Homework 5

Submission instructions.

- Submissions are due on Thursday 10/03 at 7.00pm ET == 4.00pm PT
- Please upload scans of your solution in GradeScope (via Canvas)
- Please ensure that your scans are readable.

Instructions

- Please show all necessary steps to get the final answer. However, there is no need to be overly elaborate. Crisp and complete answers.
- For all MATLAB problems, include all code written to generate solutions.
- Please post all questions on the discussion board on the Piazza course website, rather than email-ing the course staff. This will allow other students with the same question to see the response and any ensuing discussion.
- **If you feel some information is missing, you are welcome to make reasonable assumptions and proceed. Sometimes the omissions are intentional. Needless to say, only reasonable assumptions will be accepted.**

1. **(3 pts)** *Implementing filtered backprojection*

   This problem starts off with a few lines of code.

   ```
   img = phantom('Modified Shepp-Logan',256);
   theta = 0:1:180;
   rad = radon(img, theta);
   ```

   Implement filtered backprojection. Your code should take in as input the variables `rad` and `theta`, and produce an estimate of the original image.

   Deliverable: MATLAB code, an image of the reconstruction, and an image of the absolute difference between your reconstruction and the original image.

   Write your own code —you are welcome to commands like `meshgrid, imrotate, fft` , but please do not use the command `iradon` for this problem.

2. **(3 pts)** *(Convolutions and Radon Transforms)* Let the Radon transform of two images $i_1(x, y)$ and $i_2(x, y)$ be $r_1(\alpha, \theta)$ and $r_2(\alpha, \theta)$, respectively. Let $i_3(x, y) = (i_1 * i_2)(x, y)$ be the 2D convolution of i_1 and i_2.

   Find an expression for $r_3(\alpha, \theta)$, the Radon transform of $i_3(x, y)$, in terms of r_1 and r_2.

3. **(3.9 pts)** *(Inverting the radon transform using Algebraic reconstruction techniques)*

Given a vector of angles, in degrees, Theta $= [\theta_1\, \theta_2\, \ldots \theta_N]$ and an image img of size $M \times M$, the following MATLAB command operator produces its radon transform.

```
Aradon = @(img) radon(img, Theta);
```

The adjoint of this operation is given as follows

```
Aradon_adjoint = @(rad_img) iradon(rad_img, Theta, 'linear', 'none', 1, M);
```

For the commands above to work, you would need to define the variables Theta and the integer M.

(Deliverable A) **A MATLAB function**, that takes in as input the following: (a) radon transform of an image, (b) the list of angles Theta, and (c) size, in terms of number of rows, of the recovered square image M. The function should output the solution to the least square problem $\min_{\mathbf{x}} \|\text{rad} - \text{Aradon}(\mathbf{x})\|^2$, where rad is the radon measurement.

Next, use the Shepp-Logan phantom of size 256×256 as your test input for the remaining parts. The following command should get you the phantom

```
img = phantom('Modified Shepp-Logan',256);
```

Now, lets vary the number of angles N from 10 to 180 in steps of 10. For each value of N, first generate Θ by uniformly sampling from 0° to 180°, then simulate the radon transform measurements as well as its pseudoinverse using the code from above.

(Deliverable B) (i) **A plot** of reconstruction error as a function of N, the number of angles, and (ii) **reconstructed images** for $N = 10, 90$ and 180

Please measure reconstruction error in dB, defined as follows. If img is the true image and imgrec is the reconstructed image, then the reconstruction error in dB is given as

```
-20*log10( norm(img(:) - imgrec(:))/norm(img(:)))
```

Now, lets repeat the above except use a regularized least squares solution. Use the gradient energy regularization from HW3 where we penalized the energy in the x and y gradients of the image.

(Deliverable C) **Code**, **plot** of reconstruction error as a function of N, the number of angles, as well as **reconstructed images** for $N = 10, 90, 180$ for the regularized least square solution.

Hint: Reusing code from the previous HWs should make this a very easy problem, or so one hopes.

4. **(0.1 points)** How many hours did this homework take?

5. (No credit) *(Some elementary radon transforms and properties)* Consider an image $f(x, y)$ and its radon transform $r(\theta, \alpha)$.

   a) If $f(x, y)$ is rotationally-invariant, show that $r(\theta, \alpha) = r(\alpha)$, i.e., the radon transform is not a function of the projection angle θ.

   b) Let $r(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}}$. Derive $f(x, y)$.