PoPPy: A Point Process Toolbox Based on PyTorch

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1 Overview

1.1 What is PoPPy?

PoPPy is a **Po**int **P**rocess toolbox based on **Py**Torch, which achieves flexible designing and efficient learning of point process models. It can be used for interpretable sequential data modeling and analysis, $e.g.$, Granger causality analysis of multi-variate point processes, point process-based simulation and prediction of event sequences.

1.2 The Goal of PoPPy

Many real-world sequential data are often generated by complicated interactive mechanisms among multiple entities. Treating the entities as events with different discrete categories, we can represent their sequential behaviors as event sequences in continuous time domain. Mathematically, an event sequence s can be denoted as $\{(t_i^s, c_i^s, f_{c_i^s})\}_{i=1}^{I_s}$, where t_i^s and c_i^s are the timestamp and the event type ($i.e.$, the index of entity) of the i-th event, respectively. Optionally, each event type may be associated with a feature vector $\boldsymbol{f}_c \in \mathbb{R}^{D_c}$, $c \in \mathcal{C}$, and each event sequence may also have a feature vector $\boldsymbol{f}_s \in \mathbb{R}^{D_s}$, $s \in \mathcal{S}$. Many real-world scenarios can be formulated as event sequences, as shown in Table 1.

Table 1: Typical event sequences in practice.

Scene	Patient admission	Job hopping	Online shopping
Entities (Event types)	Diseases	Companies	Items
Sequences	Patients' admission records	LinkedIn users' job history	Buying/rating behaviors
Event feature	Diagnose records	Job descriptions	Item profiles
Sequence feature	Patient profiles	User profiles	User profiles
Task	Build Disease network	Model talent flow	Recommendation system

Given a set of event sequences $\mathcal{S}$, we aim to model the dynamics of the event sequences, capture the interactive mechanisms among different entities and predict their future behaviors. Temporal point process model provides us with a potential solution to achieve these aims. In particular, a multi-variate temporal point process can be represented by a set of counting processes $N = \{N_c(t)\}_{c\in\mathcal{C}}$, in which $N_c(t)$ is the number of type-c events occurring till time t. For each $N_c(t)$, the expected instantaneous happening rate of type-c events at time t is denoted as $\lambda_c(t)$, which is called "**intensity function**":

$$\lambda_c(t) = \frac{\mathbb{E}[dN_c(t)|\mathcal{H}_t]}{dt},\ \mathcal{H}_t = \{(t_i, c_i)|t_i < t, c_i \in \mathcal{C}\}, \tag{1}$$

where $\mathcal{H}_t$ represents historical observations before time t.

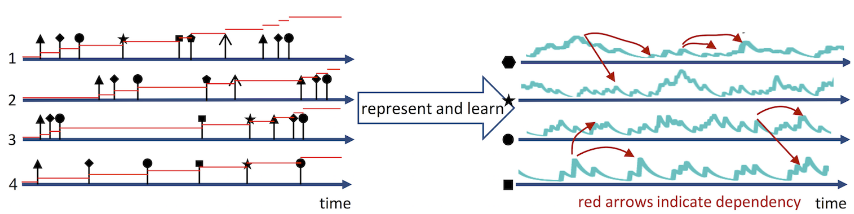


Figure 1: Event sequences and intensity functions.

As shown in Fig. 1, the counting processes can be represented as a set of intensity functions, each of which corresponds to a specific event type. The temporal dependency within the same event type and that across different event types ($i.e.$, the red arrows in Fig. 1) can be captured by choosing particular intensity functions. Therefore, the key points of point process-based sequential data modeling include

1. How to design intensity functions to describe the mechanism behind observed data?
2. How to learn the proposed intensity functions from observed data?

The goal of PoPPy is providing a user-friendly solution to the key points above and achieving large-scale point process-based sequential data analysis, simulation and prediction.

1.3 Installation of PoPPy

PoPPy is developed on Mac OS 10.13.6 but also tested on Ubuntu 16.04. The installation of PoPPy is very simple. In particular,

1. Install Anaconda3 and create a conda environment.
2. Install PyTorch0.4 in the environment.
3. Download PoPPy from `https://github.com/HongtengXu/PoPPy/` and unzip it to the directory in the environment. The unzipped folder should contains several subfolders, as shown in Fig. 2.
4. Open `dev/util.py` and change POPPY_PATH to the directory, as shown in Fig. 3.

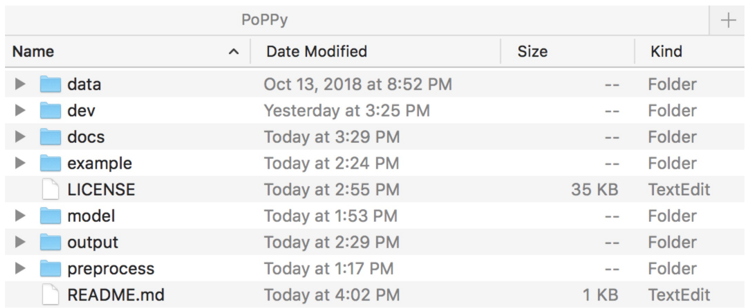


Figure 2: The subfolders in the package of PoPPy.

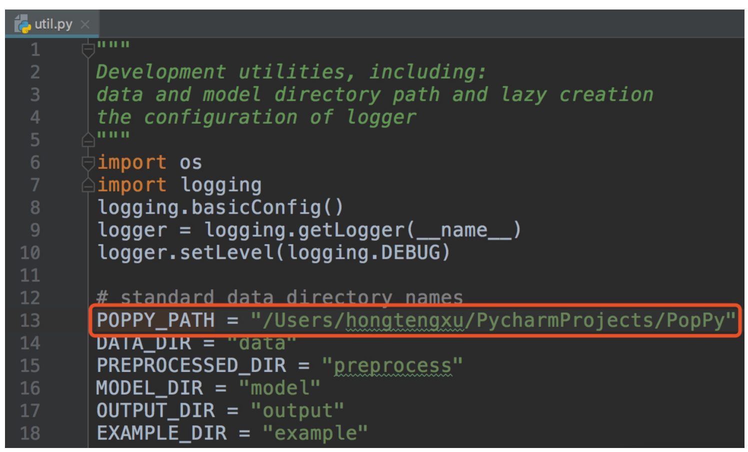


Figure 3: An example of the path of PoPPy.

The subfolders in the package include

- `data`: It contains a toy dataset in *.csv* format.
- `dev`: It contains a `util.py` file, which configures the path and the logger of the package.
- `docs`: It contains the tutorial of PoPPy.
- `example`: It contains some demo scripts for testing the functionality of the package.
- `model`: It contains the classes of predefined point process models and their modules.
- `output`: It contains the output files generated by the demo scripts in the `example` folder.
- `preprocess`: It contains the classes and the functions of data I/O and preprocessing.

In the following sections, we will introduce the details of PoPPy.

2 Data: Representation and Preprocessing

2.1 Representations of Event Sequences

PoPPy represents observed event sequences as a nested dictionary. In particular, the proposed `database` has the following structure:

```
database = {
   'event_features' : None or (De, C) float array of event features,
                      C is the number of event types.
                      De is the dimension of event feature.
   'type2idx'       : a Dict = {'event_name': event_index}
   'idx2type'       : a Dict = {event_index: 'event_name'}
   'seq2idx'        : a Dict = {'seq_name': seq_index}
   'idx2seq'        : a Dict = {seq_index: 'seq_name'}
   'sequences'      : a List = [seq_1, seq_2, ..., seq_N].
}

For the i-th sequence:
seq_i = {
   'times'          : (N,) float array of timestamps,
                      N is the number of events.
   'events'         : (N,) int array of event types.
   'seq_feature'    : None or (Ds,) float array of sequence feature.
                      Ds is the dimension of sequence feature
   't_start'        : a float number, the start timestamp of the sequence.
   't_stop'         : a float number, the stop timestamp of the sequence.
   'label'          : None or int/float number, the labels of the sequence
}
```

PoPPy provides three functions to load data from *.csv* file and convert it to the proposed `database`.

2.1.1 `preprocess.DataIO.load_sequences_csv`

This function loads event sequences and convert them to the proposed `database`. The IO and the description of this function is shown in Fig. 4.

```
def load_sequences_csv(file_name: str, domain_names: Dict):
    """
    Load event sequences from a csv file
    :param file_name: the path and name of the target csv file
    :param domain_names: a dictionary contains the names of the key columns
                         corresponding to {'seq_id', 'time', 'event'}
        The format should be
        domain_names = {'seq_id': the column name of sequence name,
                        'time': the column name of timestamps,
                        'event': the column name of events}
    :return: database: a dictionary containing observed event sequences
        database = {'event_features': None,
                    'type2idx': a Dict = {'event_name': event_index}
                    'idx2type': a Dict = {event_index: 'event_name'}
                    'seq2idx': a Dict = {'seq_name': seq_index}
                    'idx2seq': a Dict = {seq_index: 'seq_name'}
                    'sequences': a List  = [seq_1, seq_2, ..., seq_N].
                    }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                 'events': (N,) int array of event types.
                 'seq_feature': None.
                 't_start': a float number, the start timestamp of the sequence.
                 't_stop': a float number, the stop timestamp of the sequence.
                 'label': None
                 }
    """
```

Figure 4: The description of `load_sequences_csv`.

For example, the *Linkedin.csv* file in the folder `data` records a set of linkedin users's job hopping behaviors among different companies, whose format is shown in Fig. 5 Here, the column id corresponds

Linkedin

id	time	event	option1
1	29	Google	Intern Research
1	29.2521	UC Berkeley	Graduate Student Research
1	34.0849	Google	Sr Research Science
6	29	Google	Software Eng Intern
6	30	Google	Software Eng Intern
6	32.0027	Google	Software Eng Intern
6	32.2548	eBay	Software Eng Intern
6	32.5918	Baidu	Software Eng
6	33.5068	Google	Software Eng
8	31	Microsoft	Software Eng Intern
8	32.0849	LinkedIn	Software Eng Intern
8	32.9178	UCLA	Graduate Teaching Assistant

Figure 5: Some rows of *Linkedin.csv*.

to the names of sequences ($i.e.$ the index of users), the column time corresponds to the timestamps of events ($i.e.$ the ages that the users start to work), and the column event corresponds to the event types ($i.e.$, the companies). Therefore, we can define the input `domain_names` as

```
domain_names = {
   'seq_id' : 'id',
   'time'   : 'time',
   'event'  : 'event'
}
```

and `database = load_sequences_csv('Linkedin.csv', domain_names)`.

Note that the `database` created by `load_sequences_csv()` does not contain event features and sequence features, whose values in `database` are **None**. PoPPy supports to load categorical or numerical features from *.csv* files, as shown below.

2.1.2 `preprocess.DataIO.load_seq_features_csv`

This function loads sequence features from a *.csv* file and import them to the proposed `database`. The IO and the description of this function is shown in Fig. 6. Take the *Linkedin.csv* file as an example.

```
def load_seq_features_csv(file_name: str, seq_domain: str, domain_dict: Dict, database: Dict, normalize: int=0):
    """
    load sequences' features from a csv file
    :param file_name: the path and the name of the csv file
    :param seq_domain: the name of the key column corresponding to sequence index.
    :param domain_dict: a dictionary containing the names of the key columns corresponding to the features.
        The format should be
            domain_dict = {'domain_name': domain's feature type}
        Two types are considered:
        1) 'numerical': each element (row) in the corresponding domain should be a string containing D numbers
            separated by spaces, and D should be the same for various elements.
            D-dimensional real-value features will be generated for this domain.
            If each sequence has multiple rows, the average of the features will be recorded.

        2) 'categorical': each element (row) in the corresponding domain should be a strong containing N keywords
            separated by spaces, but N can be different for various elements.
            D-dimensional binary features will be generated for this domain. Here D is the number of distinguished
            keywords (vocabulary size).
            If each sequence has multiple rows, the aggregation of the binary features will be recorded.

    :param database: a dictionary of data generated by the function "load_sequences_csv()"
    :param normalize: 0 = no normalization, 1 = normalization across features, 2 = normalization across sequences
    :return: a database having sequences' features
    """
```

Figure 6: The description of `load_seq_features_csv`.

Suppose that we have already create `database` by the function `load_sequences_csv`, and we want to take the column option1 ($i.e.$, the job titles that each user had) as the categorical features of event sequences. We should have

```
domain_names = {
   'option1' : 'categorical'
}
database = load_seq_features_csv(
                        file_name = 'Linkedin.csv',
                        seq_domain = 'seq_id',
                        domain_dict = domain_names,
                        database = database)
```

Here the input `normalize` is set as default 0, which means that the features in `database['sequences'][i]['seq_feature']`, $i = 1, ..., |\mathcal{S}|$, are not normalized.

2.1.3 `preprocess.DataIO.load_event_features_csv`

This function loads event features from a *.csv* file and import them to the proposed `database`. The IO and the description of this function is shown in Fig. 7. Similarly, if we want to take the column

```
def load_event_features_csv(file_name: str, event_domain: str, domain_dict: Dict, database: Dict, normalize: int=0):
    """
    load events' features from a csv file
    :param file_name: the path and the name of the csv file
    :param event_domain: the name of the key column corresponding to event index.
    :param domain_dict: a dictionary containing the names of the key columns corresponding to the features.
        The format should be
            domain_dict = {'domain_name': domain's feature type}
        Two types are considered:
        1) 'numerical': each element (row) in the corresponding domain should be a string containing D numbers
            separated by spaces, and D should be the same for various elements.
            D-dimensional real-value features will be generated for this domain.
            If each event type has multiple rows, the average of the features will be recorded.

        2) 'categorical': each element (row) in the corresponding domain should be a strong containing N keywords
            separated by spaces, but N can be different for various elements.
            D-dimensional binary features will be generated for this domain. Here D is the number of distinguished
            keywords (vocabulary size).
            If each event type has multiple rows, the aggregation of the binary features will be recorded.

    :param database: a dictionary of data generated by the function "load_sequences_csv()"
    :param normalize: 0 = no normalization, 1 = normalization across features, 2 = normalization across event types
    :return: a database having events' features
    """
```

Figure 7: The description of `load_event_features_csv`.

option1 in *Linkedin.csv* as the categorical features of event types, we should have

```
domain_names = {
   'option1' : 'categorical'
}
database = load_event_features_csv(
                        file_name = 'Linkedin.csv',
                        event_domain = 'event',
                        domain_dict = domain_names,
                        database = database)
```

2.2 Operations for Data Preprocessing

Besides basic sequence/feature loaders and converters mentioned above, PoPPy contains multiple useful functions and classes for data preprocessing, including sequence stitching, superposing, aggregating and batch sampling. Fig. 8 illustrates the corresponding data operations.

2.2.1 `preprocess.DataOperation.stitching`

This function stitches the sequences in two `database` randomly or based on their `seq_feature` and time information (`t_start`, `t_stop`). Its description is shown in Fig. 9.

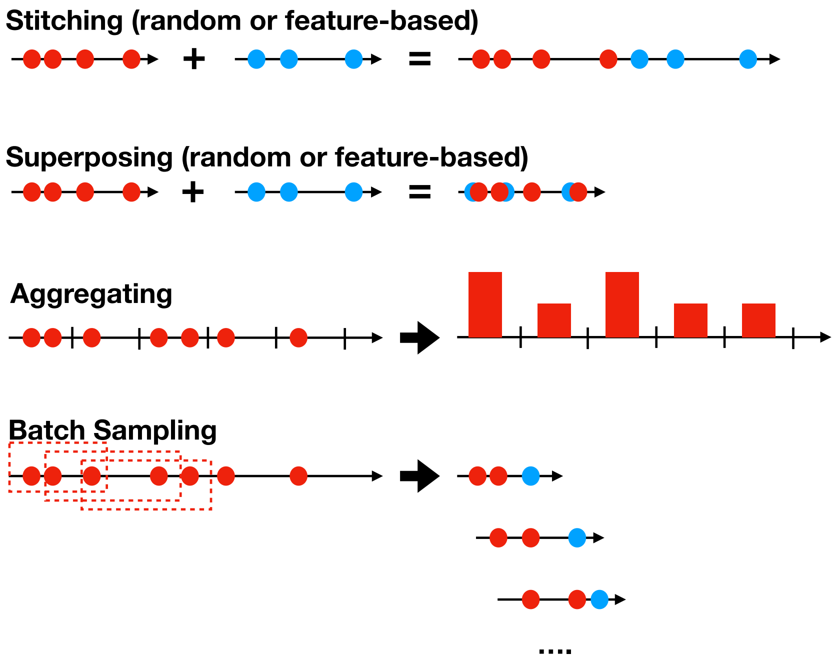


Figure 8: The illustration of four data operations.

When `method = 'random'`, for each sequence in `database1` the function randomly selects a sequence in `database2` as its follower and stitches them together. When `method = 'feature'`, the similarity between the sequence in `database1` and that in `database2` is defined by the multiplication of a temporal Gaussian kernel and a sequence feature's Gaussian kernel, and the function selects the sequence in `database2` yielding to a distribution defined by the similarity. The stitching method has been proven to be useful for enhancing the robustness of learning results, especially when the training sequences are very short [9, 4].

```
def stitching(database1: Dict, database2: Dict, method: str ='random') -> Dict:
    """
    Stitch each sequence in database2 to the end of one sequence of database1
    :param database1: the observed event sequences
    :param database2: another observed event sequences
        database = {'event_features': None or (De, C) float array of event's static features,
                                  C is the number of event types.
                    'type2idx': a Dict = {'event_name': event_index}
                    'idx2type': a Dict = {event_index: 'event_name'}
                    'seq2idx': a Dict = {'seq_name': seq_index}
                    'idx2seq': a Dict = {seq_index: 'seq_name'}
                    'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                    }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                 'events': (N,) int array of event types.
                 'seq_feature': None or (Ds,) float array of sequence's static feature.
                 't_start': a float number indicating the start timestamp of the sequence.
                 't_stop': a float number indicating the stop timestamp of the sequence.
                 'label': None or int/float number indicating the labels of the sequence}

    :param method: a string indicates stitching method:
        "random": stitch the seq_j in sequences2 to the seq_i in sequences1 for j ~ {1,...,N}, i=1,...,N and
                  time-shifting is applied to sequences2.
                  This method is suitable for the sequences generated by a same stationary point process.

        "feature": stitch the seq_j in sequences2 to the seq_i in sequences1 for j ~{1,...,N}, i=1,...,N and
                   j is sampled according to the similarity between two sequences.
                   The similarity is calculated by the Gaussian kernel of seq_features, labels and times.
                   When seq_features/labels are not available, only timestamp information are taken into account.

    :return:
        the output sequences are with the same format as database1.
    """
```

Figure 9: The description of `stitching`.

2.2.2 `preprocess.DataOperation.superposing`

This function superposes the sequences in two `database` randomly or based on their `seq_feature` and time information (`t_start`, `t_stop`). Its description is shown in Fig. 10.

When `method = 'random'`, for each sequence in `database1` the function randomly selects a sequence in `database2` as its follower and stitches them together. When `method = 'feature'`, the similarity between the sequence in `database1` and that in `database2` is defined by the multiplication of a temporal Gaussian kernel and a sequence feature's Gaussian kernel, and the function selects the sequence in `database2` yielding to a distribution defined by the similarity.

Similar to the stitching operation, the stitching method has been proven to be useful for learning linear Hawkes process robustly. However, it should be noted that different from stitching operation, which stitches similar sequences with a high probability, the superposing operation would like to superpose the dissimilar sequences with a high probability. The rationality of such an operation can be found in my paper [8, 5].

```
def superposing(database1: Dict, database2: Dict, method: str ='random') -> Dict:
    """
    Superpose each sequence in database2 to one sequence of database1
    :param database1: the observed event sequences
    :param database2: another observed event sequences
        database = {'event_features': None or (C, De) float array of event's static features,
                                  C is the number of event types.
                    'type2idx': a Dict = {'event_name': event_index}
                    'idx2type': a Dict = {event_index: 'event_name'}
                    'seq2idx': a Dict = {'seq_name': seq_index}
                    'idx2seq': a Dict = {seq_index: 'seq_name'}
                    'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                    }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                 'events': (N,) int array of event types.
                 'seq_feature': None or (Ds,) float array of sequence's static feature.
                 't_start': a float number indicating the start timestamp of the sequence.
                 't_stop': a float number indicating the stop timestamp of the sequence.
                 'label': None or int/float number indicating the labels of the sequence}

    :param method: a string indicates superposing method:
        "random": superpose the seq_j in sequences2 to the seq_i in sequences1 for j ~ {1,...,N}, i=1,...,N and
                  time-shifting is applied to sequences2.
                  This method is suitable for the sequences generated by a same stationary point process.

        "feature": superpose the seq_j in sequences2 to the seq_i in sequences1 for j ~{1,...,N}, i=1,...,N and
                   j is sampled according to the similarity between two sequences.
                   The similarity is calculated by the Gaussian kernel of seq_features, labels and times.
                   When seq_features/labels are not available, only timestamp information are taken into account.

                   Different from stitching operation, to enlarge the power of superposition,
                   the sequences with large dissimilarity are more likely to be superposed together

    :return:
        the output sequences are with the same format as database1.
    """
```

Figure 10: The description of `superposing`.

2.2.3 `preprocess.DataOperation.aggregating`

This function discretizes each event sequence into several bins and counts the number of events with specific types in each bin. Its description is shown in Fig. 11.

2.2.4 `preprocess.DataOperation.EventSampler`

This class is a subclass of `torch.utils.data.Dataset`, which samples batches from `database`. For each sample in the batch, an event (*i.e.*, its event type and timestamp) and its history with length `memorysize` (*i.e.*, the last `memorysize` events and their timestamps) are recorded. If event and/or sequence features are available, the sample will record these features as well.

```python
def aggregating(database, dt):
    """
    Count the number of events in predefined time bins,
    and convert event sequences to aggregate time series
    :param database: the observed event sequences
    :param dt: a float number indicating the length of time bin.
    :return:
        the output's format is shown as follows:

          output = {'event_features': None or (De, C) float array of event's static features,
                              C is the number of event types.
                    'type2idx': a Dict = {'event_name': event_index}
                    'idx2type': a Dict = {event_index: 'event_name'}
                    'seq2idx': a Dict = {'seq_name': seq_index}
                    'idx2seq': a Dict = {seq_index: 'seq_name'}
                    'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                    }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of discrete timestamps,
                          N = [(t_stop - t_start)/dt] is the number of bins.
                 'events': (N, C) int array of event types,
                           events[n, c] counts the number of type-c events in the n-th bin
                 'seq_feature': None or (Ds,) float array of sequence's static feature.
                 't_start': a float number indicating the start timestamp of the sequence.
                 't_stop': a float number indicating the stop timestamp of the sequence.
                 'label': None or int/float number indicating the labels of the sequence}
    """
```

Figure 11: The description of `aggregate`.

```python
class EventSampler(Dataset):
    """Load event sequences via minbatch"""
    def __init__(self, database, memorysize):
        """
        :param database: the observed event sequences
            database = {'event_features': None or (C, De) float array of event's static features,
                                  C is the number of event types.
                        'type2idx': a Dict = {'event_name': event_index}
                        'idx2type': a Dict = {event_index: 'event_name'}
                        'seq2idx': a Dict = {'seq_name': seq_index}
                        'idx2seq': a Dict = {seq_index: 'seq_name'}
                        'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                        }

            For the i-th sequence:
            seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                     'events': (N,) int array of event types.
                     'seq_feature': None or (Ds,) float array of sequence's static feature.
                     't_start': a float number indicating the start timestamp of the sequence.
                     't_stop': a float number indicating the stop timestamp of the sequence.
                     'label': None or int/float number indicating the labels of the sequence}
        :param memorysize: how many historical events remembered by each event
        """
```

Figure 12: The description of `EventSampler`.

3 Temporal Point Process Models

3.1 Modular design of point process model

PoPPy applies a flexible strategy to build point process's intensity functions from interpretable modules. Such a modular design strategy is very suitable for Hawkes process and its variants. Fig. 13 illustrates the proposed modular design strategy. In the following sections, we take Hawkes process and its variants as examples, and introduce the modules ($i.e.$, the classes) in PoPPy.

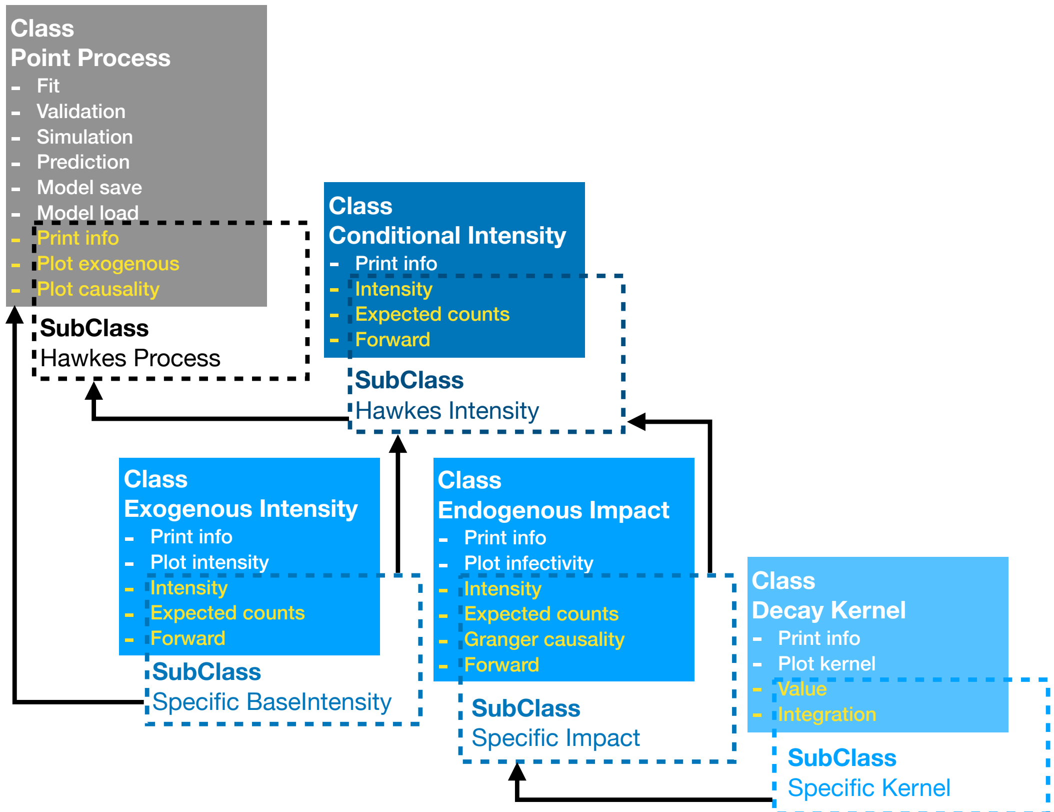


Figure 13: An illustration of proposed modular design strategy. Each color block represents a class with some functions. For each block, the dotted frame represents one of its subclass, which inherits some functions (the white ones) while overrides some others or creates new ones (the yellow ones). The black arrow means that the destination class will call the instance of the source class as input.

3.2 `model.PointProcess.PointProcessModel`

This class contains basic functions of a point process model, including

- `fit`: learn model's parameters given training data. It description is shown in Fig. 14
- `validation`: test model given validation data. It description is shown in Fig. 15
- `simulation`: simulate new event sequences from scratch or following observed sequences by Ogata's thinning algorithm [3]. It description is shown in Fig. 16
- `prediction`: predict expected counts of the events in the target time inteveral given learned model and observed sequences. It description is shown in Fig. 17
- `model_save`: save model or save its parameters. It description is shown in Fig. 18
- `model_load`: load model or load its parameters. It description is shown in Fig. 19

```
def fit(self, dataloader, optimizer, epochs: int, scheduler=None, sparsity: float=None, nonnegative=None,
        use_cuda: bool=False, validation_set=None):
    """
    Learn parameters of a generalized Hawkes process given observed sequences
    :param dataloader: a pytorch batch-based data loader
    :param optimizer: the sgd optimization method defined by PyTorch
    :param epochs: the number of training epochs
    :param scheduler: the method adjusting the learning rate of SGD defined by PyTorch
    :param sparsity: None or a float weight of L1 regularizer
    :param nonnegative: None or a float lower bound, typically the lower bound = 0
    :param use_cuda: use cuda (true) or not (false)
    :param validation_set: None or a validation dataloader
    """
```

Figure 14: The description of `fit`.

```
def validation(self, dataloader, use_cuda):
    """
    Compute the avaraged loss per event of a generalized Hawkes process
    given observed sequences and current model
    :param dataloader: a pytorch batch-based data loader
    :param use_cuda: use cuda (true) or not (false)
    """
```

Figure 15: The description of `validation`.

- `print_info`: print basic information of model
- `plot_exogenous`: print exogenous intensity.

In PoPPy, the instance of this class actually implements an inhomogeneous Poisson process, in which the exogenous intensity is used as the intensity function.

An important subclass of this class is `model.HawkesProcess.HawkesProcessModel`. This subclass inherits most of the functions above except `print_info` and `plot_exogenous`. Additionally, because Hawkes process considers the triggering patterns among different event types, this subclass has a new function `plot_causality`, which plots the adjacency matrix of the event types' Granger causality graph. The typical visualization results of the exogenous intensity of different even types and the Granger causality among them are shown in Fig. 20

Compared with its parant class, `model.HawkesProcess.HawkesProcessModel` uses a specific intensity function, which is defined in the class `model.HawkesProcess.HawkesProcessIntensity`.

3.3 `model.HawkesProcess.HawkesProcessIntensity`

This class inherits the functions in `torch.nn.Module`. It defines the intensity function of a generalized Hawkes process, which contains the following functions:

- `print_info`: print the basic information of the intensity function.
- `intensity`: calculate $\lambda_{c_i}(t_i)$ of the i-th sample in the batch sampled by `EventSampler`.
- `expected_counts`: calculate $\int_{t_{i-1}}^{t_i} \lambda_c(s)ds$ for $c \in \mathcal{C}$ and for the i-th sample in the batch.
- `forward`: override the forward function in `torch.nn.Module`. It calculates $\lambda_{c_i}(t_i)$ and $\int_{t_{i-1}}^{t_i} \lambda_c(s)ds$ for $c \in \mathcal{C}$ for SGD.

```python
def simulate(self,
             history,
             memory_size: int = 10,
             time_window: float = 1.0,
             interval: float = 1.0,
             max_number: int = 100,
             use_cuda: bool = False):
    """
    Simulate one or more event sequences from given model.
    :param history: historical observations
        history = {'event_features': None or (C, De) float array of event's static features,
                      C is the number of event types.
                   'type2idx': a Dict = {'event_name': event_index}
                   'idx2type': a Dict = {event_index: 'event_name'}
                   'seq2idx': a Dict = {'seq_name': seq_index}
                   'idx2seq': a Dict = {seq_index: 'seq_name'}
                   'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                  }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                 'events': (N,) int array of event types.
                 N can be "0" (i.e., no observations)
                 'seq_feature': None or (Ds,) float array of sequence's static feature.
                 't_start': a float number indicating the start timestamp of the sequence.
                 't_stop': a float number indicating the stop timestamp of the sequence.
                 'label': None or int/float number indicating the labels of the sequence}
    :param memory_size: the number of historical events used for simulation
    :param time_window: duration of simulation process.
    :param interval: the interval size calculating the supremum of intensity
    :param max_number: the maximum number of simulated events
    :param use_cuda: use cuda (true) or not (false)
    :return:
        new_data: having the same format as history
        counts: a list of (C,) ndarray, which counts the number of simulated events for each type
    """
```

Figure 16: The description of `simulate`.

Specifically, the intensity function of type-c event at time t is defined as

$$\begin{aligned}\lambda_c(t) &= g_\lambda\left(\underbrace{\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s)}_{\text{exogenous intensity}} + \underbrace{\sum_{t_i<t}\phi_{cc_i}(t-t_i, \boldsymbol{f}_c, \boldsymbol{f}_{c_i})}_{\text{endogeneous impact}}\right) \\ &= \mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s) + \sum_{t_i<t}\sum_{m=1}^{M}\alpha_{cc_im}(\boldsymbol{f}_c, \boldsymbol{f}_{c_i})\kappa_m(t-t_i).\end{aligned} \tag{2}$$

Here, the intensity function is consist of two parts:

- **Exogenous intensity** $\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s)$: it is independent with time, which measures the intensity contributed by the intrinsic properties of sequence and event type.
- **Endogenous impact** $\sum_{t_i<t}\phi_{cc_i}(t-t_i, \boldsymbol{f}_c, \boldsymbol{f}_{c_i})$: it sums up the influences of historical events quantitatively via **impact functions** $\{\phi_{cc'}(t)\}_{c,c'\in\mathcal{C}}$, which measures the intensity contributed by the historical observations.

Furthermore, the impact function is decomposed with the help of basis representation, where $\kappa_m(t)$ is called the m-th **decay kernel** and $\alpha_{cc_im}(\boldsymbol{f}_c, \boldsymbol{f}_{c_i})$ is the corresponding **coefficient**.

$g_\lambda(\cdot)$ is an activation function, which can be

- **Identity:** $g(x) = x$.
- **ReLU:** $g(x) = \max\{x, 0\}$.
- **Softplus:** $g(x) = \frac{1}{\beta}\log(1+\exp(-\beta x))$.

PoPPy provides multiple choices to implement various intensity functions — each module can be parametrized in different ways.

```python
def predict(self,
            history,
            memory_size: int = 10,
            time_window: float = 1.0,
            interval: float = 1.0,
            max_number: int = 1e5,
            use_cuda: bool = False,
            num_trial: int = 2):
    """
    Predict the expected number of events in the proposed target time window
    :param history: historical observations
        history = {'event_features': None or (C, De) float array of event's static features,
                                      C is the number of event types.
                   'type2idx': a Dict = {'event_name': event_index}
                   'idx2type': a Dict = {event_index: 'event_name'}
                   'seq2idx': a Dict = {'seq_name': seq_index}
                   'idx2seq': a Dict = {seq_index: 'seq_name'}
                   'sequences': a List  = {seq_1, seq_2, ..., seq_N}.
                  }

        For the i-th sequence:
        seq_i = {'times': (N,) float array of timestamps, N is the number of events.
                 'events': (N,) int array of event types.
                 N can be "0" (i.e., no observations)
                 'seq_feature': None or (Ds,) float array of sequence's static feature.
                 'dyn_features': None or (N, Dt) float array of event's dynamic features.
                 't_start': a float number indicating the start timestamp of the sequence.
                 't_stop': a float number indicating the stop timestamp of the sequence.
                 'label': None or int/float number indicating the labels of the sequence}
    :param memory_size: the number of historical events used for simulation
    :param time_window: duration of simulation process.
    :param interval: the interval size calculating the supremum of intensity
    :param max_number: the maximum number of simulated events
    :param use_cuda: whether use cuda or not
    :param num_trial: the number of simulation trials.
    """
```

Figure 17: The description of `predict`.

```python
def save_model(self, full_path, mode: str='entire'):
    """
    Save trained model
    :param full_path: the path of directory
    :param mode: 'parameter' for saving only parameters of the model,
                 'entire' for saving entire model
    """
```

Figure 18: The description of `model_save`.

3.3.1 `model.ExogenousIntensity.BasicExogenousIntensity`

This class and its subclasses in `model.ExogenousIntensityFamily` implements several models of exogenous intensity, as shown in Table 2.

Table 2: Typical models of exogenous intensity.

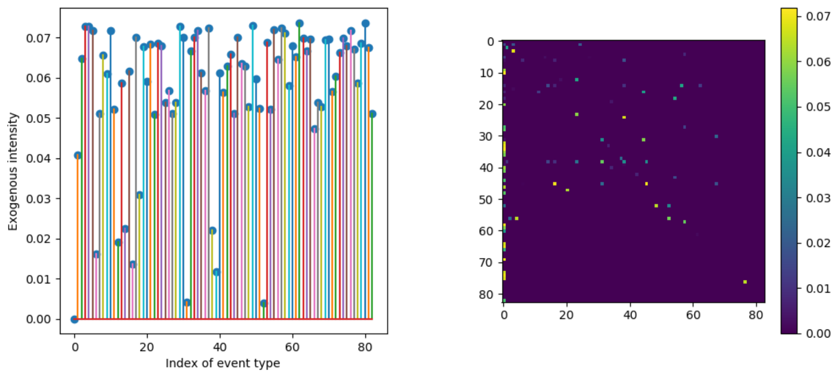
Class	Formulation
`ExogenousIntensity.BasicExogenousIntensity`	$\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s) = \mu_c$
`ExogenousIntensityFamily.NaiveExogenousIntensity`	$\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s) = g(\mu_c)$
`ExogenousIntensityFamily.LinearExogenousIntensity`	$\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s) = g(\boldsymbol{w}_c^\top \boldsymbol{f}_s)$
`ExogenousIntensityFamily.NeuralExogenousIntensity`	$\mu_c(\boldsymbol{f}_c, \boldsymbol{f}_s) = NN(\boldsymbol{f}_c, \boldsymbol{f}_s)$

Here, the activation function $g(\cdot)$ is defined as aforementioned g_λ.

Note that the last two models require event and sequence features as input. When they are called while the features are not given. PoPPy will add one more embedding layer to generate event/sequence features from their index, and learn this layer during training.

```python
def load_model(self, full_path, mode: str='entire'):
    """
    Load pre-trained model
    :param full_path: the path of directory
    :param mode: 'parameter' for saving only parameters of the model,
                 'entire' for saving entire model
    """
```

Figure 19: The description of `model_load`.



(a) exogenous intensity (b) Granger causality

Figure 20: Typical visualization results.

3.3.2 `model.EndogenousImpact.BasicEndogenousImpact`

This class and its subclasses in `model.EndogenousImpactFamily` implements several models of the coefficients of the impact function, as shown in Table 3.

Table 3: Typical models of endogenous impact's coefficient.

Class	Formulation
`EndogenousImpact.BasicEndogenousImpact`	$\alpha_{cc'm}(\boldsymbol{f}_c, \boldsymbol{f}_{c'}) = \alpha_{cc'm}$
`EndogenousImpactFamily.NaiveEndogenousImpact`	$\alpha_{cc'm}(\boldsymbol{f}_c, \boldsymbol{f}_{c'}) = g(\alpha_{cc'm})$
`EndogenousImpactFamily.FactorizedEndogenousImpact`	$\alpha_{cc'm}(\boldsymbol{f}_c, \boldsymbol{f}_{c'}) = g(\boldsymbol{u}_{cm}^{\top}\boldsymbol{v}_{c'm})$
`EndogenousImpactFamily.LinearEndogenousImpact`	$\alpha_{cc'm}(\boldsymbol{f}_c, \boldsymbol{f}_{c'}) = g(\boldsymbol{w}_{cm}^{\top}\boldsymbol{f}_{c'})$
`EndogenousImpactFamily.BiLinearEndogenousImpact`	$\alpha_{cc'm}(\boldsymbol{f}_c, \boldsymbol{f}_{c'}) = g(\boldsymbol{f}_c^{\top}\boldsymbol{W}_m\boldsymbol{f}_{c'})$

Here, the activation function $g(\cdot)$ is defined as aforementioned g_{λ}.

Note that the last two models require event and sequence features as input. When they are called while the features are not given. PoPPy will add one more embedding layer to generate event/sequence features from their index, and learn this layer during training.

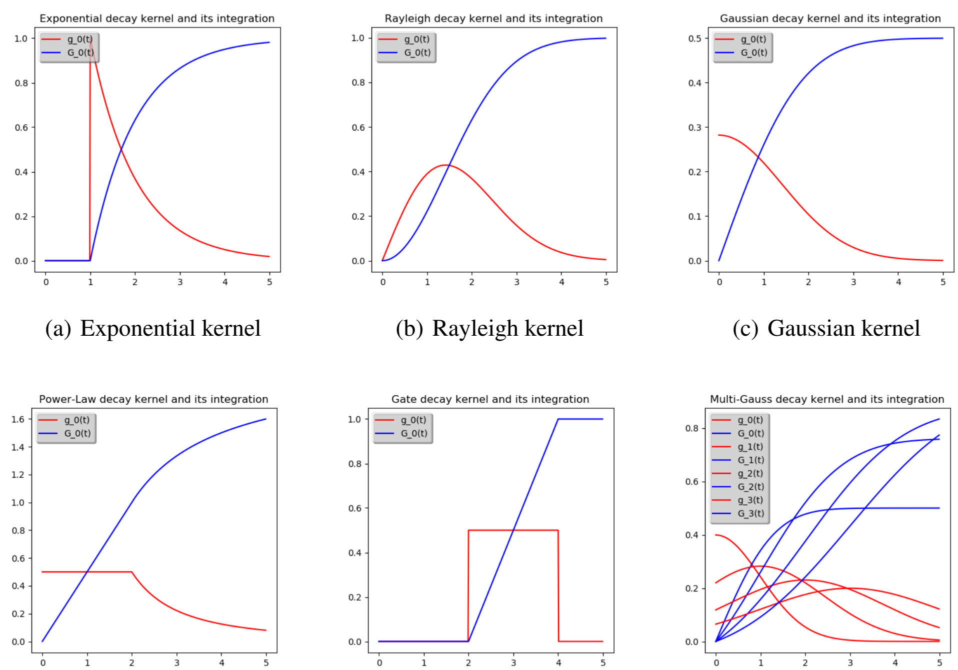
3.3.3 `model.DecayKernel.BasicDecayKernel`

This class and its subclasses in `model.DecayKernelFamily` implements several models of the decay kernel, as shown in Table 4.

Fig. 21 visualizes some examples.

Table 4: Typical models of decay kernel.

Class	M	Formulation
`DecayKernelFamily.ExponentialKernel` [13]	1	$\kappa(t) = \begin{cases} \omega \exp(-\omega(t-\delta)), & t \geq \delta, \\ 0, & t < \delta \end{cases}$
`DecayKernelFamily.RayleighKernel`	1	$\kappa(t) = \omega t \exp(-\omega t^2/s)$
`DecayKernelFamily.GaussianKernel`	1	$\kappa(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{t^2}{2\sigma^2})$
`DecayKernelFamily.PowerlawKernel` [12]	1	$\kappa(t) = \begin{cases} (\omega-1)\delta^{\omega-1}t^{-\omega}, & x \geq \delta, \\ (\omega-1)/\delta, & 0 < x < \delta \end{cases}$
`DecayKernelFamily.GateKernel`	1	$\kappa(t) = \frac{1}{\delta},\ t \in [\omega, \omega+\delta]$
`DecayKernelFamily.MultiGaussKernel` [6]	>1	$\kappa_m(t) = \frac{1}{\sqrt{2\pi}\sigma_m} \exp(-\frac{(t-t_m)^2}{2\sigma_m^2})$



(a) Exponential kernel (b) Rayleigh kernel (c) Gaussian kernel

(d) Powerlaw kernel (e) Gate kernel (f) Multi-Gaussian kernel

Figure 21: Examples of decay kernels and their integration values.

4 Learning Algorithm

4.1 Loss functions

With the help of PyTorch, PoPPy learns the point process models above efficiently by stochastic gradient descent on CPU or GPU [2].[1] Different from existing point process toolboxes, which mainly focuses on the maximum likelihood estimation of point process models, PoPPy integrates three loss functions to learn the models, as shown in Table 5.

Table 5: A list of loss functions.

Maximum Likelihood Estimation [13, 6]
- Class: `OtherLayers.MaxLogLike`
- Formulation: $L(\theta) = -\sum_{i\in\mathcal{D}}\left(\log\lambda_{c_i}(t_i) - \sum_{c\in\mathcal{C}}\int_{t_{i-1}}^{t_i}\lambda_c(s)ds\right)$
Least Square Estimation [8, 7]
- Class: `OtherLayers.LeastSquare`
- Formulation: $L(\theta) = \sum_{i\in\mathcal{D}}\Vert\int_{t_{i-1}}^{t_i}\boldsymbol{\lambda}(s)ds - \mathbf{1}_{c_i}\Vert_2^2$
Conditional Likelihood Estimation [10]
- Class: `OtherLayers.CrossEntropy`
- Formulation: $L(\theta) = -\sum_{i\in\mathcal{D}}\log p(c_i\vert t_i,\mathcal{H}_i) = -\sum_{i\in\mathcal{D}}\log softmax\left(\int_{t_{i-1}}^{t_i}\boldsymbol{\lambda}(s)ds\right)$.

Here $\boldsymbol{\lambda}(t) = [\lambda_1(t), ..., \lambda_{\vert\mathcal{C}\vert}(t)]$ and $\mathbf{1}_c$ is an one-hot vector whose the c-th element is 1.

4.2 Stochastic gradient decent

All the optimizers and the learning rate scheduler integrated in PyTorch are applicable to PoPPy. A typical configuration is using Adam + Exponential learning rate decay strategy, which should achieve good learning results in most situations. The details can be found in the demo scripts in the folder `example`.

Trick: Although most of optimizers are applicable, generally Adam achieves the best performance in our experiments [2].

4.3 Optional regularization

Besides the L2-norm regularizer integrated in the optimizers of PyTorch, PoPPy provides two more regularizers when learning models.

1. **Sparsity:** L1-norm of model's parameters can be applied to the models, which helps to learn structural parameters.
2. **Nonnegativeness:** If it is required, PoPPy can ensure the parameters to be nonnegative during training.

Trick: When the activation function of impact coefficient is softplus, you'd better close the nonnegative constraint by setting the input `nonnegative` of the function `fit` as **None**.

[1] Currently, the GPU version is under development.

5 Examples

As a result, using PoPPy, users can build their own point process models by combining different modules with high flexibility. As shown in Fig. 22, Each point process model can be built by selecting different modules and combining them together. The red dots represent the module with learnable parameters, the blue dots represent the module without parameters, and the green dots represent loss function modules. Moreover, users can add their own modules and design specific point process models for their applications easily, as long as the new classes override the corresponding functions.

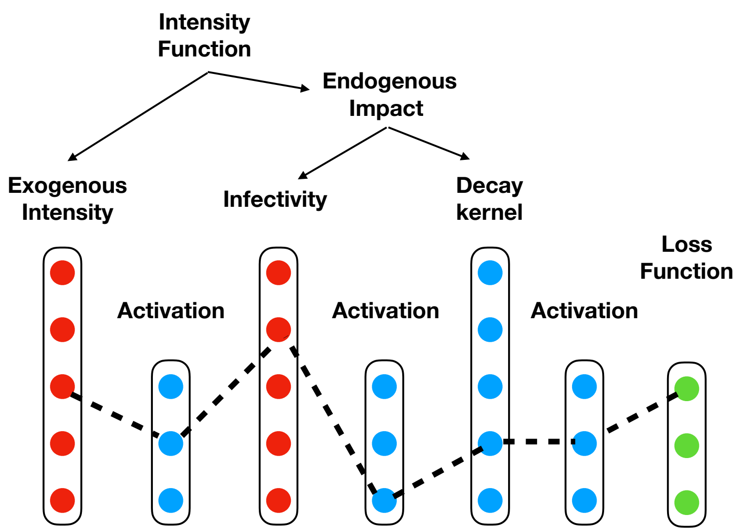


Figure 22: Illustration the contruction of a point process model.

Finally, we list some typical models implemented by PoPPy in Table 6 [2].

[2]It should be noted that our implementations may be different from the methods in the references in the aspect of model and learning algorithm so the results in the references may not be reproduced by PoPPy.

Table 6: Typical models implemented by PoPPy.

Model	Linear Hawkes process [13]
Exogenous Intensity	`NaiveExogenousIntensity`
Endogenous Impact	`NavieEndogenousImpact`
Decay Kernel	`ExponentialKernel`
Activation g_λ	Identity
Loss	`MaxLogLike`
Model	Linear Hawkes process [6, 5]
Exogenous Intensity	`NaiveExogenousIntensity`
Endogenous Impact	`NavieEndogenousImpact`
Decay Kernel	`MultiGaussKernel`
Activation g_λ	Identity
Loss	`MaxLogLike`
Model	Linear Hawkes process [8]
Exogenous Intensity	`NaiveExogenousIntensity`
Endogenous Impact	`NavieEndogenousImpact`
Decay Kernel	`MultiGaussKernel`
Activation g_λ	Identity
Loss	`LeastSquares`
Model	Factorized point process [7]
Exogenous Intensity	`LinearExogenousIntensity`
Endogenous Impact	`FactorizedEndogenousImpact`
Decay Kernel	`ExponentialKernel`
Activation g_λ	Identity
Loss	`LeastSquares`
Model	Semi-Parametric Hawkes process [1]
Exogenous Intensity	`LinearExogenousIntensity`
Endogenous Impact	`NavieEndogenousImpact`
Decay Kernel	`MultiGaussKernel`
Activation g_λ	Identity
Loss	`MaxLogLike`
Model	Parametric self-correcting process [11]
Exogenous Intensity	`LinearExogenousIntensity`
Endogenous Impact	`LinearEndogenousImpact`
Decay Kernel	`GateKernel`
Activation g_λ	Softplus
Loss	`MaxLogLike`
Model	Mutually-correcting process [10]
Exogenous Intensity	`LinearExogenousIntensity`
Endogenous Impact	`LinearEndogenousImpact`
Decay Kernel	`GaussianKernel`
Activation g_λ	Softplus
Loss	`CrossEntropy`

References

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