

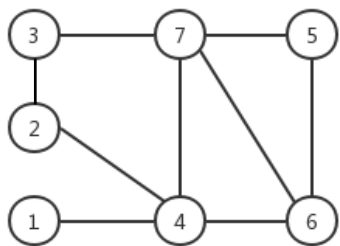
Data Structures and Algorithms
INFO 6205
Homework 9
Due: March 31, 2019

Put all your java, compiled class files and documentation files into a zip file Homework9.zip and submit it via the dropbox on the blackboard before the END of due date. Put your name on all .java files. There will be a short Quiz on this homework.

1. a) Draw the undirected graph that is represented as follows:

Vertices: 1, 2, 3, 4, 5, 6, 7

Edges: (5, 6), (4, 6), (3, 7), (6, 7), (5, 7), (1, 4), (2, 4), (2, 3), (4, 7)

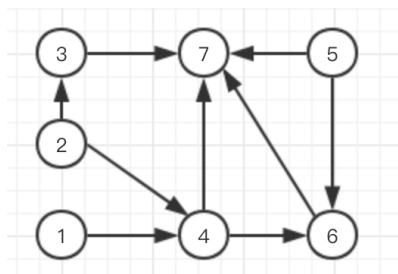


- b) Is the graph connected? Is it complete? Explain.

This graph is connected. Because if a graph is connected, then for any two vertices i, j in the graph, they can connect to each other through a certain path. In this diagram, any two vertices can reach each other through a path. So, this graph is connected.

This graph is not complete, because some vertices are not directly connected. Like 3-4, 1-2..etc.

- c) Draw the Directed graph using the same data as above.

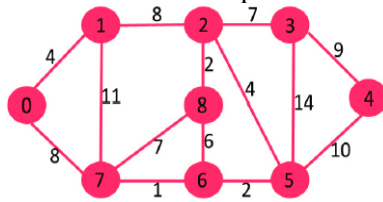


- d) Is the directed graph connected? Is it complete? Explain.

This graph is not connected. Because if a graph is connected, then for any two vertices i, j in the graph, they can connect to each other through a certain path. In this diagram, some two vertices can't reach each other through a path. So, this graph is not connected.

This graph is not complete, because some vertices are not directly connected. Like 3-4, 1-2..etc.

2. Consider this Undirected Graph:



a) What is the shortest-path of this graph, show **step-by-step** algorithm

sptSet: shortest path tree set

INF: Infinite

S: Record the vertex of the shortest path (and the corresponding shortest path length)

U: Record the vertex of the shortest path (and the distance from the vertex to the starting point s)

Show Step	Step Detail
	Select the top vertex 0 $S = \{0(0)\}$ $U = \{1(4), 2(\text{INF}), 3(\text{INF}), 4(\text{INF}), 5(\text{INF}), 6(\text{INF}), 7(8), 8(\text{INF})\}$ $\text{sptSet} = \{0\}$
	Select the next vertex 1 $S = \{0(0), 1(4)\}$ $U = \{2(12), 3(\text{INF}), 4(\text{INF}), 5(\text{INF}), 6(\text{INF}), 7(8), 8(\text{INF})\}$ $\text{sptSet} = \{0, 1\}$
	Select the next vertex 7 $S = \{0(0), 1(4), 7(8)\}$ $U = \{2(12), 3(\text{INF}), 4(\text{INF}), 5(\text{INF}), 6(9), 8(15)\}$ $\text{sptSet} = \{0, 1, 7\}$
	Select the next vertex 6 $S = \{0(0), 1(4), 7(8), 6(9)\}$ $U = \{2(12), 3(\text{INF}), 4(\text{INF}), 5(11), 8(15)\}$ $\text{sptSet} = \{0, 1, 7, 6\}$
	Select the next vertex 5 $S = \{0(0), 1(4), 7(8), 6(9), 5(11)\}$ $U = \{2(12), 3(24), 4(21), 8(15)\}$ $\text{sptSet} = \{0, 1, 7, 6, 5\}$
	Select the next vertex 2 $S = \{0(0), 1(4), 7(8), 6(9), 5(11), 2(12)\}$ $U = \{3(19), 4(21), 8(15)\}$ $\text{sptSet} = \{0, 1, 7, 6, 5, 2\}$
	Select the next vertex 8 $S = \{0(0), 1(4), 7(8), 6(9), 5(11), 2(12), 8(14)\}$ $U = \{3(19), 4(21)\}$ $\text{sptSet} = \{0, 1, 7, 6, 5, 2, 8\}$

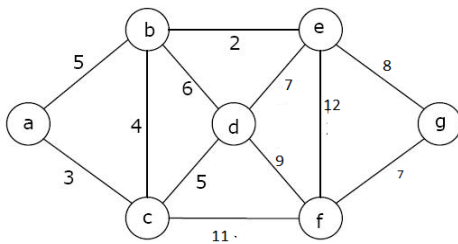
	Select the next vertex 3 $S = \{0(0), 1(4), 7(8), 6(9), 5(11), 2(12), 8(14), 3(19)\}$ $U = \{4(21)\}$ $sptSet = \{0, 1, 7, 6, 5, 2, 8, 3\}$
	Select the next vertex 4 $S = \{0(0), 1(4), 7(8), 6(9), 5(11), 2(12), 8(14), 3(19), 4(21)\}$ $U = \{\}$ $sptSet = \{0, 1, 7, 6, 5, 2, 8, 3, 4\}$

b) What is the space and time complexity of this algorithm?

Time Complexity of the implementation is $O(V^2)$. If the input graph is represented using adjacency list, it can be reduced to $O(E \log V)$.

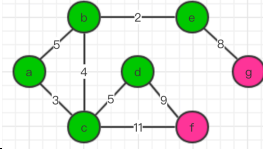
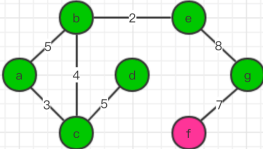
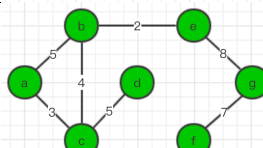
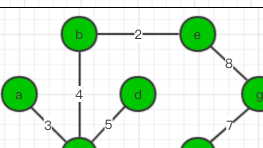
c) Write Java code, compile and test.

3. Solve the Minimum Spanning Tree for the following Graph,



a) Prim's Algorithm step-by-step

Show Step	Step Detail
	Select the top vertex a $S = \{a(0)\}$ $U = \{b(5), c(3), d(INF), e(INF), f(INF), g(INF)\}$ $mstSet = \{a\}$
	Select the next vertex c $S = \{a(0), c(3)\}$ $U = \{b(4), d(5), e(INF), f(11), g(INF)\}$ $mstSet = \{a, c\}$
	Select the next vertex b $S = \{a(0), c(3), b(4)\}$ $U = \{d(5), e(2), f(11), g(INF)\}$ $mstSet = \{a, c, b\}$
	Select the next vertex e $S = \{a(0), c(3), b(4), e(2)\}$ $U = \{d(5), f(11), g(8)\}$ $mstSet = \{a, c, b, e\}$

	Select the next vertex d $S = \{a(0), c(3), b(4), e(2), d(5)\}$ $U = \{f(9), g(8)\}$ $mstSet = \{a, c, b, e, d\}$
	Select the next vertex g $S = \{a(0), c(3), b(4), e(2), d(5), g(8)\}$ $U = \{f(7)\}$ $mstSet = \{a, c, b, e, d, g\}$
	Select the next vertex g $S = \{a(0), c(3), b(4), e(2), d(5), g(8), f(7)\}$ $U = \{\}$ $mstSet = \{a, c, b, e, d, g, f\}$
	$mstSet = \{a, c, b, e, d, g, f\}$ The order is a - c - b - e - d - g - f

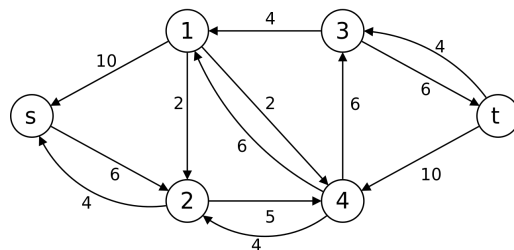
b) Write Java code for algorithm (a)

c) Space and Time complexity?

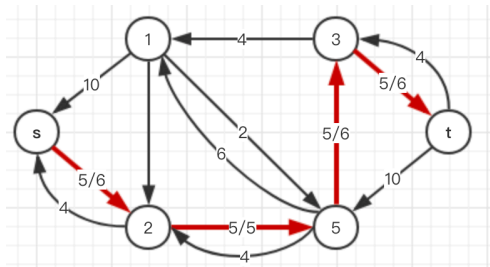
Time Complexity of the above program is $O(V^2)$. If the input graph is represented using adjacency list, then the time complexity of Prim's algorithm can be reduced to $O(E \log V)$.

4. Consider Capacity, Flow, Residual Capacity, and Augmented Path in the following

Ford-Fulkerson Graph:



a) What is the Maximum Flow in the graph, show ALL augmented paths step-by-step



The max flow is 5.

route	capability	flow	
s-2	6	5	0
2-4	5	5	1
4-3	6	5	0
3-t	6	5	1

b) What is the Time complexity and Space of the algorithm?

The time complexity of the above implementation is $O(EV^3)$

c) What is the Edmonds-Karp contribution to Fulkerson method, and Why?

The Edmonds-Karp Algorithm is a specific implementation of the Ford-Fulkerson algorithm. Like Ford-Fulkerson, Edmonds-Karp is also an algorithm that deals with the max-flow min-cut problem.

Ford-Fulkerson is sometimes called a method because some parts of its protocol are left unspecified. Edmonds-Karp, on the other hand, provides a full specification. Most importantly, it specifies that breadth-first search should be used to find the shortest paths during the intermediate stages of the program.

Edmonds-Karp improves the runtime of Ford-Fulkerson, which is $O(|E|f^*)$, to $O(|V||E|^2)$. This improvement is important because it makes the runtime of Edmonds-Karp independent of the maximum flow of the network.

d) Write the Java code for the algorithm?

Note: see example at the bottom of this page:

<http://www-groups.mcs.st-and.ac.uk/~vince/teaching/summer04/flow.pdf>

5. Read the following links on Genomics. Nothing to report except it would be on next quiz.

sSlides Attached

<https://ghr.nlm.nih.gov/primer/basics/gene>

<https://www2.le.ac.uk/projects/vgec/highereducation/topics/dna-genes-chromosomes>

<https://ghr.nlm.nih.gov/primer/mutationsanddisorders/possiblemutations>