Homework 7

Submission instructions.

- Submissions are due on Thursday 10/17 at 7.00pm ET == 4.00pm PT
- Please upload scans of your solution in GradeScope (via Canvas)
- Please ensure that your scans are readable.

Instructions

- Please show all necessary steps to get the final answer. However, there is no need to be overly elaborate. Crisp and complete answers.
- For all MATLAB problems, include all code written to generate solutions.
- Please post all questions on the discussion board on the Piazza course website, rather than email-ing the course staff. This will allow other students with the same question to see the response and any ensuing discussion.
- **If you feel some information is missing, you are welcome to make reasonable assumptions and proceed. Sometimes the omissions are intentional. Needless to say, only reasonable assumptions will be accepted.**

1. **(3 pts)** *(Time-frequency bandwidth product and scaling)*

   Consider a window function $a(t) \xleftrightarrow{FT} A(f)$.

   The time bandwidth σ_t and frequency bandwidth σ_f are defined as

   $$\sigma_t^2(a) = \int t^2 |a(t)|^2 dt, \qquad \sigma_f^2(a) = \int f^2 |A(f)|^2 df,$$

   respectively.

   Now consider a second window $a_s(t)$ obtained by scaling $a(t)$ as follows,

   $$s > 0, \qquad a_s(t) = \frac{1}{\sqrt{s}}\, a\left(\frac{t}{s}\right).$$

   Show that

   $$\sigma_t(a_s)\sigma_f(a_s) = \sigma_t(a)\sigma_f(a).$$

2. **(3 pts)** *(Gaussian and time-frequency bandwidth product)* Prove that the Gaussian is the only window for which the time-frequency bandwidth product is equal to it lower bound, i.e., $\sigma_t^2\sigma_f^2 = 1/16\pi^2$.

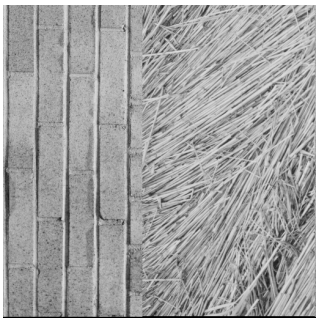

Figure 1: One.tiff

3. **(3.9 pts)** *(Gabor!)* Consider the image "one.tiff" given as part of the assignment (and shown below). You can see two distinct textures in the image.

   Deliverable # 1. Design the parameters — scale, orientation, frequency — of a Gabor window say g_1 such that the magnitude of the image convolved with g_1 is high in the left half of the image.

   Deliverable # 2. Design the parameters — scale, orientation, frequency — of a Gabor window say g_2 such that the magnitude of the image convolved with g_1 is high in the right half of the image.

   Deliverable # 3. Design a simple image segmentation scheme that uses the outputs of the convolutions as a feature to assign a per-pixel label that captures the texture membership.

   Deliverable #4 (no credit). Design a scheme to segment the textures in "two.tiff" (shown below)

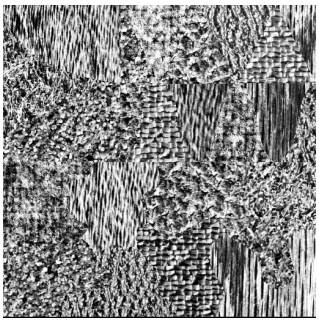

Figure 2: Two.tiff

   Hint: `https://www.mathworks.com/help/images/texture-segmentation-using-gabor-filters.html`

4. **(0.1 points)** How many hours did this homework take?