Exercises from Chapter 3

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Problem 12

Let $N = \min\{n: \sum_{i=0}^{n} U_i > 1\}$, where $U_i \sim U(0,1)$ for $i \geq 1$. Thus N is a random number and we want to estimate $E(N)$ by generating 100,1000 and 10000 values of N.

To Generate N

The idea is to generate one random uniform number U_i at a time and add them together exactly until the sum exceed 1, then we count how many of those have been added.

Below is the code to generate the random number N*. The function* ***RN12****, takes as input the sample size, and returns the sample mean which is an estimation for* $E(N)$*.*

```
RN12<-function(n){
  N2<-rep(0,n) #storage for the n random numbers.
  for(i in 1:n){

    N=0 # There is no sum in the begining, thus no uniform random number.
    S=0 # The sum is 0 in the begining.

    while(S<=1){ # Check wether the sum of the uniform random numbers exceed
1.
      U<-runif(1)
      S=S+U
      N=N+1
    }

  N2[i]=N # Once the sum exceed 1 store the random number N and repeat the
experiment.
  }

return(mean(N2))
}
```

(a) Using a sample of size 100 yields

```
RN12(100)

## [1] 2.75
```

(b) Using a sample of size 1000 yields

```
RN12(1000)

## [1] 2.71
```

(c) Using a sample of size 10000 yields

```
RN12(10000)

## [1] 2.7223
```

(d) I think that this is a cool way to estimate e. :-)

Problem 13

Let $\mathrm{N} = \max\{n: \prod_{i=0}^{n} \mathrm{U_i} \geq e^{-3}\}$, where $\mathrm{U_i} \sim \mathrm{U}(0,1)$ for $i \geq 1$. Thus N is a random number and we want to estimate $\mathrm{E(N)}$ by simulation.

To Generate N

Similar approach as in problem 12.

*Below is the code for generating the desired random numbers. The function **RN12**, takes as input the sample size, and returns the sample mean which is an estimation for* $\mathrm{E(N)}$.

```
RN13<-function(n){
  N2<-rep(0,n)

  for(i in 1:n){
    P=1  # The product is 1, when N is 0.
    N=0  # N the number of uniform random number is 0 in the begining.

    while(P>exp(-3)){ #Check whether the product exceed e^-3.
      U<-runif(1)
      P=P*U
      N=N+1            #the number of uniform random number in the product is
increased by 1.
    }

    N2[i]=N
  }

  return(mean(N2))
}
```

(a) Below an estimation for $\mathrm{E(N)}$ using a sample of size 10000

```
RN13(10000)

## [1] 4.0066
```

(b) Estimation for $\mathrm{P(N = i)}$ for $i = 0, \ldots, 6$, with sample of size 10000. The idea is to generate 10000 random numbers N, then the probability $\mathrm{P(N = i)} \approx \frac{\#\{N=i\}}{10000}$, where $\#\{N = i\}$ is the number of the simulated random numbers N that attained value i.

```
n<- 10000
N2<-rep(0,n)
```

```r
for(i in 1:n){
    P=1
    N=0
    while(P>exp(-3)){
      U<-runif(1)
      P=P*U
      N=N+1
    }
    N2[i]=N
}
P0= length(N2[N2==0])/10000
P1= length(N2[N2==1])/10000
P2= length(N2[N2==2])/10000
P3= length(N2[N2==3])/10000
P4= length(N2[N2==4])/10000
P5= length(N2[N2==5])/10000
P6= length(N2[N2==6])/10000
```

Estimation for $P(N = 0)$

```r
P0

## [1] 0
```

Estimation for $P(N = 1)$

```r
P1

## [1] 0.0473
```

Estimation for $P(N = 2)$

```r
P2

## [1] 0.1484
```

Estimation for $P(N = 3)$

```r
P3

## [1] 0.2259
```

Estimation for $P(N = 4)$

```r
P4

## [1] 0.2236
```

Estimation for $P(N = 5)$

```r
P5

## [1] 0.1665
```

Estimation for $P(N = 6)$

```
P6

## [1] 0.1049
```