Variation of Hodge structure consists of the following data

1. A connected complex manifold M
2. A flat complex vector bundle $H_{\mathbb{C}} \to M$ with a flat real structure $H_{\mathbb{R}}$, and a flat bundle of lattice $H_{\mathbb{Z}} \subset H_{\mathbb{R}}$, i.e. transition functions of $H_{\mathbb{C}}$ a locally constant and real-valued. The flat connection of $H_{\mathbb{C}}$ has real valued connection matrix
3. An integer k
4. A flat, nondegenerate bilinear form S on $H_{\mathbb{C}}$ which is rational with respect to $H_{\mathbb{Z}}$, *i.e* restricting to each fibre, S is a locally constant matrix of rational coefficient with repect to the chosen basis $H_{\mathbb{Z}}$
5. A decreasing filtration
$$H_{\mathbb{C}} \supset F^1 \supset \cdots \supset 0$$
by holomorphic subbundle.

The objects need to satisfy the following

1. Over each point t, $H_{\mathbb{C}}, F, S$ restrict to a polarized Hodge structure of weight k.
2. $\nabla(F^p) \subset F^{p-1}$

1 Classifying space for Hodge Structure

Let $H_{\mathbb{Z}}$ be some lattice over $\mathbb{Z}$, and let $H_{\mathbb{C}}$ be its complexification. Fix an integer k, and a collection of nonzero integers $h^{p,q}$ such that $p+q=k$. First objective is to give a manifold structure to the parameter space of all Hodge structure of weight k on $H_{\mathbb{C}}$. Let $D_p = dimF^p = \sum_{i\geq p} h^{p,q}$. Then, we consider the product of Grassmannian
$$G = G(H_{\mathbb{C}}, D_0) \times G(H_{\mathbb{C}}, D_1) \times \cdot \times$$
A possible filtration on $H_{\mathbb{C}}$ can be considered as a point in G. All possible filtrations on $H_{\mathbb{C}}$ can be identified with the incidence variety I on G(why incidence variety is smooth)
The general linear subgroup of $H_{\mathbb{C}}$ operates on I transitively. The subset $F \subset I$ satisfying
$$H_{\mathbb{C}} = F^p \oplus \bar{F}^{k-p+1}$$
is an Zariski open subset of I. So F parametrizes the Hodge structure on $H_{\mathbb{C}}$.
Let S be a nondegenerate bilinear form on $H_{\mathbb{C}}$, symmetric or skew, depending on the parity of k, such that
$$\begin{aligned} S(H^{p,q}, H^{r,s}) = & \quad 0,\text{unless} p = s, q = r \\ i^{p-q} S(v, \bar{v}) > & \quad 0,\text{if} v \in H^{p,q}, v \neq 0 \end{aligned}$$

Let $D \subset I$ be the subset of filtration satisfy the above two conditions. D is a open subvariety. The orthogonal group of S

$$O(S) = \{g \in GL(H_{\mathbb{C}}) | S(gu, gv) = S(u, v)\}$$

acts transitively on D. Therefore, D is smooth. D parametrize Hodge structure of weight k on $H_{\mathbb{C}}$ with the polarization S