CS221 Section 3: Search

DP, UCS and A*

Contents

1. **Uniform Cost Search**
2. Defining States
3. Dynamic Programming
4. A* Search

Uniform Cost Search

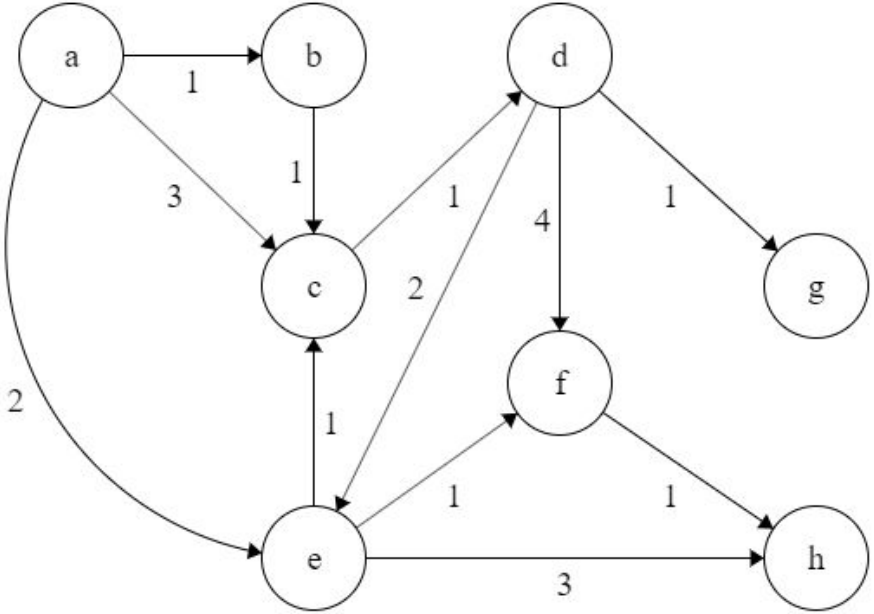
Idea: In UCS, we find the shortest cost to a node by using the fact we already know the shortest path to a set of nodes.

Recall: We have the following three sets

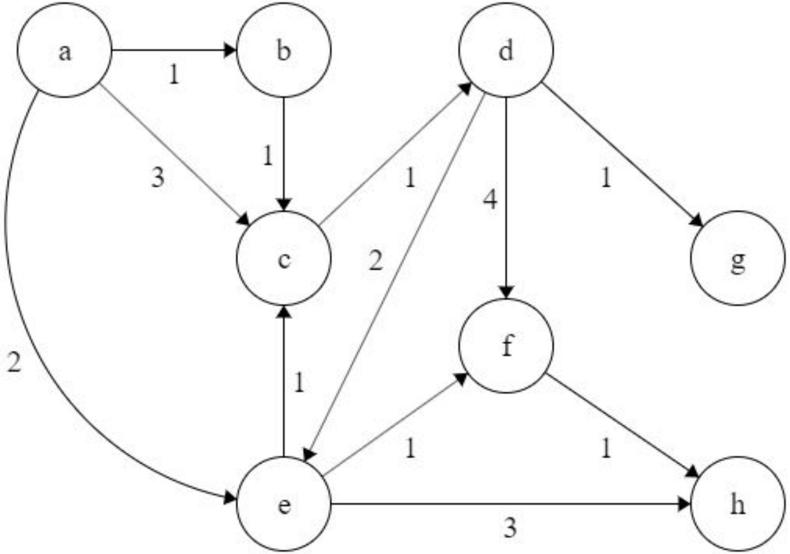
- Explored Set: contains nodes we know the path length to
- Frontier Set: contains nodes that are neighbors of those in the explored set, but we don't know their costs yet
- Unexplored Set: Nodes in the graph we haven't encountered

Problem - UCS

In the following graph, find the costs to reach each node given that we start on node **a**.



Problem - UCS



Explored

[a : 0]

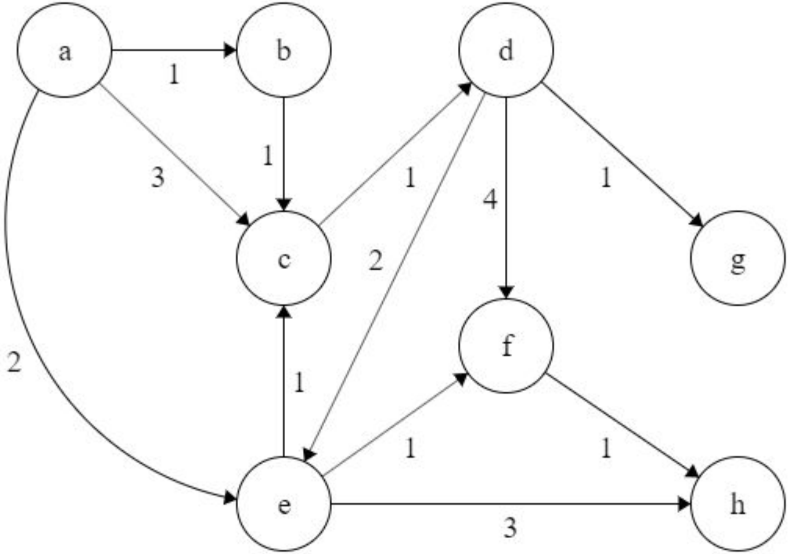
Frontier

[b : 0 + 1, e : 0 + 2, c : 0 + 3]

Unexplored

We start with node **a**. We add all neighbors of **a** to the frontier. Note: [a : 0] means it takes 0 cost to get to node a.

Problem - UCS



Explored

[a : 0, **b : 1**]

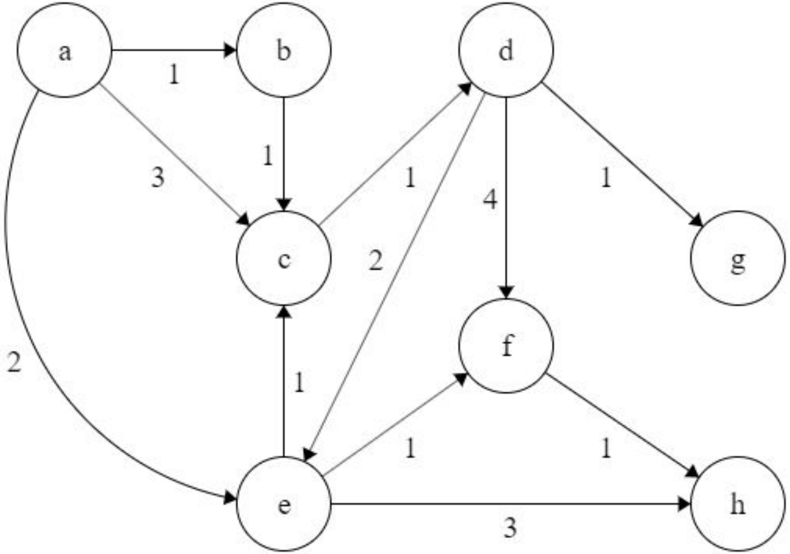
Frontier

[**c : 1 + 1**, e : 0 + 2]

Unexplored

In the frontier, **b** has the lowest cost. Thus, we can add it to the explored set. We add all neighbors of **b** to the frontier, updating costs to reach some nodes if necessary (we updated **c**).

Problem - UCS



Explored

[a : 0, b : 1, **c : 2**]

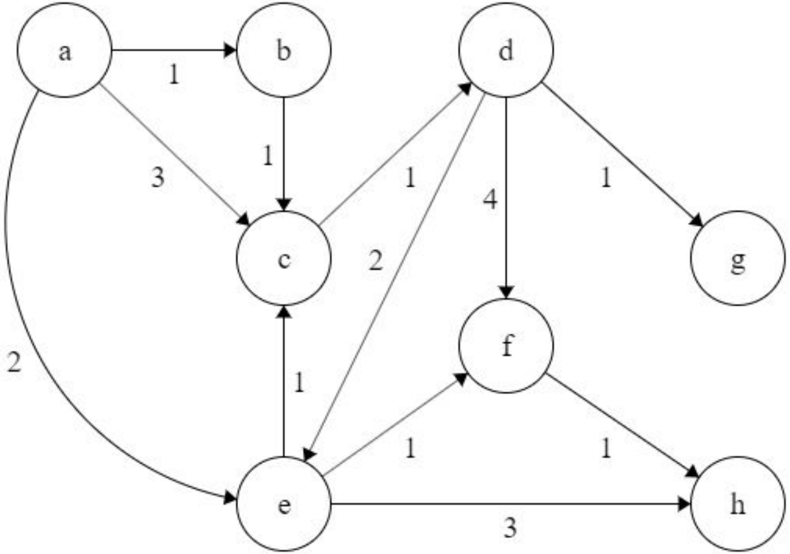
Frontier

[e : 0 + 2, **d : 2 + 1**]

Unexplored

In the frontier, **c** has the lowest cost (ties broken alphabetically here). Thus, we can add it to the explored set. We add all neighbors of **c** to the frontier, updating as necessary.

Problem - UCS



Explored

[a : 0, b : 1, c : 2, **e : 2**]

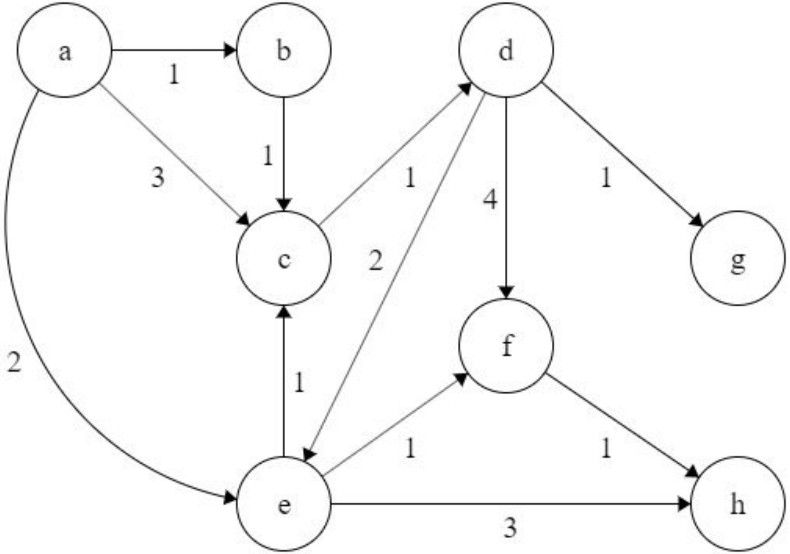
Frontier

[d : 2 + 1, **f : 2 + 1, h : 2 + 3**]

Unexplored

In the frontier, **e** has the lowest cost. Thus, we can add it to the explored set. We add all neighbors of **e** to the frontier, updating as necessary.

Problem - UCS



Explored

[a : 0, b : 1, c : 2, e : 2, **d : 3**]

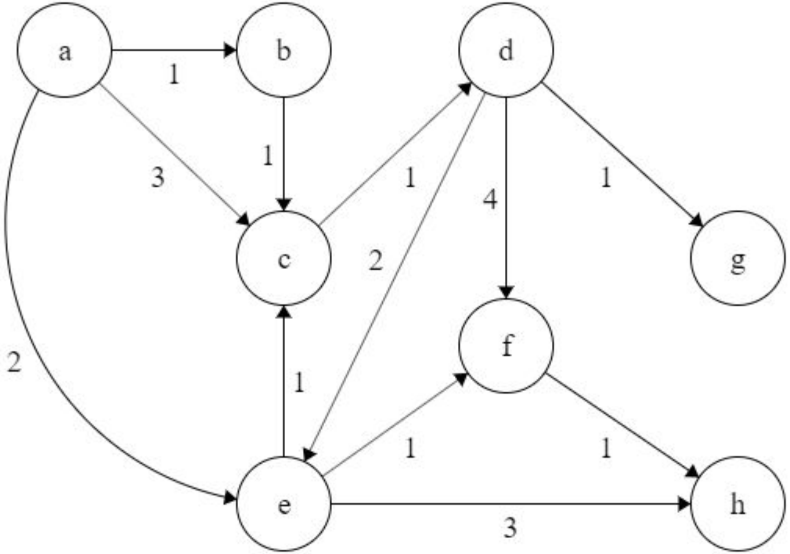
Frontier

[f : 2 + 1, **g : 3 + 1,** h : 2 + 3]

Unexplored

In the frontier, **d** has the lowest cost. Thus, we can add it to the explored set. We add all neighbors of **d** to the frontier, updating as necessary.

Problem - UCS



Explored

[a : 0, b : 1, c : 2, e : 2, d : 3, **f : 3**]

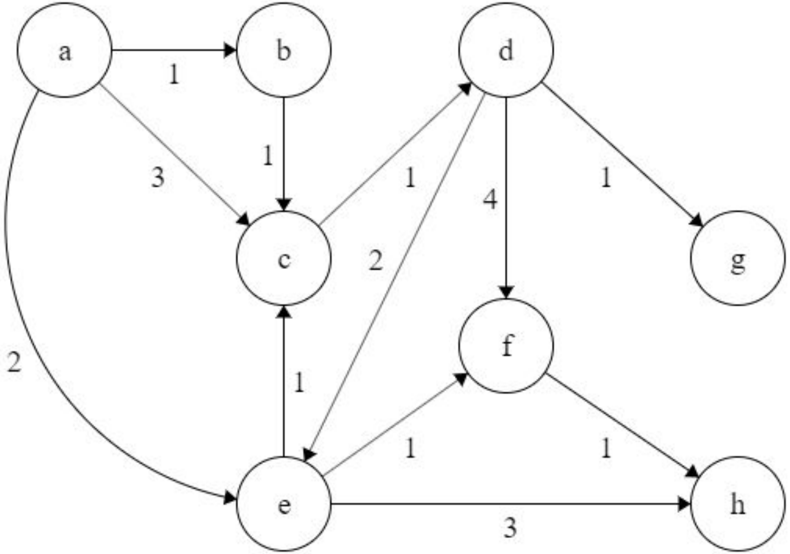
Frontier

[g : 3 + 1, **h : 3 + 1**]

Unexplored

In the frontier, **f** has the lowest cost. Thus, we can add it to the explored set. We add all neighbors of **f** to the frontier, updating as necessary.

Problem - UCS



Explored

[a : 0, b : 1, c : 2, e : 2, d : 3, f : 3, **g : 4**]

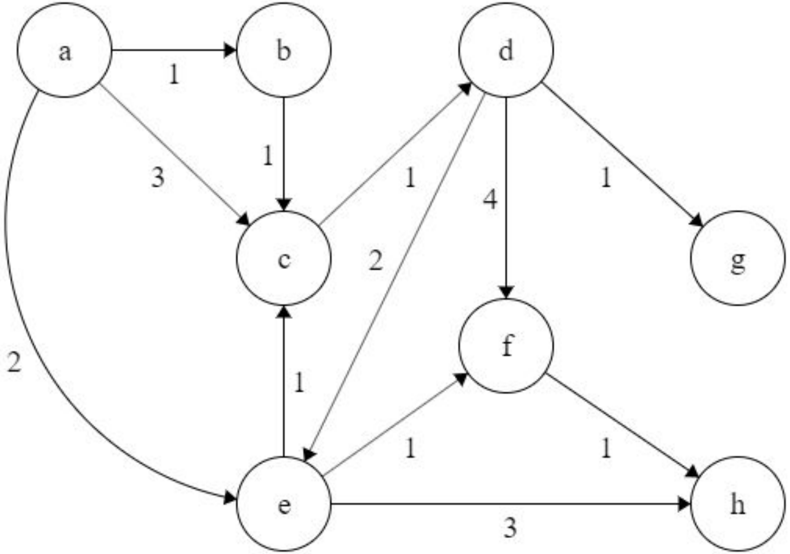
Frontier

[h : 3 + 1]

Unexplored

In the frontier, **g** has the lowest cost. Thus, we can add it to the explored set. We add all neighbors of **f** to the frontier, updating as necessary.

Problem - UCS



Explored

[a : 0, b : 1, c : 2, e : 2, d : 3, f : 3, g : 4, **h : 4**]

Frontier

Unexplored

In the frontier, **h** has the lowest cost. Thus, we can add it to the explored set. There are no more nodes in the frontier, so we are done.

Uniform Cost Search

Algorithm: uniform cost search [Dijkstra, 1956]

Add s_{start} to **frontier** (priority queue)
Repeat until frontier is empty:
    Remove s with smallest priority p from frontier
    If $\text{IsEnd}(s)$: return solution
    Add s to **explored**
    For each action $a \in \text{Actions}(s)$:
        Get successor $s' \leftarrow \text{Succ}(s, a)$
        If s' already in explored: continue
        Update **frontier** with s' and priority $p + \text{Cost}(s, a)$

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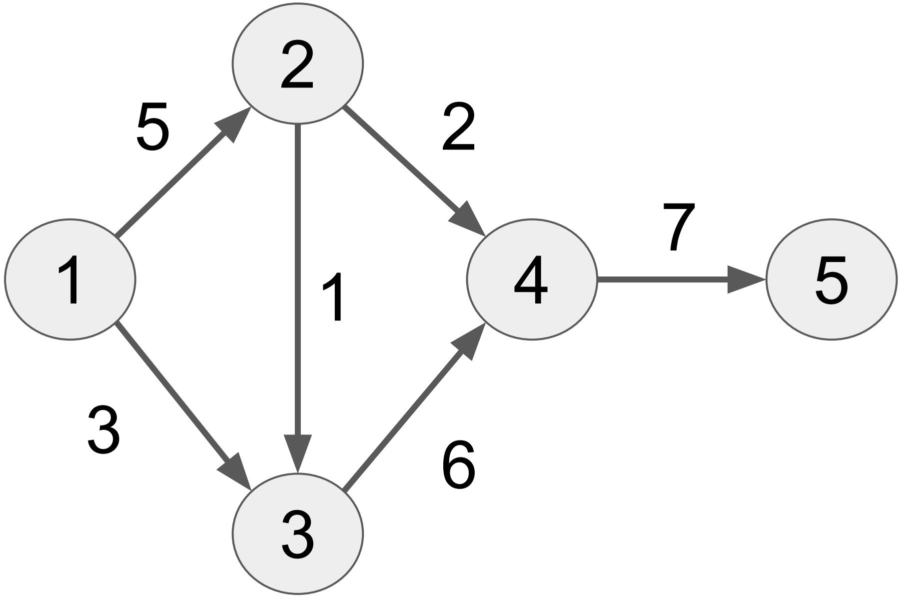
Problem

There exists $\mathbf{N}$ cities, conveniently labelled from $\mathbf{1}$ to $\mathbf{N}$.

There are roads connecting some pairs of cities. The road connecting city $\mathbf{i}$ and city $\mathbf{j}$ takes $\mathbf{c(i,j)}$ time to traverse. However, one can only travel from a city with smaller label to a city with larger label (i.e. each road is one-directional).

From city $\mathbf{1}$, we want to travel to city $\mathbf{N}$. What is the shortest time required to make this trip, given the additional constraint that we should visit more odd-labeled cities than even labeled cities?

Example



Best path is [1, 3, 4, 5] with cost 16.

[1, 2, 4, 5] has cost 14 but visits equal number of odd and even cities.

State Representation

Key idea: state

A **state** is a summary of all the past actions sufficient to choose future actions **optimally**.

State Representation

We need to know where we are currently at: **current_city**

We need to know how many odd and even cities we have visited thus far: **#odd, #even**

State Representation: **(current_city, #odd, #even)**

Total number of states: $O(N^3)$

Can We Do Better?

Check if all the information is really required

We store **#odd** and **#even** so that we can check whether **#odd** - **#even** > 0 at (N, **#odd**, **#even**)

Why not store **#odd - #even** directly instead?

(current_city, #odd - #even) -- $\mathbf{O(N^2)}$ states

Solving the Problem

Since we are computing shortest path, which is some form of optimization, we consider DP and UCS.

Recall:

- DP can handle negative edges but works only on DAGs
- UCS works on general graphs, but cannot handle negative edges

Since we have a DAG and all edges are positive, both work! We already went through UCS, so we solve this with DP.

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Solving the Problem: Dynamic Programming

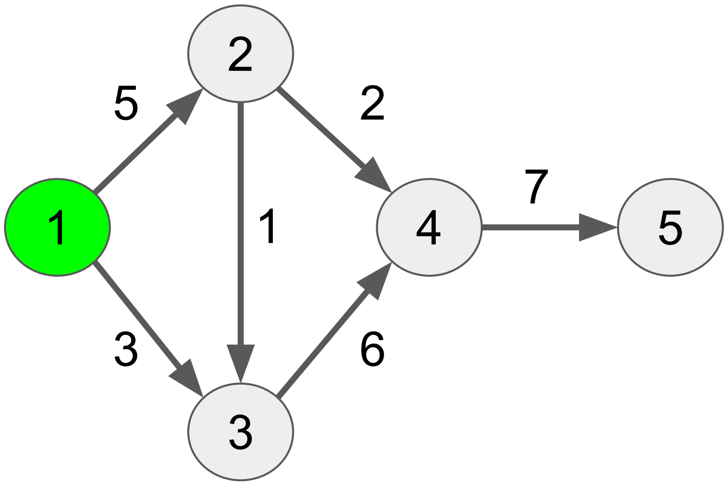
$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s, a) + \text{FutureCost}(\text{Succ}(s, a))] & \text{otherwise} \end{cases}$$

If s has no successors, we set it as undefined

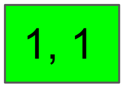
Simulation of DP

Visiting	Successors	Completed

Regular Graph



State Graph



Cache	
Key	Value

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s, a) + \text{FutureCost}(\text{Succ}(s, a))] & \text{otherwise} \end{cases}$$

Simulation of DP

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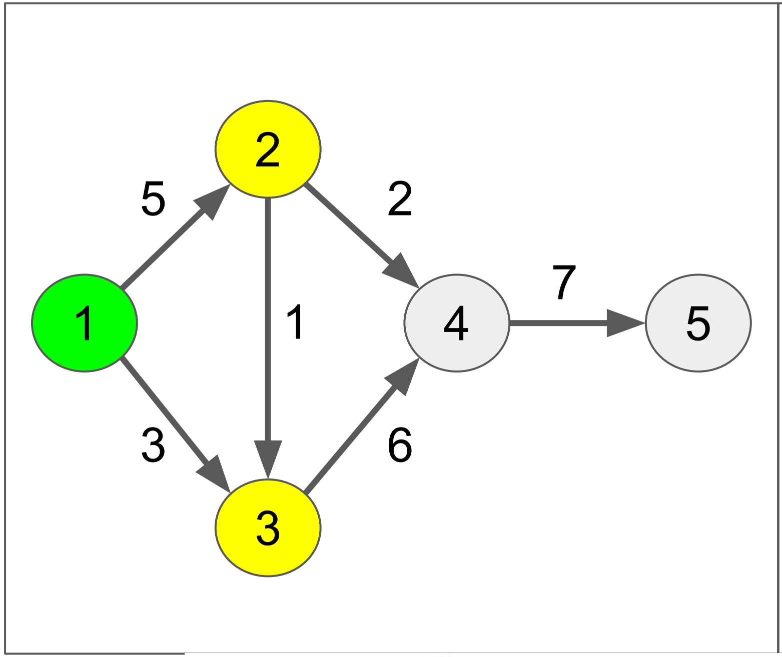
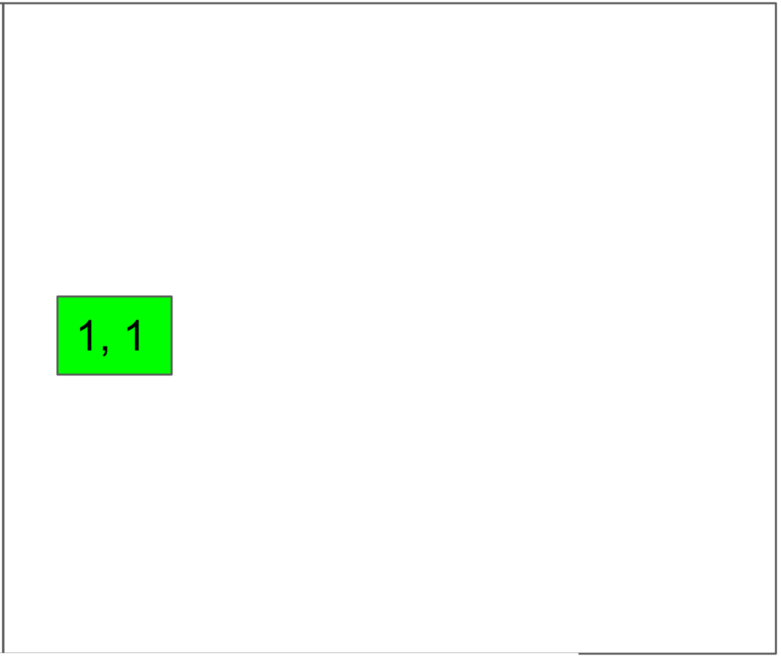
Regular Graph



State Graph



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Simulation of DP

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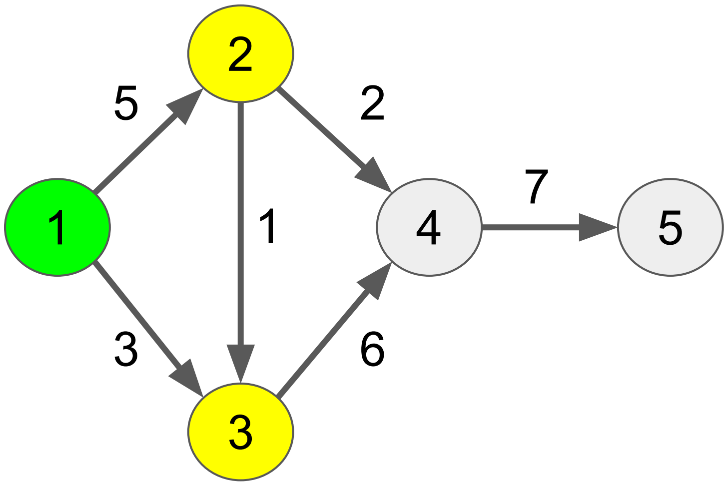

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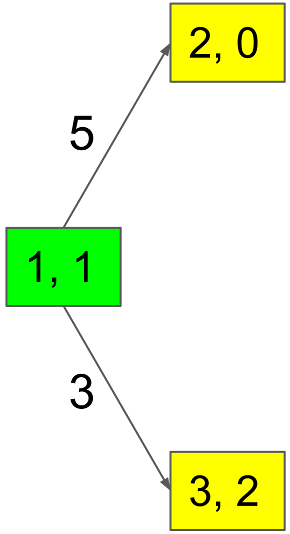


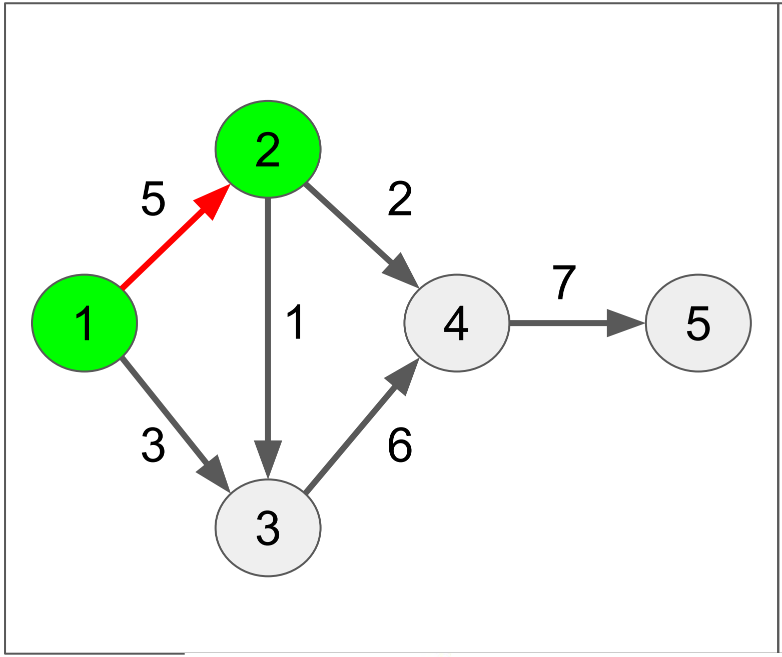
Cache	
Key	Value

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [{\color{red}\text{Cost}(s,a)} + {\color{blue}\text{FutureCost}(\text{Succ}(s,a))}] & \text{otherwise} \end{cases}$$

Simulation of DP

Visiting	Successors	Completed

Regular Graph



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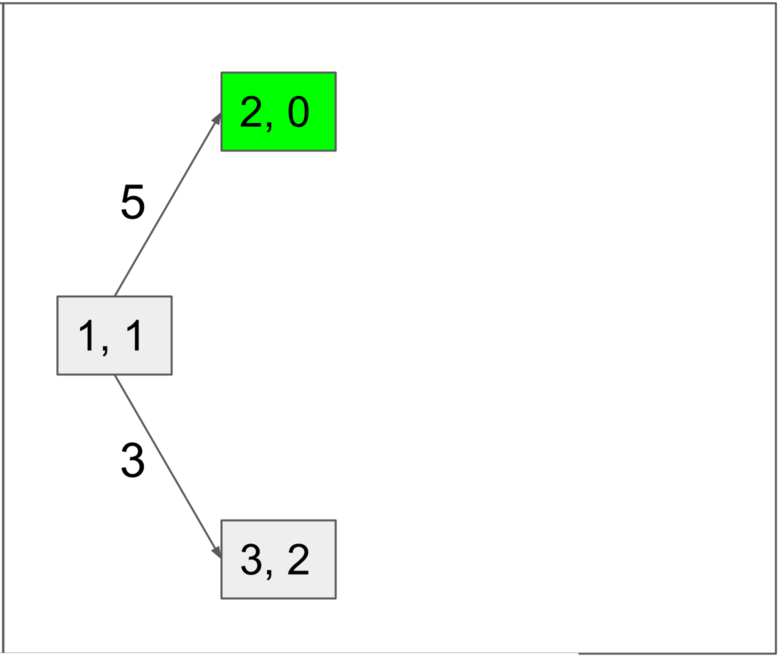

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Simulation of DP

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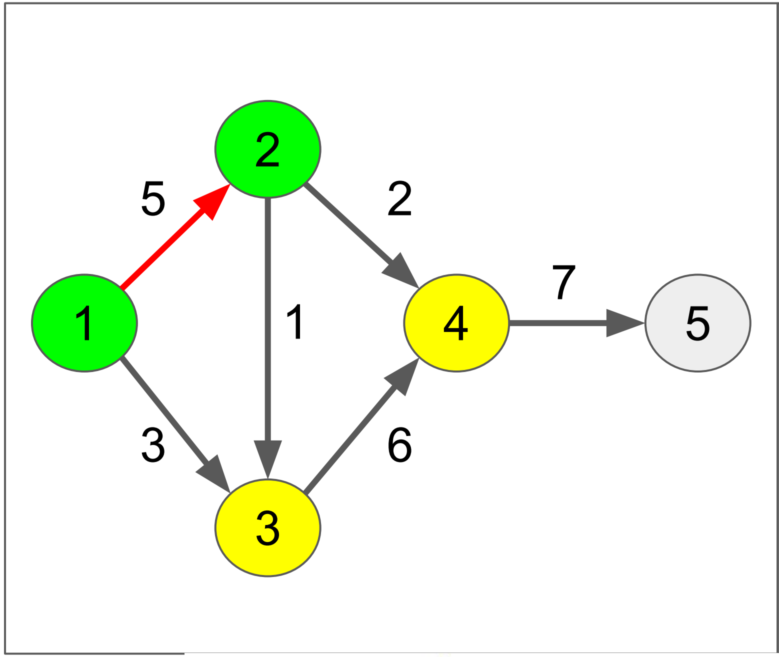
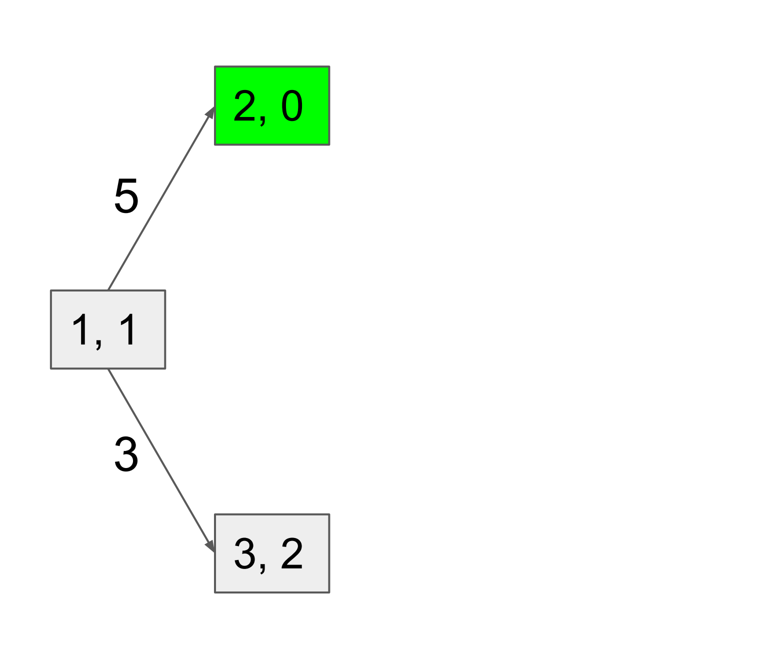
Regular Graph

State Graph





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Simulation of DP

Visiting | **Successors** | **Completed**

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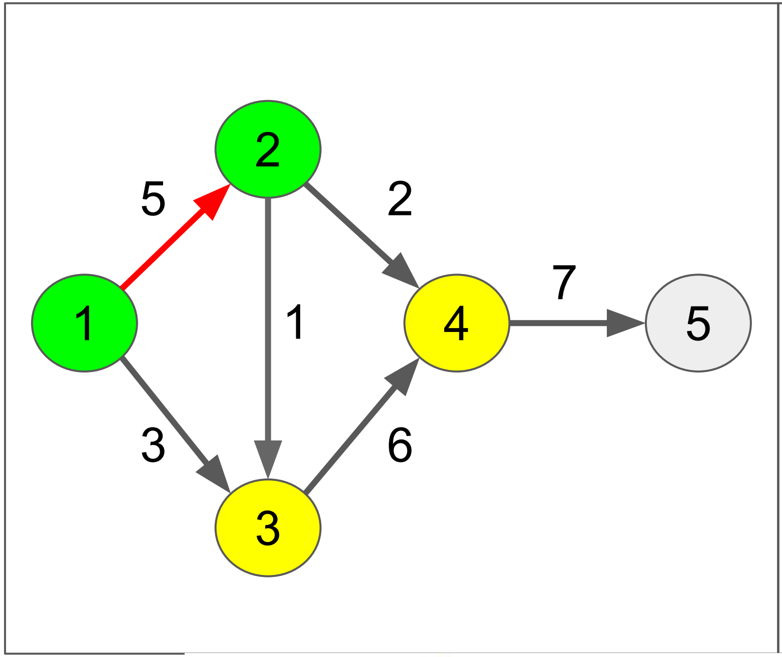

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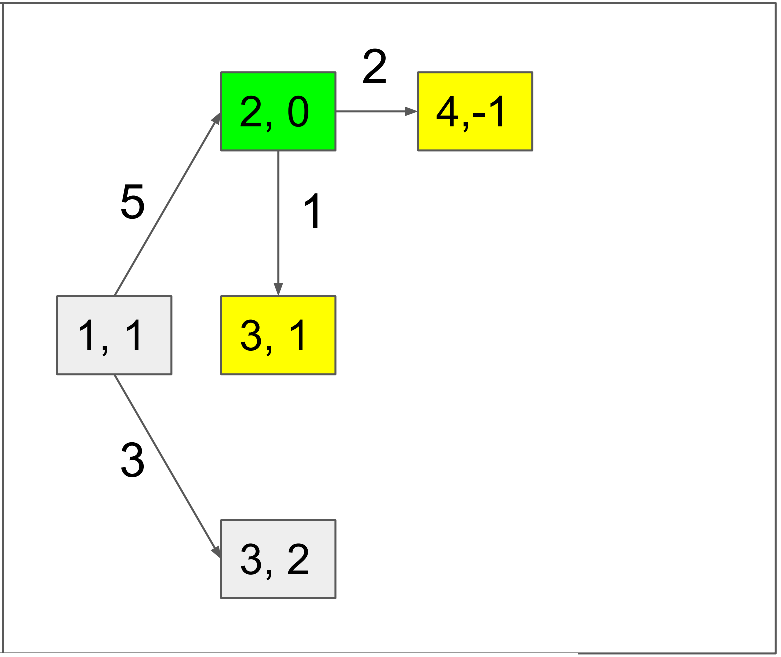

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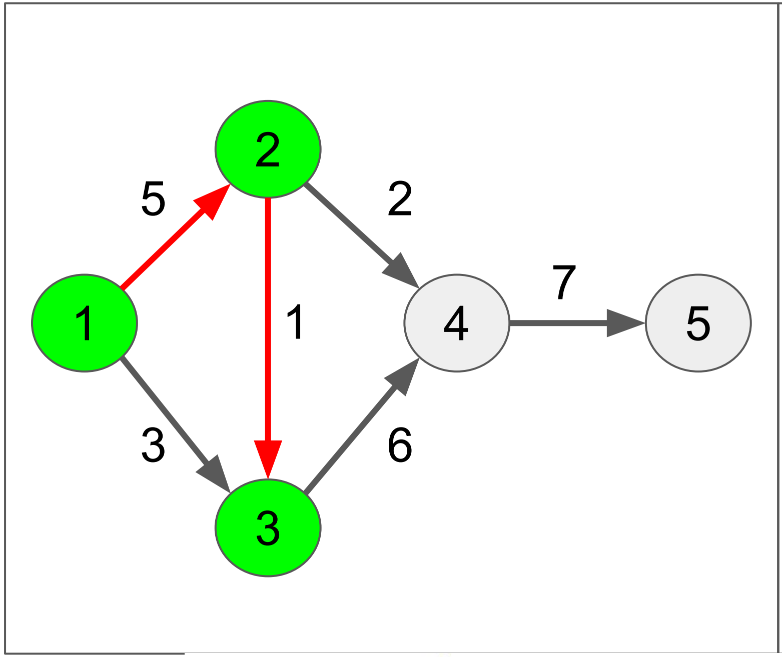

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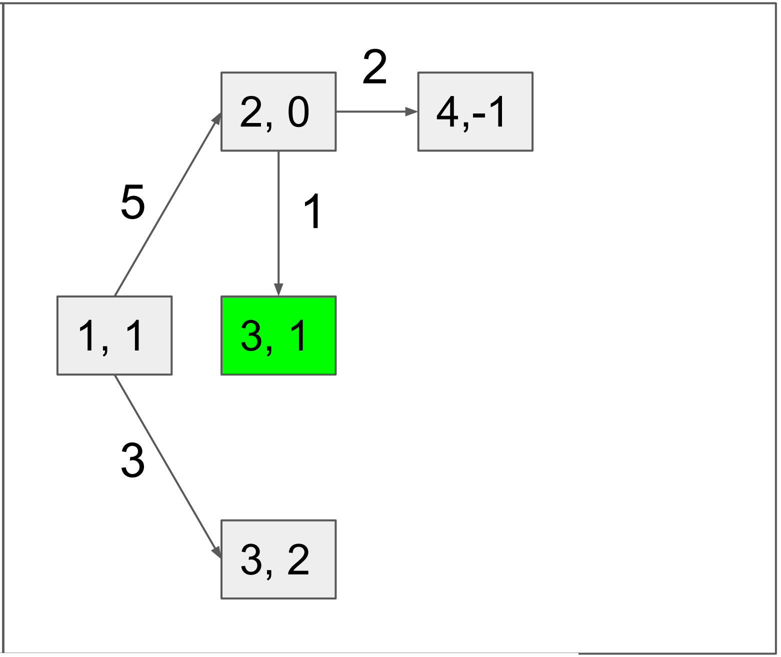

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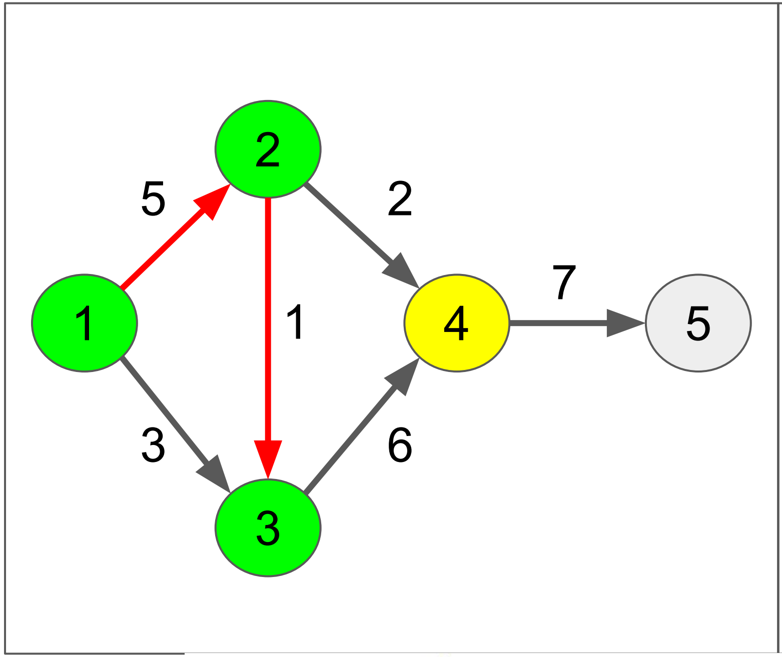

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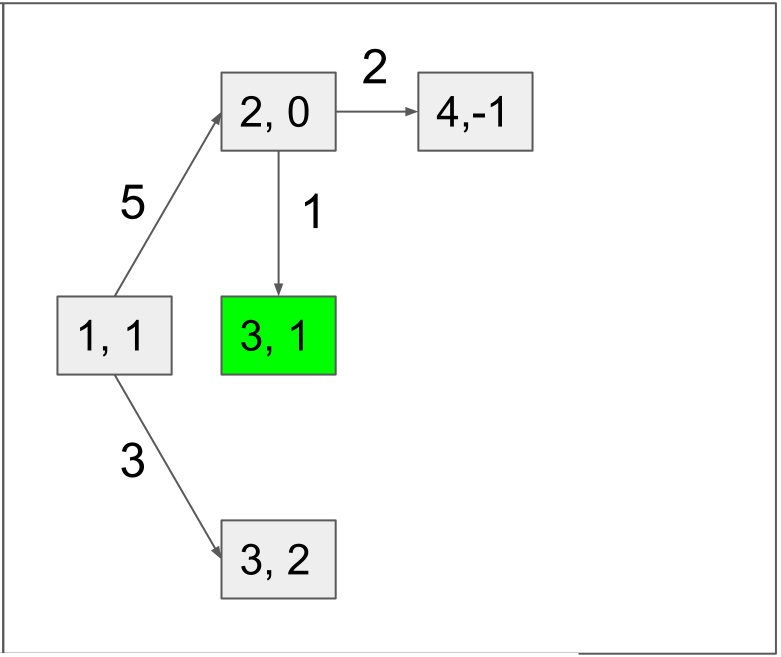

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Simulation of DP

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Cache	
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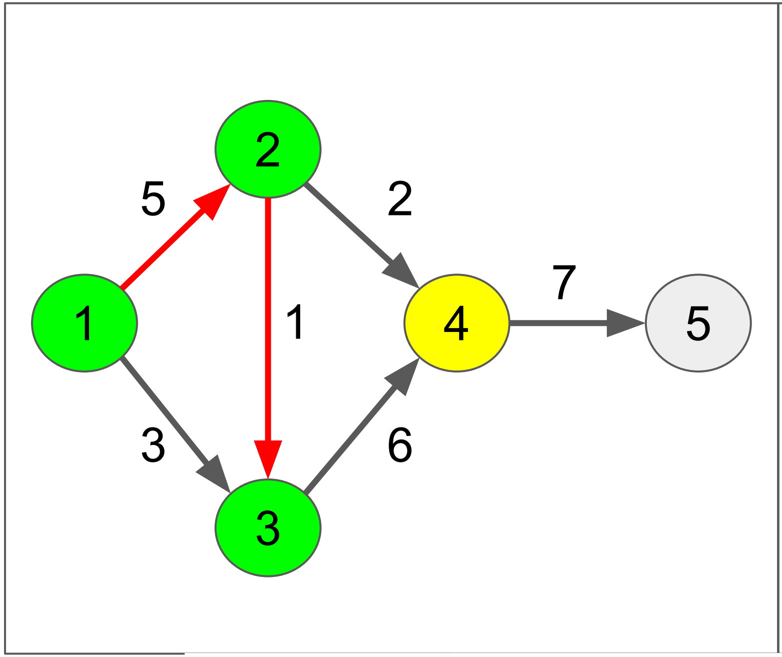
Regular Graph



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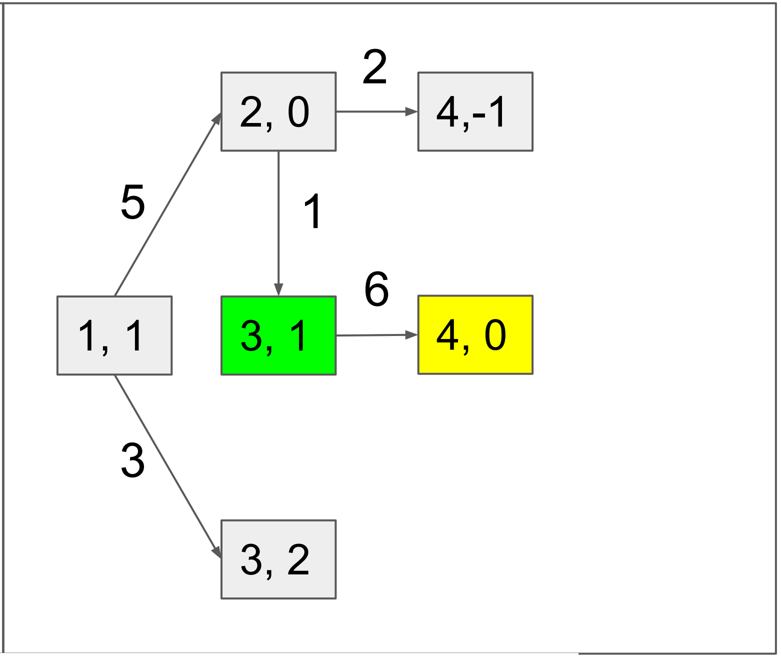

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Simulation of DP

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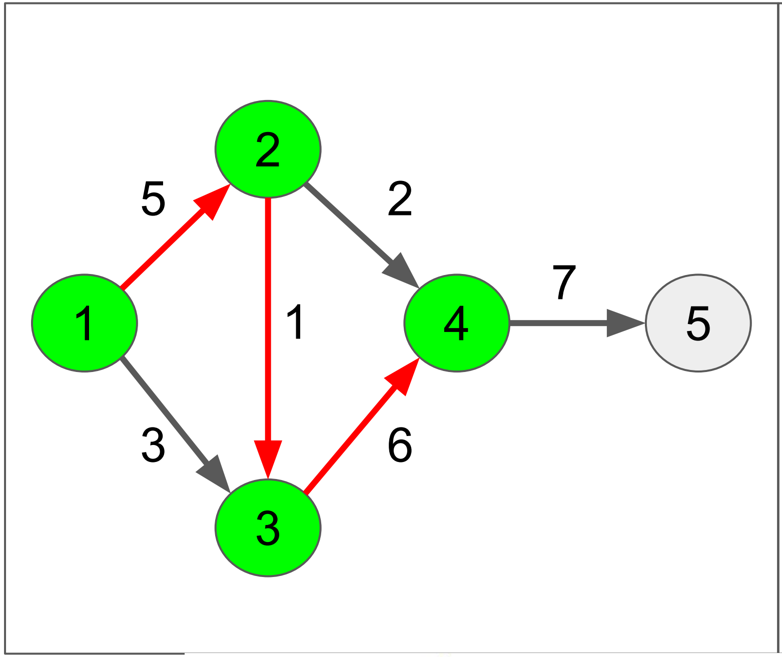
Regular Graph



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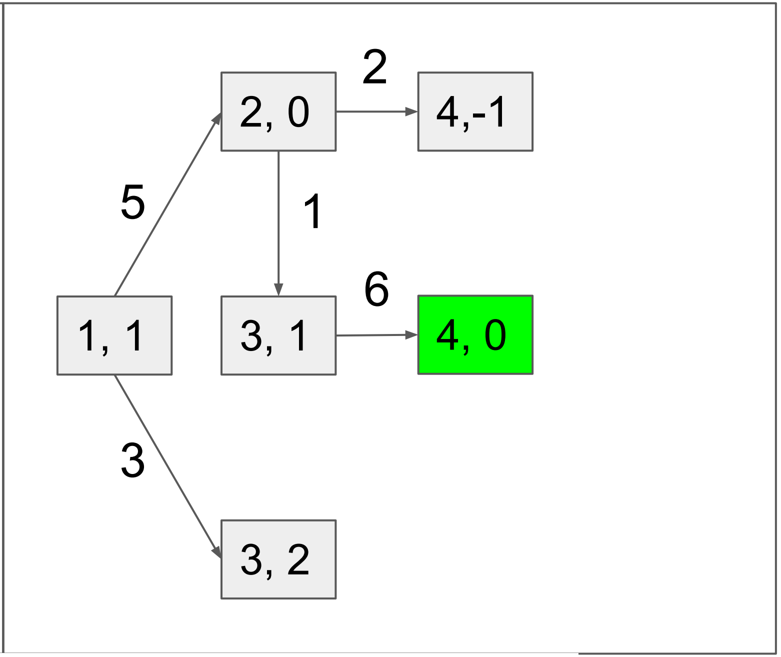


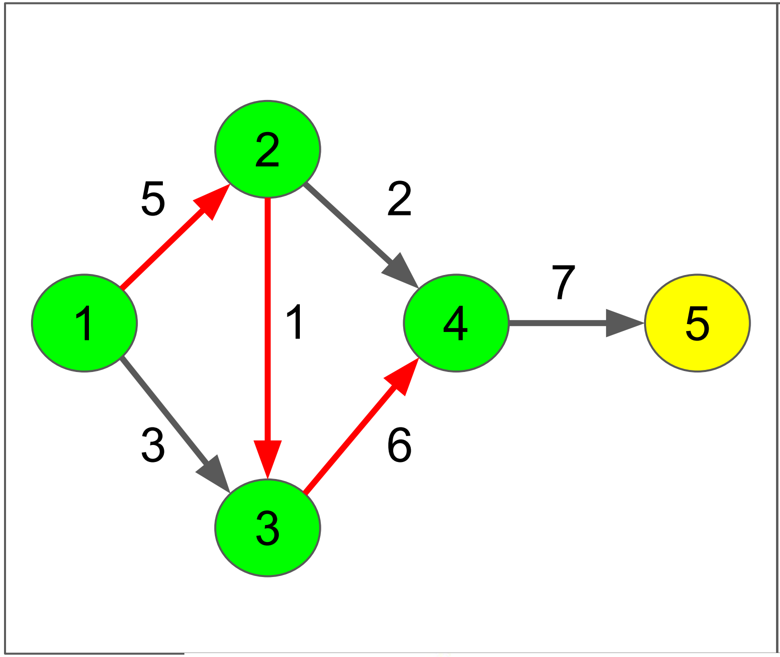
$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if } \text{IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s, a) + \text{FutureCost}(\text{Succ}(s, a))] & \text{otherwise} \end{cases}$$

Cache	
Key	**Value**

Simulation of DP

Visiting	Successors	Completed

Regular Graph



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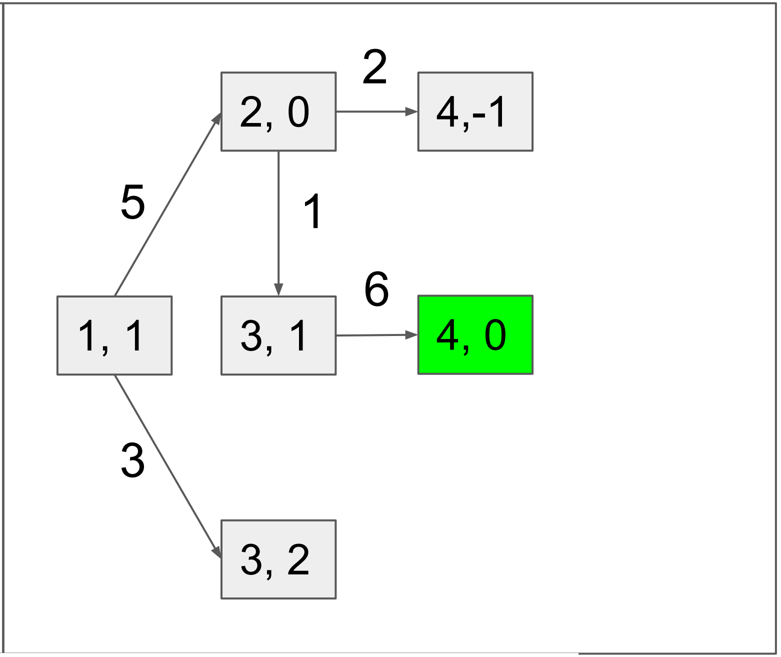

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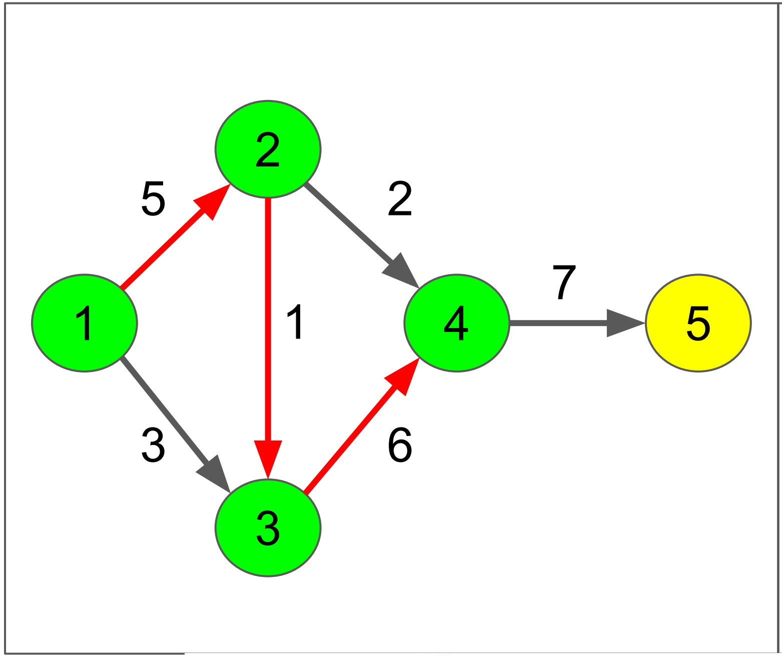

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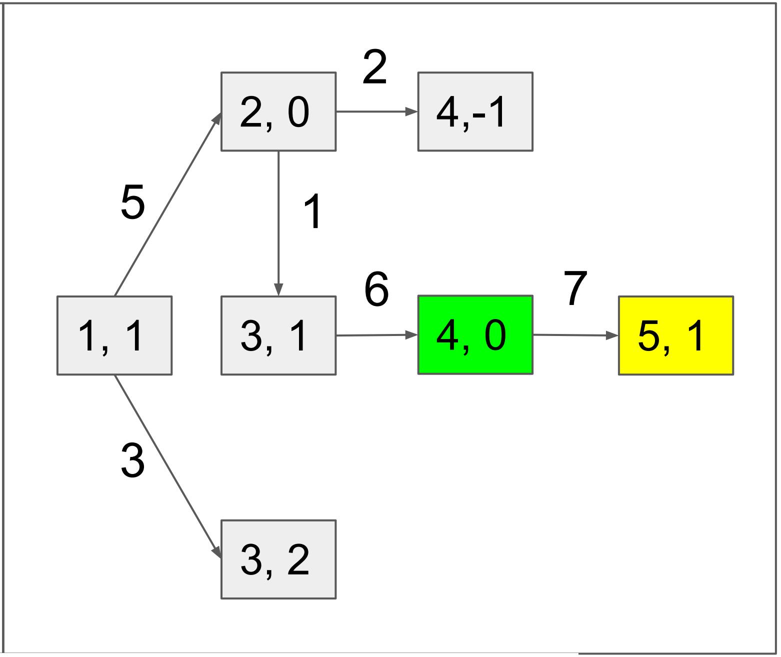

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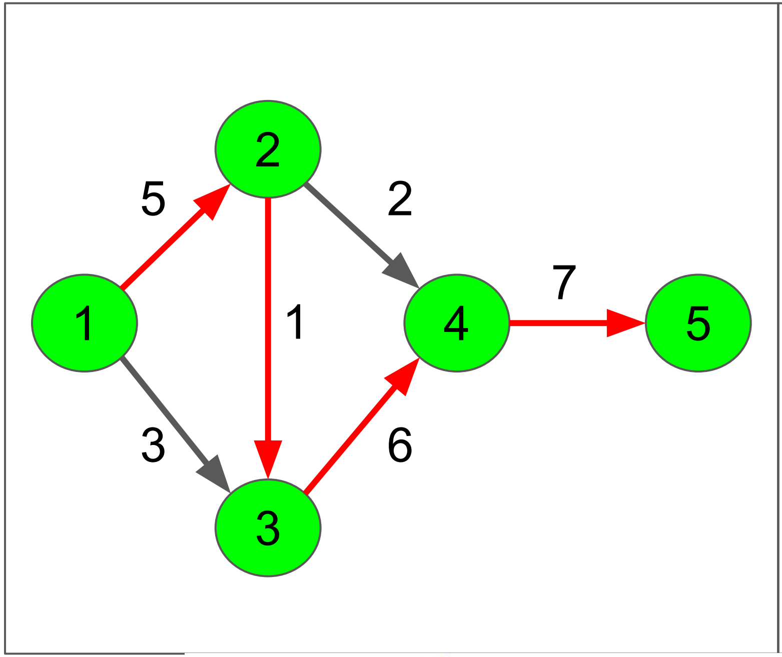
Cache

Key	Value

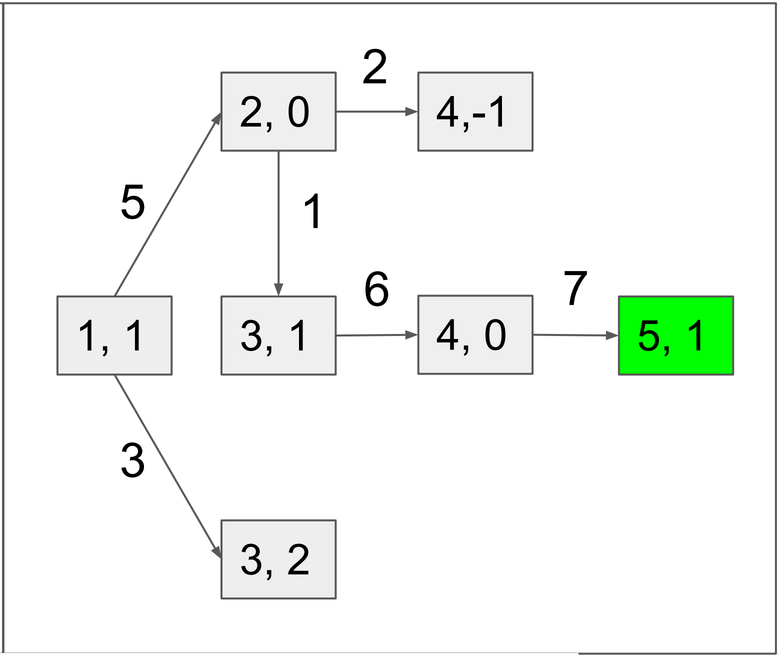
Simulation of DP

Visiting	Successors	Completed

Regular Graph



State Graph



Cache

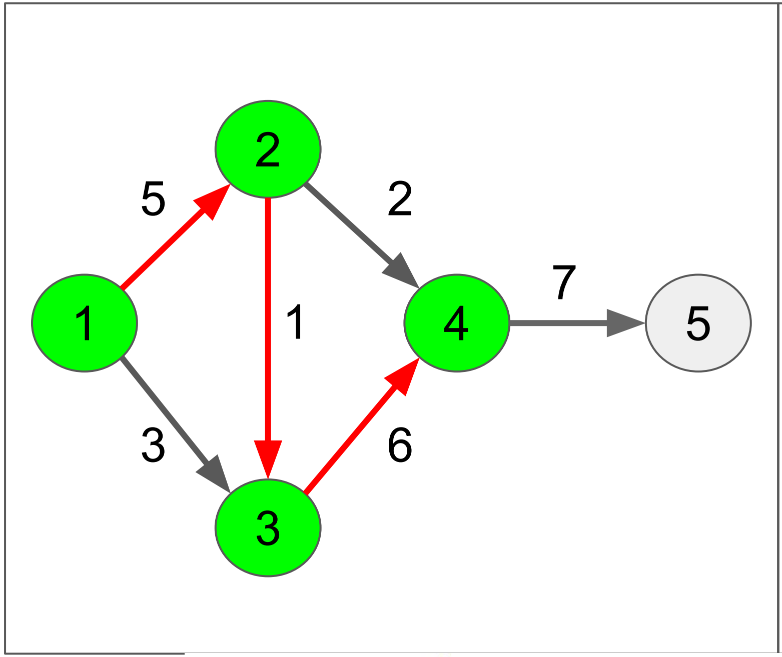
Key	Value
(5, 1)	0

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a\in\text{Actions}(s)}[\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

Visiting	Successors	Completed

Regular Graph



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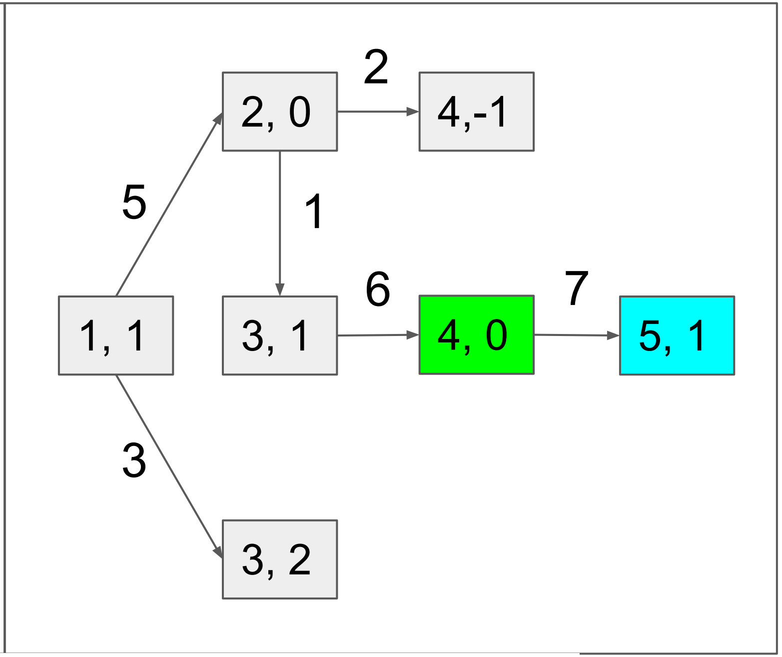


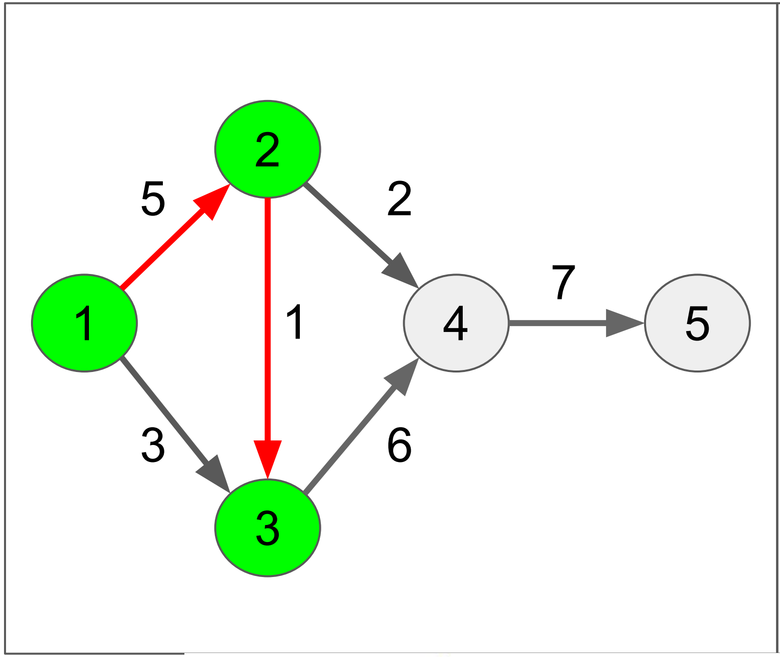
Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s, a) + \text{FutureCost}(\text{Succ}(s, a))] & \text{otherwise} \end{cases}$$

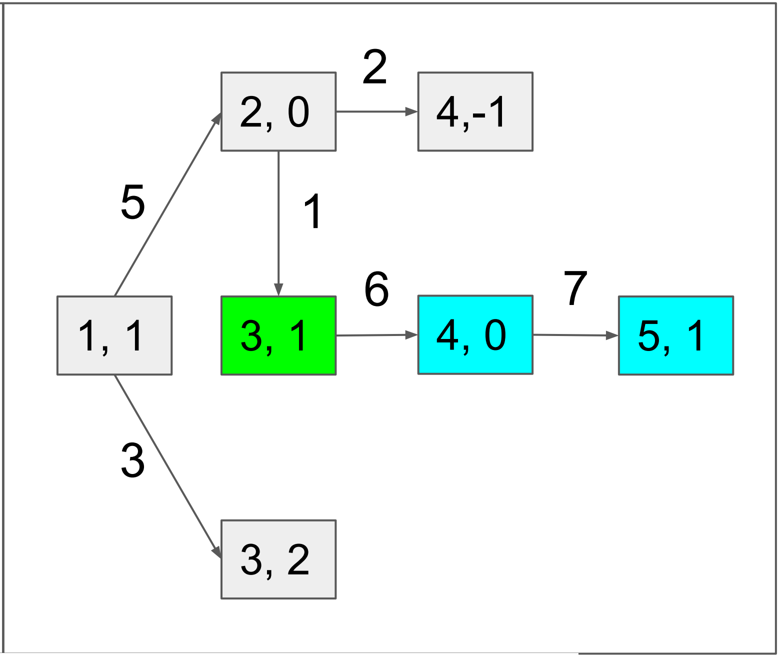
Simulation of DP

Visiting	Successors	Completed

Regular Graph



State Graph



Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7
(3, 1)	13

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

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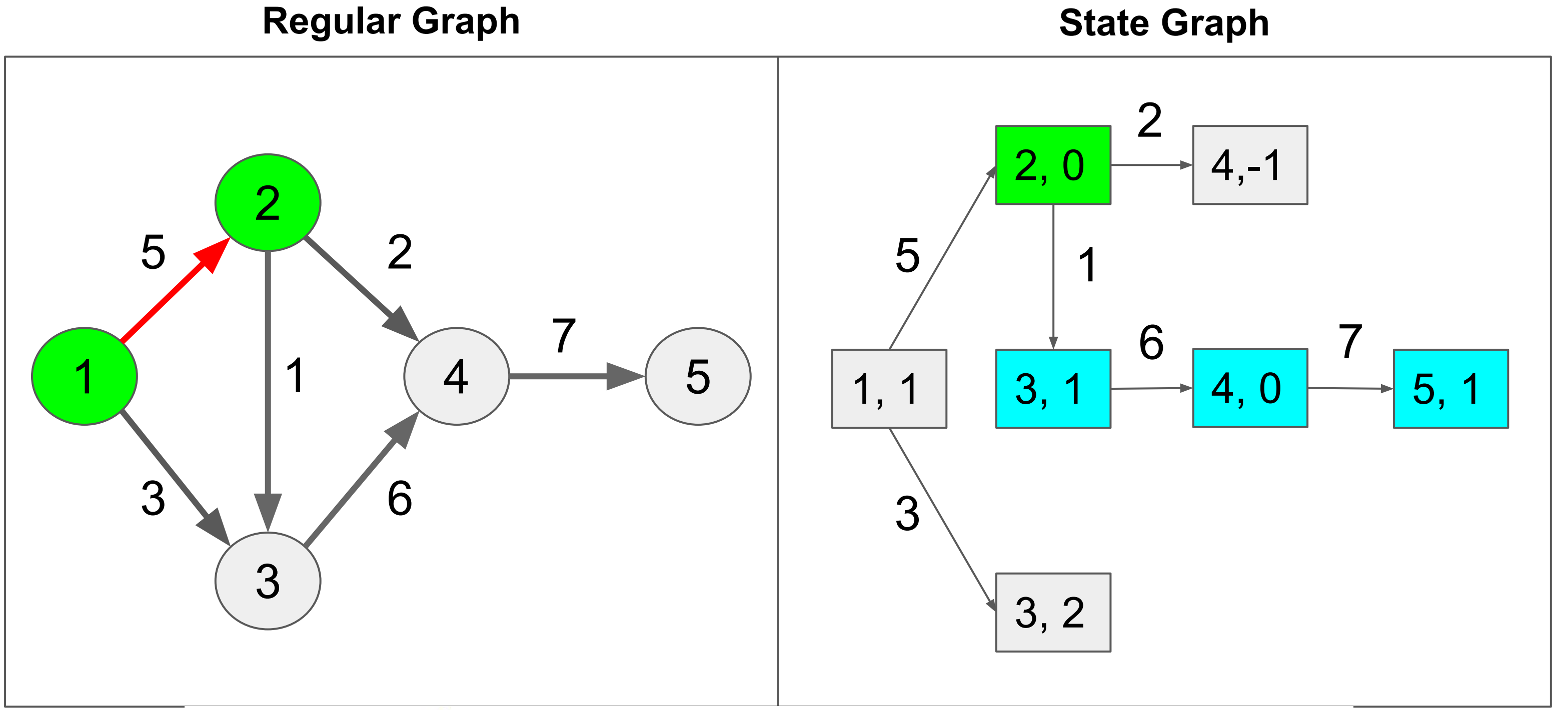


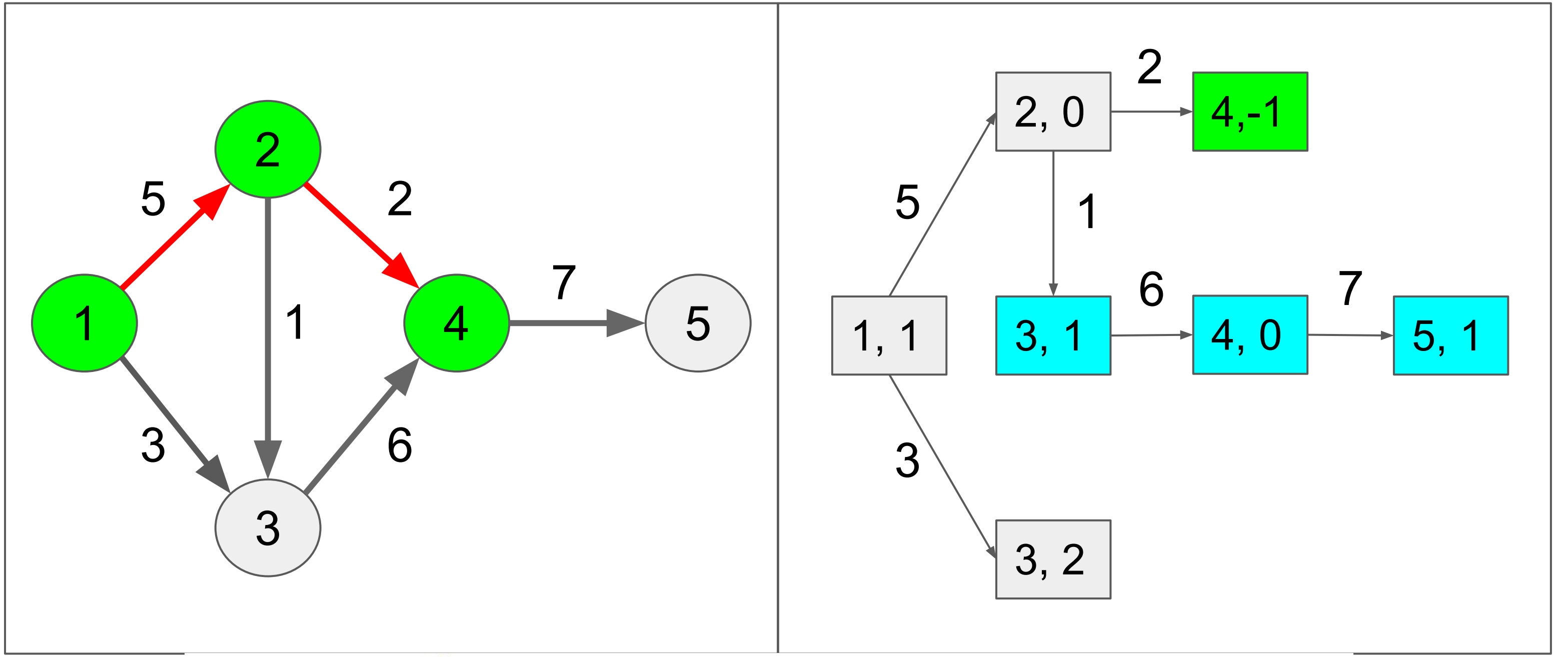
Cache	
Key	Value
(5, 1)	0
(4, 0)	7
(3, 1)	13

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

Visiting	Successors	Completed

Regular Graph **State Graph**

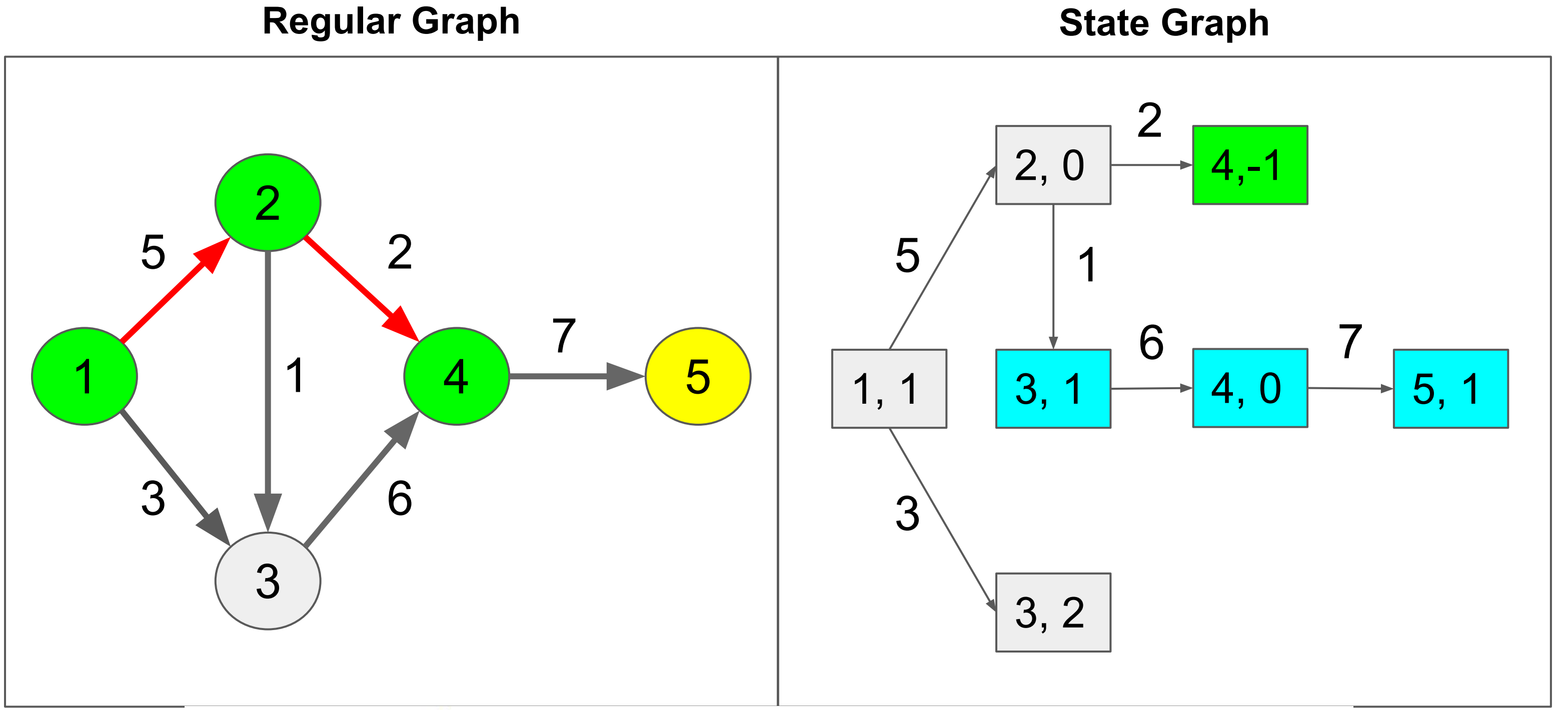


Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7
(3, 1)	13

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

Visiting	Successors	Completed



Cache	
Key	Value
(5, 1)	0
(4, 0)	7
(3, 1)	13

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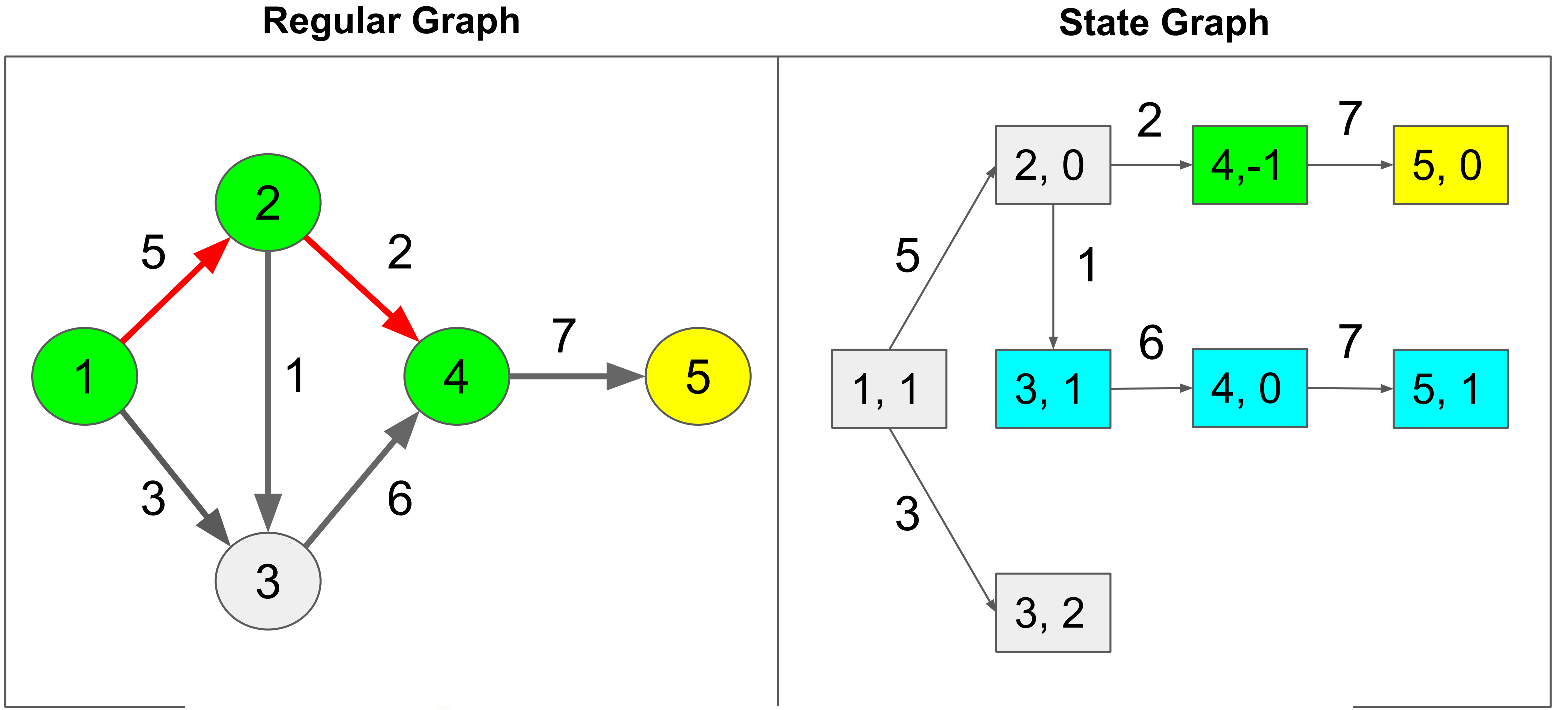

Cache	
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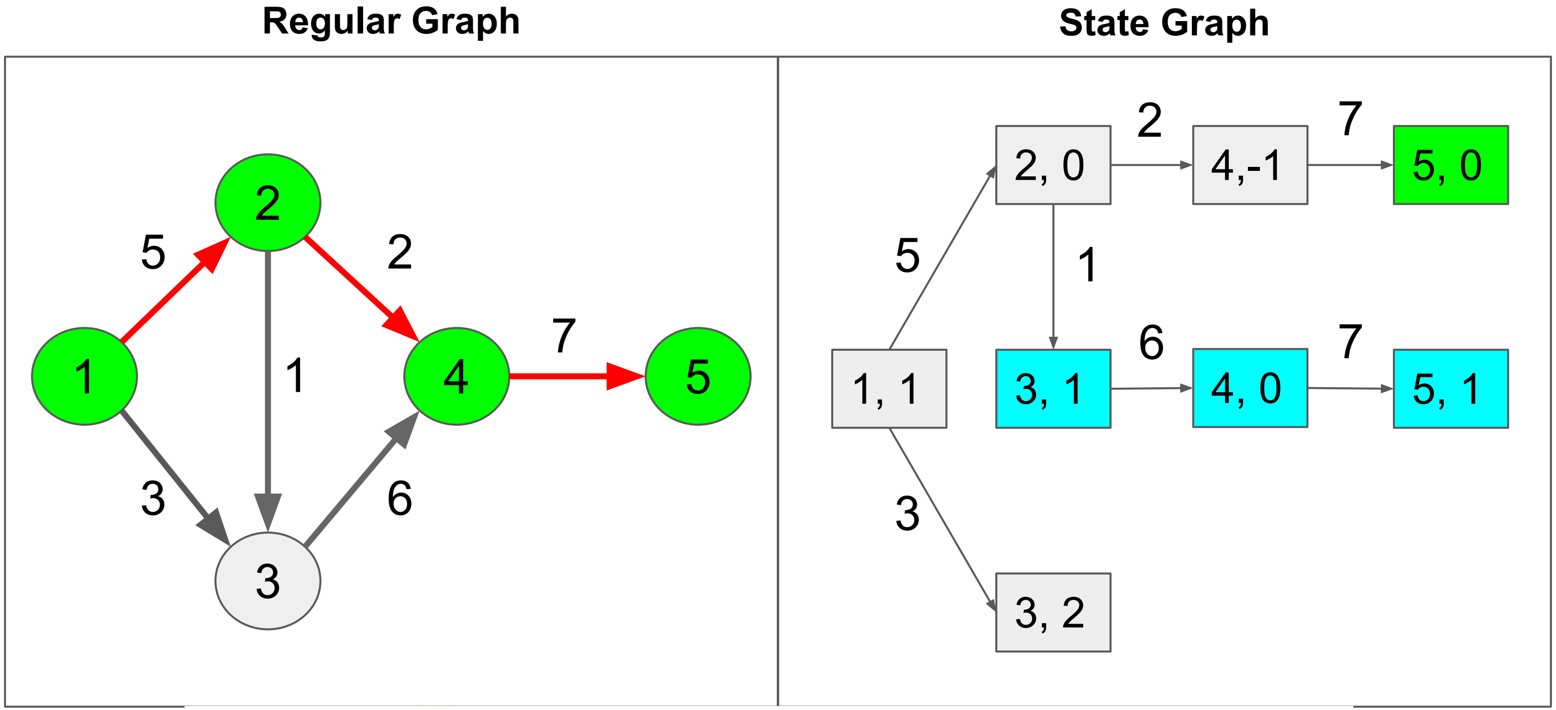
Simulation of DP



Cache	
Key	**Value**
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(4, 0)	7
(3, 1)	13
(5, 0)	∞

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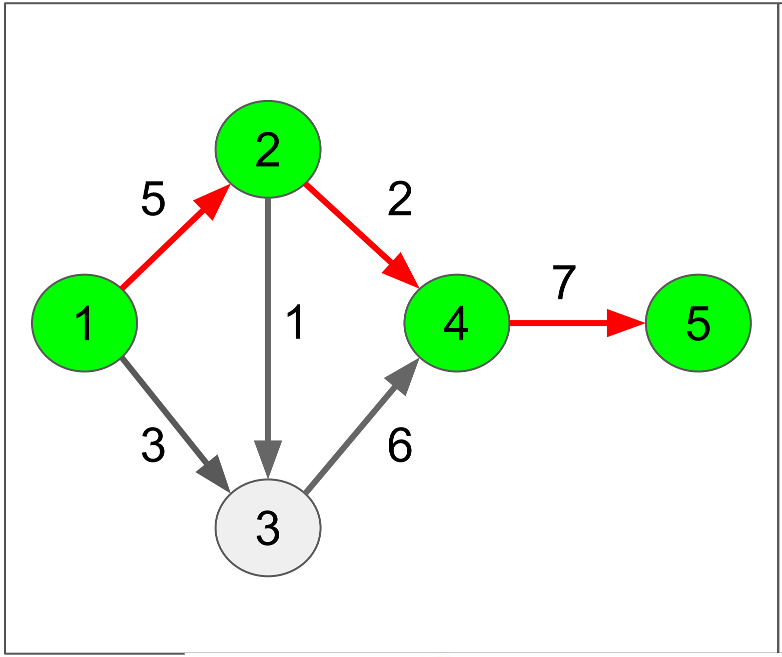
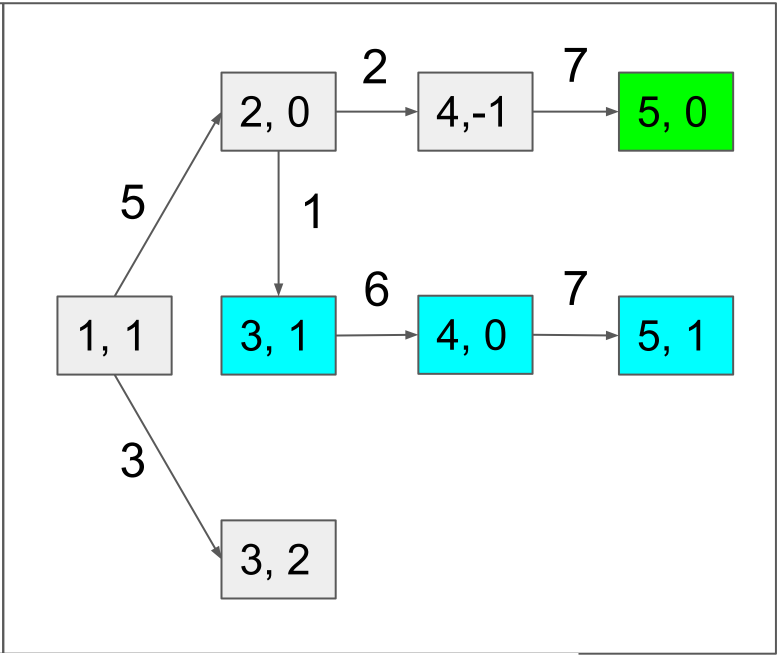
Simulation of DP

Visiting | **Successors** | **Completed**

Regular Graph



State Graph



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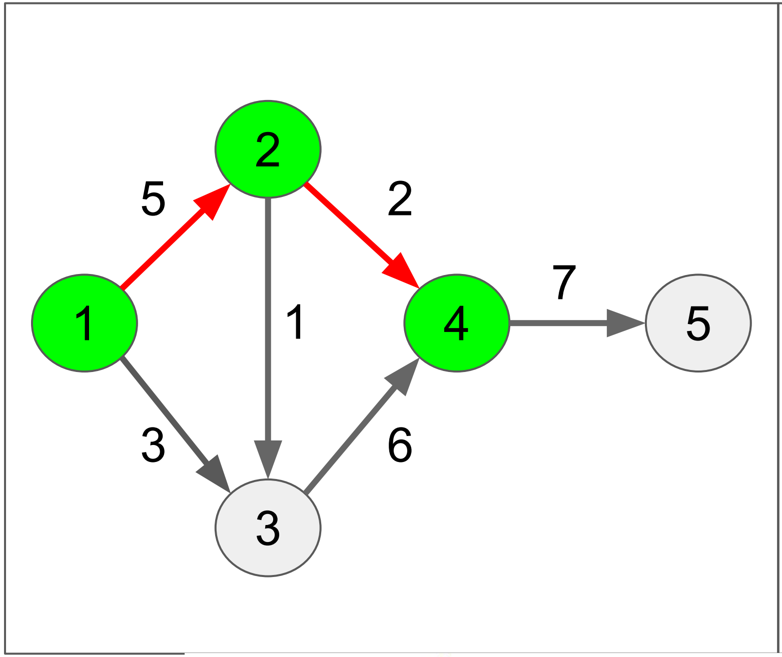
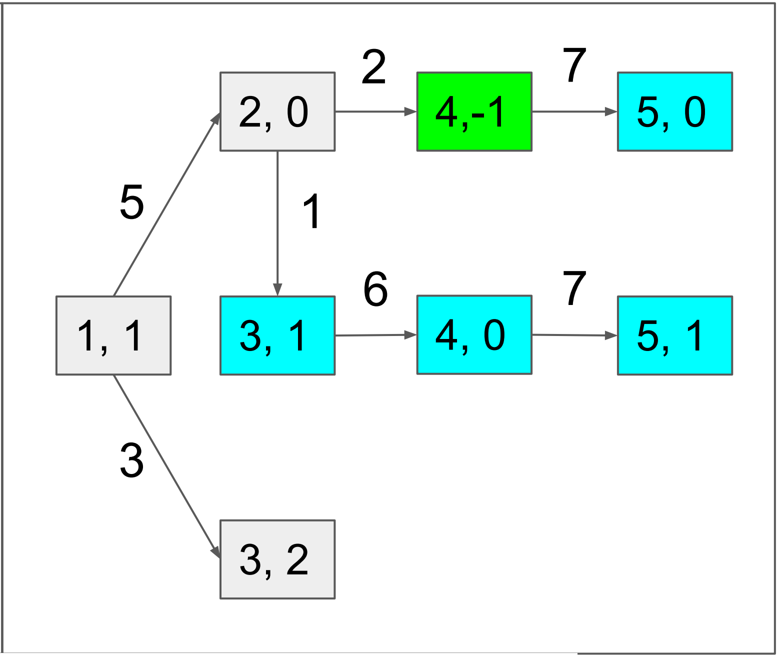
Simulation of DP

Visiting	Successors	Completed

Regular Graph



State Graph



Cache	
Key	**Value**
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(3, 1)	13
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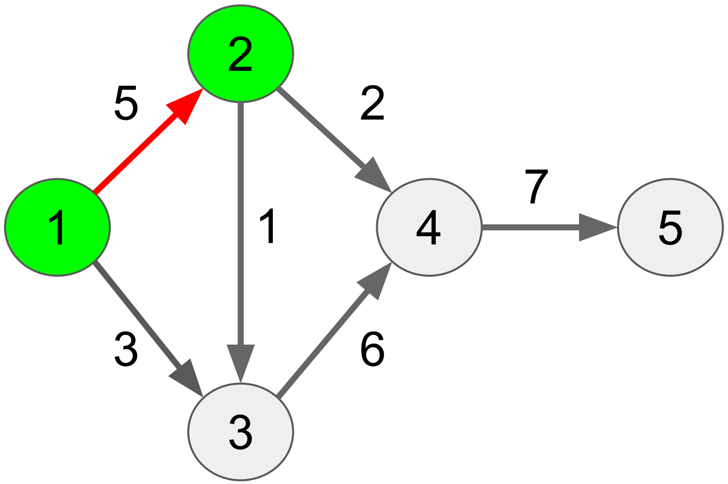
$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

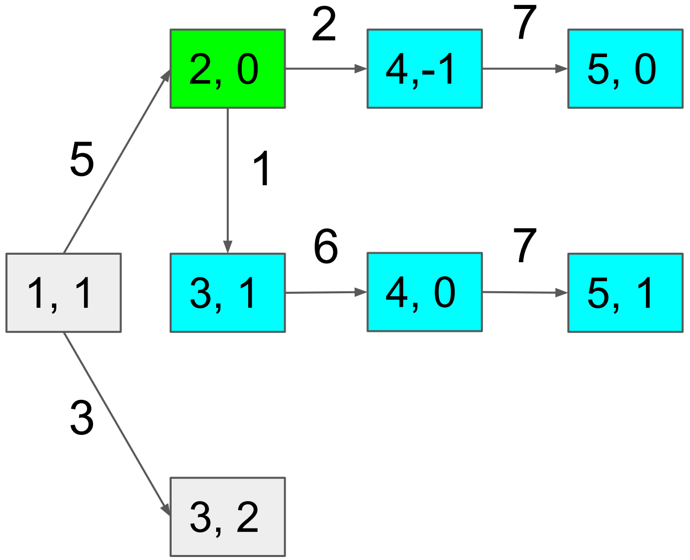
Simulation of DP

Visiting | Successors | Completed

Regular Graph



State Graph



Cache	
Key	**Value**
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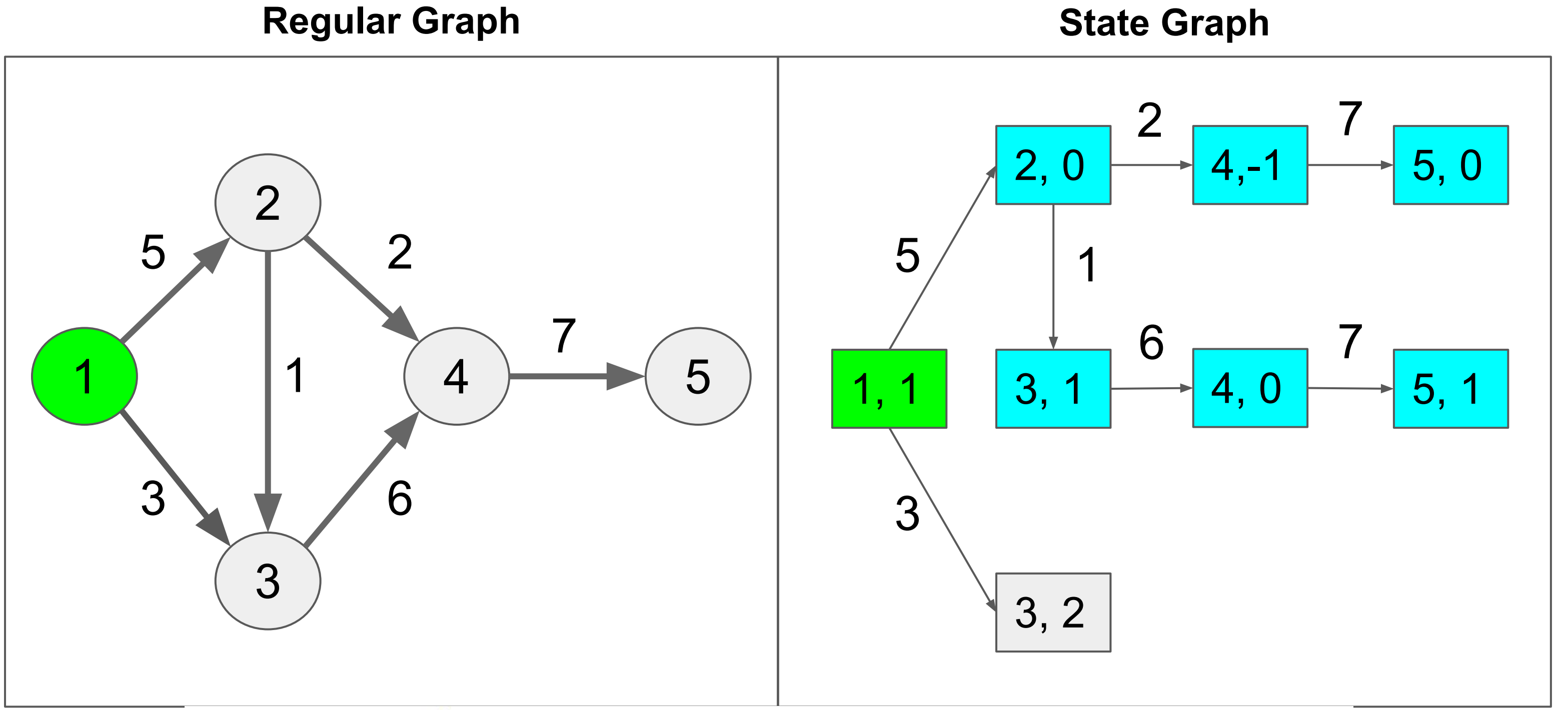
Simulation of DP



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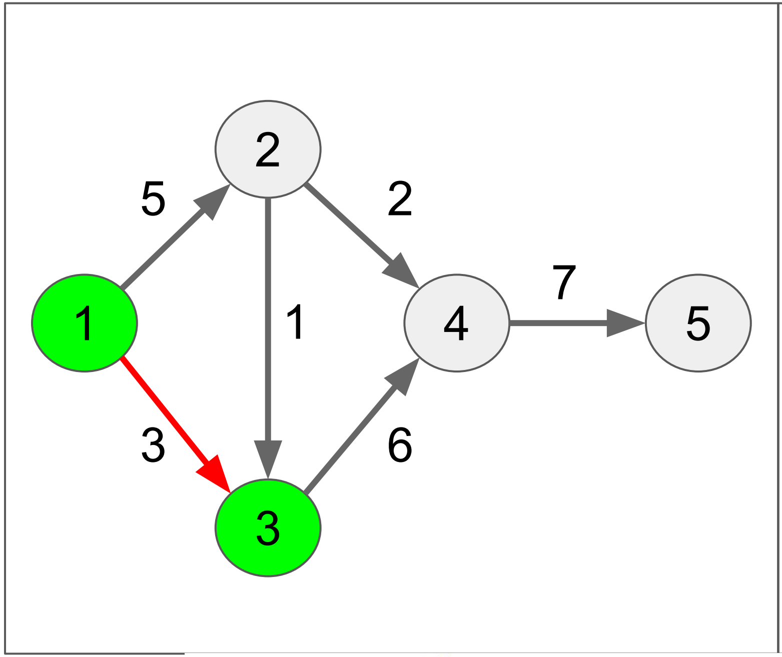
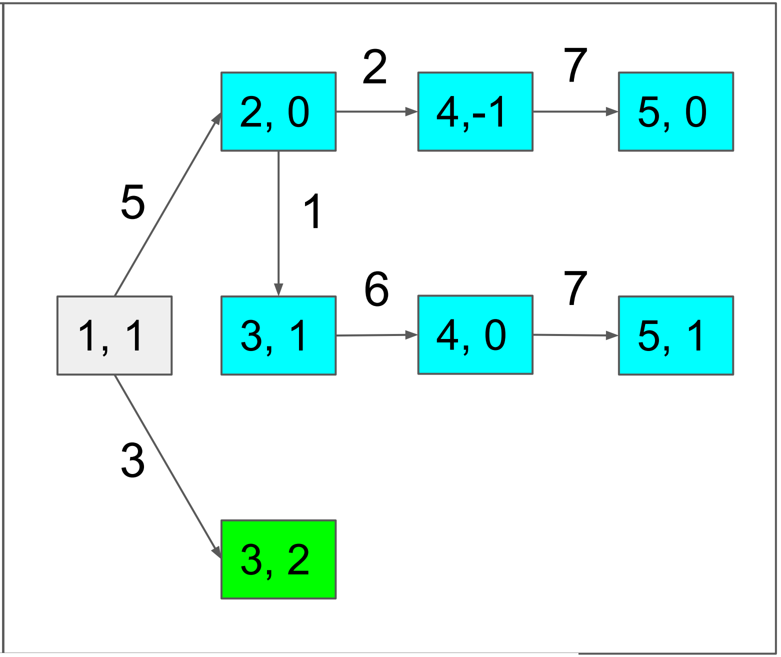
Simulation of DP

Visiting	Successors	Completed

Regular Graph



State Graph



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Simulation of DP

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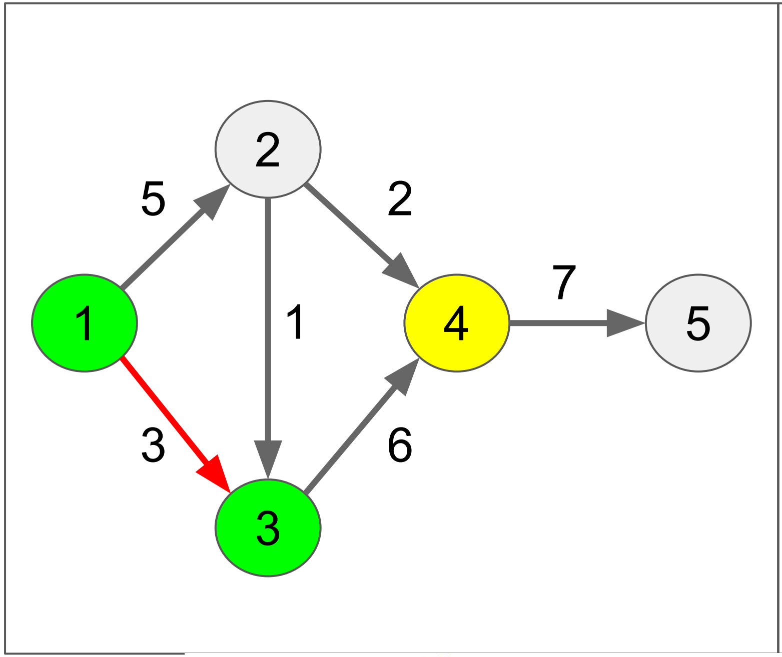

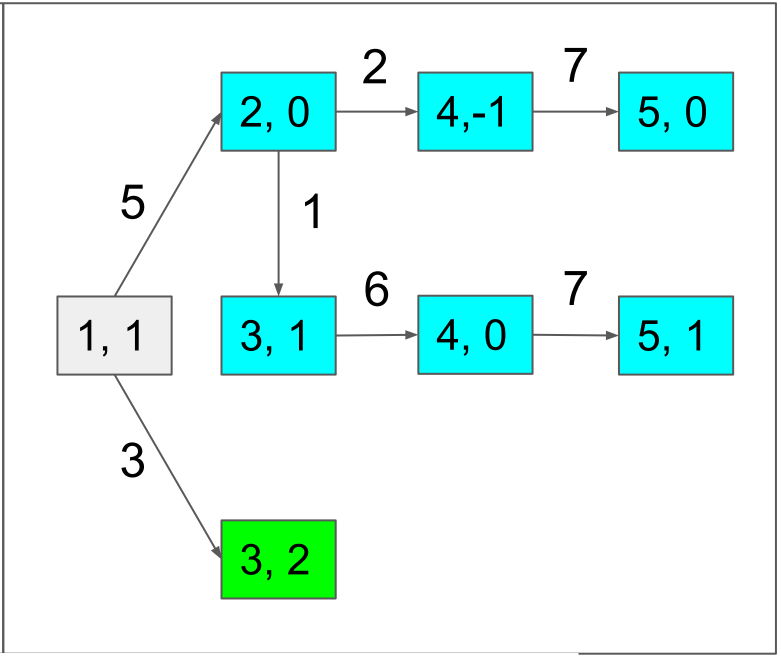

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Simulation of DP

Visiting | Successors | Completed

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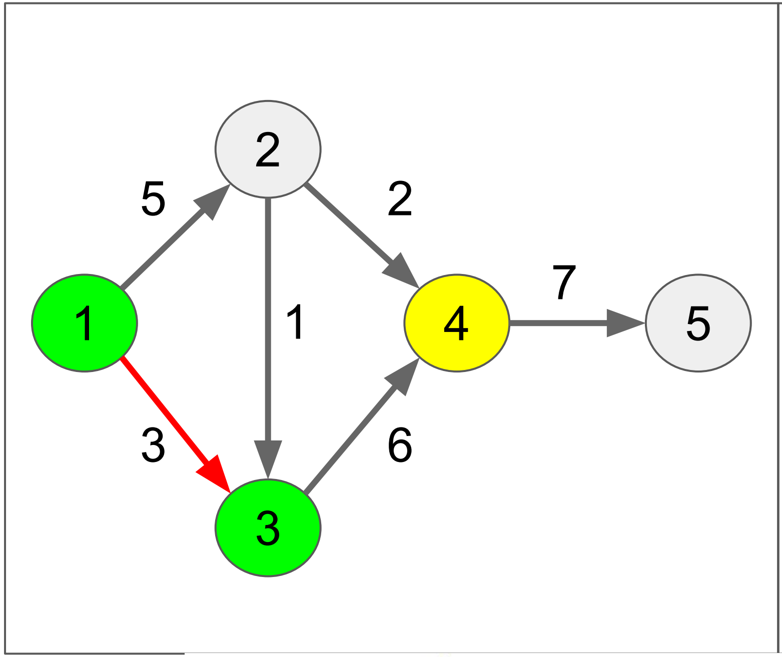

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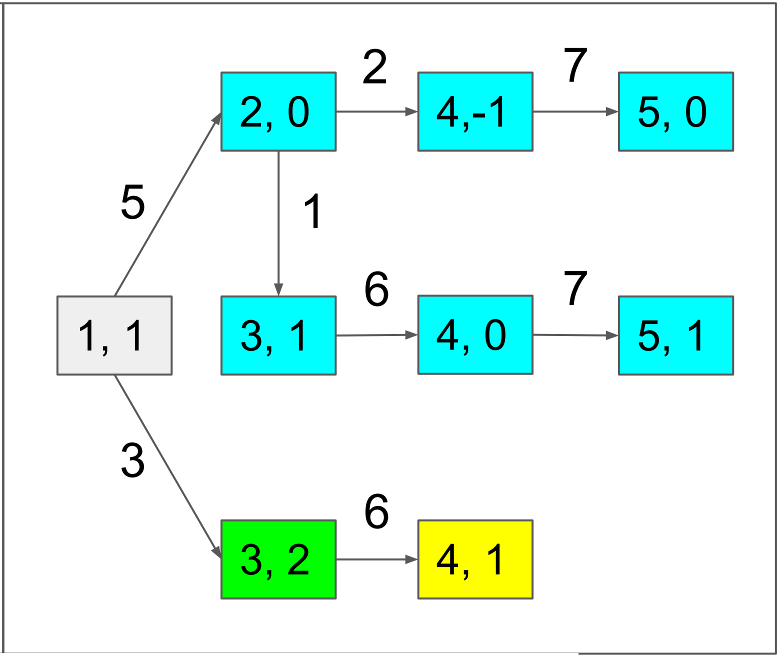

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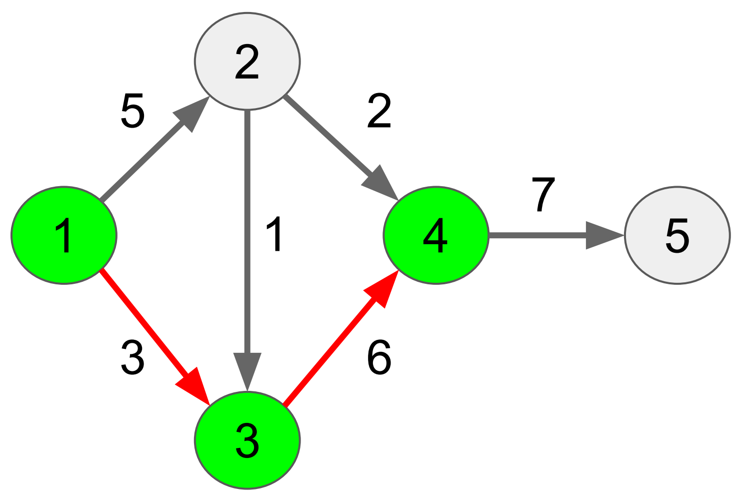
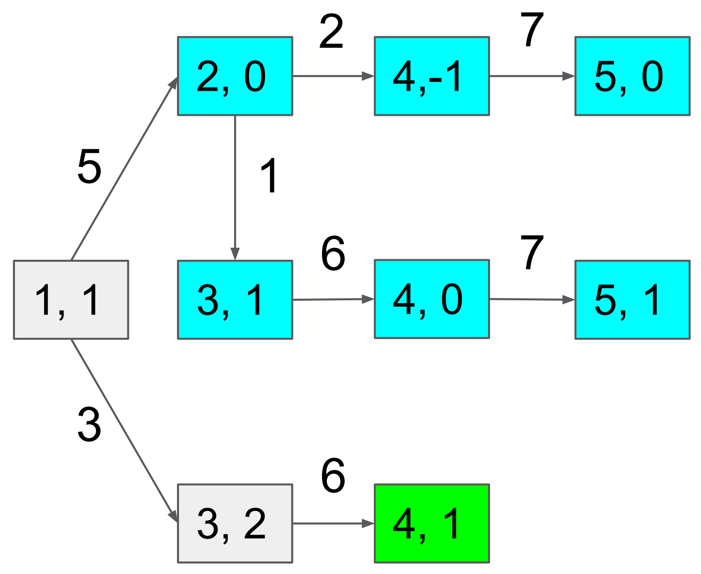
Regular Graph



State Graph



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Simulation of DP

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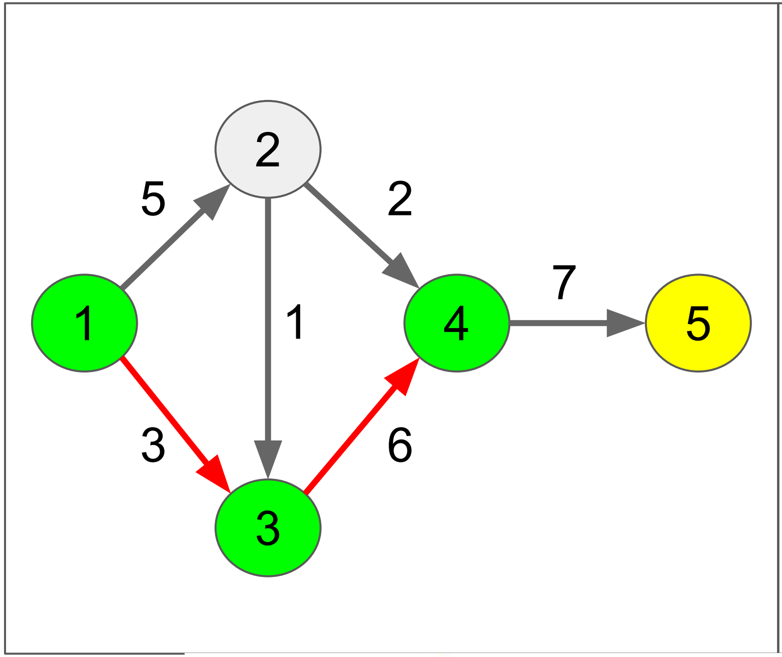

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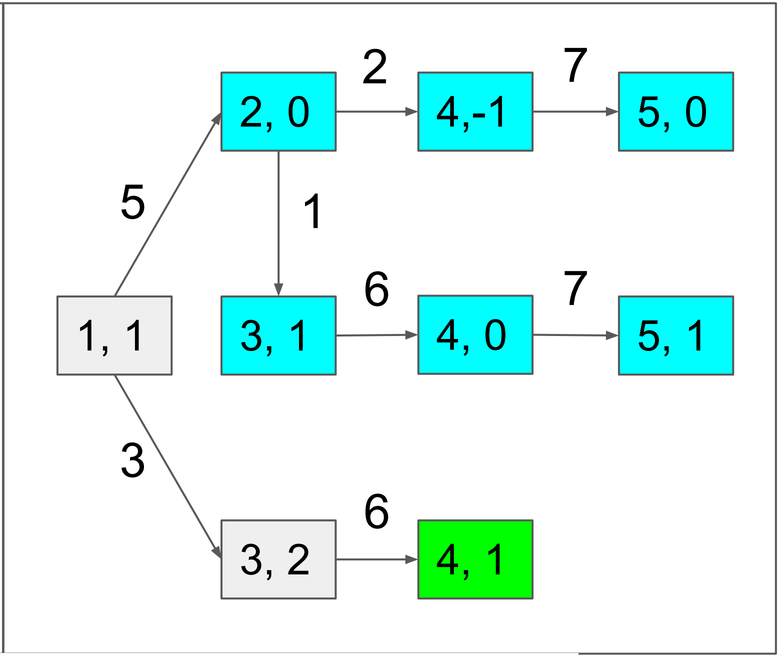

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Key	**Value**
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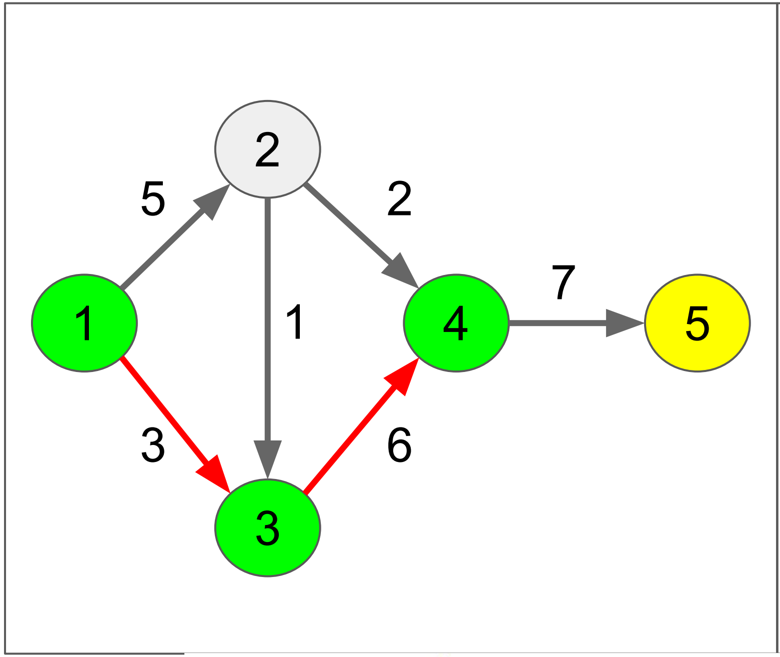
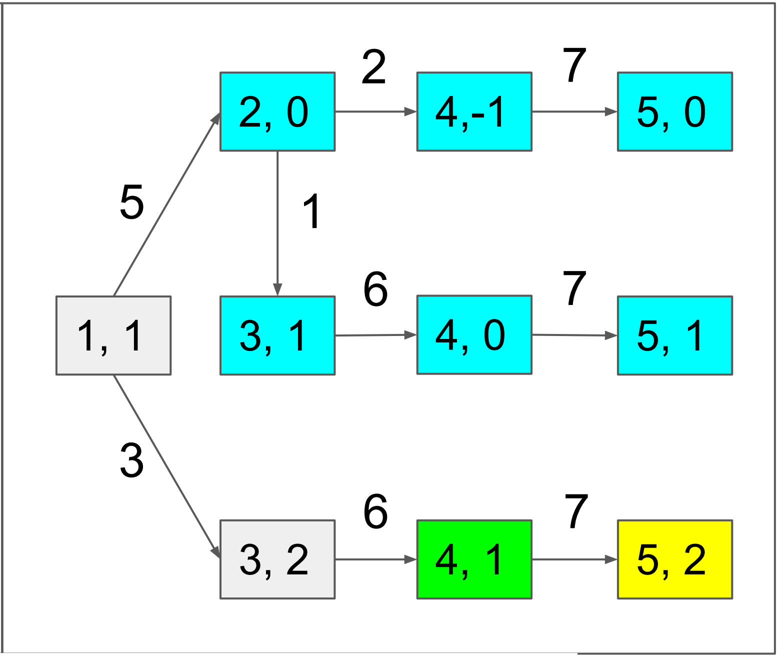
Simulation of DP

Visiting | **Successors** | **Completed**

Regular Graph



State Graph



Cache	
Key	**Value**
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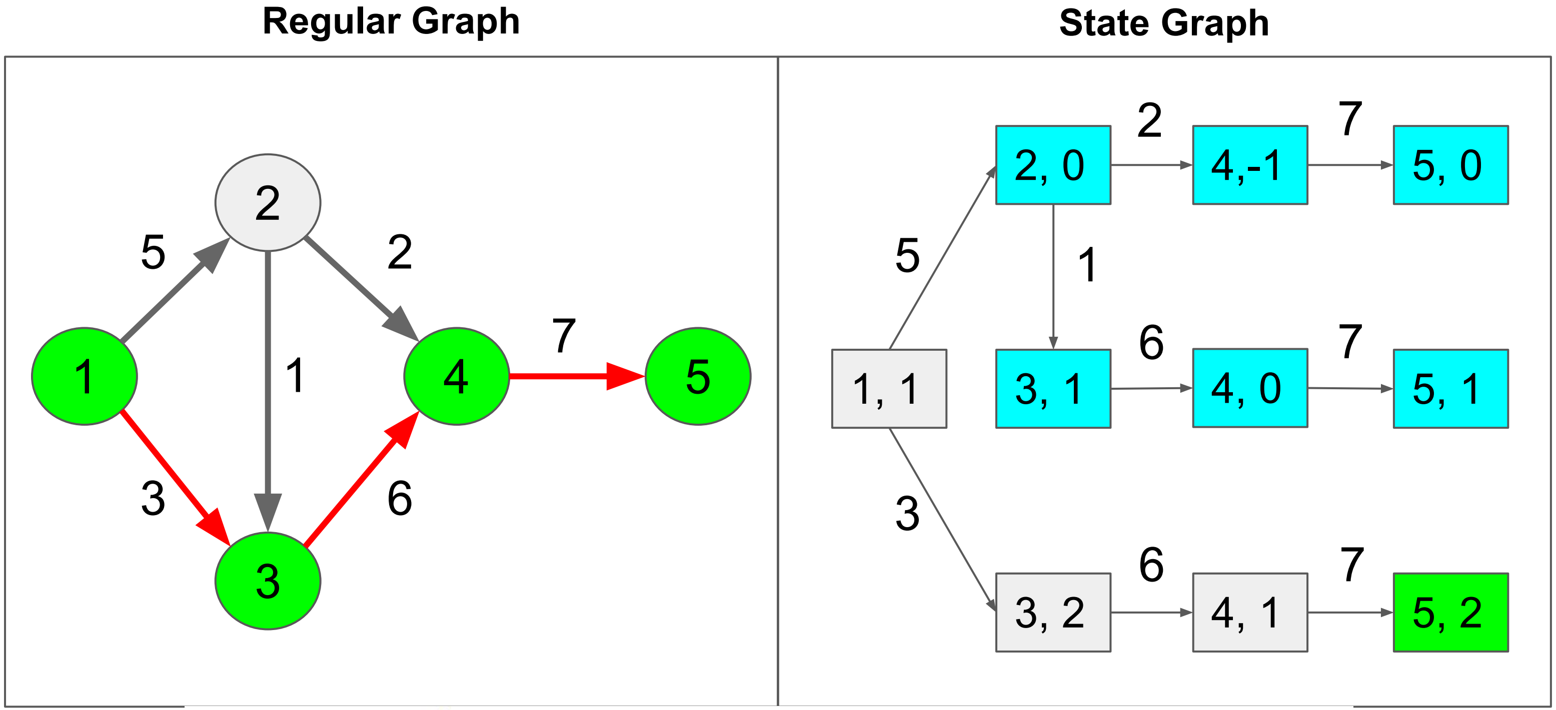
Simulation of DP

Visiting	Successors	Completed



Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7
(3, 1)	13
(5, 0)	∞
(4, -1)	∞
(2, 0)	14
(5, 2)	0

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

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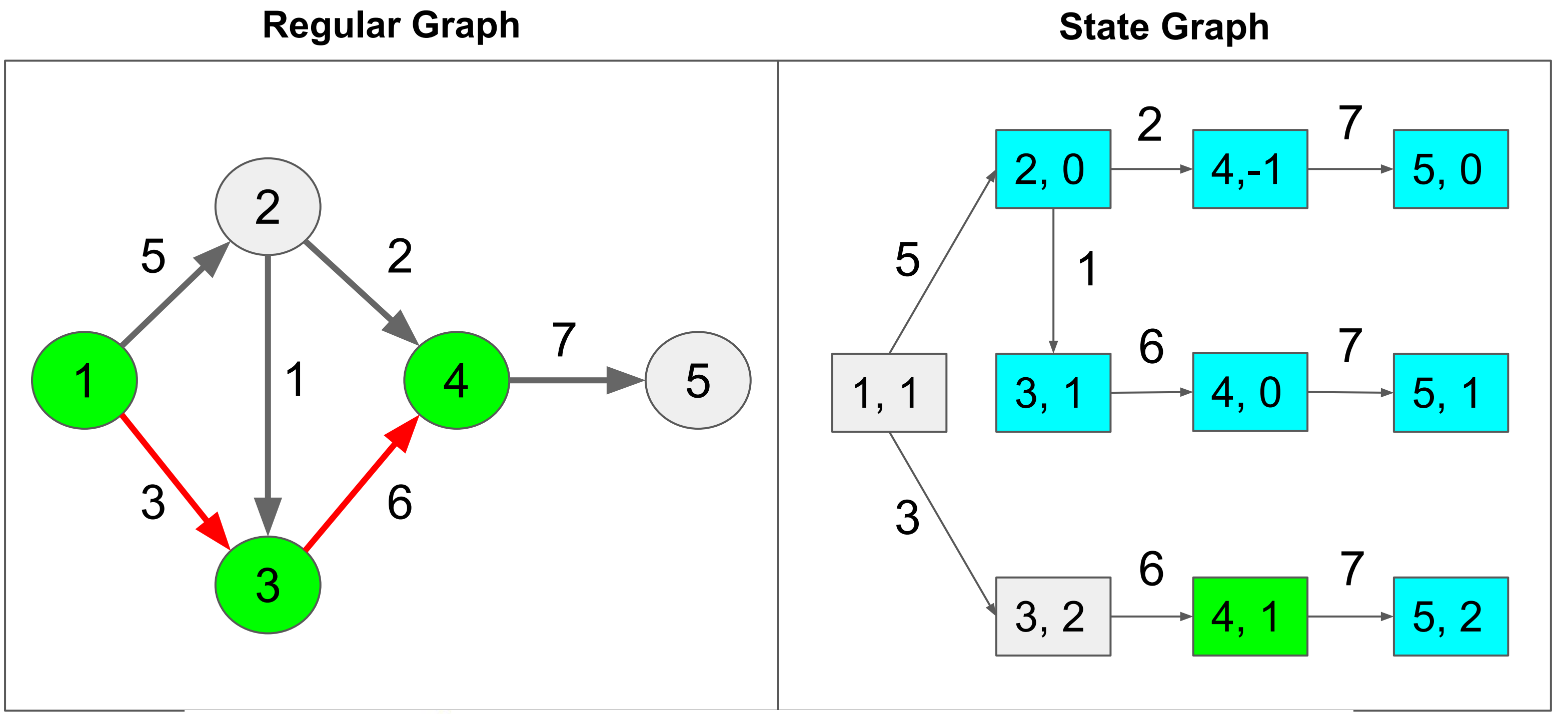

Cache	
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(5, 1)	0
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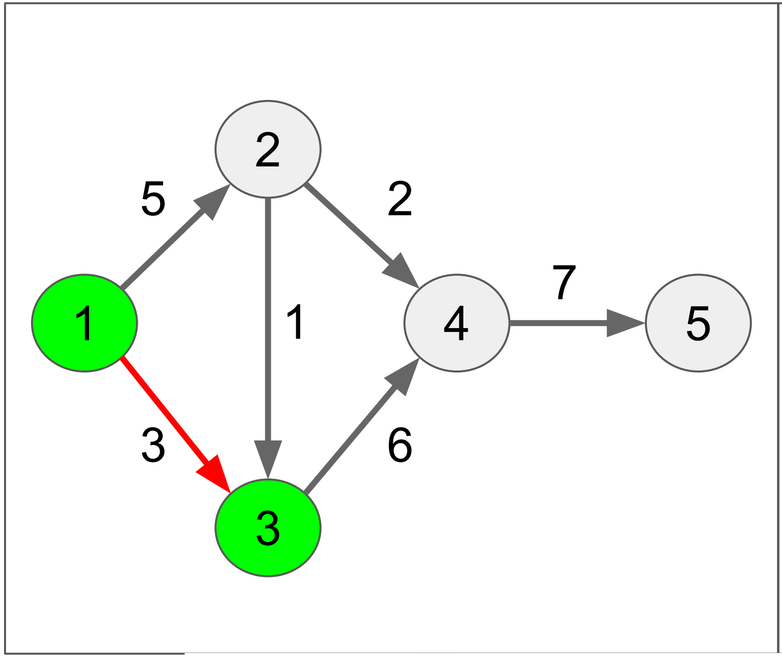
(3, 1)	13
(5, 0)	∞
(4, -1)	∞
(2, 0)	14
(5, 2)	0
(4, 1)	7

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s, a) + \text{FutureCost}(\text{Succ}(s, a))] & \text{otherwise} \end{cases}$$

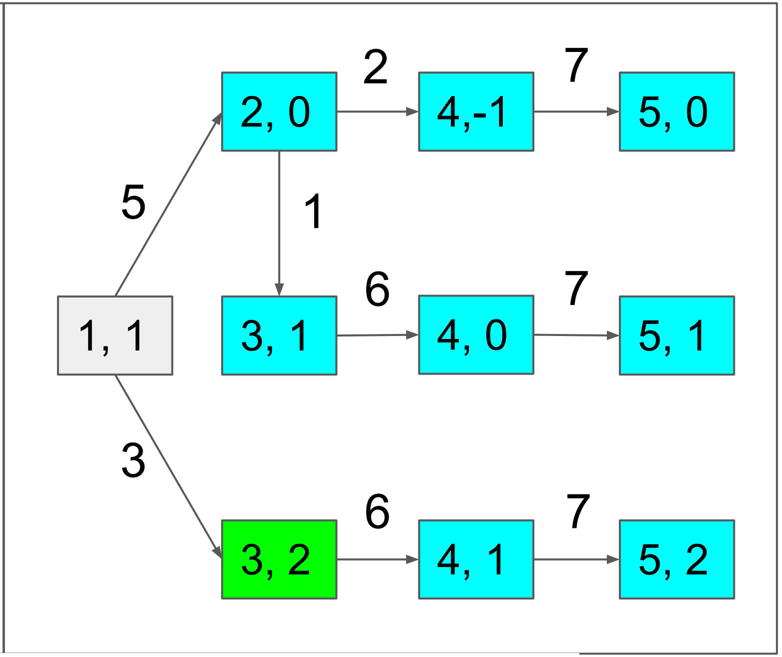
Simulation of DP

Visiting | Successors | Completed

Regular Graph



State Graph



Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7
(3, 1)	13
(5, 0)	∞
(4, -1)	∞
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(5, 2)	0
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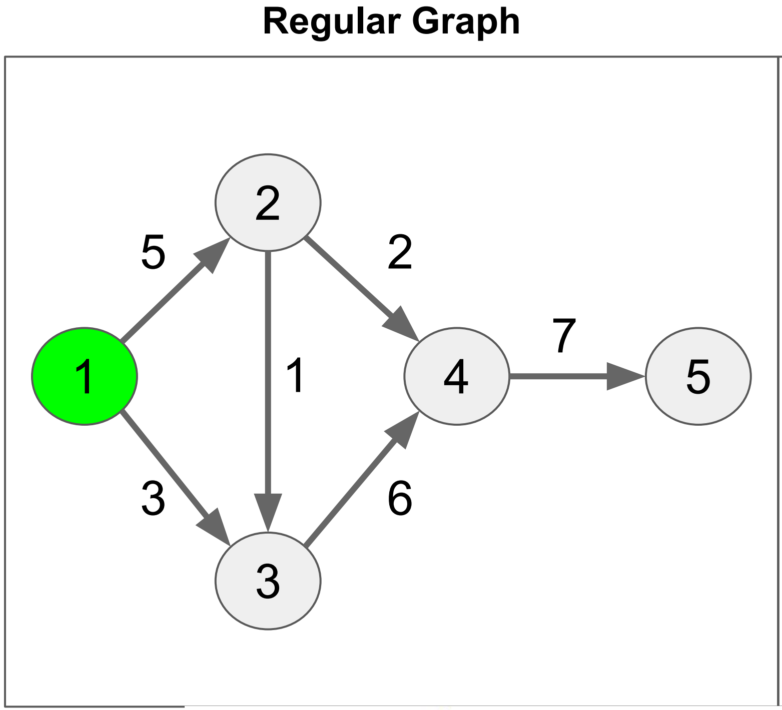
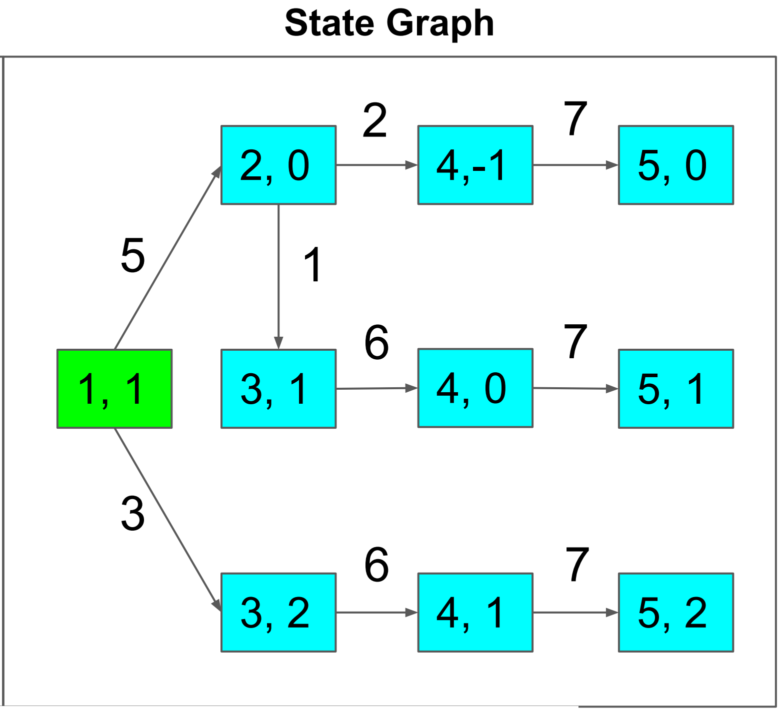
(3, 2)	13

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

Visiting	Successors	Completed



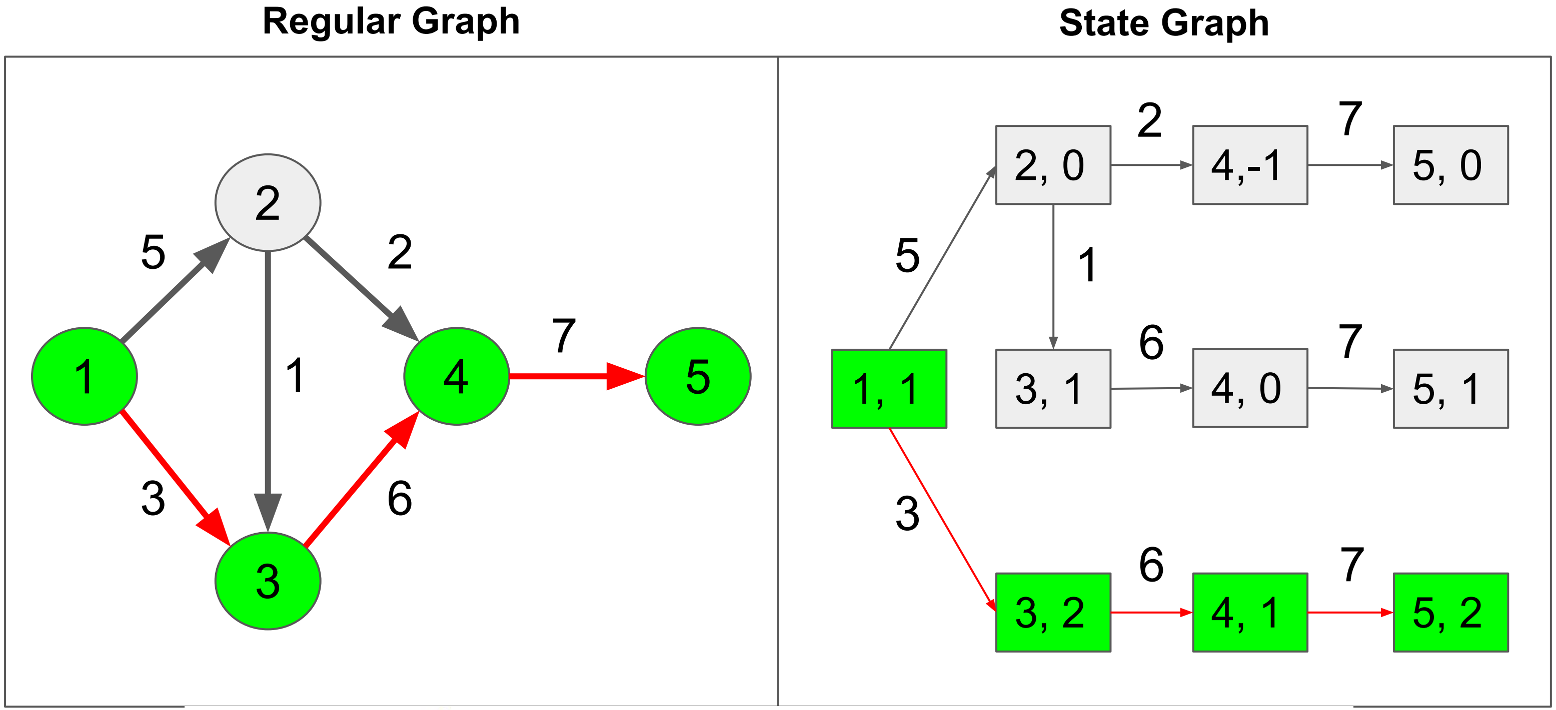


Cache	
Key	**Value**
(5, 1)	0
(4, 0)	7
(3, 1)	13
(5, 0)	∞
(4, -1)	∞
(2, 0)	14
(5, 2)	0
(4, 1)	7
(3, 2)	13
(1, 1)	16

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Simulation of DP

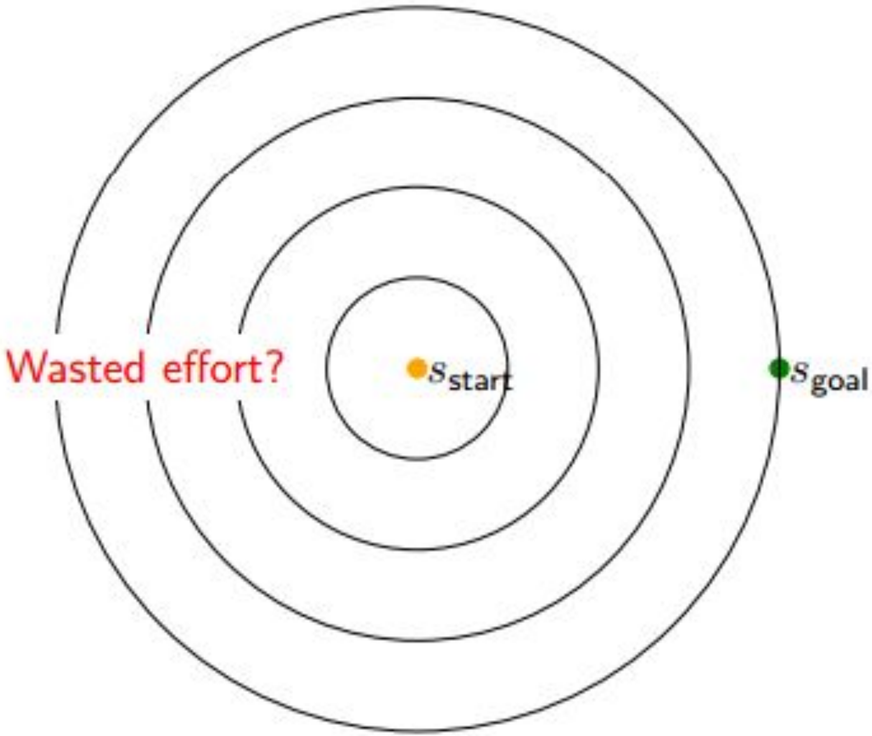
Visiting	Successors	Completed



Cache	
Key	Value
(5, 1)	0
(4, 0)	7
(3, 1)	13
(5, 0)	∞
(4, -1)	∞
(2, 0)	14
(5, 2)	0
(4, 1)	7
(3, 2)	13
(1, 1)	16

$$\text{FutureCost}(s) = \begin{cases} 0 & \text{if IsGoal}(s) \\ \min_{a \in \text{Actions}(s)} [\text{Cost}(s,a) + \text{FutureCost}(\text{Succ}(s,a))] & \text{otherwise} \end{cases}$$

Improve UCS: A* Search



Contents

1. Uniform Cost Search
2. Defining States
3. Dynamic Programming
4. **A* Search**

A* Search

- We want to avoid wasted effort (to go from SF to LA, we probably don't want to end up looking at roads to Seattle, for example).
- To do this, we can use a heuristic to estimate how far is left until we reach our goal.
- The heuristic **must be optimistic**. It must underestimate the true cost. Why?

Recap of A* Search

- Modify the cost of edges and run UCS on the new graph
  - Cost'(s, a) = Cost(s, a) + h(Succ(s, a)) - h(s)
- h(s) is a heuristic that is our estimate of FutureCost(s)
- If h(s) is consistent then the modified edge weights will return min cost path
- You can find a good consistent h by performing relaxation
- If c is min cost on original graph, c' is min cost on modified graph, then c' = c + h(s_goal) - h(s_start)

Relaxation

A good way to come up with a reasonable heuristic is to solve an easier (less constrained) version of the problem

For example, we can remove the constraint that we visit more odd cities than even cities.

h(s) = h((i, d)) = length of shortest path from city i to city N

Note on Relaxation

The main point of relaxation is to attain a problem that **can be solved more efficiently**.

In our case, the modified shortest path problem has O(N) states instead of O(N^2) can thus can be solved more efficiently

Checking consistency

- $Cost(s, a) + h(Succ(s, a)) - h(s) \geq 0$ (Triangle Inequality)
  - Suppose $s = (i, d)$ and $Succ(s, a) = (j, d')$
  - Note that $h((i, d)) - h((j, d')) \leq c(i, j) = Cost(s, a)$
- $h((N, d)) = 0$

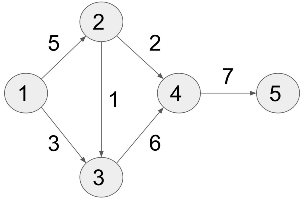
How to compute h?

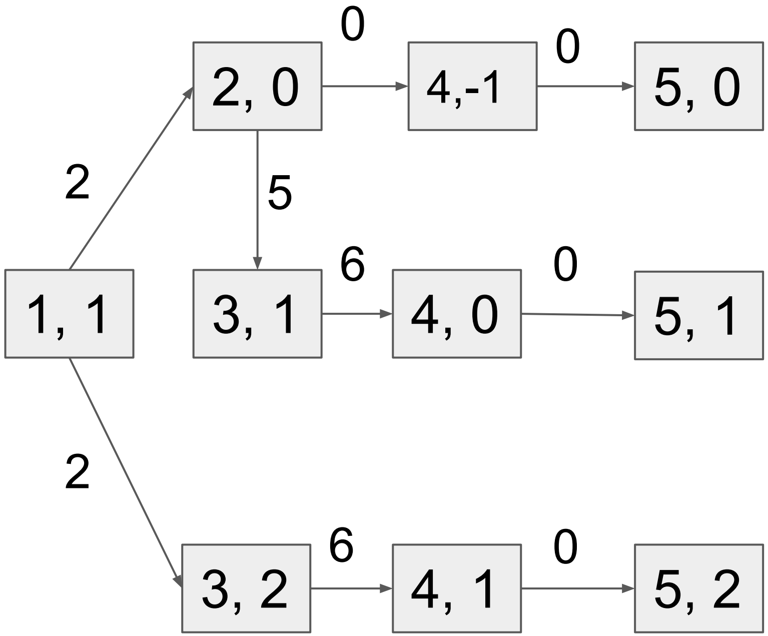
We can reverse the direction of all edges, and then perform UCS starting from city 1, and our goal state is city 5.

This takes O(n log n) time, where n is the number of states whose distance to city N is no farther than the distance of city 1 to city N

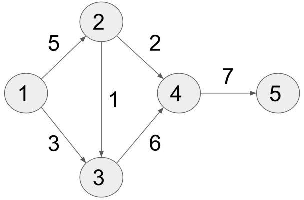
city	1	2	3	4	5
h	14	9	13	7	0

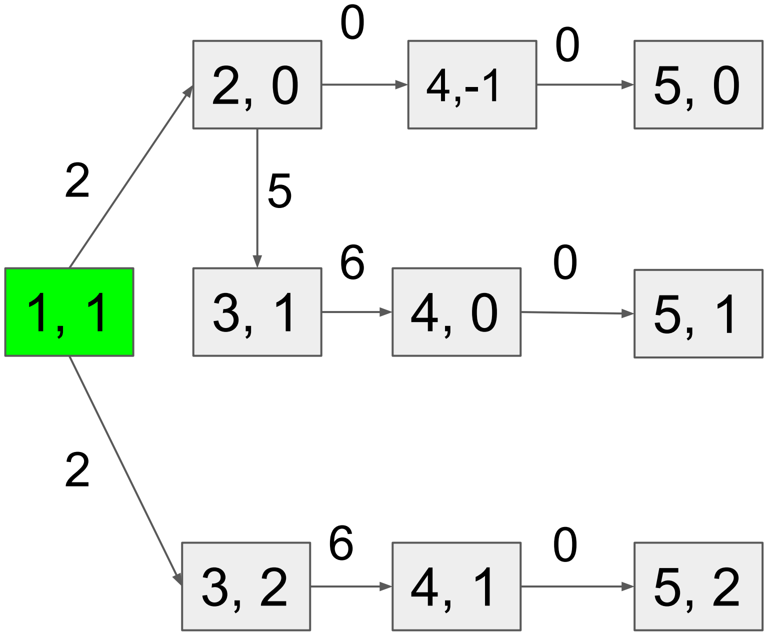
Modified State Graph





Simulation of UCS (A*)

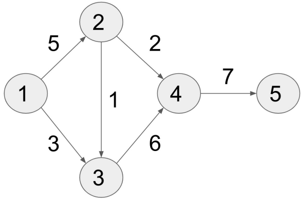


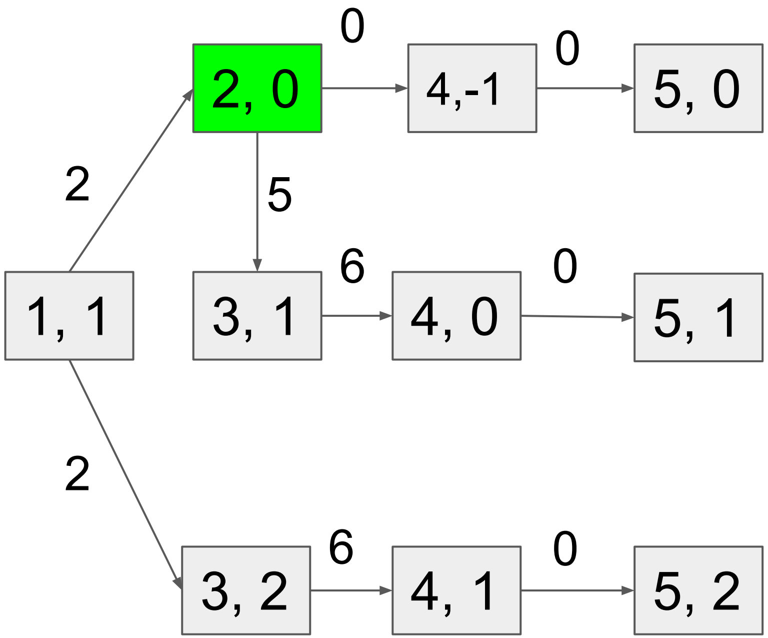


Explored:
(1, 1) : 0

Frontier:
(2, 0) : 5 + 9
(3, 2) : 3 + 13

Simulation of UCS (A*)

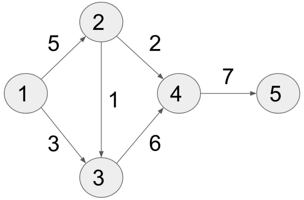


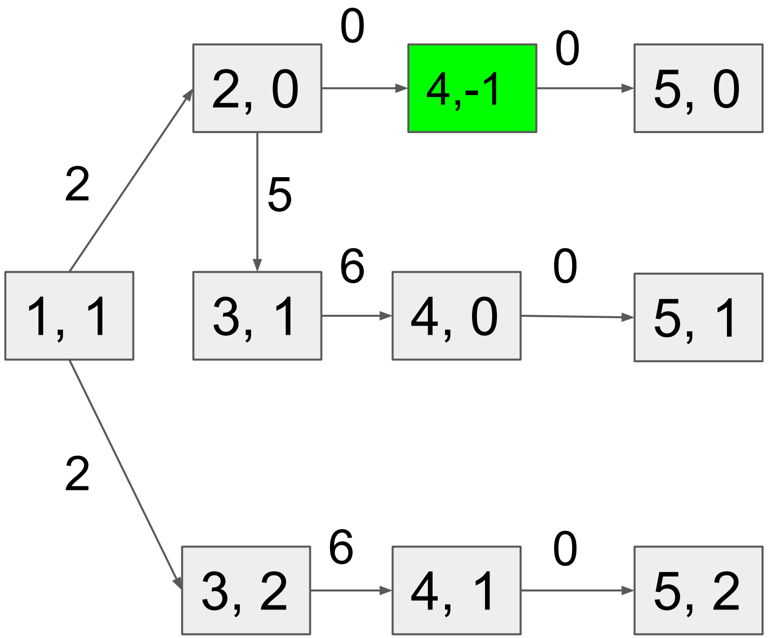


Explored:
(1, 1) : 0
(2, 0) : 5

Frontier:
(3, 2) : 3 + 13
(3, 1) : 6 + 13
(4, -1) : 7 + 7

Simulation of UCS (A*)





Explored:

(1, 1) : 0

(2, 0) : 5

(4, -1) : 7

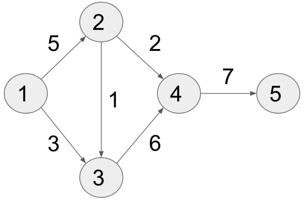
Frontier:

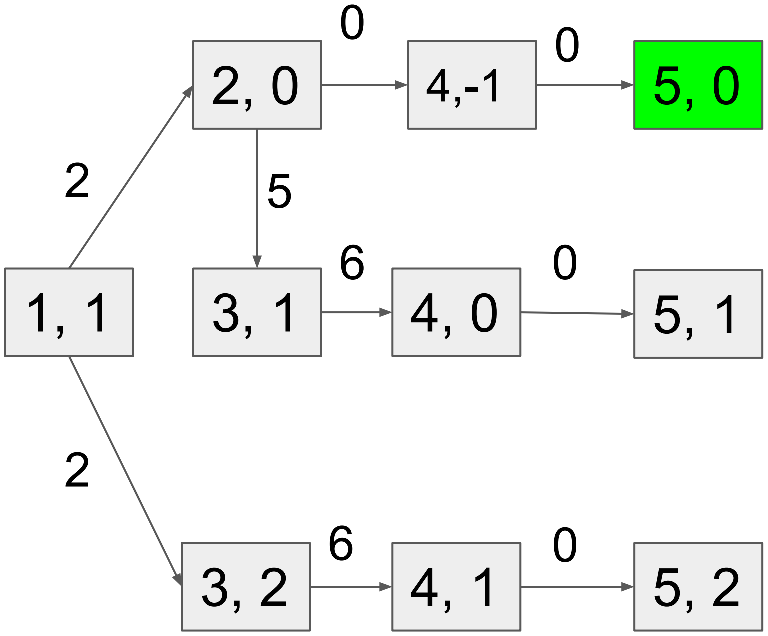
(3, 2) : 3 + 13

(3, 1) : 4 + 13

(5, 0) : 7 + 7

Simulation of UCS (A*)

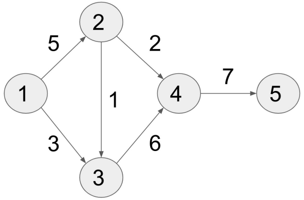


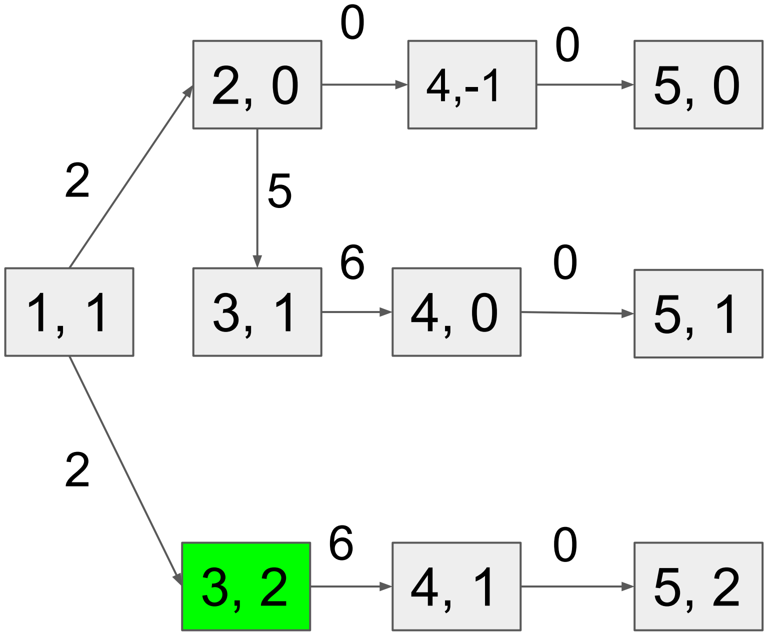


Explored:
(1, 1) : 0
(2, 0) : 5
(4, -1) : 7
(5, 0) : 14

Frontier:
(3, 2) : 3 + 13
(3, 1) : 4 + 13

Simulation of UCS (A*)

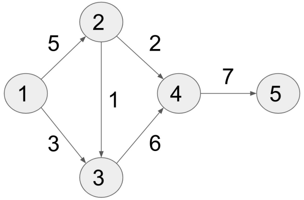


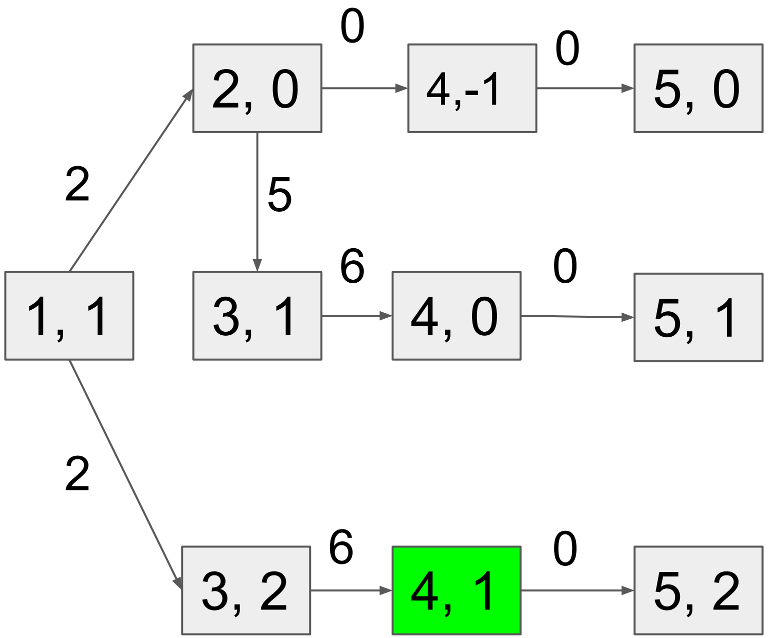


Explored:
(1, 1) : 0
(2, 0) : 5
(4, -1) : 7
(5, 0) : 14
(3, 2) : 3

Frontier:
(3, 1) : 3 + 13
(4, 1) : 9 + 7

Simulation of UCS (A*)

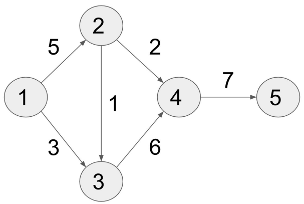


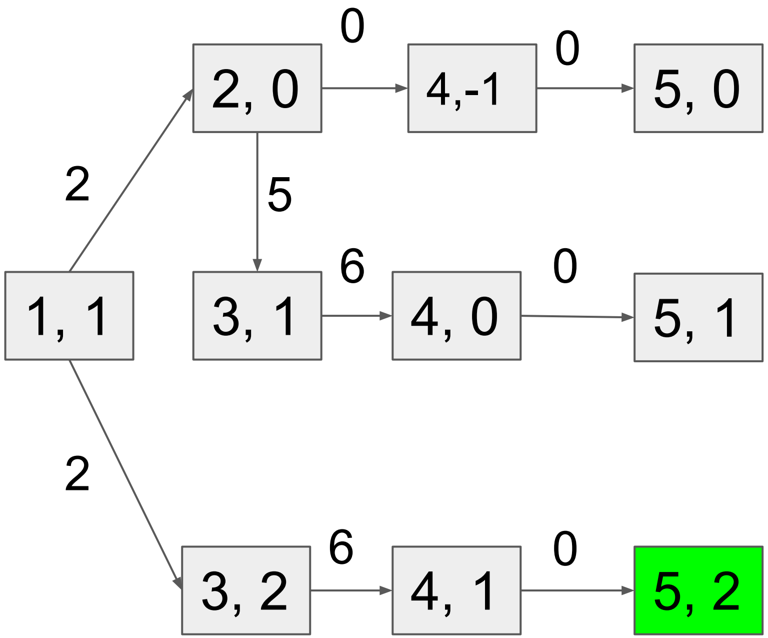


Explored:
(1, 1) : 0
(2, 0) : 5
(4, -1) : 7
(5, 0) : 14
(3, 2) : 3
(4, 1): 9

Frontier:
(3, 1) : 3 + 13
(5, 2) : 16 + 0

Simulation of UCS (A*)





Explored:
(1, 1) : 0
(2, 0) : 5
(4, -1) : 7
(5, 0) : 14
(3, 2) : 3
(4, 1): 9
(5, 2) : 16

Frontier:
(3, 1) : 3 + 13

STOP!

Comparison of States visited

UCS

Explored:	Frontier:
(1, 1) : 0	(5, 1) : 19
(3, 2) : 3	
(2, 0) : 5	
(3, 1) : 6	
(4, -1) : 7	
(4, 1) : 9	
(4, 0) : 12	
(5, 0) : 14	
(5, 2) : 16	

UCS(A*)

Explored:	Frontier:
(1, 1) : 0	(3, 1) : 3
(2, 0) : 0	
(4, -1) : 0	
(5, 0) : 0	
(3, 2) : 2	
(4, 1) : 2	
(5, 2) : 2	

Summary

- States Representation/Modelling
  - make state representation as compact as possible, remove unneccesary information
- DP
  - underlying graph cannot have cycles
  - visit all reachable states, but no log overhead
- UCS
  - actions cannot have negative cost
  - visit only a subset of states, log overhead
- A*
  - ensure that relaxed problem can be solved more efficiently