Project Week04

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February 2022

1 Problem 1

1.1 Question

Calculate and compare the expected value and standard deviation of price at time $t(P_t)$, given each of the 3 types of price returns, assuming$r_t \sim N(0, \sigma^2)$. Simulate each return equation using$r_t \sim N(0, \sigma^2)$and show the mean and standard deviation match your expectations.

1.2 Answer

1.2.1 Classical Brownian Motion

First we discuss the generic solution for multiple time periods.

$$P_t = P_{t-1} + rt = P_{t-2} + r_{t-1} + r_t = \cdots = P_0 + \sum_{i=1}^{t} r_i$$
$$\Rightarrow$$
$$E(P_t) = P_0$$
$$Var(P_t) = t\sigma^2$$

When the time period is only 1, the special solution can be obtained from the above equation.

$$E(P_{t+1}) = P_t$$
$$Var(P_{t+1}) = \sigma^2$$

1.2.2 Arithmetic Return System

As in the previous subsection, we first discuss the generic solution for multiple time phases.

$$P_t = P_{t-1}(1 + r_t) = P_{t-2}(1 + r_{t-1})(1 + rt) = \cdots = P_0 \prod_{i=1}^{t}(1 + r_i)$$
$$\Rightarrow$$
$$E(P_t) = P_0 \prod_{i=1}^{t} E(1 + r_i) = P_0$$
$$Var(P_t) = P_0^2 Var(\prod_{i=1}^{t}(1 + r_i))$$
$$= P_0^2 Var(\prod_{i=1}^{t}(1^2 + \sigma^2) - \prod_{i=1}^{t} 1^2)$$
$$= P_0^2((1 + \sigma^2)^t - 1)$$

When the time period is only 1, the special solution can be obtained from the above equation.

$$E(P_{t+1}) = P_t$$
$$Var(P_{t+1}) = \sigma^2 P_t$$

1.2.3 Log Return Motion

In this section, we directly use the properties of the log-normal distribution.

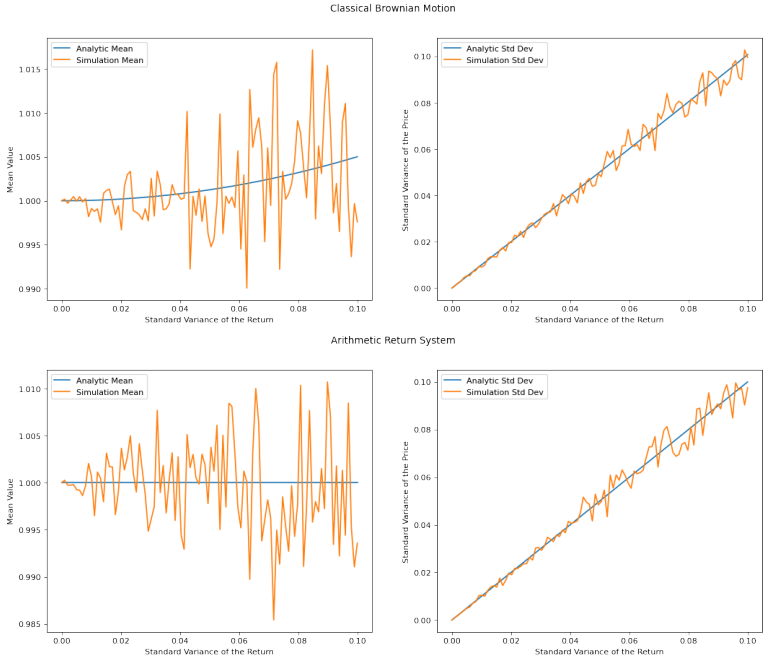
$$E(P_t) = e^{ln(P_0)+t\sigma^2/2}$$
$$Var(P_t) = (e^{t\sigma^2} - 1)e^{2ln(P_0+t\sigma^2)}$$

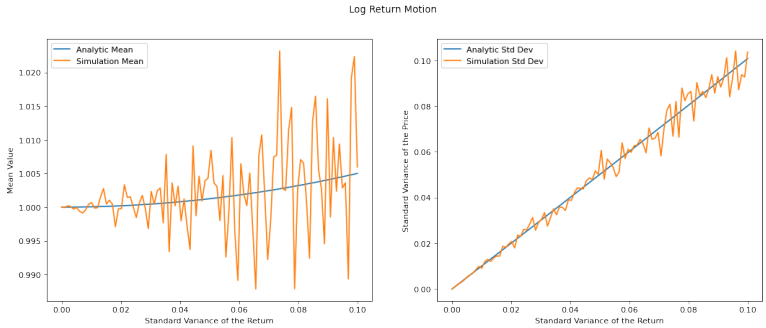
When the time period is only 1, the special solution can be obtained from the above equation.

$$E(P_{t+1}) = e^{ln(P_t)+\sigma^2/2}$$
$$Var(P_{t+1}) = (e^{\sigma^2} - 1)e^{2ln(P_t+\sigma^2)}$$

1.2.4 Monte Carlo Simulation

Use the random number generator to check the difference between the analytic solution and the simulated solution.





2 Problem 2

2.1 Question

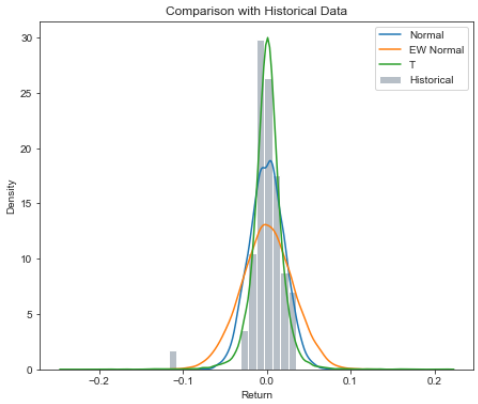
Implement a function similar to the "$return_calculate()$" in this week's code. Allow the user to specify the method of return calculation. Use DailyPrices.csv. Calculate the arithmetic returns for INTC. Remove the mean from the series so that the mean(INTC)=0
Calculate VaR
1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a Historic Simulation.
Compare the 4 values. Look at the empirical distribution of returns, in sample. Download from Yahoo! Finance the prices since the end of the data in the CSV file (about 2 weeks). Look the empirical distribution of returns, out of sample. Discuss the ability of these models to describe the risk in this stock.

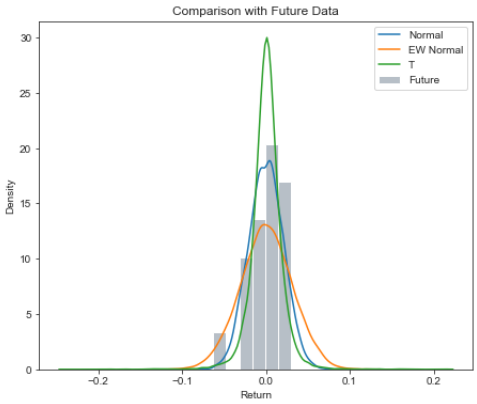
2.2 Answer

The following table shows the VaRs calculated by different methods.

Method	VaR
$Historical$	2.07%
$Normal$	3.40%
$EWNormal$	5.04%
T	2.70%

The following figure shows the fit of the different methods to the historical and future data. It can be seen that among the methods other than the historical method, the T-distribution fits best for the past data and the normal distribution predicts best for the future data.





3 Problem 3

3.1 Question

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.
This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings).
Discuss your methods, why you chose those methods, and your results.

3.2 Answer

In this topic, I did not use the rate of return to calculate VaR, but directly used the asset portfolio PL for the calculation. The profit and loss formula is as follows.

$$P\&L = \sum \Delta Price_i Holding_i$$

The results are presented in the following table

Portfolio	VaR
A	6469.05
B	6073.01
C	3679.06
$Total$	16221.13