

# Project Week03

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## 1 Problem 1

First, I use the following formula to get the weights for different times. The further back the time, the greater the weight, and the sum of the weights is 1.

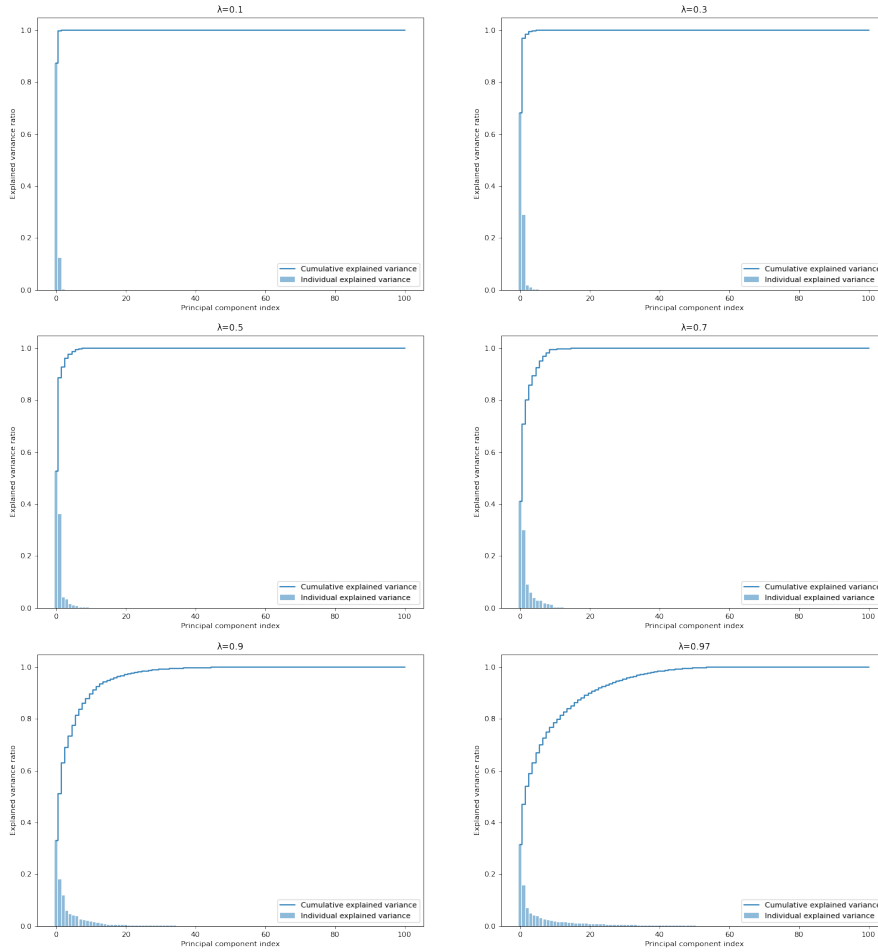
$$w_{t-i} = (1 - \lambda)\lambda^{i-1}$$

$$\hat{w}_{t-i} = \frac{w_{t-i}}{\sum_{j=1}^n w_{t-j}}$$

After that, the following formula of to the exponentially weighted covariance system is used. This formula is very similar to the covariance formula, the only difference is that there is an additional weighting factor.

$$Cov(\hat{x}, \hat{y}) = \sum_{i=1}^n \hat{w}_{t-i} (x_i - \bar{x})(y_i - \bar{y})$$

Finally, I use the PCA method to calculate the cumulative covariance explained by eigenvalue for each chosen  $\lambda$ . Here are the charts. Obviously, the higher the  $\lambda$  is, the fewer eigenvalues we need to explain.



## 2 Problem 2

I am using the following table to compare the two different PSD methods.

Method	n	Norm	Time
<i>nearPSD</i>	500	0.39378	0.02287s
<i>higham<sub>n</sub>nearestPSD</i>	500	0.00804	1.77003s
<i>nearPSD</i>	1000	0.79297	0.15064s
<i>higham<sub>n</sub>nearestPSD</i>	1000	0.00815	13.73630s

By comparing the operation time and the result error, we can conclude that Higham method has better accuracy but spends more time, while Rebonato and Jackels method is faster but less accurate.

### 3 Problem 3

The following tables and graph shows the run time and Frobenius Norm of the four simulation methods applied on the covariance matrix generated by:

1. EWMA correlation and standard deviation
2. EWMA correlation and Pearson standard deviation
3. Pearson correlation and standard deviation
4. Pearson correlation and EWMA standard deviation

	Name	Simulation	Norm	RunTime
0	EWMA	Full	5.808264e-07	0.174197
1	EWMA_COR_PEARSON_STD	Full	7.721642e-07	0.116791
2	PEARSON	Full	1.003231e-06	0.093920
3	PEARSON_COR_EWMA_STD	Full	7.392774e-07	0.094156
4	EWMA	PCA=1	4.577813e-03	0.062893
5	EWMA_COR_PEARSON_STD	PCA=1	5.339868e-03	0.045191
6	PEARSON	PCA=1	6.758644e-03	0.075414
7	PEARSON_COR_EWMA_STD	PCA=1	5.921737e-03	0.078212
8	EWMA	PCA=0.75	4.588553e-03	0.013578
9	EWMA_COR_PEARSON_STD	PCA=0.75	5.356165e-03	0.011057
10	PEARSON	PCA=0.75	6.782469e-03	0.010833
11	PEARSON_COR_EWMA_STD	PCA=0.75	5.933658e-03	0.010802
12	EWMA	PCA=0.5	4.588952e-03	0.006829
13	EWMA_COR_PEARSON_STD	PCA=0.5	5.361222e-03	0.006244
14	PEARSON	PCA=0.5	6.783191e-03	0.006089
15	PEARSON_COR_EWMA_STD	PCA=0.5	5.943695e-03	0.006090

It can be seen that the accuracy is negatively related to the run time.