In this note, we will consider the canonical extension of a quasi-unipotent local syetem of rank 1, *i.e.* root of unity. We will use a simple example to justify the idea that the equivariant bundle on the branched cover is the canonical extension of the pullback of the local system.

Let $X = \Delta$ be the unit disk, $D \subset X$ be the origin. Consider the local system $\mathcal{V}$ on Δ^* given by $(\mathbb{Z}, T)$, where $T = \epsilon_{10}^3 = \exp \frac{6\pi i}{10}$. Let x be the coordinate on Δ. Then, the canonical extension (V, ∇) of $\mathcal{V}$ is $L = O_X \otimes x^{3/10}$. The connection matrix of ∇ is given by $\frac{3}{10}\frac{dx}{x}$, and the inclusion map

$$\mathcal{V} \to L$$

is given by $e_1 \mapsto x^{-1/3} \otimes x^{1/3}$.

Let $p : Y \to X$ be the branched cyclic cover defined by $y^1 0 = x$. $p^{-1}\mathcal{V}$ is the trivial local system, so its canonical extension is the trivial line bundle with trivial connection. I will recover the trivial line bundle on Y using Biswas's construction of orbifold bundle.

Use Biswas's notation. The parabolic structure of L is given by

$$L = F_1(L) \subset F_2(L) = L \otimes O_X(-D)$$

the weight is $3/10$.

Let $f_2 : (10-3)\tilde{D} \to n\tilde{D}$ be the inclusion of schemes. Let $\bar{V}_2$ be the restriction of $p^*(L|_D / F_2(L|_D)) \otimes O_X(D)$ to the scheme $(10-3)\tilde{D}$. But in our case, $F_2(L|_D) = 0$. So $\bar{V}_2$ is the restriction of $p^*(L|_D \otimes O_X(D))$ to the scheme $(10-3)\tilde{D}$. We will be done once we write down the generators of $\bar{V}_2$ as a $\mathbb{C}$-vector space.

$L|_D = \mathbb{C} \otimes x^{3/10}$.

$p^*L|_D = \mathbb{C} \otimes y^3$.

$p^*O_X(D) = O_Y \otimes y^{-10}$.

Restriction of $p^*O_X(D)$ to $(10-3)\tilde{D}$ is $\mathbb{C} < 1 \otimes y^{-10}, y \otimes y^{-10}, \cdots, y^6 \otimes y^{-10} >$

So $\bar{V}_2 = \mathbb{C} < 1 \otimes y^{-7}, y \otimes y^{-7}, \cdots, y^6 \otimes y^{-7} >$

Now, consider the exact sequence

$$0 \to V_2 \to V \to \bar{V} \to 0$$

Use Biswas's algorithm in our case, we know that V_2 is the orbifold bundle corresponding to L. And according to our description of $\bar{V}_2$, we know that

$$V_2 = O_Y y^7 \otimes y^{-7}$$

Then, it is obvious how to think of V_2 as a trivial line bundle.