TU/ASSIGNMENT: RK2/4 ODE-IVP

Part 1: 1st order ODE,   Single equation

Problem

Solve for the response of an RC circuit with a sinusoidal input

$$f(t,v)=\frac{dv}{dt}=-\frac{1}{\tau}v(t)+\frac{1}{\tau}V_m\cos(2\pi ft)$$

```
tau=1; T=1/tau; f=10; Vm=1; w=2*pi*f; a=0; b=0.1; h=0.001;
```

Exercise 1

- Write a function for a general form of RK2, with C1, C2, alpha, beta.

```
void odeRK2(double odeFunc(const double t, const double y), double y[], double t0, double tf, double h, double y0);
```

$$y_{i+1}=y_i+\left(C_1K_1+C_2K_2\right)h$$

$$C_1=1-C_2,\ C_2=\frac{1}{2\alpha}\ \text{ and }\ \begin{cases}K_1=f(t,y)\\K_2=f(t+\alpha h,y+\beta K_1h)\end{cases}$$

Exercise 2

- Write a function for the standard form of RK4
- `void odeRK4(double odeFunc(const double t, const double y), double y[], double t0, double tf, double h, double y0);`

$$y_{i+1}=y_i+\frac{1}{6}(K_1+2K_2+2K_3+K_4)h$$

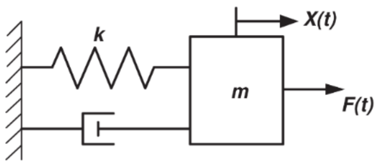
$$\begin{cases}K_1=f(x_i,y_i)\\K_2=f(x_i+\frac{1}{2}h,\ \underbrace{y_i+\frac{1}{2}K_1h}_{y_m^{mid}})\\K_3=f(x_i+\frac{1}{2}h,\ \underbrace{y_i+\frac{1}{2}K_2h}_{y_m^{mid}})\\K_4=f(x_i+h,\ y_i+K_3h)\end{cases}$$

Part 2: 2nd order ODE

A mass-spring-damper (m-c-k) system is a 2^{nd} order ODE

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky(t)=F(t)$$

where F(t) is the input force, y(t) is the displacement.



Create a program that gives the values of y(t), dy/dt(t) for the following response

- Harmonic Response (F(t)=Acos(2*pi*f*t)
- (Exercise) Free vibration (i.e. F(t)=0) from the initial condition
- (Exercise) Step Response (F(t)=A)

- Parameters/Initial condition

  m=1kg, k=6.9N/m, c=7 N/m/s, A=2 N, f=5Hz,

  t=0 to 1 sec,   h=0.01 sec

  Initial Condition: y(0)=0.0 m,   dy/dt $_{|t=0}$=0.2 m/s

Exercise 3

- Write a function for the governing equation for m-c-k system.

```
void odeFunc_mck(const double t, const double Y[], double dYdt[])
{
    double m = 1;
    double c = 7;
    double k = 6.9;
    double f = 5;

    double Fin = 2 * cos(2 * PI * f * t);

    dYdt[0] = Y[1];

    // EXERCISE: MODIFY HERE
    dYdt[1] = 0;
}
```

HINT:

$$\begin{cases} \dot{y} = z(t) \\ \ddot{y} = \dot{z} = \frac{1}{m}\left(-ky(t) - cz(t) + u(t)\right) \end{cases} \Rightarrow \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} z(t) \\ -ky(t) - cz(t) + u(t) \end{bmatrix}$$

- Create a function of RK2 for 2^{nd} order system

```
void sys2RK2(void odeFunc_sys2(const double t, const double Y[], double dYdt[]),
            double y1[], double y2[], double t0, double tf, double h, double y1_init,
            double y2_init);
```

- Compare the answer with MATLAB's ODE solver by copy-pasting your outputs in MATLAB.

Exercise 4

- Create a function of RK4 for 2^{nd} order system

```
void sys2RK4(void odeFunc_sys2(const double t, const double Y[], double dYdt[]),
        double y1[], double y2[], double t0, double tf, double h, double y1_init,
        double y2_init);
```