18793 HW8

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1.

1. $W_1(\tau, s) = \int_t i_1(t)\,\phi_{s,\tau}(t)\,dt \qquad \phi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\phi_0\left(\frac{t-\tau}{s}\right)$

a. $i_2(t) = i_1(t - t_0)$

$$W_2(\tau, s) = \int_t i_2(t)\,\phi_{s,\tau}(t)\,dt = \int_t i_1(t-t_0)\,\phi_{s,\tau}(t)\,dt$$

$$= \int_t i_1(t-t_0)\frac{1}{\sqrt{s}}\phi_0\left(\frac{t-\tau}{s}\right)dt$$

let $z = t - t_0 \quad t = z + t_0 \quad dz = dt$ substitute, we have:

$$W_2(\tau, s) = \int_t i_1(z)\frac{1}{\sqrt{s}}\phi_0\left(\frac{z-(\tau-t_0)}{s}\right)dz$$

$$= \int_t i_1(z)\,\phi_{s,\tau-t_0}(z)\,dz$$

$$= W_1(\tau - t_0, s)$$

b. $i_3(t) = i_1(at)$, $a > 0$

$$W_3(\tau, s) = \int_t i_3(t)\,\phi_{s,\tau}(t)\,dt = \int_t i_1(at)\,\phi_{s,\tau}(t)\,dt$$

$$= \int_t i_1(at)\frac{1}{\sqrt{s}}\phi_0\left(\frac{t-\tau}{s}\right)dt$$

let $z = at$, $t = \frac{z}{a}$, $dz = a\,dt$, substitute, we have:

$$W_3(\tau, s) = \int_t i_1(z)\frac{1}{\sqrt{s}}\phi_0\left(\frac{\frac{z}{a}-\tau}{s}\right)\frac{1}{a}\,dz$$

$$= \int_t i_1(z)\frac{1}{\sqrt{s}}\phi_0\left(\frac{z-a\tau}{as}\right)\frac{1}{a}\,dz$$

$$= \int_t i_1(z)\frac{1}{\sqrt{a}\sqrt{s}}\phi_0\left(\frac{z-a\tau}{as}\right)\frac{1}{\sqrt{a}}\,dz$$

$$= \frac{1}{\sqrt{a}}\int_t i_1(z)\,\phi_{as,a\tau}(z)\,dz$$

$$= \frac{1}{\sqrt{a}}W_1(a\tau, as)$$

2.

2.a.

Time complexity of convolution $X[n]$ of length N with $g[n]$, $h[n]$ of length L

$$O(\text{conv}) = NL$$

Time complexity of downsampling

$$O(\text{downsample}) = \frac{N}{2}$$

$$T(N) = \text{convolution} + \text{downsampling} + T(\tfrac{N}{2})$$

since half the frequencies of the signal ~~have been~~ can be discarded according to Nyquist's rule.

$$\begin{aligned} O(T(N)) &= O(\text{convolution}) + O(\text{downsample}) + O(T(\tfrac{N}{2})) \\ &= NL + \frac{N}{2} + \frac{N}{2}L + \frac{N}{4} + \cdots \\ &= NL(1 + \tfrac{1}{2} + \tfrac{1}{4} + \cdots) + N(\tfrac{1}{2} + \tfrac{1}{4} + \cdots \tfrac{1}{2^k}) \\ &= NL \cdot 2(1-(\tfrac{1}{2})^{k}) + N(1-(\tfrac{1}{2})^k) \end{aligned}$$

since $L \ll N$

$$O(T(N)) = O(N)$$

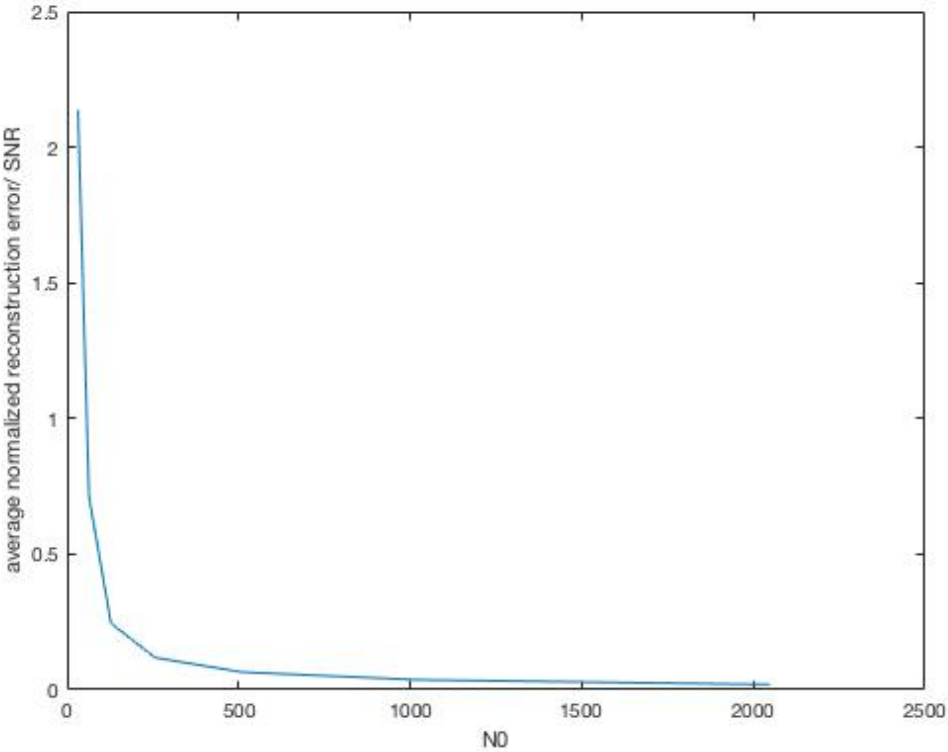
b. $L = N$, time complexity of convolution $O(\text{conv}) = N\log N$

$$\begin{aligned} O(T(N)) &= O(\text{conv}) + O(\text{downsample}) + O(T(\tfrac{N}{2})) \\ &= N\log N + \frac{N}{2} + \frac{N}{2}\log\frac{N}{2} + \frac{N}{4} + \cdots \\ &= N(\tfrac{1}{2} + \tfrac{1}{4} + \cdots) + N\log N + \tfrac{N}{2}\log N - \tfrac{N}{2}\log 2 + \tfrac{N}{4}\log N - \tfrac{N}{4}\log 4 + \cdots \\ &= N(1-(\tfrac{1}{2})^k) + \log N(N + \tfrac{N}{2} + \tfrac{N}{4} + \cdots + \tfrac{N}{2^k}) - \tfrac{N}{2}\log 2 \cdot (k-1) \\ &= N(1-(\tfrac{1}{2})^k) + N\log N \cdot 2(1-(\tfrac{1}{2})^k) - \tfrac{N}{2}\log 2 \cdot (k-1) \\ &= O(N\log N) \end{aligned}$$

3

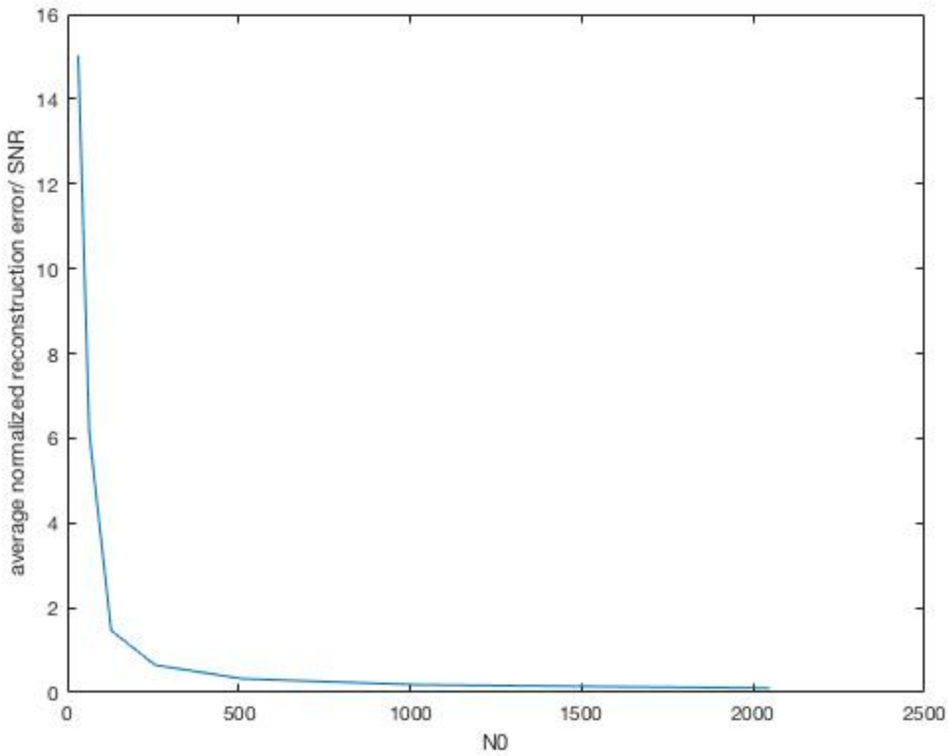
a.

b.



As the resolution of the image increases, the signal to noise ratio/ average normalized reconstruction error decreases. This is because, as we increase the size of down-sampled image, we are able to capture more information and details, such as localized change in intensity, shape edges, etc. As a result, even though we keep the fraction of wavelet coefficients used constant, the reconstructed image is more similar to the original image.

c.



Compared with result in b, the average normalized reconstruction error for smaller fraction of wavelet coefficients is smaller. This is intuitive because, we use less wavelet functions to reconstruct the image and the result will be less accurate. In other words, the reconstruction accuracy increases as we uses more wavelet coefficients.

Code:

```
clear
close all
%% Load image
targetSize = [2048 2048];
Kfrac = 0.01;
ss = [32, 64, 128, 256, 512, 1024, 2048];
error = zeros(16,length(ss));

for i = 1:16
    img = imread(sprintf('%01d.jpg', i));
    img = rgb2gray(img);
    img = double(img)/255;
    %[x,y,w,h] = center(size(img),targetSize);
    I = imcrop(img,[1,1,targetSize(1)-1,targetSize(2)-1]);
    %processed(1,i) = I;
    %subplot(4,4,i);
    %imshow(I);

    for j = 1:length(ss)
        N0 = ss(j);
        %downsample
```

```
        I1 = imresize(I,[N0 N0]);
%         figure;
%         imshow(I1);

        dwtmode("per");
        [s0, cbook] = wavedec2(I1, log2(N0), 'db4');

        s = abs(sort(s0, 'descend'));
        K = int32(Kfrac*N0*N0);
        shat = s(K);

        s0(s0<shat & s0>-shat) = 0;
        imghat = waverec2(s0, cbook, 'db4');

        error(i,j) = norm(I1-imghat,2)/norm(I1,2);
        disp(i)
        disp(j)
    end
end

err = sum(error);
figure;
plot(ss,err);
xlabel('N0')
ylabel('average normalized reconstruction error/ SNR')

function [x, y, w, h] = center(a,b)
    x = int16(a(1)/2 - b(1)/2);
    y = int16(a(2)/2 - b(2)/2);
    w = int16(b(1) - 1);
    h = int16(b(2) - 1);
end
```

4. 4 hours + 2 office hours