Chapter 3
DOA Estimation of Coherent Resources Based on Matrix Reconstruction

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Abstract This study presents a direction-of-arrival (DOA) estimation method for coherent signals. DOA estimation is an important part in the target precision locating. But, when the signals are correlated, conventional algorithm will fail in DOA estimation such as MUSIC. A new de-correlation DOA estimation algorithm is proposed for this. It is based on the cross-correlation vector matrix reconstruction. For wide coherent signals, its performance is superior to ISM and CSM. Simulation results are presented to illustrate the efficiency and accuracy of this method.

Keywords Direction of arrival (DOA) · MUSIC algorithm · Conjugate data rearrangement · Matrix reconstruction · Coherent resources

3.1 Introduction

Signals are coherent at a high degree in the military environment, existing deception jamming or multi-path interference; this will lead to the loss of the rank of covariance matrix [1], and the conventional MUSIC [2] algorithm will fail in the direction-of-arrival (DOA) estimation. So, it is important to study an improved algorithm for mixed resources in the coherent environment.

Modified MUSIC (MMUSIC) algorithm based on the spatial smoothing is effective for the coherent resources, but its ability of de-correlation is obtained by sacrificing the effective number of arrays. There is a certain degree of loss of the array aperture, so a modified DOA estimation algorithm is proposed in this study. Because the actual observations matrix is a complex matrix, the proposed method

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makes the original data and its anti-conjugated matrix [3] together. This is equivalent to double the length of the data being used. It greatly improves the performance of the DOA estimation by taking full use of the self-correlation and cross-correlation information between the original data matrix and the conjugate matrix.

3.2 Signals Model

Consider a uniform linear array of N sensors receiving $K(K<N)$, and signals with same frequency are independent. The first sensor is reference sensor, the distance of sensors is d, and the incident angles are θ_k $(k = 1, \ldots, K)$, respectively, then the sensor output vector can be represented by

$$X(t) = A(t)S(t) + N(t) \tag{3.1}$$

where $X(t)$ is the $N \times 1$ array output vector, $N(t)$ is the $N \times 1$ sensor noise vector. So, we can get:

$$X(t) = [x_1(t), \ldots, x_N(t)]^T \tag{3.2}$$

$$N(t) = [n_1(t), \ldots, n_N(t)]^T \tag{3.3}$$

$S(t)$ is the $K \times 1$ signal vector

$$S(t) = [s_1(t), \ldots, s_k(t)]^T \tag{3.4}$$

$A(\theta)$ is the $N \times K$ response matrix of the uniform linear array. Suppose every sensor is consistent, then

$$A(\theta) = \left[1, e^{-j2\pi fd \sin\theta_i/c}, \ldots, e^{-j2\pi fd(M-1)\sin\theta_i/c}\right]^T. \tag{3.5}$$

Suppose the receiving noise vector is independent zero-mean vector with identical second-order moment. The noise vector and the signal vector are uncorrelated, then

$$E\left[n(t_1)n^H(t_2)\right] = \sigma^2 I \delta_{t_1 t_2} \tag{3.6}$$

$$E\left[n(t_1)n^H(t_1)\right] = 0 \tag{3.7}$$

The covariance matrix of the output vector can be represented by

$$R = \sum_{k=1}^{K} P_k A(\theta) A^H(\theta) + \sigma_n^2 I \tag{3.8}$$

where P_k is the output power of the output vector, $A(\theta)$ is the response matrix of the uniform linear array, and $\sigma^2{}_n I$ is the covariance matrix of the noise vector, in which I is the identity matrix.

3.3 Modified Algorithm

Suppose the snapshots matrix is $X(k)$, in the modified MUSIC algorithm [3, 4], $Y(k) = J_M X^*(k)$, where $X^*(k)$ is conjugate of $X(k)$. Suppose J is a $M \times M$ replaced matrix in which all elements are zeros except counter diagonal elements are ones. So, $J_M \cdot J_M = I_M$, in which I is the identity matrix. We can get the correlation matrix of $Y(k)$ as following: $R_{yy} = E[Y(k)Y^H(k)] = J_M R^*_{xx} J_M$.

Taking the average of covariance matrix R_{xx} and R_{yy}, we can obtain the matrix R, that is, $R = (R_{XX} + R_{YY})/2 = (R_{XX} + J_M R^*_{xx} J_M)/2$.

For coherent sources, the new matrix usually has full rank, solving the problem of matrix rank loss, so as to achieve the purpose of de-correlation. Therefore, MMUSIC algorithm is a special case of space-smoothing technology. It is equivalent that the space array is divided into two sub-arrays. The number of each sub-array is equal to the number of original array element. The number of array element is not lost, so the estimation performance for the relevant sources or irrelevant sources is better than original algorithm.

A modified algorithm is proposed which makes use of the matrix reconstruction based on the direct data snapshot matrix. The algorithm flow can be summarized as the following steps:

Step 1: Get the snapshots matrix by the antenna element $X(k)$, then calculate its correlation matrix R_{xx};
Step 2: Take the conjugate inversion matrix of $X(k)$, denote $Y(k)$, $Y(k) = J_M X^*(K)$, construct the new matrix $Z = [X(k)Y(k)]$;
Step 3: Calculate the correlation matrix of Z. Construct the covariance matrix $\hat{R} = [R_{xx} R_{zz}]$;
Step 4: Take the singular value decomposition of covariance matrix and isolate the signal subspace and the noise subspace;
Step 5: Complete the DOA estimation based on MUSIC algorithm.

3.4 Simulation for Narrowband Signals

Suppose that the array is an isotropy uniform linear array, wavelength $\lambda = 1$, array distance $D = 0.4\lambda$, snapshots $T = 1,024$, digital sensor channels $K = 8$. Receiving signals are long-range coherent narrowband signals, and noise is additive white Gauss noise (AWGN). The number of signals $M = 3$. Adopt MUSIC method,

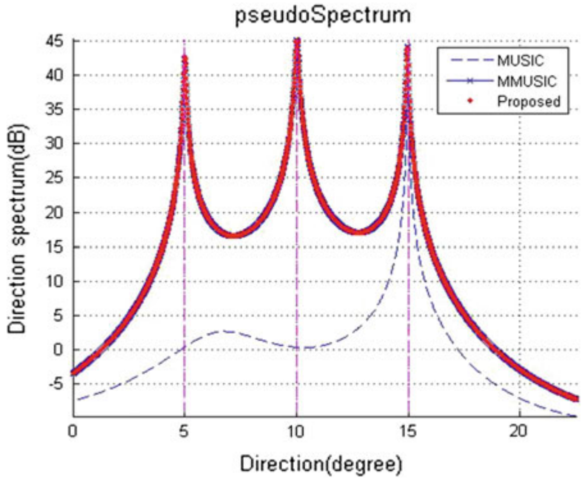

Fig. 3.1 Estimation for far-angle interval at SNR = 20 dB

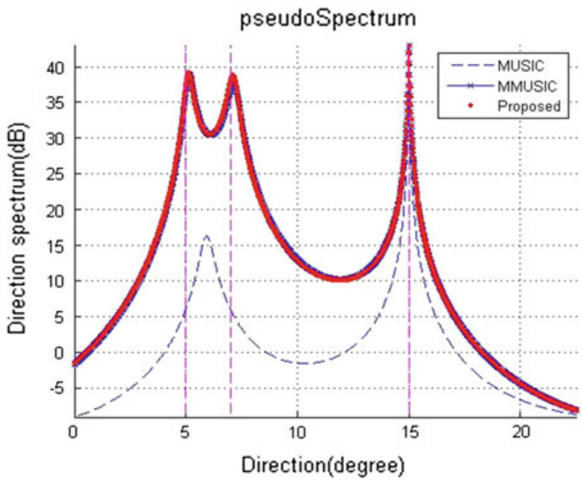

Fig. 3.2 Estimation for close-angle interval at SNR = 20 dB

MMUSIC method, and method proposed in this article to estimate the DOA of the signals. The simulation results are as follows.

Experiment 1: The angles of incidence are 5°, 10°, and 15°, respectively. The first two sources are coherent and signal-to-noise ratio for each array element is 20 dB. The simulation result is shown in Fig. 3.1.

Experiment 2: Change the incident angles into 5°, 7°, and 15°, other parameters not changed. The simulation result is shown in Fig. 3.2.

Experiment 3: Reduce signal-to-noise ratio of each array element to 0 dB and keep other parameters not change. The simulation result is shown in Fig. 3.3.

From Figs. 3.1 and 3.2, it can be seen that MUSIC algorithm cannot correctly estimate the DOA for both far or close coherent sources. The modified MUSIC

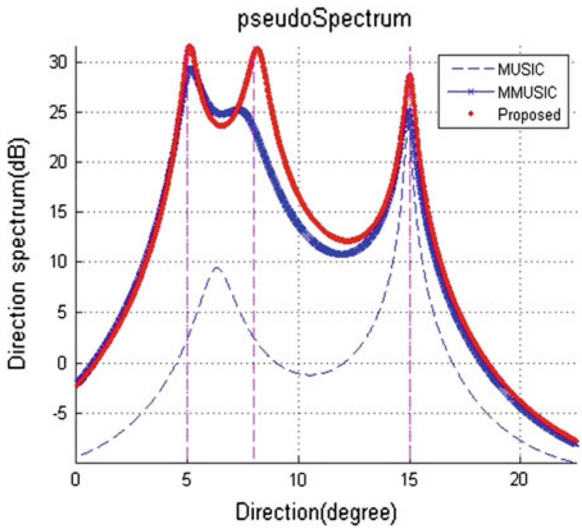

Fig. 3.3 Estimation for close-angle interval at SNR = 0 dB

Table 3.1 Estimation mean and standard deviation (Std) of MMUSIC algorithm and this article algorithm

SNR (dB)	MMUSIC algorithm				Algorithm in the article			
	5°		15°		5°		15°	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
10	5.12	0.0182	15.16	0.0243	4.96	0.0175	15.08	0.0124
5	5.21	0.0843	15.25	0.0612	4.92	0.0723	15.14	0.0536
0	5.47	0.3046	15.47	0.3017	4.83	0.2634	15.23	0.2415
−5	5.81	0.7392	15.78	0.8192	4.64	0.6028	15.42	0.5897

algorithm and the algorithm of this article can estimate the DOA better under the condition of relatively high signal-to-noise ratio. When the signal-to-noise ratio turns worse, for the signal sources with relatively little angle interval, as shown in Fig. 3.3, MMUSIC algorithm is also powerless, and the algorithm in the article is still able to differentiate two close coherent signals sources.

Table 3.1 is the comparison result about the estimation mean and standard deviation of two algorithms [3]. From it, we can see that whether the coherent sources (5°) or non-coherent sources (15°), the DOA estimation performance of the algorithm is better than that of MMUSIC algorithm. Therefore, this algorithm has a good performance in the narrowband signal DOA estimation. In particular, when the signal-to-noise ratio changes worse, it can still identify very close coherent sources.

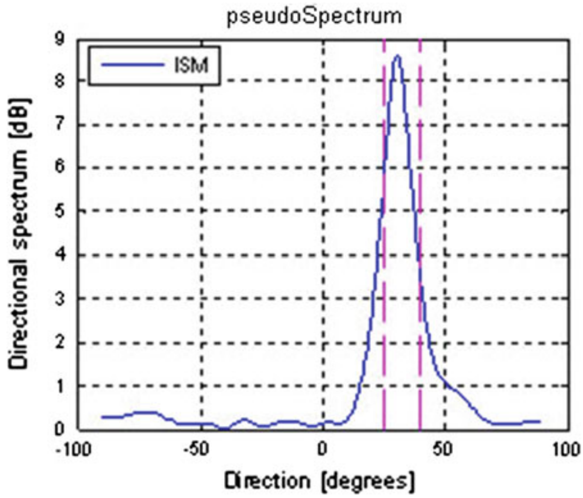

Fig. 3.4 DOA estimation of ISM

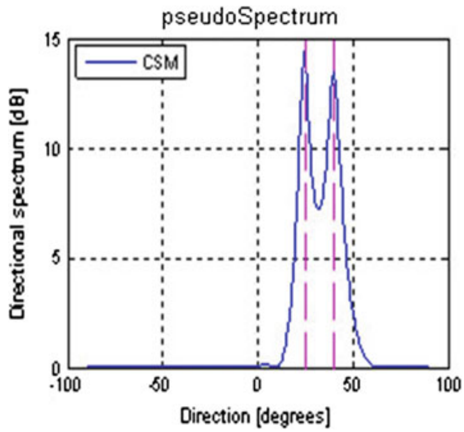

Fig. 3.5 DOA estimation of CSM

3.5 DOA Estimation for Wideband Signals

Adopting array is isotropy uniform linear array, wavelength $\lambda = 1$, array distance $D = 0.4\lambda$, snapshots $T = 1,024$, digital sensor channels $K = 8$. Receiving signals are long-range signals, and noise is normal random noise. The source signals are coherent wideband signals, which have same carrierFreq $= 3e9$ Hz and bandwidth $= 2e9$ Hz. The number of signals $M = 2$ and the angles of incidence are 25° and 40°, respectively, and the sensors SNR is 0 dB. Adopt ISM method, CSM method, and modified ISM method to estimate the DOA of the signals. The simulation results show as follows (Figs. 3.4, 3.5, 3.6, 3.7).

Simulation results show that both CSM and modified ISM can estimate the DOA well except ISM when the signals are coherent wideband signals. The capability of

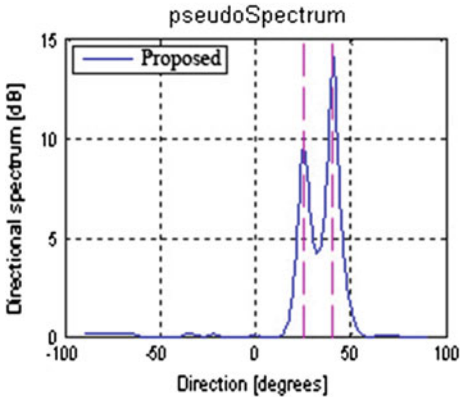

Fig. 3.6 DOA estimation of proposed ISM

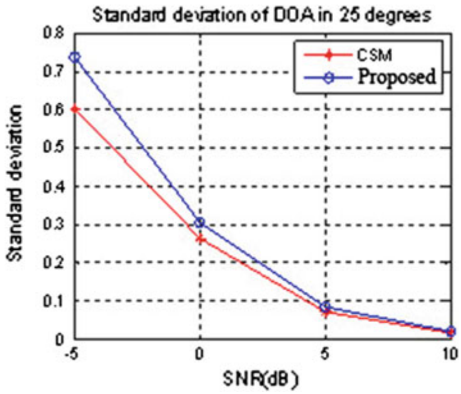

Fig. 3.7 Estimation results comparison between CSM and proposed ISM

the CSM is better than the modified ISM, but it needs direction pre-estimation and consistent focusing matrix. And it is also easy affected by the signals and needs more computation, so the CSM cannot process the signals in real time. The algorithm proposed in this study does not need these steps and has less computation. The algorithm has the same capability as CSM when the SNR is high enough.

3.6 Conclusion

A de-correlation method is proposed in this study which is based on the snapshot matrix reconstruction. The accuracy is evaluated through simulation in comparison with some conventional methods, and we found that the proposed method is as accurate as the conventional methods and needs smaller computational cost. The simulation results prove that it can be used to complete the DOA estimation for narrowband and wideband coherent signals. For narrowband signals, the algorithm is superior to the MMUSIC algorithm. And for wideband signals, the performance is below that of CSM algorithm, but because of its less computation, it is more practical to estimate the DOA of wideband coherent signals by the modified ISM in terms of real time. How to realize the algorithm by hardware should be improved as one of future studies.

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