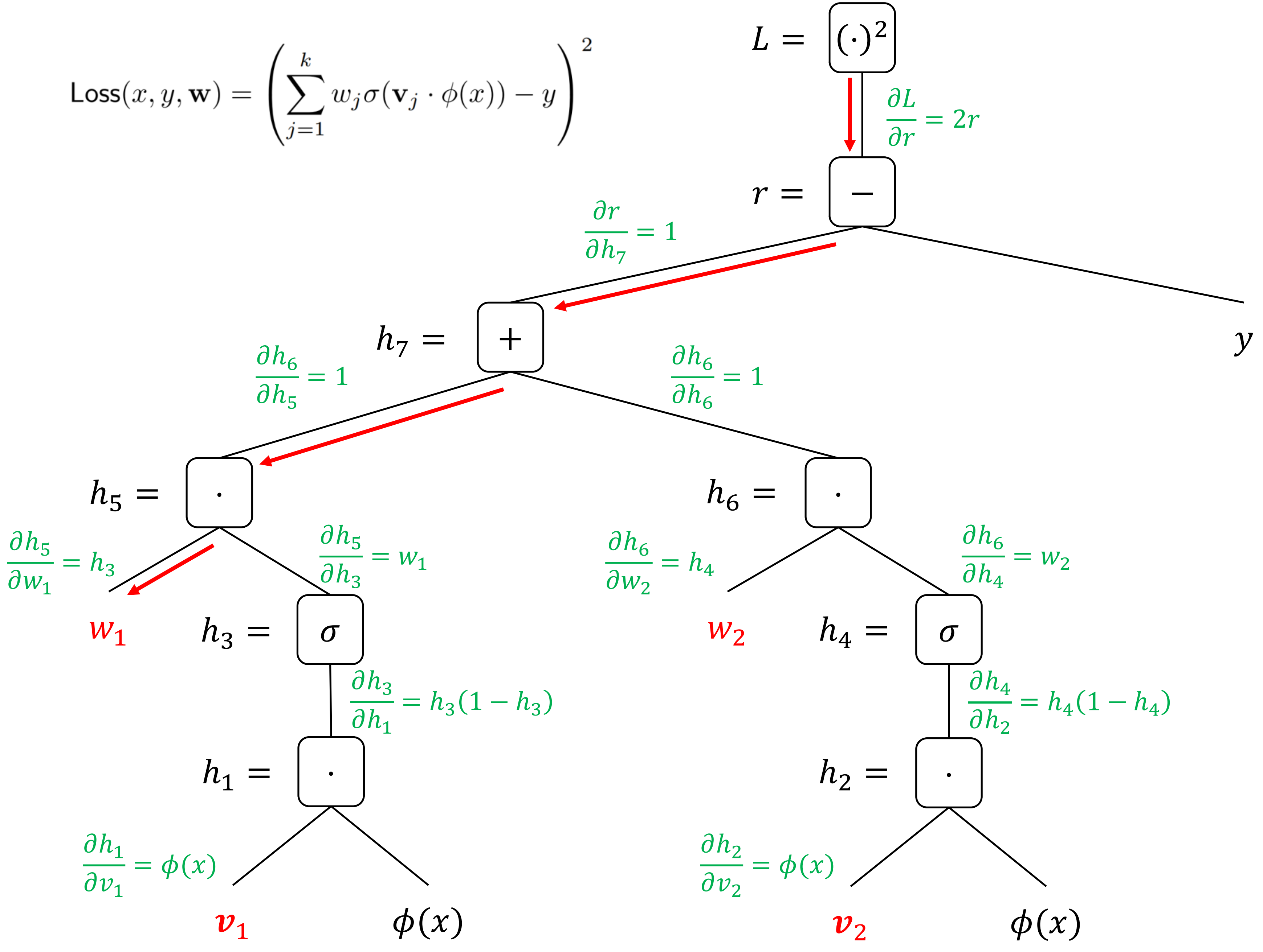
Chain rule: $\frac{\partial \text{out}}{\partial \text{in}} = \frac{\partial \text{out}}{\partial \text{mid}} \frac{\partial \text{mid}}{\partial \text{in}}$

$$\text{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$



[Whiteboard]

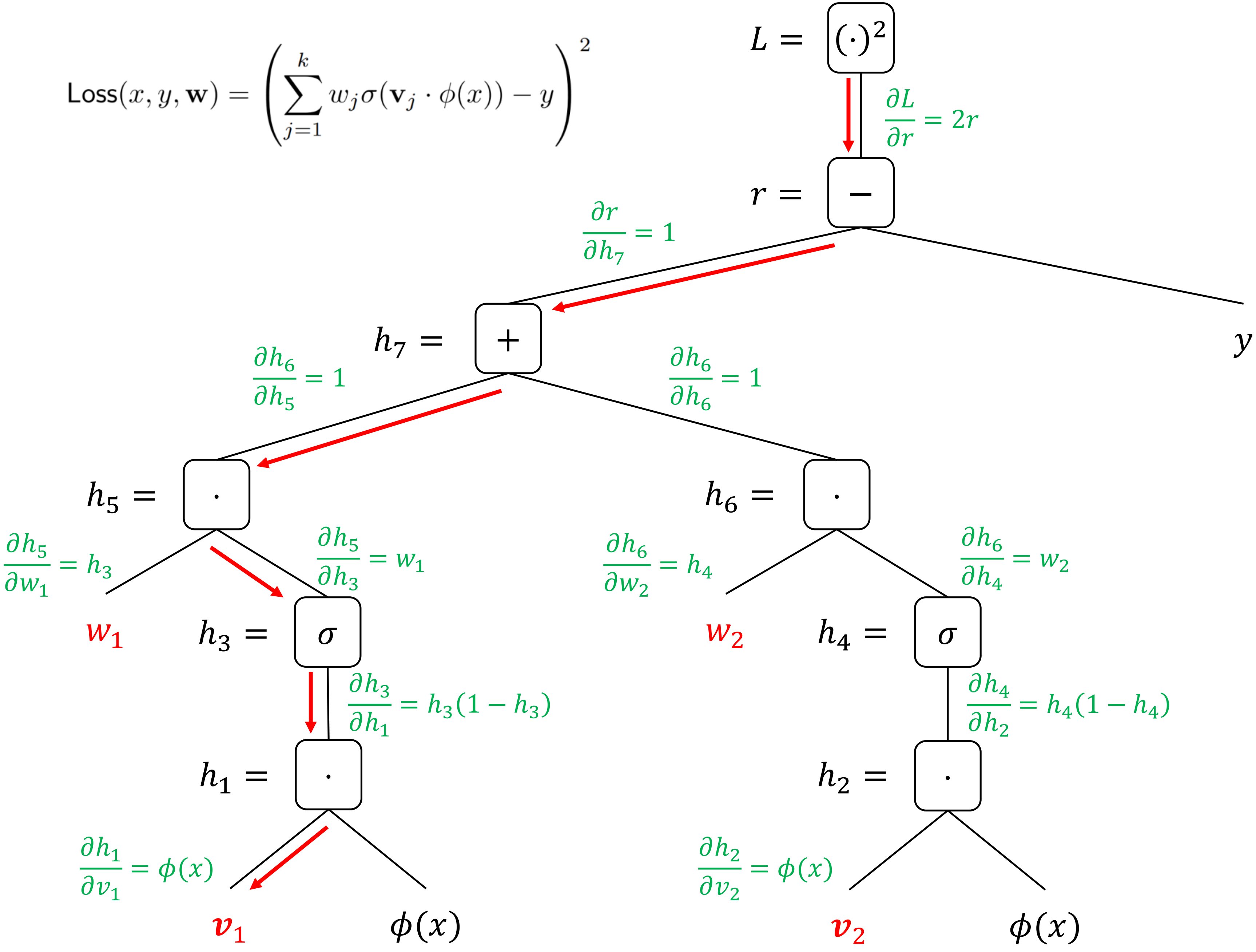
$$\text{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$



$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial r} \cdot \frac{\partial r}{\partial h_7} \cdot \frac{\partial h_7}{\partial h_5} \cdot \frac{\partial h_5}{\partial w_1}$$

$$\frac{\partial L}{\partial w_1} = (2r)(1)(1)(h_3)$$

$$\text{Loss}(x, y, \mathbf{w}) = \left(\sum_{j=1}^{k} w_j \sigma(\mathbf{v}_j \cdot \phi(x)) - y \right)^2$$



$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial r} \cdot \frac{\partial r}{\partial h_7} \cdot \frac{\partial h_7}{\partial h_5} \cdot \frac{\partial h_5}{\partial w_1}$$

$$\frac{\partial L}{\partial w_1} = (2r)(1)(1)(h_3)$$

$$\frac{\partial L}{\partial v_1} = \frac{\partial L}{\partial r} \cdot \frac{\partial r}{\partial h_7} \cdot \frac{\partial h_7}{\partial h_5} \cdot \frac{\partial h_5}{\partial h_3} \cdot \frac{\partial h_3}{\partial h_1} \cdot \frac{\partial h_1}{\partial v_1}$$

$$\frac{\partial L}{\partial w_1} = (2r)(1)(1)(w_1)(h_3(1 - h_3))(\phi(x))$$

Backpropagation

Backprop: http://cs231n.github.io/optimization-2/

Vector, Matrix, and Tensor Derivatives: http://cs231n.Stanford.edu/vecDerivs.pdf

Nearest Neighbors

Nearest neighbors

Algorithm: nearest neighbors

Training: just store $\mathcal{D}_{\text{train}}$

Predictor $f(x')$:

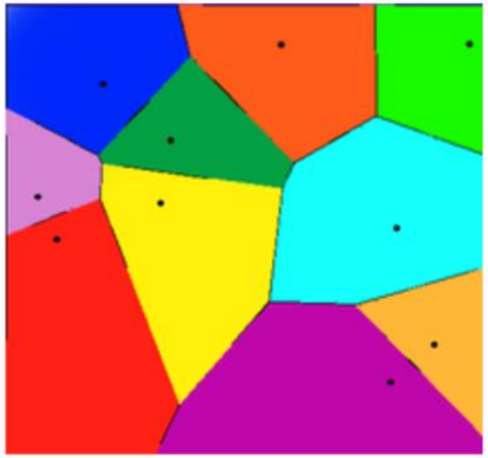
- Find $(x, y) \in \mathcal{D}_{\text{train}}$ where $\|\phi(x) - \phi(x')\|$ is smallest
- Return y

Key idea: similarity

Similar examples tend to have similar outputs.

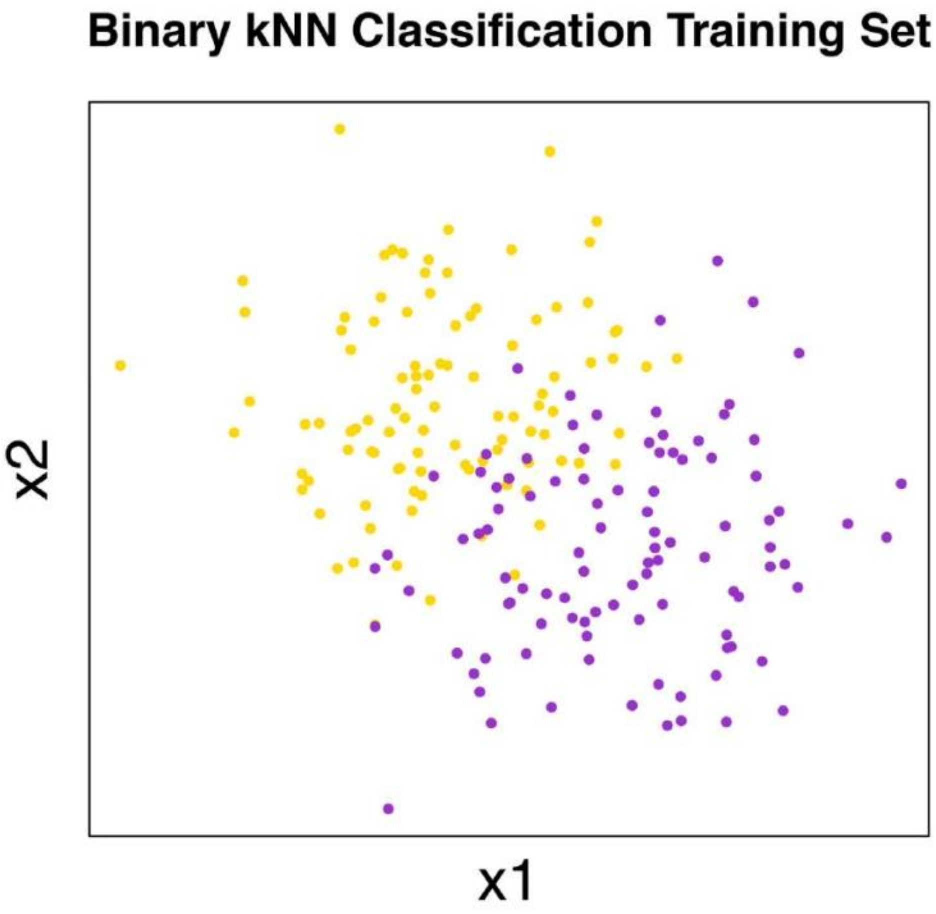
Expressivity of nearest neighbors

Decision boundary: based on Voronoi diagram



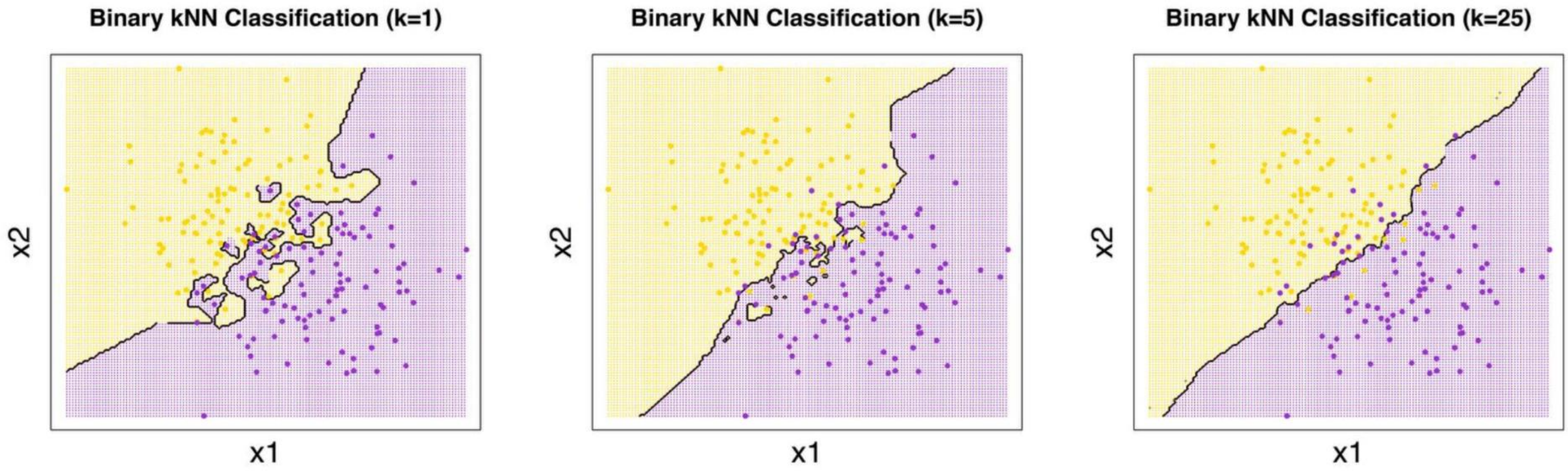
- Much more expressive than quadratic features
- **Non-parametric**: the hypothesis class adapts to number of examples
- Simple and powerful, but kind of brute force

k-nearest neighbors

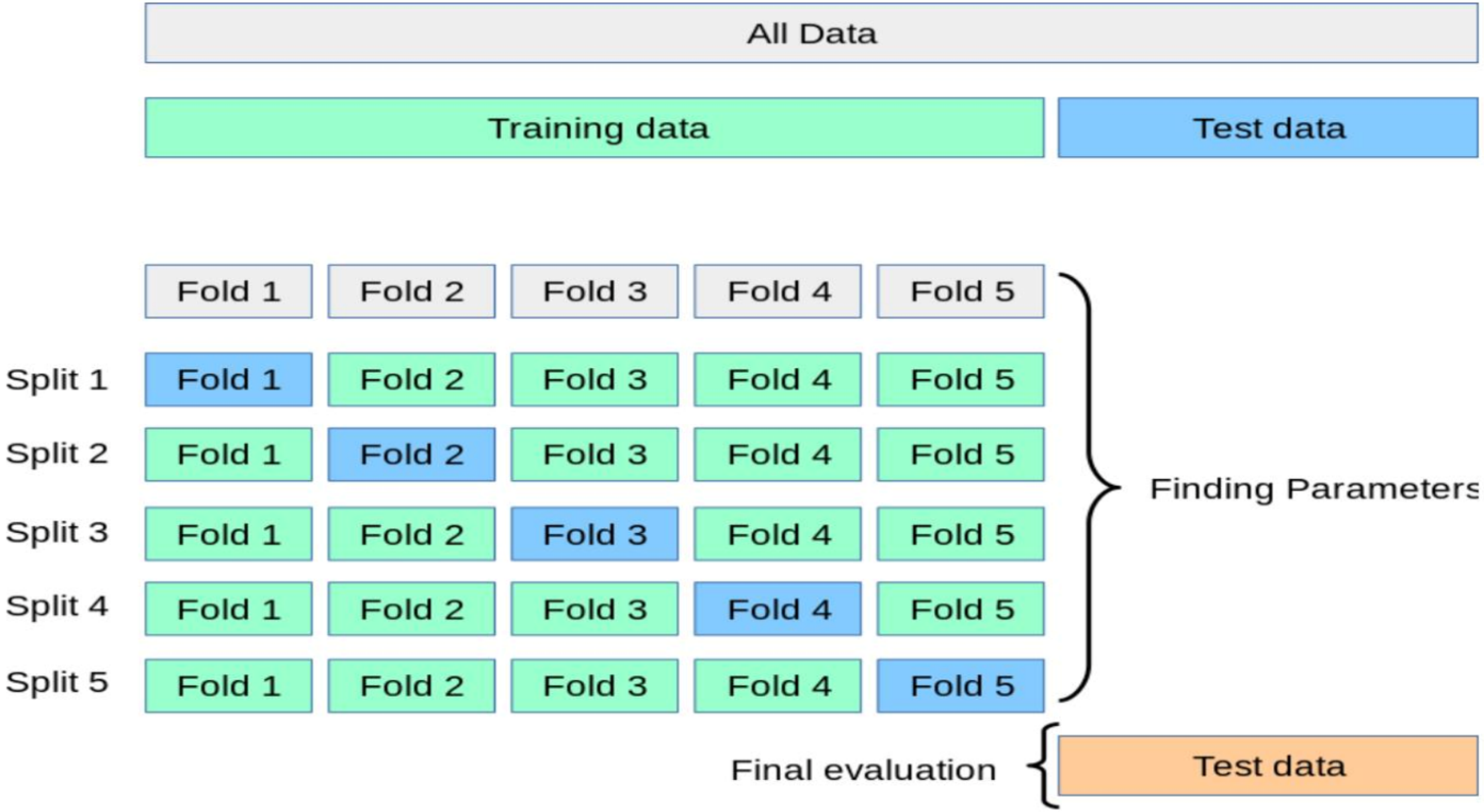


k-nearest neighbors

Effect of *k*:



k-fold cross-validation



https://scikit-learn.org/stable/modules/cross_validation.html

Scikit-learn Tutorial

[Switch to ipython notebook]