Let D be a smooth divisor of a smooth projective variety X. If I want a cyclic cover branched over D, then I can just through in n-th root of local equations of D in the function field $\mathbb{C}(X)$ then normalize X in $\mathbb{C}(X)$(n-th root of local equation of D).

$$\pi : \tilde{X} \to X$$

And hope that D is the only place that π branches. But local equation of D is just a rational function on X. Somewhere on X it will pick up a pole, and naturally π branches over the pole just as branches over the zeros of the rational function. That is why during the construction of cyclic cover, one picks up some other branch divisors.

This is why in Kawamata's construction of cyclic cover, we cannot just throw in f_1^{1/n_1}, where f_1 is the local equation of D_1, because f_1 might have a pole on D_2.

Let $D = D_1 + D_2 + \cdots + D_n$ be a normal crossing divisor. If we want a cyclic cover

$$\pi : \tilde{X} \to X$$

that branches over D_i of degree n_i. Then, the naive idea would be to throw in f_i^{1/n_i}, where f_i is the local equation of D_i. Obviously, this is a stupid idea, because π will be generically etale and it cannot be generically etale of degree n_1 and n_2 at the same time.

Consider the following map ϕ

$$\begin{aligned} \Delta \times \Delta \to & \qquad \Delta \times \Delta \\ y_1, y_2 \mapsto & \qquad y_1^n, y_2 \end{aligned}$$

Let D_1 and D_2 be divisors downstairs defined by $y_1 = 0$ and $y_2 = 0$. Then, $\phi^* D_1 = n_1 D_1$, and $\phi^* D_2 = D_2$. To make a cyclic cover branched over D_2, we fix the first coordinate, and raise the power of the second coordinate. Call the following map ϕ

$$\begin{aligned} \Delta \times \Delta \to & \qquad \Delta \times \Delta \\ y_1, y_2 \mapsto & \qquad y_1^{n_1}, y_2^{n_2} \end{aligned}$$

Then

$$\phi^* D_1 = n_1 D_1, \phi^* D_2 = n_2 D_2$$