Notes on Ergodic Theory

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1 Dynamical Systems

1.1 Smooth Dynamical Systems on the 2-Dimensional Torus

Let

$$\mathrm{Tor}^2 = \frac{\mathbb{R}^2}{\mathbb{Z}^2} \tag{1}$$

with cyclic coordinates (u, v) and Lebesgue measure $\mathrm{d}u\mathrm{d}v$ and consider the system of differential equations

$$\frac{\mathrm{d}u}{\mathrm{d}t} = A(u, v) \tag{2}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = B(u, v) \tag{3}$$

where $A, B \in C^r$ where $r \geqslant 2$. Since this system satisifes uniqueness and existence conditions we can let $\{T^t\}$ be the one-parameter group of translations along its solutions. [1, 16.1]

Theorem 1. *If λ is irrational then the flow $\{T^t\}$ is metrically isomorphic to the special flow constructed from the automorphism T_1 of rotation of the circle S_1 by a certain irrational angle α of the form*

$$\alpha = \frac{m\lambda + n}{p\lambda + q} \tag{4}$$

where $m, n, p, q \in \mathbb{Z}$ and

$$\det\left(\begin{Vmatrix} m & n \\ p & q \end{Vmatrix} \right) = \pm 1 \tag{5}$$

and a function $F \in C^5(S^1)$

$$F: S^1 \to \mathbb{R}^1 \tag{6}$$

[1, Theorem 16.1]

Bibliography

[1] I.P. Kornfel′d, S.V. Fomin, and J.G. Sinaj. *Ergodic Theory*. Grundlehren der mathematischen Wissenschaften : a series of comprehensive studies in mathematics. Springer-Verlag, 1982.