

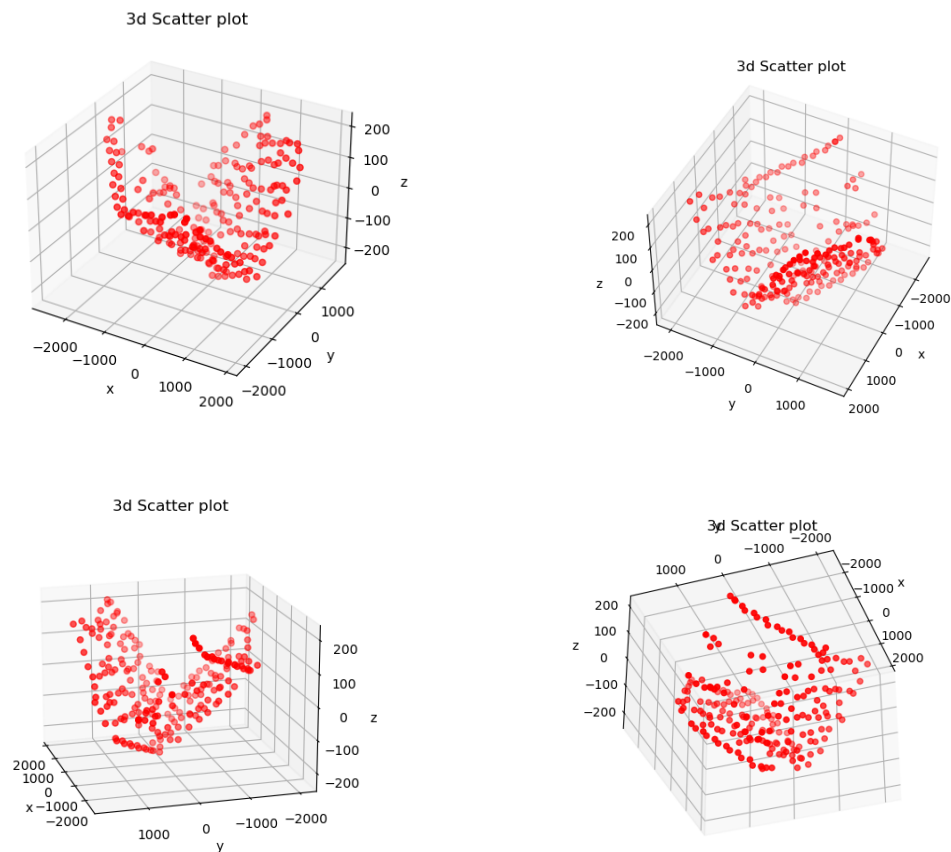
CS543/ECE549 Assignment 5

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Part 1: Affine factorization

A: Display the 3D structure (you may want to include snapshots from several viewpoints to show the structure clearly). Report the Q matrix you found to eliminate the affine ambiguity. Discuss whether or not the reconstruction has an ambiguity.



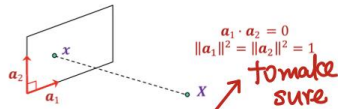
Q matrix:

```
[[ 7.94778741e-03  0.00000000e+00  0.00000000e+00]
 [-4.08314266e-18  8.53955215e-03  0.00000000e+00]
 [-5.96818241e-18 -1.58703020e-18  2.53757864e-02]]
```

Ambiguity problem:

If q is full rank 4×4 matrix, the construct may have a objective ambiguity; if we separate $A(3 \times 3$ full rank matrix) from Q in the way mentioned in the lecture, the reconstruction up to an affine ambiguity. However, by using method showing below, we can eliminate the ambiguity.

- Let a_1 and a_2 be the rows of a 2×3 orthographic projection matrix. Then



- This translates into $3m$ constraints on the 9 entries of Q :

$$(A_i Q)(A_i Q)^T = A_i (Q Q^T) A_i^T = I_{2 \times 2}, \quad i = 1, \dots, m$$

- Are the constraints linear?
- First, solve for $L = Q Q^T$
- Recover Q from L by Cholesky decomposition
- Update M to MQ , S to $Q^{-1}S$

algorithm:

1. Compute the singular-value decomposition $\tilde{W} = O_1 \Sigma O_2^T$.
2. Define $\hat{R} = O_1 (\Sigma)^{1/2}$ and $\hat{S} = (\Sigma')^{1/2} O_2^T$, where the primes refer to the block partitioning defined in (13).
3. Compute the matrix Q in equations (15) by imposing the metric constraints (equations (16)).
4. Compute the rotation matrix R and the shape matrix S as $R = \hat{R}Q$ and $S = Q^{-1}\hat{S}$.
5. If desired, align the first camera reference system with the world reference system by forming the products RR_0 and $R_0^T S$, where the orthonormal matrix $R_0 = [i_1 \ j_1 \ k_1]$ rotates the first camera reference system into the identity matrix.

how to deal with Q ?

$$\text{Now } \hat{W} = O_1' \Sigma' O_2'$$

$$\text{init, } \hat{R} = O_1' [\Sigma']^{1/2} \quad \hat{S} = [\Sigma']^{1/2} O_2'$$

$$\hat{W} = \hat{R} \hat{S} = \hat{R} \cdot Q \cdot Q^T \hat{S} \rightarrow \text{shape matrix}$$

rotation matrix

$$A_i (2 \times 3) \quad A_i^T (3 \times 2)$$

$$Q (3 \times 3) \quad Q^T (3 \times 3)$$

$$A (2m \times 3) \quad Q \cdot Q^T \cdot A^T (3 \times 2m) = I (2m \times 2m)$$

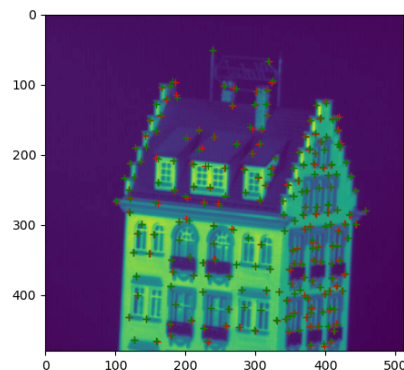
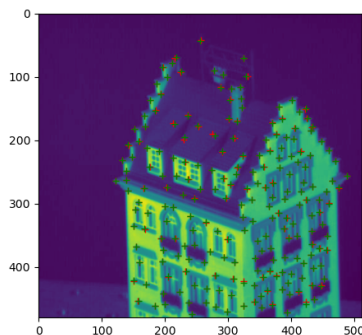
3x3

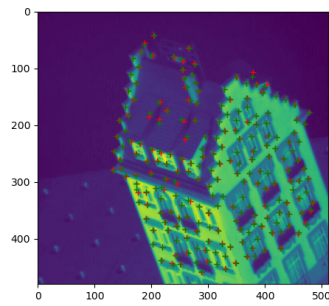
$$\therefore L = A^{-1} \cdot I \cdot A^{-T}$$

$$Q^{-1} \cdot S$$

(3x3) (3x2, 5)

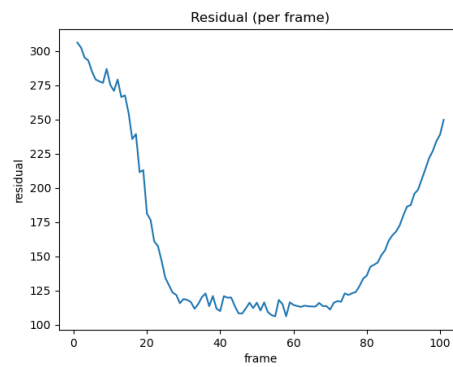
B: Display three frames with both the observed feature points and the estimated projected 3D points overlaid.





C: Report your total residual (sum of squared Euclidean distances, in pixels, between the observed and the reprojected features) over all the frames, and plot the per-frame residual as a function of the frame number.

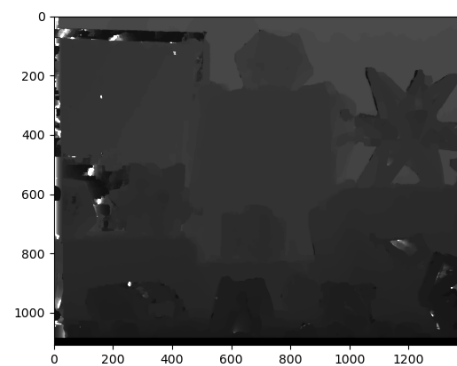
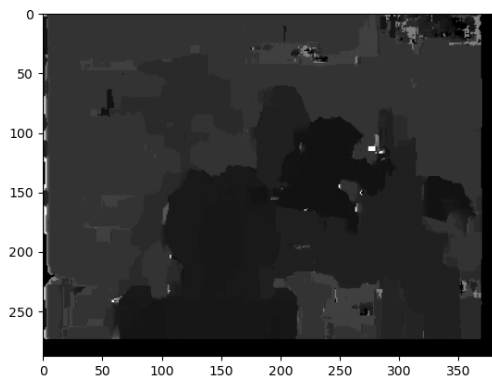
total residual: 16428.33206302818



Part 2: Binocular stereo

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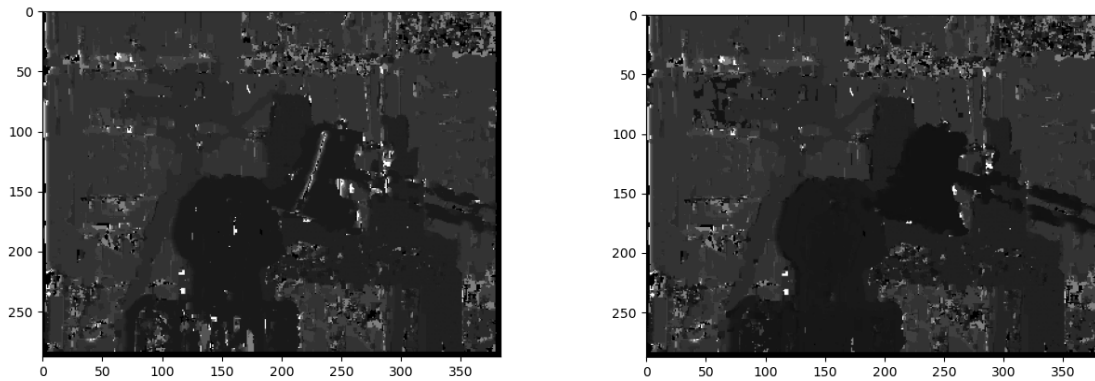
A: Display best output disparity maps for both pairs.



B: Study of implementation parameters:

1. **Search window size:** show disparity maps for several window sizes and discuss which window size works the best (or what are the tradeoffs between using different window sizes). How does the running time depend on window size?

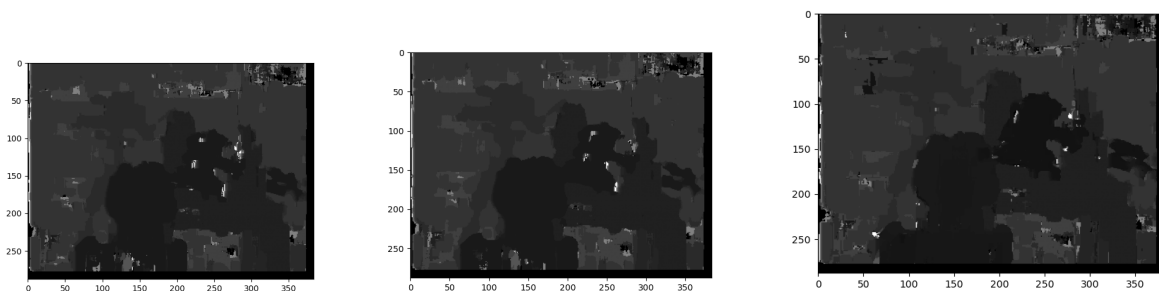
For the tsukuba image: when the window size is small ($w = 5$), there are a lot of noise in there:



	SSD	SAD	Normalized correlation
Running time(s)	12.3014	11.88446	86.435249

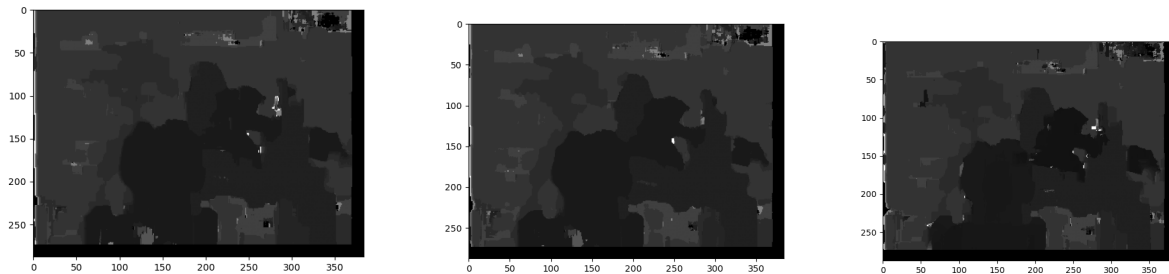
The left one is using SSD matching function while the right one is using normalized correlation. The right one has a relative better performance, however, both of them have a lot of noise.

When the window size goes to 11, the disparity map has less noise but also has less details. (SSD, SAD, normalized correlation from left to right).



	SSD	SAD	Normalized correlation
Running time(s)	11.690	11.845	84.889

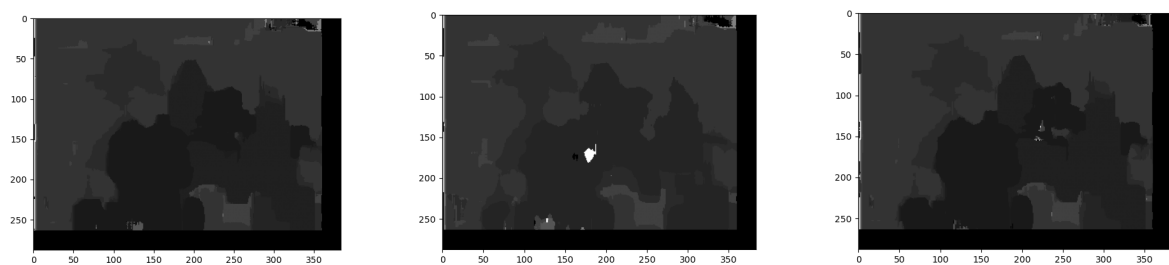
When the window size is 15, (SSD, SAD, normalized correlation from left to right).



	SSD	SAD	Normalized correlation
Running time(s)	12.328	11.9849	87.927

I think now it has a perfect performance, has acceptable noise and sufficient depth details.

When the window size is 25, as shown below, the disparity maps are smooth but with less details.(SSD, SAD, normalized correlation from left to right)



	SSD	SAD	Normalized correlation
Running time(s)	8.973	12.248	60.6528

All in all, I think window size = 15 works the best.

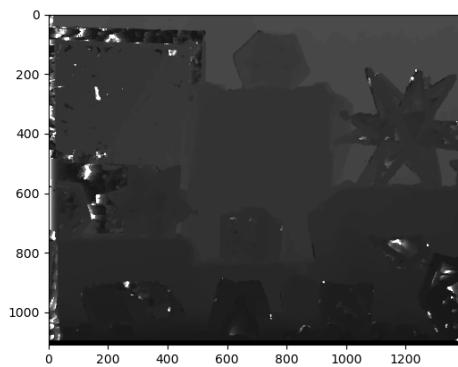
It may not have significant difference in running time when window size goes from 5 to 25, however, we can see that there is only significant difference between different matching methods.

I think the reason why there is no significant difference between different window sizes is that when the window size increases, the time of each window to compute match value increases, however, the number of window that needs to compute decreases. In that case, there is no significant difference in running time between different window sizes.

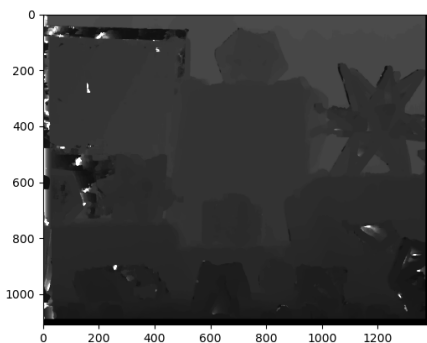
For moebius image:

Since the moebius image is too large, it's compute time is also so big.

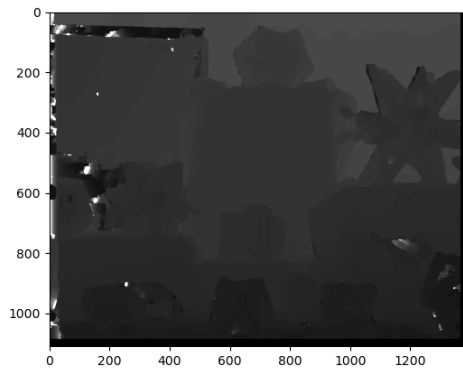
$W = 15, d = 111, m = 1$:



$W = 21, d = 111, m = 1$:



$W = 25, d = 151, m = 1$



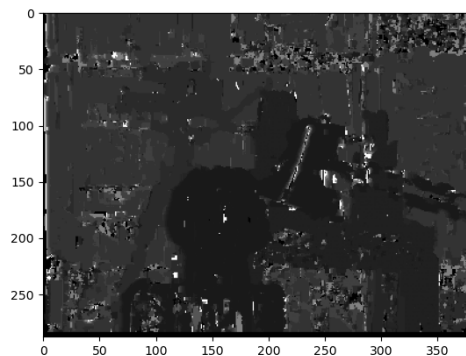
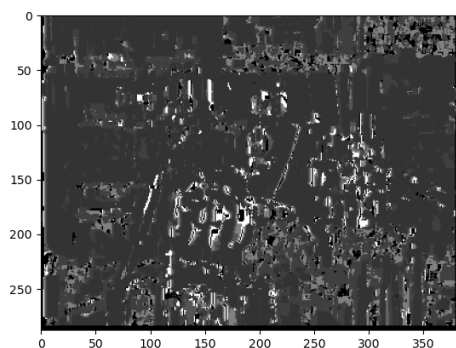
2. **Disparity range:** what is the range of the scanline in the second image that should be traversed in order to find a match for a given location in the first image? Examine the stereo pair to determine what is the maximum disparity value that makes sense, where to start the search on the scanline, and which direction to search in. Report which settings you ended up using.

Since the pictures are rectified, which means match points will be in the same row, the direction we need to search in is only from left to right.

For the tsukuba image:

The disparity range of SSD and SAD is at least 21 when window size=5,11,15 and at least 15 when window size = 25.

As shown below: the left img is SSD(window size = 5) when disparity = 11 and the right image is SSD when disparity = 21.



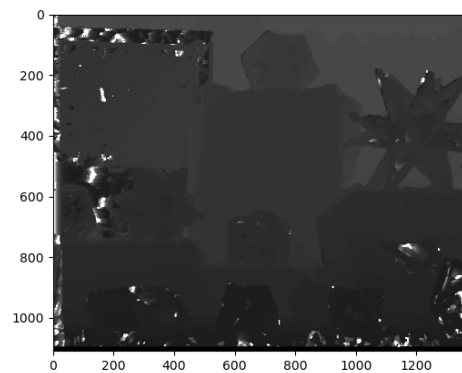
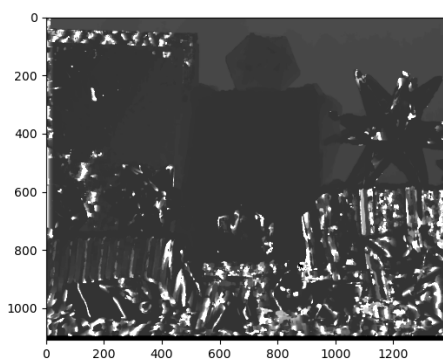
The disparity range of normalized correlation is at least 31.

If the disparity range is too small, it will cause the algorithm to fail to find the match pattern in picture B, if the disparity range is too large, it will make the computing time of the algorithm increase.

In that case, I choose 21 for SSD and SAD and 31 for normalized correlation.

For the moebius:

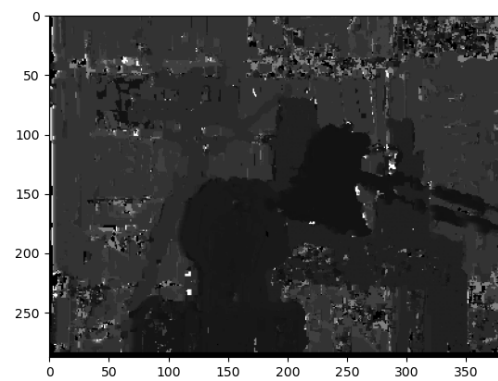
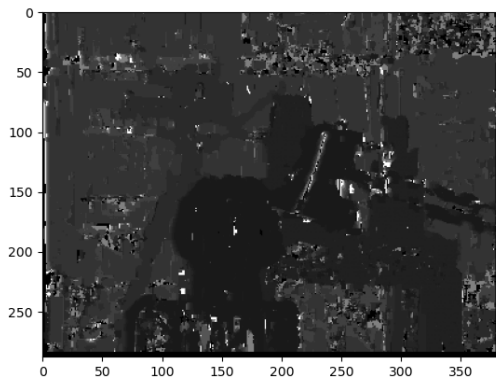
When the window size = 15, the left image is disparity range = 51, the right image is disparity range = 91. We can see that the left disparity range is too small, so there are some places with blank, which means image A in that space does not find appropriate window in image B.



3. **Matching function:** try sum of squared differences (SSD), sum of absolute differences (SAD), and normalized correlation. Discuss whether there is any difference between using these functions, both in terms of quality of the results and in terms of running time.

Normalized correlation needs more disparity range than other two methods, and it also uses more time than other methods with the same search window size and disparity range. For instance, for the tsukuba image, when window size = 5, SSD and SAD needs disparity range = 21 to have a good performance while normalized correlation needs disparity range = 31 to have a good performance. When disparity range = 21, the running time of SSD is 12.314s while the running time of normalized correlation is 58.170s.

However, normalized correlation has a better performance than SSD or SAD by having less noise under the same window size, as shown below. The left one is using SSD matching function while the right one is using normalized correlation.



C: Discuss the shortcomings of your algorithm. Where do the estimated disparity maps look good, and where do they look bad? What would be required to produce better results? Also discuss the running time of your approach and what might be needed to make stereo run faster.

For different match methods, SSD and SAD have fewer computing time than normalized correlation while normalized correlation has better performance than those two. If the image size is too large, we have to use SSD and SAD, but we may have a little bit worse result. It is a trade off between computing time and performance.

When we have a good disparity range and windows, results show well. However, there are too many parameters needed to set, which will cause lots of effort and time.

If we want faster running time, we need to use SSD or SAD instead of normalized correlation; we need to use a disparity range as small as possible which is large enough to have good performance.

Part 3: Extra Credit

Post any extra credit for parts 1 or 2 here. Don't forget to include references, an explanation, and outputs to receive credit. Refer to the assignment for suggested outputs.

Study notes for MP5:

https://drive.google.com/file/d/1MPLFzSFmtXwuwLzyYDoiFOm3leRT9CvJ/view?usp=share_link