Let $L := O_X \cdot s$ be line bundle with connection ∇ such that

$$\nabla(s) = \frac{i}{n}\frac{dx}{x}s$$

And let $\mathcal{L}$ be local system of flat sections of L Let $p : Y \to X$ be defined by $y^n = x$. The equivariant bundle on Y should be the Deligne extension of the pullback of the local system corresponding to L. In this case, the equivariant bundle should be the vector bundle $V_Y = O_Y \cdot e$ with the trivial connection. Let Γ be the Galois group of p. Γ is generated by the n-th root of unity μ. The Γ-action on V should encode the local system on X. Last time I defined the action to be

$$\mu \cdot e = \mu^{-i} e$$

which was a consequence of having the knowledge that $s = x^{i/n}$ in mind. But this time I have a better reason to justify this action, and we will see the jump happens at i/n.

First, consider $(V_1 := p^*L, \nabla_1 = p^*\nabla)$. This sheaf carries a natural Γ-action. Write $V_1 = O_Y \cdot e_1$. Then, ∇_1 can be represented as

$$\nabla_1(e_1) = i\frac{dy}{y}e_1$$

The flat sections of V_1 is in fact $y^{-i}e_1$, which lives in $V_1(i\tilde{D})$. This means we can define an O_Y-isomorphism

$$\begin{aligned} \alpha : V_1(i\tilde{D}) &\to V_Y \\ y^{-i}e_1 &\mapsto e \end{aligned}$$

sending flat sections to flat sections. There is a natural Γ-action on $V_1(i\tilde{D})$. So we can define a Γ-action on V_Y via α, *i.e.*

$$\mu \cdot e = \mu^{-i} \times e$$

Next, we will see how

$$(p_* V_Y \otimes O_Y(\lfloor -nt \rfloor \tilde{D}))^\Gamma$$

recovers the parabolic structure of L. First, we need a map

$$\beta : (p_* V_Y)^\Gamma \to L$$

As a O_X-module, $p_* O_Y$ looks like

$$\sum_{j=0}^{n-1} O_X \cdot y^j$$

and it has an algebra structure given by

$$y^n = x$$

We write

$$p_* V_Y = \sum_{j=0}^{n-1} O_X \cdot y^j \otimes e$$

Then, we know how Γ acts on each direct summand. Obviously, the Γ-invariant part is

$$O_X \cdot y^i \otimes e$$

So the map β is

$$\begin{aligned} \beta : (p_* V_Y)^\Gamma &\to L \\ y^i \otimes e &\mapsto s \end{aligned}$$

Write $V_Y \otimes O_Y(-k\tilde{D}) = O_Y \cdot y^k \otimes e$. Then, we write

$$p_* V_Y \otimes O_Y(-j\tilde{D}) = \sum_{j=0}^{n-1} O_X \cdot y^j \otimes y^k \otimes e$$

we can see the invariant part is

$$O_X \cdot y^j \otimes y^k \otimes e$$

such that $j + k = i, i + n, i + 2n, \cdots$. That why as soon as k is bigger than i, we will jump to

$$L(-D)$$