Problem1

```
Stiffness Matrix:
[[ 19120879.12087912   7767857.14285714  -2390109.89010989
    -597527.47252747  -9560439.56043956  -7767857.14285714
   -7170329.67032967    597527.47252747]
 [  7767857.14285714  34656593.4065934     597527.47252747
   13145604.3956044   -7767857.14285714 -17328296.70329671
    -597527.47252747 -30473901.0989011 ]
 [ -2390109.89010989    597527.47252747  19120879.12087912
   -7767857.14285714  -7170329.67032967   -597527.47252747
   -9560439.56043956   7767857.14285714]
 [  -597527.47252747  13145604.3956044   -7767857.14285714
   34656593.4065934     597527.47252747 -30473901.0989011
    7767857.14285714 -17328296.70329671]
 [ -9560439.56043956  -7767857.14285714  -7170329.67032967
     597527.47252747  19120879.12087912   7767857.14285714
   -2390109.89010989   -597527.47252747]
 [ -7767857.14285714 -17328296.70329671   -597527.47252747
  -30473901.0989011    7767857.14285714  34656593.4065934
     597527.47252747  13145604.3956044 ]
 [ -7170329.67032967   -597527.47252747  -9560439.56043956
    7767857.14285714  -2390109.89010989    597527.47252747
   19120879.12087912  -7767857.14285714]
 [   597527.47252747 -30473901.0989011    7767857.14285714
  -17328296.70329671   -597527.47252747  13145604.3956044
   -7767857.14285714  34656593.4065934 ]]

Unknown deformation
[ 4.59770115e-04  4.59770115e-04 -6.89655172e-05 -6.89655172e-05]

Unknown force
[-5.00000000e+03  0.00000000e+00  4.54747351e-13 -5.00000000e+03]

Deformation vector:
[ 0.00000000e+00  0.00000000e+00  4.59770115e-04  0.00000000e+00
  4.59770115e-04 -6.89655172e-05  0.00000000e+00 -6.89655172e-05]
Force vector:
[-5.00000000e+03  0.00000000e+00  5.00000000e+03  4.54747351e-13
  5.00000000e+03  0.00000000e+00 -5.00000000e+03  0.00000000e+00]
```

1.The displacement at node 3

ANS: 4.5977e-04 in at x direction and -6.89655172e-05 in at y direction

2.Stress and strain at the center of the element

```
stress
[ 6.66666667e+03 -4.54747351e-13 -1.13372102e-13]

strain
[ 2.29885057e-04 -6.89655172e-05 -1.01643954e-20]
```

$$\sigma_x = 6.666e3\ psi, \sigma_y = \tau_{xy} = 0$$

$$\varepsilon_x = 2.29885e-4\ psi, \varepsilon_y = -6.89655e-5\ psi, \gamma_{xy} = 0$$

3.Analytical solution

$$\sigma_x = \frac{p}{A} = \frac{10{,}000}{1.5*1} = 6666.67\ psi,\ \ and\ Error = \frac{|6666.67-6666.67|}{6666.67} * 100 = 0\%,$$

$$\sigma_y = 0\ psi,\ \ and\ Error = 0\%,$$

$$\tau_{xy} = 0\ psi,\ \ and\ Error = 0\%,$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{6666.67}{29e6} = 0.0002298,\ \ and\ Error = \frac{|0.0002398-0.0002298|}{0.0002398} * 100 = 0\%,$$

$$\varepsilon_y = \nu\varepsilon_x = 0.3 * 0.00023 = 0.000069,\ \ and\ Error = \frac{|0.00069-0.00069|}{0.000069} * 100 = 0\%,$$

$$\gamma_{xy} = 0,\ \ and\ Error = 0\%,$$

$$\delta_x = \frac{pL}{EA} = \frac{10{,}000*2}{29e6*1.5} = 0.00046\ in, and\ Error = \frac{|0.00046-0.00046|}{0.00046} * 100 = 0\%.$$

All solutions are same with the answer of FEM

4.What is wrong with Gauss integration order=1

For the 2D shape function, it requires two variables of natural coordinate system. But 1st order only has one variable. Thus, algorithm makes dimension error when I try to make shape function.

```
N[0,0] = (1/4)*(1-xi[0])*(1-xi[1])
IndexError: list index out of range
```

5.What goes wrong if the nodal coordinates are ordered in a clockwise manner

```
Check what happens in clockwise manner
Clockwise manner nodes
[[0 0]
 [0 1]
 [2 1]
 [2 0]]

Stiffness Matrix:
[[-19120879.12087912   -7767857.14285714    7170329.67032967
    -597527.47252747    9560439.56043956    7767857.14285714
    2390109.89010989     597527.47252747]
 [ -7767857.14285714 -34656593.4065934      597527.47252747
   30473901.0989011    7767857.14285714  17328296.70329671
    -597527.47252747 -13145604.3956044 ]
 [  7170329.67032967     597527.47252747 -19120879.12087912
    7767857.14285714    2390109.89010989    -597527.47252747
    9560439.56043956   -7767857.14285714]
 [  -597527.47252747   30473901.0989011     7767857.14285714
  -34656593.4065934      597527.47252747 -13145604.3956044
   -7767857.14285714   17328296.70329671]
 [  9560439.56043956    7767857.14285714    2390109.89010989
    597527.47252747 -19120879.12087912   -7767857.14285714
    7170329.67032967    -597527.47252747]
 [  7767857.14285714   17328296.70329671    -597527.47252747
  -13145604.3956044    -7767857.14285714 -34656593.4065934
    597527.47252747   30473901.0989011 ]
 [  2390109.89010989    -597527.47252747    9560439.56043956
   -7767857.14285714    7170329.67032967     597527.47252747
  -19120879.12087912    7767857.14285714]
 [   597527.47252747 -13145604.3956044    -7767857.14285714
   17328296.70329671    -597527.47252747   30473901.0989011
    7767857.14285714 -34656593.40659341]]

Unknown deformation
[ 7.71804300e-06  3.38401404e-04 -7.66835602e-05  1.21368710e-04]

Unknown force
[ 2925.07204611  1462.53602305  2074.92795389 -1462.53602305]
```

Jacobian matrix's order fit for counter clockwise manner. So if it is changed to clockwise manner, it will give wrong answer. Above figure is the result of clockwise. Ans its answer is wrong.

*Problem1.py is code for problem1.

Problem2

```
Stiffness Matrix:
[[ 2.46666657e+09  1.31111147e+08 -2.48973055e+09  2.65726379e+07
  -1.25020900e+09 -1.32863189e+08  1.22273343e+09 -2.48205958e+07]
 [ 1.31111147e+08  7.37230097e+08 -2.68646449e+07 -7.00028404e+08
  -1.34323224e+08 -3.68717251e+08  2.48205958e+07  3.31515558e+08]
 [-2.48973055e+09 -2.68646449e+07  2.53685040e+09 -1.36075266e+08
   1.25765014e+09  2.86166869e+07 -1.25020900e+09  1.34323224e+08]
 [ 2.65726379e+07 -7.00028404e+08 -1.36075266e+08  7.37638907e+08
  -2.86166869e+07  3.31106748e+08  1.32863189e+08 -3.68717251e+08]
 [-1.25020900e+09 -1.34323224e+08  1.25765014e+09 -2.86166869e+07
   2.53685040e+09  1.36075266e+08 -2.48973055e+09  2.68646449e+07]
 [-1.32863189e+08 -3.68717251e+08  2.86166869e+07  3.31106748e+08
   1.36075266e+08  7.37638907e+08 -2.65726379e+07 -7.00028404e+08]
 [ 1.22273343e+09  2.48205958e+07 -1.25020900e+09  1.32863189e+08
  -2.48973055e+09 -2.65726379e+07  2.46666657e+09 -1.31111147e+08]
 [-2.48205958e+07  3.31515558e+08  1.34323224e+08 -3.68717251e+08
   2.68646449e+07 -7.00028404e+08 -1.31111147e+08  7.37230097e+08]]

Unknown deformation
[ 1.06213161e-02  1.05122859e-02 -1.08216700e-06  1.05122859e-02
 -1.01752994e-03  1.06213161e-02 -1.01861211e-03]

Unknown force
[-6.98491931e-10]

Deformation vector
[ 1.06213161e-02  0.00000000e+00  1.05122859e-02 -1.08216700e-06
  1.05122859e-02 -1.01752994e-03  1.06213161e-02 -1.01861211e-03]

Force vector
[ 3.14159265e+04 -6.98491931e-10  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  3.14159265e+04  0.00000000e+00]
```

1.The displacement at node 3

ANS: 1.051222859e-02 in at x direction and -1.01752994e-03in at y direction

2.Stress and strain at the center of the element

```
stress
[-1.62361189e+03 -2.30853196e-01  3.86976775e-11  9.99840698e+04]

strain
[-1.09030219e-03 -1.01752994e-03  3.46944695e-18  3.46452492e-03]
```

$$\sigma_r = -1.6236e3\ psi, \sigma_z = -2.3085e-1\ psi, \tau_{rz} = 0\ psi, \sigma_\theta = 9.9984e4\ psi$$

$$\varepsilon_r = -1.09e-3, \varepsilon_z = -1.0175e-3, \gamma_{rz} = 0\ , \varepsilon_\theta = 3.4645e-3$$

3.Analytical solution

$p = \frac{2\pi*F}{2\pi*r*t} = \frac{2\pi*10e3}{2\pi*3*1} = 3333.3psi$,

$\sigma_r = -\frac{p}{2} = -\frac{3333.3}{2} = -1666.65\ psi, and\ Error = \frac{|16666.65-16236|}{16666.65} * 100 = 2.58\%,$

$\sigma_z = \tau_{rz} = 0\ psi, and\ Error = 0\%,$

$\sigma_\theta = \frac{p*r}{t} = \frac{3333.333*3}{0.1} = 100000\ psi, and\ Error = \frac{|100000-99984|}{100000} * 100 = 0.016\%,$

All solutions are very similar with the answer of FEM

4.What is wrong with Gauss integration order=1

```
    N[0,0] = (1/4)*(1-xi[0])*(1-xi[1])
IndexError: list index out of range
```

For the 2D shape function, it requires two variables of natural coordinate system. But 1st order only has one variable. Thus, algorithm makes dimension error when I try to make shape function (same with problem1)

5.What goes wrong if the nodal coordinates are ordered in a clockwise manner

```
Check what happens in clockwise manner
Clock wise manner
[[3.  0. ]
 [3.  1. ]
 [3.1 1. ]
 [3.1 0. ]]
Stiffness Matrix:
[[ 2.46666657e+09  1.31111147e+08 -2.48973055e+09  2.65726379e+07
  -1.25020900e+09 -1.32863189e+08  1.22273343e+09 -2.48205958e+07]
 [ 1.31111147e+08  7.37230097e+08 -2.68646449e+07 -7.00028404e+08
  -1.34323224e+08 -3.68717251e+08  2.48205958e+07  3.31515558e+08]
 [-2.48973055e+09 -2.68646449e+07  2.53685040e+09 -1.36075266e+08
   1.25765014e+09  2.86166869e+07 -1.25020900e+09  1.34323224e+08]
 [ 2.65726379e+07 -7.00028404e+08 -1.36075266e+08  7.37638907e+08
  -2.86166869e+07  3.31106748e+08  1.32863189e+08 -3.68717251e+08]
 [-1.25020900e+09 -1.34323224e+08  1.25765014e+09 -2.86166869e+07
   2.53685040e+09  1.36075266e+08 -2.48973055e+09  2.68646449e+07]
 [-1.32863189e+08 -3.68717251e+08  2.86166869e+07  3.31106748e+08
   1.36075266e+08  7.37638907e+08 -2.65726379e+07 -7.00028404e+08]
 [ 1.22273343e+09  2.48205958e+07 -1.25020900e+09  1.32863189e+08
  -2.48973055e+09 -2.65726379e+07  2.46666657e+09 -1.31111147e+08]
 [-2.48205958e+07  3.31515558e+08  1.34323224e+08 -3.68717251e+08
   2.68646449e+07 -7.00028404e+08 -1.31111147e+08  7.37230097e+08]]

check Force: [31415.9265359     0.            0.            0.            0.
     0.        31415.9265359     0.        ]
Unknown deformation
[ 1.06213161e-02  1.05122859e-02 -1.08216700e-06  1.05122859e-02
 -1.01752994e-03  1.06213161e-02 -1.01861211e-03]

Unknown force
[-6.98491931e-10]
```

Jacobian matrix is fit for counter clockwise manner of node coordinate. Thus, if it is changed to clockwise manner, it will give wrong answer. Above figure is the result of clockwise. Ans its answer is wrong. (same with problem1)

*Problem2.py is code for problem1.