Chapter 8

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April 26, 2016

Exercise 1

1. For any set of numbers $x_1, ..., x_n$, prove algebraically that

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

where $\bar{x} = \frac{1}{n}\sum\limits_{i=1}^{n} x_i$.

proof.

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i^2 + \bar{x}^2 - 2x_i\bar{x}) = \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n}(\bar{x}^2 - 2x_i\bar{x})$$

$$= \sum_{i=1}^{n} x_i^2 + \bar{x}\sum_{i=1}^{n}(\bar{x} - 2x_i) = \sum_{i=1}^{n} x_i^2 + \bar{x}\left(\sum_{i=1}^{n}\bar{x} - 2\sum_{i=1}^{n} x_i\right)$$

$$= \sum_{i=1}^{n} x_i^2 + \bar{x}\left(n\bar{x} - 2n\sum_{i=1}^{n}\frac{x_i}{n}\right) = \sum_{i=1}^{n} x_i^2 + \bar{x}\left(n\bar{x} - 2n\bar{x}\right) = \sum_{i=1}^{n} x_i^2 + \bar{x}(-n\bar{x})$$

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

which is the desired result.