Summary

The compound helicopter is widely used in various fields. Its flight attitude, i.e. pitch, yaw and roll angles, is influenced by the coaxial rigid rotor, the propeller booster, the horizontal tail and the vertical tail. This paper focuses on the control relationship of these four dynamic components on the flight attitude of the helicopter.

For problem 1, the pitch moment equation is established from the data and the equation for the pitch moment M_2 given in the Annex. It was found that the solution has **a constant pitch moment** M_2 **=58.59** J .Then, according to **the angular momentum theorem**, the **second order differential relationship** between the pitch moment and the **pitching angle** θ is established, and the solution gives the pitch angle as a function of time as $\theta(\mathbf{t}) = \mathbf{0.00146t^2}$. Finally, substituting the three moments into the function to obtain $\theta(\mathbf{5}) = \mathbf{0.0366degree}$,$\theta(\mathbf{10}) = \mathbf{0.1464degree}$,and$\theta(\mathbf{20}) = \mathbf{0.5858degree}$.

For problem 2, from the information provided, the solution is the same as for problem 1. Using the equations given in the Appendix, the three moments are solved and found to be constant. We obtain the **roll moment** $\mathbf{M_1 = -35.91J}$, the **pitch moment** $\mathbf{M_2 = 58.59J}$ and the **yaw moment** $\mathbf{M_3 = 10.084J}$. Next, the **second order differential equation** is set up using **the angular momentum theorem**. By solving the equation we obtain the attitude angle time function, **roll angle** $\phi(\mathbf{t}) = \mathbf{0.00449t^2}$, **pitch angle** $\theta(\mathbf{t}) = \mathbf{0.00146t^2}$ and **yaw angle** $\psi(\mathbf{t}) = \mathbf{0.0002} * \mathbf{t^2}$.Finally, substituting time wen can gives corresponding angle.

For problem 3, we first establish the **moment equations for low and high speed flight**. Then, the problem of solving the equations is transformed into a **single objective optimisation problem**. The **maximum value of the inverse of the sum of the absolute values of the three moments** is taken as the **objective function**, and the **initial ranges of the manoeuvring** parts are taken as the **constraints**. We use the **genetic algorithm** to solve it. The peculiarity of this optimization model is that the optimal solution of the model is not one. Therefore, each optimization yields one of the countless optimization solutions. Then, using the idea of **Monte Carlo simulation**, the optimization is repeated so many times that it is highly probable that the **boundary value of the optimal solution** is reached if the number of iterations is sufficiently large. To obtain the **manoeuvring amplitude of each component** , the boundary values in the simulation results are recorded.

For problem 4, we first determine the **expression for horizontal flight speed with time**. Then,the flight process is divided into **low speed phase, medium speed phase and high speed phase.** The set of moment equations for each of the three phases follows the **genetic algorithm optimisation** model of the problem 3. The dynamic values of the helicopter manoeuvring components with respect to time are obtained.

Key word: attitude angle, moment, angular momentum theorem, genetic algorithm, Monte Carlo

Content

1. Introduction

1.1 Background

The compound helicopter is widely used in a variety of applications. Its flight attitude, i.e. pitch, yaw and roll angles, are influenced by the co-axial rigid rotor, the propeller booster, the horizontal stabilizer and the vertical stabilizer. The co-axial rigid rotor generates roll, pitch and yaw moments. The propeller booster generates thrust and roll torque. Thrust generates roll moment. The horizontal tail generates pitching torque. The vertical tail generates yaw and roll moments. Controlling these four power components controls the helicopter's flight state.

1.2 Our work

Task 1: We need to calculate the formulae for pitch moment and pitch angle for the helicopter state given in Problem 1 and give the values of pitch angle for 5s,10s,10s.

Task 2: We need to calculate the formulae for pitch, roll and yaw moments and the formulae for pitch, roll and yaw angles for the helicopter state given in Problem 2 and give their values at 5s,10s,10s respectively.

Task 3: We need to design the magnitude of change of each component so that the helicopter maintains zero attitude angle at high and low speeds respectively.

Task 4: We need to design the magnitude of change of each component so that the helicopter can accelerate at zero attitude angle.

2. Problem Analysis

2.1 Information Analysis

We analyzed the given material and got the following useful information:

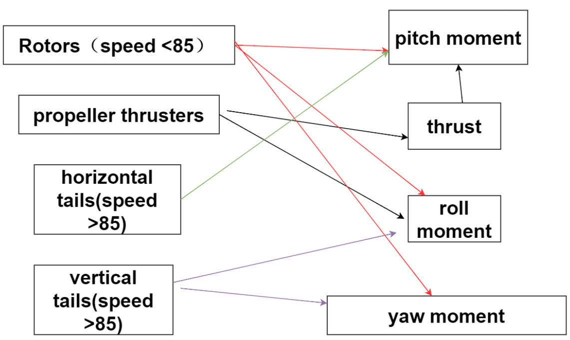

Figure 1 The image of information

2.2 Analysis of Question One

For the problem one, we need to calculate the pitch moment and the pitch angle of the helicopter under the conditions given in the problem one. The formula for the pitching moment and data from the annex can be utilized to determine the pitching moment expression. Then, using Newton's second law and the angular momentum theorem, it is possible to establish a second-order differential relationship between the pitching moment and the pitch angle. The second-order differential equation can then be solved to find the expression of pitch angle concerning time.

2.3 Analysis of Question Two

For the problem two, we need to establish the expressions for the compound helicopter's roll, pitch, and yaw moments and the variation of attitude angles.The approach to calculating roll moment, roll angle, yaw moment, and yaw angle is identical to the technique employed for pitch moment and pitch angle in Problem 1, with the only difference being the substitution of some parameters.

2.4 Analysis of Question Three

For problem 3, to achieve horizontal flight (zero attitude angle) of the vehicle,we need to design the maneuvering amplitude of each component, based on the low and high speed flight characteristics.Zero attitude angle for a helicopter means that the roll angle, pitch angle, and yaw angle are maintained at 0 degree. In addition, the attitude angle of the helicopter is affected by different factors at low and high speeds, so we should first establish the system of moment equations for the two cases for the two cases separately.

In the solution process, the problem of solving the system of equations can be transformed into an optimization problem, and the single-objective optimization model can be processed by the improved genetic algorithm. At the same time, based on the idea of Monte Carlo simulation, the optimization process of the genetic algorithm is repeated several times. The maximum and minimum values of each component in all the optimization results are compared and recorded. This will give the maneuvering range of each component when the helicopter is flying horizontally (zero attitude).

2.5 Analysis of Question Four

For problem 4, we consider the dynamics of the helicopter components under uniform acceleration and horizontal flight (zero attitude angle). Since the rotor pitch ratio is proportional to the flight speed, we consider time as the independent variable. We divide the flight process into a low speed phase (80m/s 84m/s), a medium speed phase (85m/s 100m/s) and a high speed phase (101m/s 180m/s). Following the idea of the third question, when the time is fixed, the genetic algorithm optimisation model of the third question can be applied to derive a series of corresponding manoeuvre amplitudes for each component of the helicopter over time. Since there is a range solution for each component of the helicopter at each moment that satisfies the zero attitude angle, in order to make the changes of the helicopter at different moments as continuous as possible, we should introduce the optimised value of the previous moment when considering the optimised value of the next moment, and take the minimum of the Euclidean distance between the two optimised values as the basis of the optimisation.

3. Symbol and Assumptions

3.1 Symbol Description

Symbol	Description
I_1	roll axis moment of inertia
I_2	pitch axis moment of inertia
I_3	yaw axis moment of inertia
M_1	roll moment
M_2	pitch moment
M_3	yaw moment
m_1	rotor roll moment coefficien
m_2	rotor pitch moment coefficien
m_3	rotor yaw moment coefficien
ϕ	roll angle
θ	pitch angle
ψ	yaw angle
S_c	dotor disc area
V_o	blade centre-of-mass linear velocity
ρ	air density

3.2 Assumptions

1. Assume that the angle of attitude of the helicopter in flight relates only to the coaxial rotor, the propeller engine, the horizontal stabilizer and the vertical stabilizer as given in the question.

The question does not deal with the calculation of the various drag forces and other parameters of the other components of the helicopter, but only considers the main factors given in the question and in the appendices.

2. Assume that the propeller blades of a coaxial rotor have a regular shape and uniform mass distribution.

The shape and mass distribution of the rotor blades can affect the speed of the rotor tip and this assumption can simplify the model calculations.

3. Assume that we do not consider the effect of weather factors on helicopter flight.

Weather is a complex factor and this question is a theoretical analysis of the ideal state, so this factor is ignored.

4. Model

4.1 Model of Question One

4.1.1 Data Preparation

Directly useable parameters:

From the annex and the question 1, directly useable parameters can be extracted as shown in Table 1 below:

Table 1 Directly useable parameters used in the question one

Parameters	I_2	u_c	u_e	u_{cd}	S_c	V_o
Value	20000	0	-3.4817	-2.1552	36π	180
Unit	kilogram-meter-square	degree	degree	degree	meter-square	meter per second

where I_2 represents pitch axis moment of inertia, S_c represents dotor disc area, V_o represents blade centre-of-mass linear velocity,coaxial rotor overall distance is u_c ,coaxial rotor differential total distance is $u_c d$ and coaxial rotor longitudinal cycle pitch is u_e .

Parameters needed to be solved:

1.Rotor Advance Ratio

The formula for the Rotor Advance Ratio: **Rotor Advance Ratio = Flight Speed / Rotor Blade Tip Speed** can be found in the Appendix: Rotor moment factor.

The **flight speed** v can be calculated from the speed synthesis formula:

$$v = \sqrt{v_f^2 + v_v^2} \tag{1}$$

where v_f is the forward component 80 m/s, v_v is the vertical ascent component 2 m/s, and by substituting the data, the flight speed v is approximately equal to the flight speed forward component ,$v = 80m/s$.

The **Rotor Blade Tip Speed** $V_c$is related to the Coaxial Rigid Rotor Speed V_o, which can be viewed as the linear velocity of the blade's centre of mass. Assuming that the blade is a cylinder with uniformly distributed mass, the centre of mass is located at the geometric centre of the blade. Then there is the following relationship:

$$R_c = 2R_o \tag{2}$$

where R_c is the distance of the tip from the rotor centre and R_o is the distance of the centre of mass from the rotor centre. According to the formula of linear velocity :

$$\begin{aligned} V_o &= \omega R_o \\ V_c &= \omega R_c \end{aligned} \tag{3}$$

where ω is the angular velocity, we can get $V_c = 2V_o = 360m/s$. Then,

$$RotorAdvanceRatio = \frac{FlightSpeed}{RotorBladeTipSpeed} = \frac{v}{V_c} = 0.22 \tag{4}$$

2.Pitch deviation value,Longitudinal variable pitch coefficient,Total pitch coefficient and Differential total pitch coefficient

Pitch deviation value a_2, Longitudinal variable pitch coefficient b_2, Total pitch coefficient c, and Differential total pitch coefficient d_2, these four parameters are solved in the same way.

For instance, to obtain a_2, we utilize the weighted average method with relevant data from the appendix.The correlation between the Pitch deviation value and rotor advance ratio can be located in the Rotor moment factor appendix,described as $(Pitchdeviationvalue, rotoradvanceratio)$. Two correlations must be utilized: $(0.0029, 0.2)$ and$(0.0042, 0.3)$. The method of weighted averages was implemented to determine a2.And the weights were determined based on the value of the difference in rotor advance ratio.The formula is as follows:

$$a_2 = \frac{0.0029 * (0.22 - 0.2) + 0.0042 * (0.3 - 0.22)}{0.3 - 0.2} = 0.0032 \tag{5}$$

Similarly,

$$b_2 = \frac{0.00045 * (0.22 - 0.2) + 0.0005 * (0.3 - 0.22)}{0.3 - 0.2} = 0.00046 \tag{6}$$

$$c = \frac{0.00034 * (0.22 - 0.2) + 0.00043 * (0.3 - 0.22)}{0.3 - 0.2} = 0.00036 \tag{7}$$

$$d_2 = \frac{-0.000001 * (0.22 - 0.2) - 0.000001 * (0.3 - 0.22)}{0.3 - 0.2} = -0.0000008 \tag{8}$$

3.Air density

By checking the relevant information[3], we can obtain the relationship between air density $\rho$and altitude h:

$$\rho = 1.2097 - 9.799 * 10^{-5} * h \tag{9}$$

where h=3000 meters.we obtain $\rho = 0.9091kg/m^3$.

The above available parameters are summarised in Table 2 below:

Table 2 parameters used in the question one

Parameters	a_2	b_2	c	d_2	V_c	ρ
Value	0.0032	0.00046	0.00036	-8e-7	360m/s	0.9091 kg/m^3

4.1.2 Model

1.Pitch Moment M_2

According to the information given in question A and relevant material[2], the rotor and propeller thruster generate the pitch moment M_2. Because the flight speed at this point, $v = 80m/s$, is relatively low, only the impact of the rotor and propeller thrust is taken into account.Since the Propeller thruster operating capacity $u_t = 0$, the pitch momentM_2 in this question is generated by the rotor only. The pitch moment generated by the coaxial rigid rotors can be approximated as a function proportional to the air densityρ, rotor disc area S_c, and rotor blade tip velocity V_c , the positive scale factor is called the rotor moment factor m_2 with the following equation:

$$M_2 = m_2 \rho S_c V_c \tag{10}$$

where the values of ρ ,S_c,and V_c can be found in Table 1 and Table 2.

The equations given in the appendix for m_2 is Rotor pitch moment coefficient = pitch deviation value + longitudinal variable pitch coefficient * longitudinal cycle pitch + total pitch coefficient * overall distance + differential total pitch coefficient * differential total distance .Express the formula as letters:

$$m_2 = a_2 + b_2 * u_e + c * u_c + d_2 * u_{cd} \tag{11}$$

where the values of a_2 ,b_2, c ,d_2,u_e,u_c and u_{cd} can be found in Table 1 and Table 2. We obtain $m_2 = 0.0016$.

Then,substituting the datas we solve for the pitch moment $M_2 = 58.59J$.

2.Pitch Angle Variation

Using Newton's second law and the angular momentum theorem[4], it is possible to establish a second-order differential relationship between the pitching momentM_2 and the pitch angle θ. The second-order differential equation is:

$$M_2 = I_2 * \frac{d^2\theta}{dt^2} \tag{12}$$

Assuming the initial value of fo the pitch angle is 0 and the initial change rate of the pitch angle is 0, we obtain:

$$\theta(t) = \frac{1}{2}\frac{M_2}{I_2}t^2 = 0.00146t^2 \tag{13}$$

The attitude angles of the flying machine at 5s, 10s, and 20s respectively are:

$$\theta(5) = 0.0366 degree$$

$$\theta(10) = 0.1464 degree$$

$$\theta(20) = 0.5858 degree$$

The positive θ represents a positive pitch moment, which causes the nose of the helicopter to stick up.

The image of the function of $\theta(t)$ is shown below in Figure 2 :

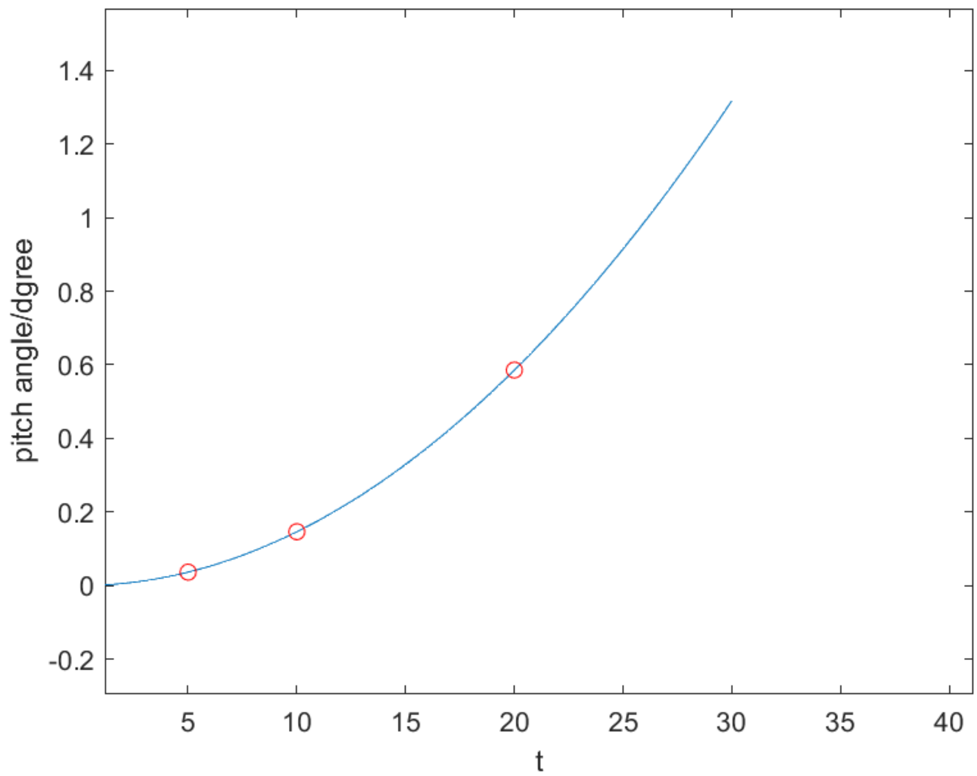

Figure 2 The image of pitch angle $\theta(t)$

From the graph shown in Figure 2, it is evident that the pitch angle θ steadily increases over time, with a corresponding increase in the rate of change. However, this trend is limited in practice as the pitch angle cannot reach infinity in real-world driving situations.

4.2 Model of Question Two

The helicopter remains in the same state as it was in Problem 1 while solving Problem 2. Therefore, the pitch moment and pitch angle remain the same as in Problem 1. The only variables that need to be solved in Problem 2 are the roll moment M_1, roll angle ϕ, yaw moment M_3, and yaw angle Ψ.

4.2.1 Data Preparation

Firstly, solve for roll deflection value a_1,lateral pitch roll factor b_1 ,differential total distance roll coefficient d_1,yaw deviation value a_3 ,and differential total pitch yaw coefficient d_3 . The solution is identical to that of a_2 in Problem 1, merely replace the relevant data in the formula for a_2 with the appropriate datas from the Annex.Their equation is as follows:

$$a_1 = \frac{0.0004 * (0.22 - 0.2) + 0.0004 * (0.3 - 0.22)}{0.3 - 0.2} = 0.0004 \tag{14}$$

$$b_1 = \frac{0.0004 * (0.22 - 0.2) + 0.00044 * (0.3 - 0.22)}{0.3 - 0.2} = 0.000409 \tag{15}$$

$$d_1 = \frac{-0.00024 * (0.22 - 0.2) - 0.00025 * (0.3 - 0.22)}{0.3 - 0.2} = -0.000242 \tag{16}$$

$$a_3 = \frac{0.0001 * (0.22 - 0.2) + 0.0001 * (0.3 - 0.22)}{0.3 - 0.2} = 0.0001 \tag{17}$$

$$d_3 = \frac{0.00008 * (0.22 - 0.2) + 0.00008 * (0.3 - 0.22)}{0.3 - 0.2} = 0.00008 \tag{18}$$

The above available parameters are summarised in Table 3 below:

Table 3 parameters used in the question two

Parameters	a_1	b_1	d_1	a_3	d_3	u_a	I_1	I_3
Value	0.0004	0.000409	0.000242	0.0001	0.00008	-2.70743	8000	25000

where u_a is coaxial rotor lateral cycle pitch, I_1 is Roll Axis Moment of Inertia and I_2 is Yaw Axis Moment of Inertia .

4.2.2 Roll Moment M_1 and Rll Angle ϕ

Consistent with Problem 1, at this point, the helicopter is at low speed and only the effects of the rotor and propeller thrusters are considered.Since the Propeller thruster operating capacity $u_t = 0$, the pitch momentM_2 in this question is generated by the rotor only. The pitch moment generated by the coaxial rigid rotors can be approximated as a function proportional to the air densityρ, rotor disc area S_c, and rotor blade tip velocity V_c , the positive scale factor is called the rotor moment factor m_1 with the following equation:

$$M_1 = m_1 \rho S_c V_c \tag{19}$$

where m_1 is rotor roll moment coefficient = roll deviation value + lateral pitch roll factor *lateral cycle pitch + differential total distance roll coefficient * differential total distance .Express the dormula as letters:

$$m_1 = a_1 + b_1 * u_a + d_1 * u_{cd} \tag{20}$$

Then,substituting the datas we solve for the **roll moment** $M_1 = -35.91J$ **.**

Then,using Newton's second law and the angular momentum theorem, it is possible to establish a second-order differential relationship between the roll momentM_1 and the roll angle ϕ. The second-order differential equation is:

$$M_1 = I_1 * \frac{d^2\phi}{dt^2} \tag{21}$$

Assuming the initial value of fo the roll angle is 0 and the initial change rate of the pitch angle is 0, we obtain:

$$\phi(t) = \frac{1}{2}\frac{M_1}{I_1}t^2 = 0.00449t^2 \tag{22}$$

The attitude angles of the flying machine at 5s, 10s, and 20s respectively are:

$$\phi(5) = -0.0561 degree$$

$$\phi(10) = -0.2245 degree$$

$$\phi(20) = -0.8979 degree$$

The negative ϕ represents a negative roll moment, which causes the helicopter rotates clockwise looking back from the nose of the helicopter.

The image of the function of $\theta(t)$ is shown below in Figure 3 :

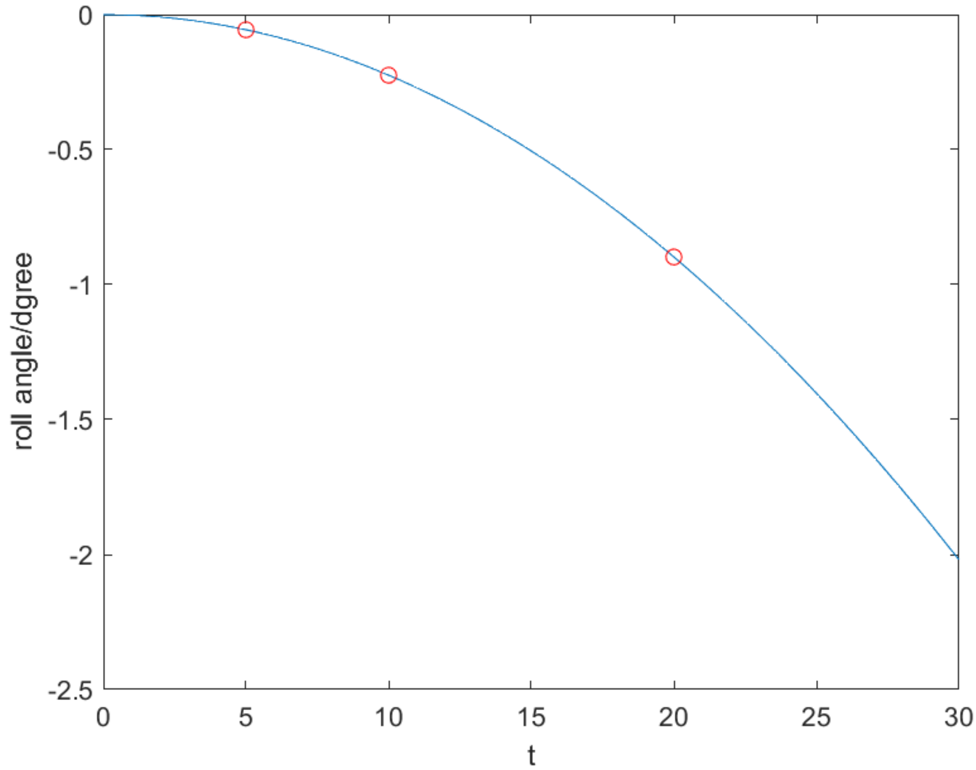

Figure 3 The image of roll angle $\phi\,(t)$

From the graph shown in Figure 3, it is evident that the roll angle ϕ steadily declines over time, with a increase in the rate of change.

4.2.3 Yaw Moment M_3 and Yaw Angle Ψ

Solving for the yaw moment and yaw angle is identical to solving for the pitch moment and pitch angle. By combining the information provided in the annex and Problem A, we directly list and solve the following equations:

$$\begin{cases} M_3 = m_3 \rho S_c V_c \\ m_3 = a_3 + d_3 * u_{cd} \\ M_3 = I_3 * \dfrac{d^2\psi}{dt^2} \end{cases} \tag{23}$$

where M_3 is yaw moment and yaw angle Ψ We obtain:

$$M_3 = 10.084J$$

$$\psi\,(t) = 0.0002 * t^2$$

$$\psi\,(5) = 0.00504degree$$

$$\psi\,(10) = 0.02017degree$$

$$\psi\ (20) = 0.08067degree$$

The image of the function of $\psi\ (t)$ is shown below in Figure 4 :

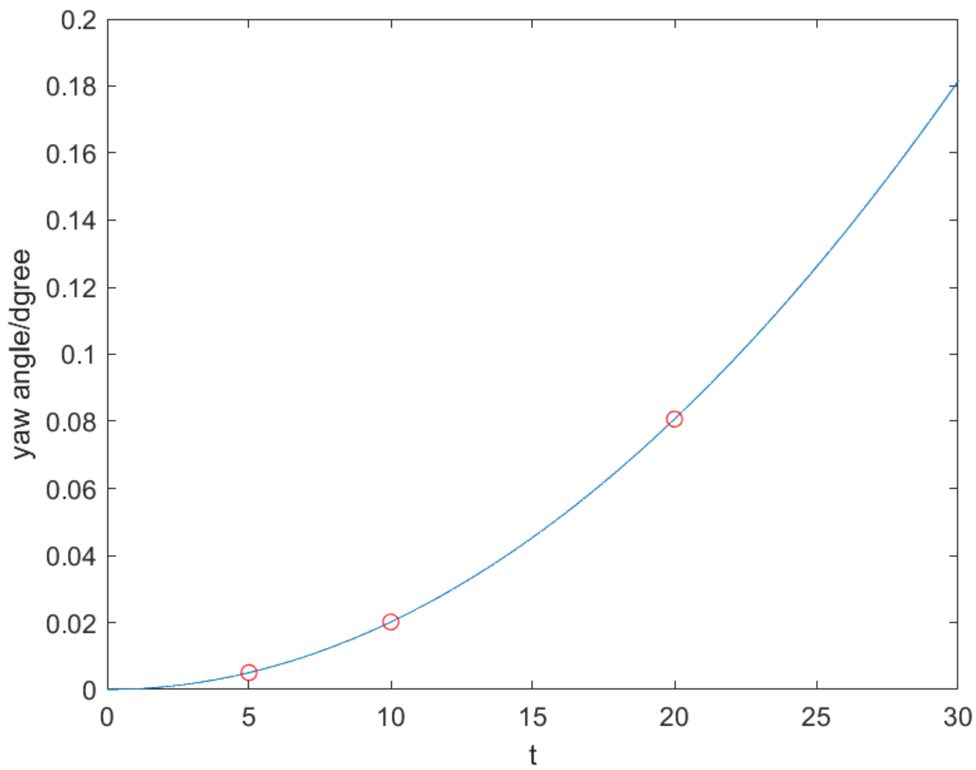

Figure 4 The image of yaw angle $\phi\ (t)$

From the graph shown in Figure 4, it is evident that the yaw angle ψ steadily increases over time, with a increase in the rate of change.

4.3 Model of Question Three

4.3.1 Establishment of Moment equations

First of all, the moments are analyzed when the helicopter is flying **at low speeds**.

According to the information on Problem A and relevant material[5], we know that the attitude angle control of the aircraft in low-speed flight is mainly realized by the coaxial rotor and the propeller thruster. Based on Problem 1 and Problem 2, we can just add the moment component brought by the propeller thruster to the original moment equation system.

It is known that the propeller thruster generates thrust and rotational moment by rotation. The thrust$F$ is parallel to the longitudinal axis where the center of mass of the helicopter is located. The longitudinal distance from the propeller to the center of mass is$D$, which generates the pitch moment component m_{p2}, calculated as:

$$m_{p2} = F \times D \tag{24}$$

The direction is opposite to the pitch moment component generated by the rotor blades.
The graphic description is shown below Figure 4:

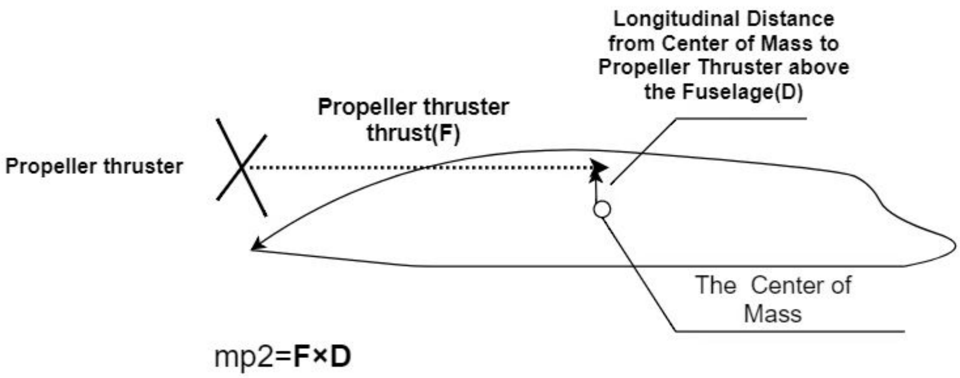

Figure 5 The image of thrust $\phi\ (t)$

In addition, the rotational torque m_{p1} generated by the propeller is part of the roll moment, and its direction is opposite to the pitch moment component generated by the rotor.

The moment equations as follows:

$$\begin{cases} M_1 = m_1 * \rho * S_c * V_c - m_{p1} \\ M_2 = m_2 * \rho * S_c * V_c - m_{p2} \\ M_3 = m_3 * \rho * S_c * V_c \end{cases} \tag{25}$$

The following is an analysis of the helicopter in **high speed** flight (forward speed component of 180m/s, vertical climb component of 0.2m/s).

From the information in Problem A, we know that the control of the attitude angle of the aircraft in low speed flight is mainly achieved by the propeller, elevator and rudder. The analysis of the pitch and roll moments generated by the propeller is the same as above.

The pitching moment component generated by the horizontal tail rudder is:

$$M_h = m_h * P * S_h * L_h \tag{26}$$

$$m_h = T_1 + e_1 * u_{eh} \tag{27}$$

$$P = \frac{1}{2} * \rho * V^2 \tag{28}$$

The roll moment component generated by the vertical tail rudder is:

$$M_{v1} = m_v * P * S_v * L_{vo} \tag{29}$$

$$m_v = T_2 + e_2 * u_{av} \tag{30}$$

The yaw moment generated by the vertical tail rudder is:

$$M_{v2} = m_v * P * S_v * L_{va} \tag{31}$$

where M_h is the pitching moment component generated by the elevator and rudder,
M_{v1} and M_{v2} are the roll moment and yaw moment generated by the rudder,
m_h and m_v are the horizontal tail moment coefficient and the vertical tail moment coefficient respectively,
P is the dynamic pressure,
S_h and S_v are the horizontal tail area and the vertical tail area respectively,
L_{vo} and L_{va} are the longitudinal distance from the centre of gravity to the tail and the transverse distance from the centre of gravity to the tail,
T_1 and T_2 are the horizontal tail moment coefficient and the vertical tail moment coefficient respectively,
L_{vo} and L_{va} are the longitudinal distance from the centre of gravity to the tail and the transverse distance from the centre of gravity to the tail, respectively,
e_1 and e_2 are the elevator and rudder coefficients, respectively,
L_h is transverse distance from the center of mass to the horizontal tailplane.

The system of moment equations for the helicopter at high speed after collation is as follows (the equations take the direction of the moment generated by the propeller as positive):

$$\begin{cases} M_1 = m_{p1} + M_{v1} \\ M_2 = m_{p2} + M_h \\ M_3 = M_{v2} \end{cases} \tag{32}$$

4.3.2 Transformation of the Target Problem

Since the helicopter components have a certain manoeuvre range, this means that if we directly set the three moment equations in the system of equations equal to zero, there will be an infinite number of solutions, which cannot be calculated directly by Matlab. In this case, we convert the original manoeuvre range solution problem into a singleobjective optimisation model, and repeat the optimisation process several times according to the idea of the Monte Carlo algorithm, and use the improved genetic algorithm to deal with the singleobjective optimisation model in this problem.

4.3.3 Determination of the Objective Function and its Constraints

The initial solution is to make all three moment equations in the system of moment equations equal to 0 at the same time, in the optimisation model we take the maximum value of the inverse of the sum of the absolute values of the three moments as the objective function, i.e:

$$max\{-(|M_1| + |M_2| + |M_3|)\} \tag{33}$$

where the constraints of the objective function are the initial ranges of each component of the helicopter in the Appendix.

4.3.4 Improvement of Genetic Algorithms

Genetic algorithm (GA) is an intelligent technique for solving practical problems by modelling biological evolutionary processes and mechanisms. It is also known as a heuristic algorithm, a search heuristic algorithm used to solve optimisation problems in the field of artificial intelligence in computer science, and is a type of evolutionary algorithm. Genetic algorithms use heuristics to generate useful solutions to optimisation and search problems. Genetic algorithms were originally based on a number of phenomena in evolutionary biology, such as heredity, mutation, natural selection and hybridisation. A basic genetic algorithm (also known as a standard genetic algorithm or a simple genetic algorithm) is a population-based operation that targets all individuals in a population and uses only the basic genetic operators: selection, crossover, and mutation. This evolutionary genetic operation process is simple and easy to understand, and is the basis for some other genetic algorithms, which not only provides a basic framework for various genetic algorithms, but also has some application value. In genetic algorithms, selection, crossover and mutation are the three main operators that make up the genetic operation and give the genetic algorithm some features not found in other methods. The representation of the genetic algorithm is as follows:

$$SGA = (C, E, P_0, M, \phi, \psi, T) \tag{34}$$

1.Chromosome coding

In genetic algorithms, the solutions in the solution space are described by a representation. This representation maps the solution to the problem onto the genotype, a process known as encoding. Encoding is the method of converting the feasible solution of the problem from the solution space to the search space of the genetic algorithm. Before performing the search, the genetic algorithm represents the solution in the solution space as the genetic algorithm's genotype string (also known as a chromosome) structured data, where different combinations of genotype strings form different points. Common coding methods include binary coding, gray code coding, floating point coding, cascade coding for each parameter, and multi-parameter cross-coding. These coding methods are chosen according to the characteristics and requirements of the specific problem, so that they can effectively represent the solution space of the problem, and can be manipulated and optimised during the search process of the genetic algorithm. By appropriately selecting and designing the coding methods, the quality of the search efficiency and solution of the genetic algorithm can be improved.

Let the value range of a parameter is $[U_1, U_2]$.we use the binary coding notation of the length as to represent the parameter, then it produces a total of 2k different types of coding, which can make the parameter coding when the correspondence:

$$\begin{aligned} 000000\cdots0000 &= 0 \rightarrow U_1 \\ 000000\cdots0001 &= 1 \rightarrow U_1+\delta \\ 000000\cdots0010 &= 2 \rightarrow U_1+2\delta \end{aligned} \tag{35}$$

where

$$\delta = \frac{U_2-U_1}{2^k-1}$$

2.Chromosome decoding

Genetic algorithm chromosome to solve the problem. Given the coding of a given individual, the corresponding decoding equation is:

$$X = U_1 + \left(\sum_{i=1}^{k} b_i \cdot 2^{i-1}\right) \cdot \frac{U_2-U_1}{2^k-1}$$

3.Initial stock formation

Set the maximum number of evolutionary generations T, population size M, crossover probability P_c, variation probability P_m, and randomly generate M individuals as the initialisation population P_0.

4. Evaluating and determining the fitness value

The fitness function indicates the superiority or inferiority of an individual or a solution. For different problems, the fitness function is defined in different ways. According to the specific problem, the fitness of each individual in the population $P(t)$ is calculated.

Since the traditional genetic algorithm has the disadvantages of slow convergence speed and easy to fall into local optimum, in order to improve the efficiency, we made the following improvements to the traditional genetic algorithm[1] model:

(1) Retention of elite individuals In each iteration, the optimal individuals of the previous generation are directly retained in the next generation to prevent the optimal solution from being most damaged.

(2) Adaptive scaling By linearly scaling the fitness, the fitness value is mapped to a more suitable range of the selection algorithm, thus speeding up the convergence process. Let the original fitness be F , then the fitness function after linear scaling is:

$$F' = A * F + B \tag{36}$$

where A and B are the scaling coefficients.

The general flow of the algorithm for this problem is shown below:

Step1: Initialisation. Set the evolution iterator epoch=M, set the maximum number of evolutionary generations G, and take the freight volume change line on the day of the first normal operation as the initial population. and will later be screened, mutated and evolved on the basis of this population.

Step 2: Evaluation of the first generation. Calculate the sum of the absolute values of the three helicopter moments in the first generation as the degree of adaptation of the first generation population.

Step3: Selection operation. According to the number of sums of the absolute values of the moments of several helicopters at the time of the first generation, the first generation population is divided into good individuals and bad individuals, where good individuals, i.e. the sums of the absolute values of the forces, are closer to zero and bad individuals, i.e. the sums of the absolute values of the moments, are further away from zero.

Step 4: Crossover operator. The crossover operator is applied to the target group for site selection, and the inferior individuals will operate with the superior individuals, which can be close to the superior individuals to some extent after the operation.

Step5: Variation operation. The variation operator will be applied to the optimisation target group, the inferior individuals will have a higher probability of generating variation, where the smaller probability is left to prevent entering the local optimal solution, and the superior individuals will have a smaller probability of generating variation. Both groups will produce mutation, the difference in addition to the probability is not the same, the inferior individuals before and after the mutation of the relatively large difference, while the excellent individuals will be less change.

Step6: Cyclic operation. The target population of helicopter component parameters will get the next generation population after selection, crossover and mutation operations, the fitness value of this target population of helicopter component parameters will be calculated and compared with the fitness value of the target population of helicopter component parameters of the previous generation. If it is better, the optimal result is replaced; if it is worse, it is prepared for the next genetic operation.

Step7: Judgement of the termination condition. If the number of iterations has reached the target number of iterations, the computation can be terminated and the algorithm can be exited, otherwise it will continue to step3. Among them, the fitness function and the selection method are the most important parts of the genetic algorithm. The fitness function is used to measure the fitness of individuals in order to select better individuals for crossover and mutation. The selection method selects individuals based on their fitness values, so that individuals with higher fitness have a higher probability of being selected, thus achieving the purpose of evolutionary optimisation, and the flowchart of the whole algorithm is shown in the following Figure 6:

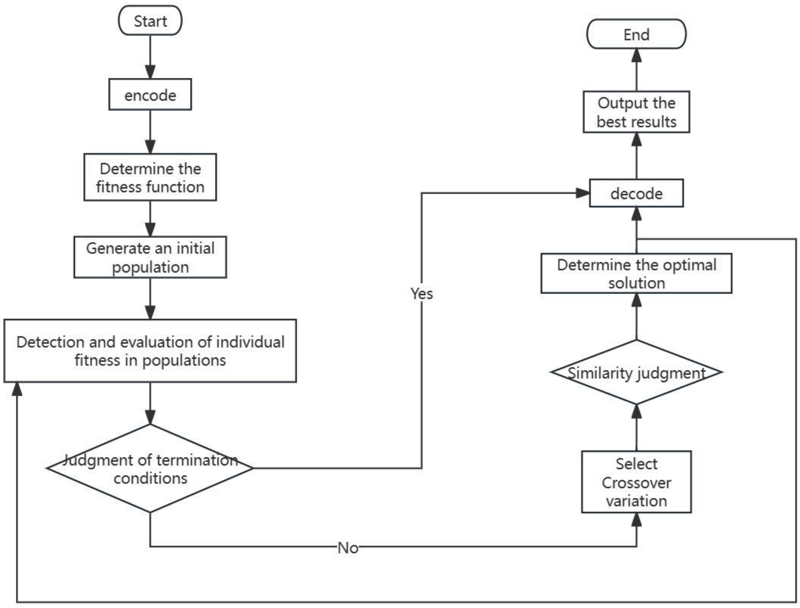

Figure 6 Schematic diagram of the genetic algorithm process

4.3.5 Application of Monte Carlo thinking

Since the system of moment equations has an infinite number of solutions, with a large enough number of iterations, the value of the objective function can definitely be brought to zero, and there is an infinite number of corresponding cases. So the optimisation process is repeated many times to aggregate the different cases, based on the idea of Monte Carlo. As long as the number of iterations is large enough, there is a high probability that the boundary value that satisfies the condition can be taken. As long as we compare the values of each helicopter component in each optimisation result, superimposing the update of the maximum and minimum values, we can finally obtain the manoeuvring range of the helicopter components.

4.3.6 Result

At **low speeds** only the role of the rotor and propeller need be considered, hence only coaxial rotor overall distance u_c , coaxial rotor longitudinal cycle pitch u_e , coaxial rotor lateral cycle pitch u_a , propeller thruster operating capacity u_t and coaxial rotor differential total distance u_{cd}. The iteration process of the genetic algorithm at low speed is shown in Figure 7:

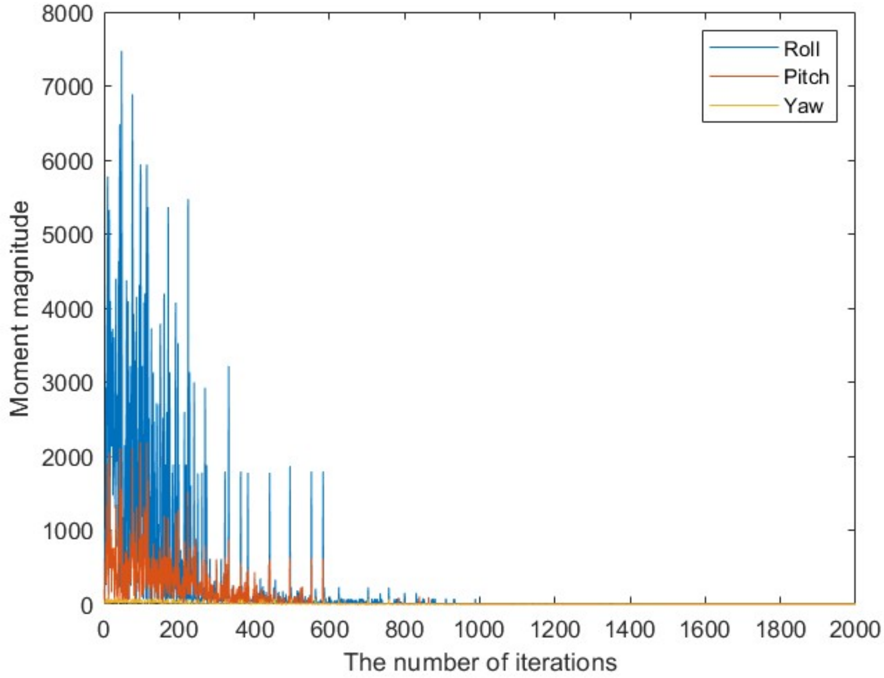

Figure 7 The iteration process of the genetic algorithm at low speed

The results are as follows in Table 4:

Table 4 parameters Range at low speeds

Parameters	u_c	u_e	u_a	u_t	u_{cd}
min	0.0001	-24.0658	-6.3786	0.0001	-20.7936
max	24.0123	0	24.8298	6.9759	19.0643
Most appropriate value	0.0001	-24.0658	-6.3786	0.0001	-20.7936

At **high speeds**, only the role of the propeller and the horizontal and vertical tail need be considered, hence propeller thruster operating capacity u_t , elevator deflection values u_{eh} , rudder deflection values u_{av} .The iteration process of the genetic algorithm at low speed is shown in Figure 8:

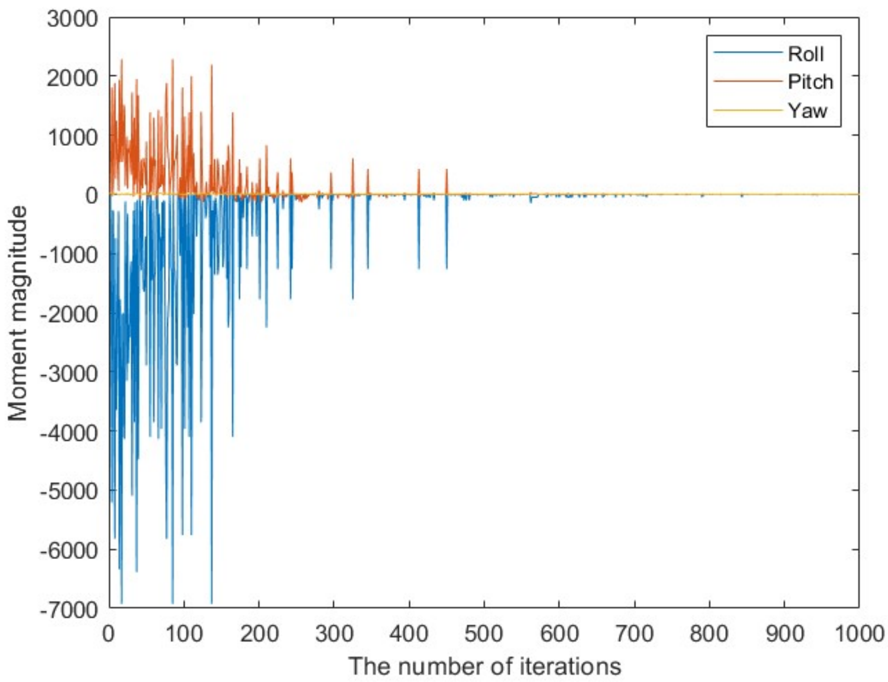

Figure 8 The iteration process of the genetic algorithm at high speed

The results are as follows in Table 5:

Table 5 parameters Range at high speeds

Parameters	u_t	u_{eh}	u_{av}
min	0	-0.7771	-8.6277
max	0.0045	0	0
Most appropriate value	0	-0.7962	-7.1429

4.4 Model of Question Four

4.4.1 Establishment of moment equations in the medium speed stage

According to the given information, the equation for the horizontal speed of the helicopter in relation to time is:

$$V = v_0 + a * t(0 <= t <= 20) \tag{37}$$

Where v_0 is the initial horizontal speed of the helicopter, which is 80m/s, a is the horizontal acceleration of the helicopter, which is $5m/s^2$.

The system of moment equations for the helicopter at low and high speeds is given in Problem 3 and will not be repeated here.

Considering that the **medium speed** phase of the helicopter is equivalent to the transition phase from low speed to high speed[6], we take into account the helicopter rotor blades, propellers and horizontal and vertical tail blades when calculating the moments. The system of moment equations is as follows (the moment components in the system of moment equations all contain directions):

$$\begin{cases} M_1 = m_1 * \rho * S_c * V_c + m_{p1} + M_{v1} \\ M_2 = m_2 * \rho * S_c * V_c + m_{p2} + M_h \\ M_3 = m_3 * \rho * S_c * V_c + M_{v2} \end{cases} \tag{38}$$

where $m_1 * \rho * S_c * S_c$,$m_2 * \rho * S_c * V_c$ and $m_3 * \rho * S_c * V_c$ are the roll moment component, pitch moment component and yaw moment component generated by the rotor blade respectively; m_{p1} and m_{p2} are the torque and pitch moment components generated by the propeller respectively; M_h is the pitch moment component generated by the horizontal tail; Mv1 and M_{v2} are the roll moment component and yaw moment component generated by the vertical tail respectively.

4.4.2 Maneuvering Range results for each component over time

Applying the genetic algorithm optimisation model of the third question in each of the three speed cases with time as the independent variable, the results of the manoeuvring ranges of the individual components of the helicopter as a function of time can be found in the Excel: Problem 4 Results table in the Supporting Materials document we submitted and **our appendix**.The result is graphically represented in the Figure 9 below:

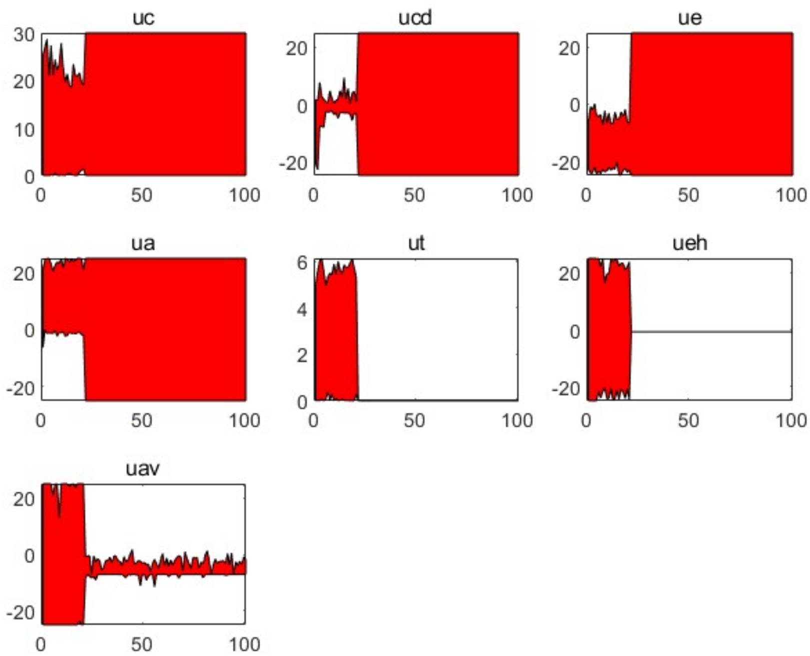

Figure 9 Maneuvering Range

4.4.3 Correlation Analysis and Model Improvement

The above is just to get the manoeuvre magnitude of each component of the helicopter at different flight moments. To make the helicopter's input parameters as continuous as possible over time, we further optimise the genetic algorithm of the third question. The single-objective optimisation problem is improved to a multi-objective optimisation problem. The size of the Euclidean distance of the optimisation results at adjacent times is used as an indicator of continuity. Assuming that Pi is the optimal result vector of the genetic algorithm at the ith instant, the modified objective function is:

$$\begin{cases} min\left\{|M_1| + |M_2| + |M_3|\right\} \\ min\left\{|P_i + 1 - P_i|^2\right\} \end{cases} \tag{39}$$

Although the model is strongly influenced by the input parameters at the initial moment, in practical situations the input parameters are usually determined at the initial moment of the helicopter. Therefore, the model has some feasibility. The results of the dynamic values of the helicopter components over time that we obtained after taking one case of the optimisation result of the genetic algorithm at the initial moment as the initial parameter can be found in the Excel: Problem 4 Results table in the Supporting Materials document we submitted and **our appendix.**

5. Strengths and Weaknesses

5.1 Strengths

1. The modelling is from specific to general, from simple to complex, which can realise the solution under specific conditions and is also applicable to general situations. 2. A combination of improved genetic algorithm and Monte Carlo simulation is used to make the solution results more robust.

5.2 Weaknesses

1. The influence of other factors such as the damping torque on the helicopter's attitude angle is not considered.

2. There is some subjectivity in the determination of the initial conditions when solving the second order differential equation for the attitude angle.

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Appendix

5.3 the result of problem 4

Horizontal speed(m/s)	uc	ucd	ue	ua
80	(0.005,23.294)	(-13.157,4.926)	(-24.546,-0.125)	(-0.147,23.693)
81	(0.001,22.709)	(-6.614,2.254)	(-24.616,-2.963)	(-0.947,24.883)
82	(0.001,24.416)	(-12.663,1.995)	(-24.253,-6.615)	(-1.585,17.631)
83	(0.135,23.361)	(-16.162,19.204)	(-23.412,0.663)	(-1.167,24.072)
84	(0.000,25.068)	(-6.769,8.339)	(-23.006,-5.193)	(-1.570,23.540)
85	(0.206,19.757)	(-2.743,3.143)	(-22.640,-5.189)	(-1.966,21.858)
86	(0.000,28.651)	(-8.129,3.605)	(-22.758,-6.900)	(-1.347,25.000)
87	(0.043,25.136)	(-2.818,1.633)	(-22.866,-6.583)	(-0.790,20.884)
88	(1.378,22.068)	(-2.801,3.495)	(-22.552,-7.614)	(-1.074,19.330)
89	(1.000,26.716)	(-2.868,2.722)	(-23.884,-6.183)	(-1.736,24.014)
90	(0.473,23.160)	(-2.918,-0.164)	(-23.793,-1.536)	(-1.121,24.893)
91	(0.003,20.958)	(-2.960,1.291)	(-22.634,-6.646)	(-1.049,23.149)
92	(0.030,24.626)	(-3.002,2.110)	(-23.030,-6.259)	(-0.263,22.428)
93	(0.502,25.377)	(-5.226,0.208)	(-23.139,-6.505)	(-0.534,25.000)
94	(0.911,24.456)	(-3.081,1.223)	(-24.712,-5.728)	(-1.046,24.906)
95	(0.237,22.077)	(-6.050,1.474)	(-23.965,-3.782)	(-1.132,25.000)
96	(0.016,20.999)	(-2.975,2.349)	(-24.071,-6.667)	(-2.330,24.445)
97	(0.000,22.758)	(-4.154,1.765)	(-24.569,-2.914)	(-0.472,25.000)
98	(0.016,20.174)	(-3.240,1.450)	(-24.388,-6.215)	(-1.548,22.026)
99	(0.001,18.938)	(-3.247,0.256)	(-21.816,-7.726)	(-0.712,18.262)
100	(0.113,20.019)	(-3.322,3.036)	(-23.564,-5.555)	(-1.466,22.586)
101	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
102	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
103	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
104	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
105	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
106	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
107	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
108	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
109	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
110	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
111	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)

112	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
113	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
114	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
115	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
116	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
117	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
118	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
119	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
120	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
121	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
122	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
123	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
124	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
125	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
126	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
127	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
128	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
129	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
130	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
131	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
132	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
133	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
134	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
135	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
136	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
137	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
138	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
139	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
140	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
141	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
142	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
143	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
144	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
145	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
146	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
147	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
148	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)

149	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
150	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
151	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
152	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
153	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
154	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
155	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
156	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
157	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
158	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
159	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
160	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
161	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
162	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
163	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
164	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
165	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
166	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
167	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
168	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
169	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
170	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
171	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
172	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
173	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
174	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
175	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
176	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
177	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
178	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
179	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)
180	(0.000,30.000)	(-25.000,25.000)	(-25.000,25.000)	(-25.000,25.000)

Horizontal speed(m/s)	ut	ueh	uav
80	(0.000,6.331)	(-25.000,25.000)	(-25.000,25.000)
81	(0.000,6.014)	(-25.000,25.000)	(-25.000,25.000)
82	(0.004,4.784)	(-25.000,25.000)	(-25.000,25.000)

83	(0.004,6.393)	(-25.000,25.000)	(-25.000,25.000)
84	(0.150,5.472)	(-25.000,25.000)	(-25.000,25.000)
85	(0.000,5.210)	(-24.067,20.659)	(-25.000,24.919)
86	(0.250,6.182)	(-24.972,14.811)	(-25.000,25.000)
87	(0.254,5.265)	(-23.895,19.005)	(-25.000,24.990)
88	(0.100,5.085)	(-24.983,17.056)	(-25.000,23.923)
89	(0.004,5.609)	(-17.823,24.801)	(-25.000,24.542)
90	(0.126,5.804)	(-21.129,22.763)	(-25.000,24.795)
91	(0.085,5.456)	(-24.808,23.382)	(-25.000,24.882)
92	(0.070,5.535)	(-17.736,19.550)	(-25.000,24.971)
93	(0.257,6.033)	(-24.801,18.511)	(-25.000,25.000)
94	(0.139,5.847)	(-20.994,24.297)	(-25.000,25.000)
95	(0.122,6.110)	(-24.421,24.837)	(-25.000,25.000)
96	(0.063,5.624)	(-24.555,22.893)	(-25.000,21.886)
97	(0.188,5.989)	(-20.835,23.901)	(-25.000,25.000)
98	(0.016,5.415)	(-22.297,23.033)	(-25.000,25.000)
99	(0.117,4.997)	(-23.203,24.990)	(-25.000,24.458)
100	(0.004,5.623)	(-24.864,18.734)	(-25.000,25.000)
101	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-7.124)
102	(0.000,0.001)	(-0.772,-0.769)	(-9.120,-2.186)
103	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-1.794)
104	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-3.279)
105	(0.000,0.000)	(-0.769,-0.769)	(-7.143,0.606)
106	(0.000,0.001)	(-0.772,-0.769)	(-8.573,-4.113)
107	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.492)
108	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-5.200)
109	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-1.748)
110	(0.000,0.001)	(-0.772,-0.769)	(-8.860,-1.917)
111	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-2.699)
112	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.700)
113	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-1.237)
114	(0.000,0.000)	(-0.770,-0.769)	(-7.540,-1.500)
115	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-5.126)
116	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-1.050)
117	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-2.435)
118	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-4.432)
119	(0.000,0.001)	(-0.772,-0.769)	(-8.825,-2.927)

120	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-7.143)
121	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-3.694)
122	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.962)
123	(0.000,0.000)	(-0.771,-0.769)	(-8.201,-0.547)
124	(0.000,0.000)	(-0.770,-0.769)	(-7.718,-3.484)
125	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-2.333)
126	(0.000,0.000)	(-0.771,-0.769)	(-7.144,-3.103)
127	(0.000,0.000)	(-0.770,-0.769)	(-7.727,-6.170)
128	(0.000,0.001)	(-0.772,-0.769)	(-9.041,-2.249)
129	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-4.748)
130	(0.000,0.000)	(-0.770,-0.769)	(-7.161,-2.652)
131	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-1.035)
132	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.725)
133	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-4.425)
134	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.069)
135	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-3.670)
136	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-5.937)
137	(0.000,0.000)	(-0.771,-0.769)	(-8.048,3.141)
138	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-6.296)
139	(0.000,0.000)	(-0.770,-0.769)	(-7.143,-3.247)
140	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.177)
141	(0.000,0.000)	(-0.770,-0.769)	(-7.143,0.041)
142	(0.000,0.000)	(-0.771,-0.769)	(-7.396,-1.378)
143	(0.000,0.000)	(-0.769,-0.768)	(-7.143,-1.003)
144	(0.000,0.000)	(-0.771,-0.769)	(-7.143,-4.858)
145	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.927)
146	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-3.977)
147	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.614)
148	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.447)
149	(0.000,0.001)	(-0.771,-0.769)	(-8.494,-0.197)
150	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.090)
151	(0.000,0.000)	(-0.769,-0.769)	(-7.143,0.081)
152	(0.000,0.000)	(-0.770,-0.769)	(-7.368,-2.184)
153	(0.000,0.003)	(-0.775,-0.769)	(-11.376,-6.329)
154	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.182)
155	(0.000,0.000)	(-0.770,-0.769)	(-7.226,-2.738)
156	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.617)

157	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-7.004)
158	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.274)
159	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-4.519)
160	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.813)
161	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.624)
162	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-3.715)
163	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.959)
164	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-6.292)
165	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.868)
166	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.463)
167	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.547)
168	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.235)
169	(0.000,0.000)	(-0.769,-0.769)	(-7.155,-1.410)
170	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.474)
171	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-2.964)
172	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.564)
173	(0.000,0.000)	(-0.769,-0.769)	(-7.143,0.992)
174	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.184)
175	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-3.007)
176	(0.000,0.002)	(-0.773,-0.769)	(-9.680,-1.307)
177	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-0.341)
178	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-1.945)
179	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-4.493)
180	(0.000,0.000)	(-0.769,-0.769)	(-7.143,-4.905)

5.4 the most Appropriate value of problem

Horizontal speed(m/s)	uc	ucd	ue
80	2.984999	-1.24962	-8.66167
81	3.576278	-1.24627	-9.40651
82	3.951469	0.233645	-9.75637
83	4.714198	0.294419	-10.6417
84	1.653987	-0.05894	-8.3456
85	1.203584	1.283763	-7.79517
86	2.501465	-0.26436	-9.41632
87	0.000186	-0.65803	-7.28085
88	2.563502	-0.33343	-9.2329

89	0.753174	-0.60739	-7.96066
90	4.022193	-0.9168	-10.3068
91	3.614001	-0.91665	-10.2157
92	7.400234	-0.89781	-13.9299
93	9.718995	-0.79182	-15.243
94	10.10754	-0.28699	-15.6394
95	8.564949	1.208105	-14.6101
96	9.007368	0.083906	-15.5435
97	12.1931	-0.27035	-18.4916
98	7.289541	-0.45638	-14.2496
99	9.854195	-0.95522	-15.9332
100	15.72853	-0.29057	-20.4391
101	0	0	0
102	0	0	0
103	0	0	0
104	0	0	0
105	0	0	0
106	0	0	0
107	0	0	0
108	0	0	0
109	0	0	0
110	0	0	0
111	0	0	0
112	0	0	0
113	0	0	0
114	0	0	0
115	0	0	0
116	0	0	0
117	0	0	0
118	0	0	0
119	0	0	0
120	0	0	0
121	0	0	0
122	0	0	0
123	0	0	0
124	0	0	0
125	0	0	0

126	0	0	0
127	0	0	0
128	0	0	0
129	0	0	0
130	0	0	0
131	0	0	0
132	0	0	0
133	0	0	0
134	0	0	0
135	0	0	0
136	0	0	0
137	0	0	0
138	0	0	0
139	0	0	0
140	0	0	0
141	0	0	0
142	0	0	0
143	0	0	0
144	0	0	0
145	0	0	0
146	0	0	0
147	0	0	0
148	0	0	0
149	0	0	0
150	0	0	0
151	0	0	0
152	0	0	0
153	0	0	0
154	0	0	0
155	0	0	0
156	0	0	0
157	0	0	0
158	0	0	0
159	0	0	0
160	0	0	0
161	0	0	0
162	0	0	0

163	0	0	0
164	0	0	0
165	0	0	0
166	0	0	0
167	0	0	0
168	0	0	0
169	0	0	0
170	0	0	0
171	0	0	0
172	0	0	0
173	0	0	0
174	0	0	0
175	0	0	0
176	0	0	0
177	0	0	0
178	0	0	0
179	0	0	0
180	0	0	0

Horizontal speed(m/s)	ua	ut	ueh	uav
80	1.58894	1.001176	0	0
81	0.3828	0.636046	0	0
82	1.322088	0.656502	0	0
83	0.151287	0.289314	0	0
84	-0.38869	0.187801	0	0
85	0.64463	0.262383	-20.2268	-25
86	-0.79081	0.100571	6.467197	-24.9999
87	-0.53449	0.250198	-0.28677	-19.2735
88	1.331292	0.768207	7.305321	-25
89	-0.16139	0.356114	0.709426	-19.726
90	1.82727	1.030554	1.850285	-13.5232
91	1.260173	0.855426	5.825732	-13.3865
92	-0.27558	0.373191	21.20216	-13.5968
93	2.108852	1.10113	7.21068	-15.3595
94	2.073687	1	3.685776	-24.0472
95	1.700371	0.611447	0.563988	-24.9999
96	-0.36633	0.164935	12.59406	-24.8774

97	-0.89377	0.062501	20.63067	-23.2921
98	0.460692	0.525923	20.88671	-19.9597
99	3.303808	1.522846	21.34658	-11.8078
100	6.348778	2.375655	19.45384	-22.024
101	0	1.49E-08	-0.76922	-7.14286
102	0	0	-0.76921	-7.14286
103	0	0	-0.76921	-7.14285
104	0	0	-0.76922	-7.14286
105	0	0	-0.76923	-7.14285
106	0	0	-0.76923	-7.14286
107	0	0	-0.76922	-7.14286
108	0	9.86E-07	-0.76923	-7.14286
109	0	0	-0.76923	-7.14286
110	0	0	-0.76923	-7.14286
111	0	0	-0.76923	-7.14285
112	0	1.49E-08	-0.76923	-7.14285
113	0	1.67E-06	-0.76923	-7.14286
114	0	2.28E-06	-0.76922	-7.14285
115	0	0	-0.76923	-7.14286
116	0	0	-0.76922	-7.14285
117	0	0	-0.76923	-7.14285
118	0	5.96E-08	-0.76923	-7.14286
119	0	1.49E-08	-0.76923	-7.14286
120	0	1.49E-08	-0.76922	-7.14286
121	0	1.49E-08	-0.76923	-7.14286
122	0	0	-0.76922	-7.14286
123	0	1.08E-06	-0.76923	-7.14286
124	0	0	-0.76923	-7.14286
125	0	0	-0.76923	-7.14285
126	0	0	-0.76922	-7.14285
127	0	0	-0.76923	-7.14285
128	0	2.02E-06	-0.76923	-7.14285
129	0	1.49E-08	-0.76923	-7.14286
130	0	2.86E-07	-0.76923	-7.14286
131	0	0	-0.76923	-7.14286
132	0	1.49E-08	-0.76923	-7.14286
133	0	0	-0.76923	-7.14285

134	0	1.32E-06	-0.76923	-7.14285
135	0	0	-0.76923	-7.14286
136	0	8.05E-07	-0.76924	-7.14286
137	0	5.96E-08	-0.76923	-7.14286
138	0	1.49E-08	-0.76923	-7.14286
139	0	1.49E-08	-0.76923	-7.14286
140	0	1.49E-08	-0.76923	-7.14286
141	0	5.96E-08	-0.76923	-7.14285
142	0	1.30E-06	-0.76923	-7.14286
143	0	4.05E-06	-0.76923	-7.14286
144	0	0	-0.76923	-7.14286
145	0	0	-0.76923	-7.14285
146	0	0	-0.76923	-7.14285
147	0	1.34E-06	-0.76922	-7.14286
148	0	1.49E-08	-0.76923	-7.14285
149	0	0	-0.76923	-7.14286
150	0	0	-0.76923	-7.14286
151	0	3.13E-07	-0.76923	-7.14286
152	0	0	-0.76923	-7.14285
153	0	0	-0.76923	-7.14286
154	0	0	-0.76922	-7.14286
155	0	0	-0.76923	-7.14286
156	0	2.93E-06	-0.76922	-7.14286
157	0	0	-0.76923	-7.14286
158	0	1.01E-06	-0.76923	-7.14286
159	0	1.49E-08	-0.76923	-7.14286
160	0	0	-0.76923	-7.14286
161	0	0	-0.76922	-7.14285
162	0	1.49E-08	-0.76923	-7.14285
163	0	1.49E-08	-0.76922	-7.14285
164	0	1.06E-06	-0.76922	-7.14286
165	0	0	-0.76923	-7.14286
166	0	6.48E-07	-0.76923	-7.14286
167	0	0	-0.76923	-7.14286
168	0	9.54E-07	-0.76923	-7.14286
169	0	0	-0.76923	-7.14286
170	0	0	-0.76923	-7.14286

171	0	2.38E-07	-0.76923	-7.14286
172	0	1.49E-08	-0.76923	-7.14285
173	0	5.96E-08	-0.76923	-7.14286
174	0	7.89E-07	-0.76923	-7.14286
175	0	1.13E-06	-0.76923	-7.14286
176	0	1.49E-08	-0.76922	-7.14286
177	0	1.49E-08	-0.76923	-7.14285
178	0	1.49E-08	-0.76923	-7.14285
179	0	5.96E-08	-0.76923	-7.14285
180	0	0	-0.76923	-7.14285