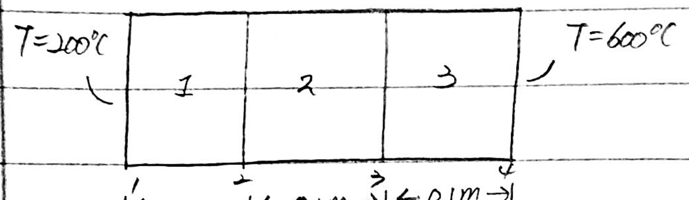
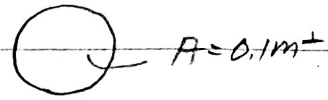
HW4.

Exercise 1.





Element	K	node
①	5	1,2
②	10	2,3
③	15	3,4

$$\frac{A}{L}\begin{array}{c} \\ s \\ f \\ f \\ s \end{array}\begin{array}{c} \begin{array}{cccc} s & f & f & s \end{array} \\ \begin{bmatrix} 5 & -5 & 0 & 0 \\ -5 & 15 & -10 & 0 \\ 0 & -10 & 25 & -15 \\ 0 & 0 & -15 & 15 \end{bmatrix} \end{array} \begin{Bmatrix} 200 \\ T_2 \\ T_3 \\ 600 \end{Bmatrix}\begin{array}{c} s \\ f \\ f \\ s \end{array} = \begin{Bmatrix} q_1 \\ 0 \\ 0 \\ q_2 \end{Bmatrix}\begin{array}{c} s \\ f \\ f \\ s \end{array}$$

f index is 2 and 3

s index is 1 and 4

$$K_{ff}^{-1}(q_f - K_{fs}T_s) = T_f$$

$$T_f = \begin{bmatrix} 15 & -10 \\ -10 & 25 \end{bmatrix}^{-1}\left(\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} -5 & 0 \\ 0 & -15 \end{bmatrix}\begin{Bmatrix} 200 \\ 600 \end{Bmatrix}\right) = \begin{Bmatrix} 418.182 \\ 527.273 \end{Bmatrix}$$

$$q_s = K_{ss}T_s + K_{sf}T_f$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 15 \end{bmatrix}\begin{Bmatrix} 200 \\ 600 \end{Bmatrix} + \begin{bmatrix} -5 & 0 \\ 0 & -15 \end{bmatrix}\begin{Bmatrix} 418.182 \\ 527.273 \end{Bmatrix} = \begin{Bmatrix} -1090.91 \\ 1090.91 \end{Bmatrix}$$

Exercise1-Coding answer

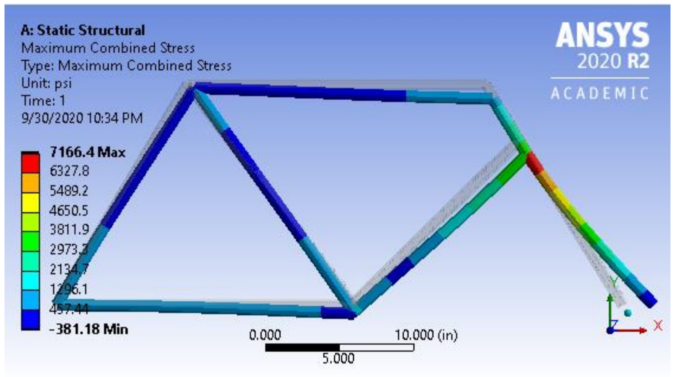
```
Global stiffness matrix K:
[[  5.  -5.   0.   0.]
 [ -5.  15. -10.   0.]
 [  0. -10.  25. -15.]
 [  0.   0. -15.  15.]]

Temperature at free index:
At node 1, temperature is 418.1818181818181
At node 2, temperature is 527.2727272727271

Heatflux at supported index:
At node 0, heatflux is -1090.9090909090905
At node 3, heatflux is 1090.9090909090928
```

*Q1.py is the code for exercise1.

Exercise2

1.Element type

Beam element is selected. Since a rigid body frame is defined as a series of beam elements rigidly connected to each other, the beam element is selected. And computer programs often refer to the frame element as a beam element with the understanding that the program is using the stiffness matrix for plane frame analysis. Lower figure is the result of the analysis.

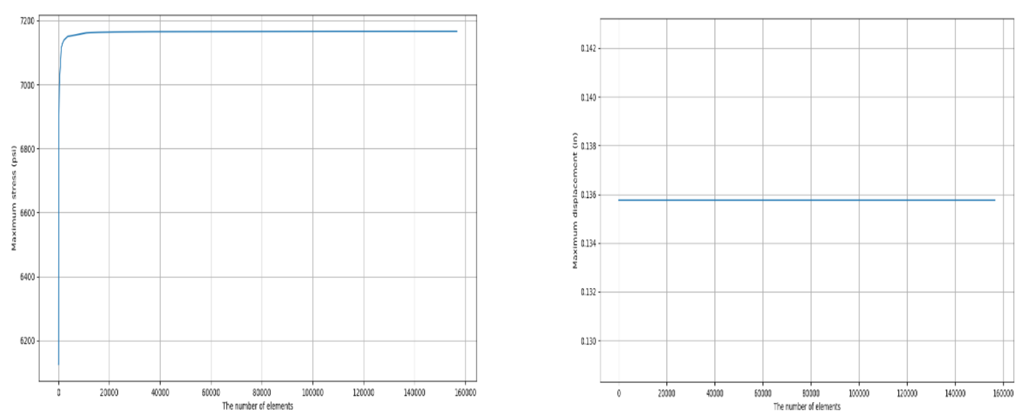


<Figure. Result of maximum combined stress at the system>

2.Convergence study for the maximum displacement and maximum stress

The value of stress quickly increases and converge to specific value. But maximum displacement is same.

3.Convergerged results

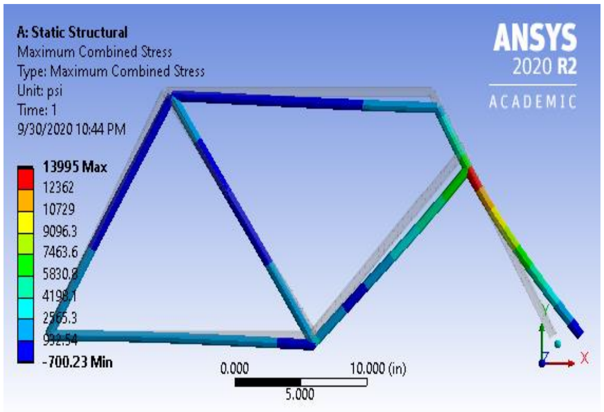


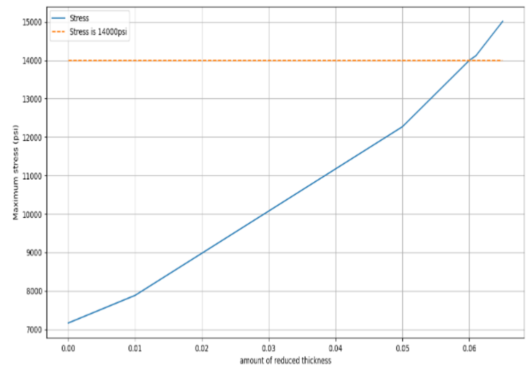
<Figure. Convergence of maximum stress and maximum displacement>

Above figure (left side) is graph between the number of elements and maximum stress. From the early stage, stress value reach around 7100psi and slowly increase until the number of elements become 109576. Then, maximum stress is converged to 7166.4psi. But displacement graph (right side) is same in all period.

4.Cracking

The maximum stress (7166.4psi) is lower than 14000psi. Thus, the system is safe from crack.

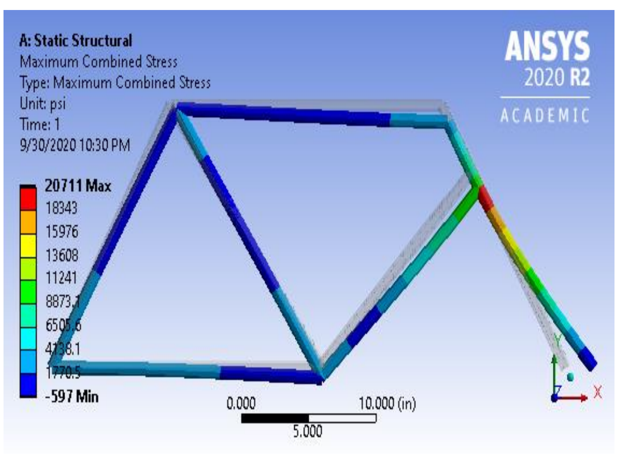


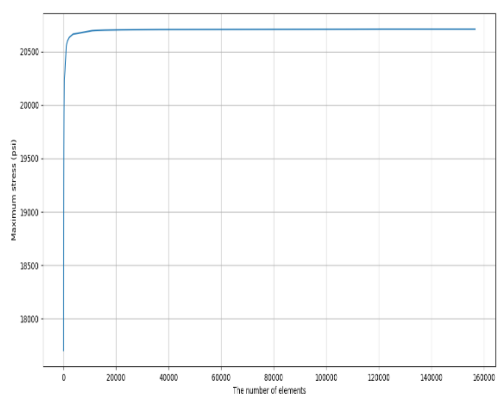


<Figure. Maximum stress graph with the thickness of material>

If the thickness is reduced 0.06inch (t = 0.09 inch), the system reaches 14000psi and start to crack.

5.





<Figure. Convergence of new design>

When new load is applied at pedal and the load of handle increases, maximum stress is over 14000psi. Thus, there will be crack. But convergence trend is totally same with original system. From the early stage, stress value reach around 20700psi and slowly increase until the number of elements become 109576. Then, maximum stress is converged to 20711psi

***Reference for Exercise2-1: A First Course in the Finite Element Method, Daryl L. Logan, Chapter 5-1.**

HW04.

Exercise 3.

$$I = \int_0^{0.8} 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5 \, dx$$

1. $2n-1 = 5 \qquad \therefore n = \underline{\underline{3}}$

2.

$$I = \frac{b-a}{2} \sum_{i=1} w_i f\left(\frac{b+a}{2} + \frac{b-a}{2}\xi_i\right)$$

1 Point

$$I = 0.4\left(w_1 f(0.4 + 0.4\xi_1)\right) = 0.4\left(2 \cdot f(0.4)\right)$$

$$= 1.9648$$

2 Point

$$I = 0.4\left(w_1 f(0.4+0.4\xi_1) + w_2 f(0.4+0.4\xi_2)\right)$$

$$= 0.4\left(1 f\left(0.4+0.4\left(-\tfrac{1}{\sqrt{3}}\right)\right) + 1 \cdot f\left(0.4+0.4\tfrac{1}{\sqrt{3}}\right)\right)$$

$$= 1.82258$$

3 Point

$$I = 0.4\left(w_1 f(0.4+0.4\xi_1) + w_2 f(0.4+0.4\xi_2) + w_3 f(0.4+0.4\xi_3)\right)$$

$$= 0.4\left(\tfrac{8}{9} f(0.4) + \tfrac{5}{9} f\left(0.4+0.4\left(-\sqrt{\tfrac{3}{5}}\right)\right) + \tfrac{5}{9} f\left(0.4+0.4\sqrt{\tfrac{3}{5}}\right)\right)$$

$$= 1.64035$$

3.

analytical solution $= 1.64053$

$$\text{error}_1 = \frac{|1.64053 - 1.96481|}{1.64053} = \underline{0.198 \ (= 19.8\%)}$$

$$\text{error}_2 = \frac{|1.82278 - 1.96481|}{1.64053} = \underline{0.111 \ (= 11.1\%)}, \quad \text{error}_3 = \underline{0\%}$$

Exercise3

1.Order of Gauss integration

ANS: 2n-1=5. Thus, n = 3

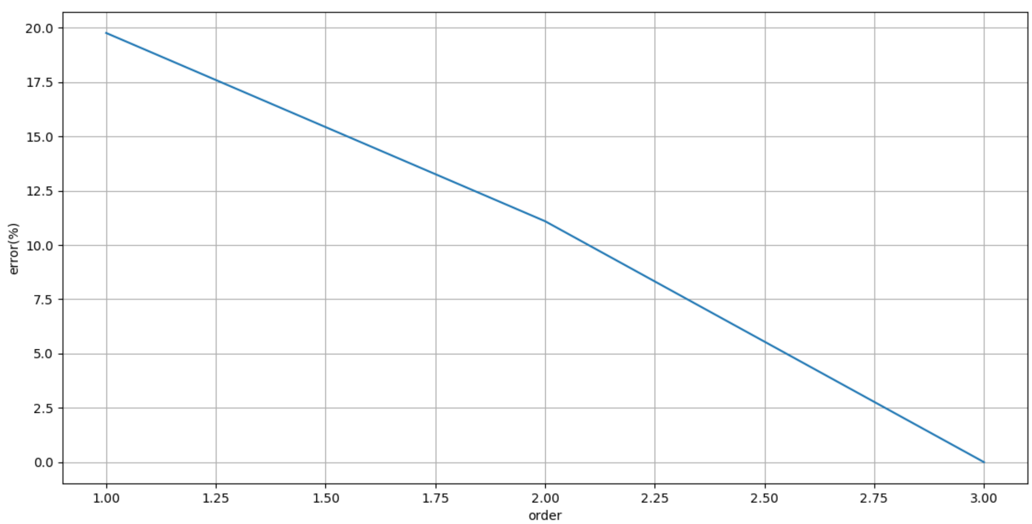
2.Evaluate using 1-point, 2-point, and 3-point Gauss integration

```
Order 1's answer is 1.9648000000000003
Order 2's answer is 1.8225777777777772
Order 3's answer is 1.6405333333333294
```

3.True percent error

```
True percent error of order 1 is 19.7659297789339%
True percent error of order 2 is 11.0966623320331%
True percent error of order 3 is 5.41396143347769E-14%
```

4.Error plot



<Figure. Errors graph>

It is easy to see the trend. As the order increases, error value decreases. In the case of order 3, it shows same answer with analytical solution so error becomes zero.