1 Normal crossing divisor with only one component

Let D be a smooth divisor of X, and let $\mathcal{V}$ be a quasi-unipotent local system of rank r. on $X - D$. Let (V, ∇) be the Degline canonical extension of $\mathcal{V}$. The quasi-unipotent structure of V along D is determined by Res $\circ \nabla$.
Let $x \in D$ and choose a small neighborhood U of x biholomorphic to a polydisk on which V is trivial. Fix a frame $< s_1, ..., s_r >$ with repect to which Res $\circ \nabla$ is in the Jordan canonical form

$$M = J_1(\alpha_1/n) \oplus J_2(\alpha_2/n) \oplus \cdots \oplus J_l(\alpha_l/n)$$

Where each $J_i(\alpha_i/n)$ means the Jordan block with eigenvalue α_i/n.
Let $p : Y \to X$ be a cyclic cover branched over D of degree kn
Let (V_Y, ∇_Y) be the Deligne canonical extension of $p^*\mathcal{V}$, and let $(\tilde{V}, \tilde{\nabla})$ be the pullback of V
Let $\tilde{D} = (p^*D)_{red}$. Let $y \in \tilde{D}$, and choose a coordinate $y_1, \cdots, y_d$ so that $\tilde{D}$ is defined by $y_1 = 0$. p is locally defined by $x_1 = y_1^{kn}$
Choose a basis of $\tilde{V}$ so that the residue of the connection matrix of $\tilde{\nabla}$ looks like

$$kn \cdot M$$

Write

$$kn \cdot M = D + N$$

Where D is the invertible part, and N is the nilpotent part. Then, with respect to the same basis, the connection matrix of ∇_Y looks like

$$N$$

The flat section f of $\tilde{\nabla}$ satisfy the differential equation

$$df + \frac{D+N}{y} f = 0$$

The solution to it is

$$f = e^{-(D+N)\log y} C$$

where C can be taken to the $< 1, 1, \cdots, 1 >$.
The flat section g of ∇_Y satisfy the differential equation

$$dg + \frac{N}{y} g = 0$$

The solution is

$$g = e^{-N \log y} C$$

The flat section of V_Y and $\tilde{V}$ differ by y^{-D}. So we know how to define an G-action on V.

2 Normal crossing divisor with multiple components

There is no consist group action, but there is step-wise group action. Let $D = D_1 + D_2 + \cdot + D_N$ be a normal crossing divisor of X. Let γ_i be the monodromy of $\mathcal{V}$ around D_i. Let N_i be an integer so that $\gamma_i^{N_i}$ is unipotent. Take Kawamata's construction of cyclic cover

$$X_N \xrightarrow{p_N} X_{N-1} \xrightarrow{p_{N-1}} X_{N-2} \cdots X_1 \xrightarrow{p_1} X$$

so that p_i is branched over D_i with degree $N_i k_i$. Let V_i the Deligne canonical extension of $p_i^{-1}\mathcal{V}$, and let G_i be the Galois group of p_i. We can define a G_i-action on V_i, so that

$$(p_{i*}V_i)^{G_i} \cong V_{i-1}$$

However, let G be the Galois group of $p_N \circ p_{N-1} \circ \cdots \circ p_1$, I don't think we can define an G-action on V_N that identifies its invariant section with V in one step. Here is the reason why. Let $N = 2$. For simplicity, let $\dim X = 2$. We will see that even locally, one cannot define an one-step G-action. The local picture of cyclic covering looks like

$$\begin{array}{lll} \Delta_1 \times \Delta_2 & \xrightarrow{p_2} \Delta_1 \times \Delta_2 & \xrightarrow{p_1} \Delta_1 \times \Delta_2 \\ (y_1, y_2) & \mapsto (y_1, y_2^{m_2}) & \mapsto (y_1^{m_1}, y_2^{m_2}) \end{array}$$

Choose a frame for V, so that the connection matrix of ∇ looks like

$$\Gamma_1 \frac{dy_1^{m_1}}{y_1^{m_1}} + \Gamma_2 \frac{dy_2^{m_2}}{y_2^{m_2}}$$

where Γ_i are in their Jordan canonical form. The eigenvalues of Γ_i are rational and lie in $[0, 1)$. Let $p = p_1 \circ p_2$. Consider pullback of the connection

$$p^*\nabla = (D_1 + N_1)\frac{dy_1}{y_1} + (D_2 + N_2)\frac{dy_2}{y_2}$$

where D_i denotes the diagonal part, and N_i denote the nilpotent part.
The connection matrix of ∇_2 will be

$$N_1 \frac{dy_1}{y_1} + N_2 \frac{dy_2}{y_2}$$

Therefore the flat sections of ∇_2 and $p^*\nabla$ differ by

$$y_1^{-D_1} y_2^{-D_2}$$

So we can see that we cannot play the ususal game to define a G-action on V_2. WELL, actually we can still define a one step action, but we need to pay a price of making the frame of the Deligne canonical extension a bit more complicated, to keep track of each divisor. In the above example, write the usual frame of V_2 this way

$$e_1^1 \otimes e_1^2, e_2^1 \otimes e_2^2, e_3^1 \otimes e_3^2, \cdots, e_r^1 \otimes e_r^2$$

where underscript means the rank of V and superscript keeps track of divisors. Let $G = G_1 \times G_2$. Let $\mu = (\mu_1, \mu_2)$ be an element of G. Then, μ_1 acts on e_1^i according to y_1^{-D} and μ_2 acts on e_2^i according to $y_2^{-D_2}$.