Problem Chosen	2023 ShuWei Cup Summary Sheet	Team Control Number
A		2023101419943

Summary

In order to meet the unity of flight ability and flexibility, the research of compound helicopter has been fully developed, and the research of coaxial helicopter has also been paid attention to.The main problem of this paper is how to optimize the control parameters by referring to the correlation coefficient of coaxial helicopter given in the attachment. In this paper, we build a control parameter optimization model under the condition that the control parameter adjustment difficulty changes with the speed to solve this problem.

For problem 1: We analyze the appendix data, calculate the torque coefficient of each component, and take the center of mass as the simplification point, respectively calculate the pitch torque generated by the coaxial rotor, tail and propeller, and solve the differential equation under the condition of absolute stability at the initial time to give the change of pitch Angle with time. The pitch angle of 5s, 10s and 20s is 0.045, 0.189 and 0.721, respectively.

For problem 2: by extending the model of problem 1 in three dimensions, the torque vector generated by coaxial rotor, tail and propeller is obtained, the moment equation about the center of mass is established, and the change of attitude angle with time is given by solving the differential equations under the condition of initial absolute stability. Five seconds, ten seconds, 20 s attitude Angle of the vector, respectively (0.00137,0.0450,0.0000757), (0.00549,0.180,0.000303), (0.0219, 0.721, 0.00121).

For problem 3: we combine the relevant control difficulty and give the "s" control difficulty curve with speed and some control parameter as variables. On the basis of model 1 and Model 2, the control parameter optimization model based on steady state is established with the minimum control parameter transformation difficulty as the main objective and zero torque as the constraint condition. Matrix element analysis and linear optimization are used to solve the problem, and the optimal control parameters under 80m/s are obtained, and the optimal control parameters under 180m/s are obtained.

For question 4: We introduced and took into account the time variable to improve the model we built earlier. Then the control parameter optimization model is obtained in a certain time, and the local optimization algorithm of adaptive step size adjustment is used to solve the model in the discrete time region.The optimal change curves of each control parameter with respect to time are obtained, which are shown and analyzed in the model solution.

Keywords:

Three-degree-of-freedom helicopter; Dynamic model; flight control; local search algorithm; dichotomy

Content

1. Introduction

1.1 Background

Traditional helicopters are vulnerable to shock waves and unstable, so composite helicopters began to develop. With the development of research, various composite helicopters emerge endlessly, which makes the research of specific composite helicopters insufficient. In order to deepen the understanding and research of composite helicopter, coaxial helicopter as its model bears the brunt. However, there are relatively few researches on three-degree-of-freedom coaxial helicopters[1], because coaxial helicopters have many external control parameters and different control difficulties at different speeds. Therefore, it is necessary to build a reasonable control parameter optimization model to achieve the optimization in two directions of stable attitude Angle and less adjustment difficulty. Then the stable flight state under certain flight tasks can be achieved.

1.2 Work

The optimization model of control parameters involves helicopter type parameters, control parameters and state parameters. Therefore, in order to build an optimization strategy that meets the needs of multiple parties, joint optimization from different perspectives should be considered in the case of coupling[2]. Therefore, in order to solve this problem, we need to solve the torque coefficient, solve the torque balance, evaluate the control difficulty of different speeds and a certain control parameter, and finally rationally analyze and design the optimization strategy. From this, the problem we have to solve is:

- How to solve the torque coefficient of each component through a certain flight state and control parameters, and then solve the torque vector of each component.
- How to combine the helicopter state and control parameters, give the control difficulty of a specific control parameter, and then make its control and control of other parameters distinguish, so that the optimal solution is unique.
- How to combine the evaluation function of regulation difficulty to compare the advantages and disadvantages of the two stable states.
- How to optimize the control parameters of a particular state by combining the torque balance and the control range of the control parameters, and extend the optimization to the unstable state.

2. Problem analysis

2.1 Data analysis

The attachment provides the coaxial-rotor torque correlation coefficient at 0.1 step with the propulsive ratio from 0 to 0.3, and the data of propulsive force and torsional moment at the thruster gear from 4 to 36 step with 4 step, as well as more detailed model parameters. Through data sorting and analysis, we find that with the increase of the advance ratio, the growth rate of most coefficients slows down and even tends to a fixed value, so the bounded function can be used to fit the continuous change results. At the same time, for the model parameters, we found that the speed and radius of the coaxial rotor, as well as the relevant coordinates of each component have been determined, which provides great help for solving.

2.2 Analysis of question 1

For problem 1, if the torque expression of a composite helicopter is to be established, according to the aircraft configuration diagram provided in the problem, As well as the table data and formulas in the appendix (namely the formulas in rotor moment factor and Horizontal and vertical moment coefficient), the specific expressions of pitching torque can be listed. Then the pitch Angle of the helicopter in different time is analyzed.

2.3 Analysis of question 2

For problem 2, if the torque expression of a composite helicopter is to be established, according to the aircraft configuration diagram provided in the problem, As well as the table data and formulas in the appendix (namely the formulas in rotor moment factor and Horizontal and vertical moment coefficient), the torque vector expression of propeller, rotor, horizontal tail and vertical tail can be written out. Then the rolling Angle, yaw Angle and pitch Angle of the helicopter produced in different time can be analyzed and calculated.

2.4 Analysis of question 3

For problem 3, the maneuver amplitude of each component should be designed according to the different flight maneuver characteristics of the helicopter at low speed and high speed, so as to meet the level flight task of the aircraft. According to the data, the multi-dimensional scatter-plot is simulated, and the S-shaped curve of the coefficient and velocity is evaluated at low and high speed, so that the floating range of each component can be narrowed, and then the optimal solution can be searched by using the local analysis solution of the established feasible region.

2.5 Analysis of question 4

For question 4, the problem is similar to question 3, with the main difference being that we just need to keep M's modules as small as possible. So, just take the equation and transform it into another objective function, multidimensional objective optimization. Then the model can be solved by using the adaptive local optimization algorithm of adjusting the step size in the discrete time region.Finally, by drawing the optimal change curve of each control parameter with respect to time, the multi-dimensional optimization is realized. Then, by discretizing the time and using the improved local search, the optimal solution of the control parameters changing with time is given.

3. Symbol and Assumptions

3. 1 Symbol Description

Parameter	Meanings
ρ	air density
Φ	roll angle
ψ	yaw angle
θ	pitch angle
$\vec{u}$	Control parameter
$\vec{\sigma}$ $\vec{\beta}$	Deviation parameter
$\vec{\varepsilon}$	*State parameter*
$\vec{\alpha}$	Model parameter
γ	Rotor advance ratio

3.2 Fundamental assumptions

1.Without considering the influence of small deformation of the aircraft material, the aircraft can be regarded as a rigid body.
2.When solving the attitude Angle, the force of the aircraft is balanced, so the simplified result of the force is always the torque, and the center of rotation can be the center of gravity.
3.The total effect of air resistance acts on the center of gravity, so when the center of gravity is the simplified center, the air resistance torque is ignored.
4.When the aircraft is flying at high speed, the additional torque caused by air compression and violent turbulence is ignored, so the torque suffered by the aircraft is only generated by the rotor, tail and propeller of the aircraft.
5.The difficulty of adjusting the tail and rotor is different, so the attitude Angle adjustment has the only solution under the same adjustment effect.

4. Model

4.1 Model of date

$$\vec{u} = (u_c, u_{cd}, u_e, u_a, u_t, u_{eh}, u_{av}) \quad (4-1-1)$$
$$\vec{\sigma} = (\sigma_\phi, \sigma_\theta, \sigma_\Psi, \sigma_V, \sigma_H) \quad (4-1-2)$$
$$\vec{\beta} = (\beta_l, \beta_\phi, \beta_V, \beta_{all}, \beta_\theta, \beta_\Psi, \beta_v, \beta_e) \quad (4-1-3)$$
$$\vec{\varepsilon} = (\rho, u, v) \quad (4-1-4)$$
$$\vec{\alpha} = (S_C, S_H, S_V) \quad (4-1-5)$$

Parameter	Meanings	Parameter	Meanings
u_c	Coaxial rotor overall distance	β_{all}	total pitch coefficient
u_{cd}	Coaxial rotor differential total distance	β_θ	differential total pitch coefficient
u_e	Coaxial rotor longitudinal cycle pitch	β_Ψ	differential total pitch yaw coefficient
u_a	Propeller thruster operating capacity	β_v	Rudder coefficient
u_t	Elevator deflection value	β_e	Elevator coefficient
u_{eh}	Rudder deflection values	u	the edge velocity of the coaxial rotor
u_{av}	Rudder deflection values	v	the speed of the helicopter
σ_ϕ	Roll deviation value	S_c	Area of the coaxial rotors
σ_θ	Pitch deviation value	S_H	Area of the area of the horizontal tail
σ_Ψ	Yaw deviation	S_V	Area of the vertical tail
σ_v	Vertical tail moment coefficient deviation	x_P	X-axis coordinates of the thruster
σ_H	Horizontal tail moment coefficient deviation	x_H	X-axis coordinates of the horizontal tail
β_l	longitudinal variable pitch coefficient	x_V	X-axis coordinates of the vertical tail
β_ϕ	differential total distance roll coefficient	z_P	Z-axis coordinates of the thruster
β_V	longitudinal variable pitch coefficient	z_v	Z-axis coordinates of the vertical tail

Based on the analysis, we can know that $\vec{u}$ is determined by external input, and $\vec{\varepsilon}$ and $\vec{\alpha}$ are uniquely determined by the current helicopter type. Uncertain $\vec{\sigma}$ and $\vec{\beta}$

satisfy the following two equations. Since the value of $\vec{u}$ has been determined by the external input, the above few vector parameters can be determined only in terms of γ.

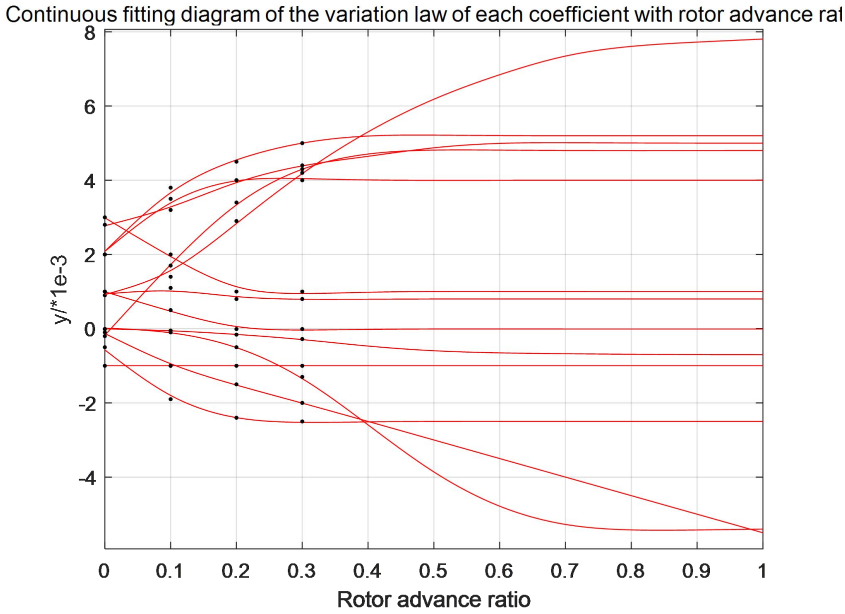


Figure 1 Continuous fitting diagram of the
variation law of each coefficient with rotor advance ratio

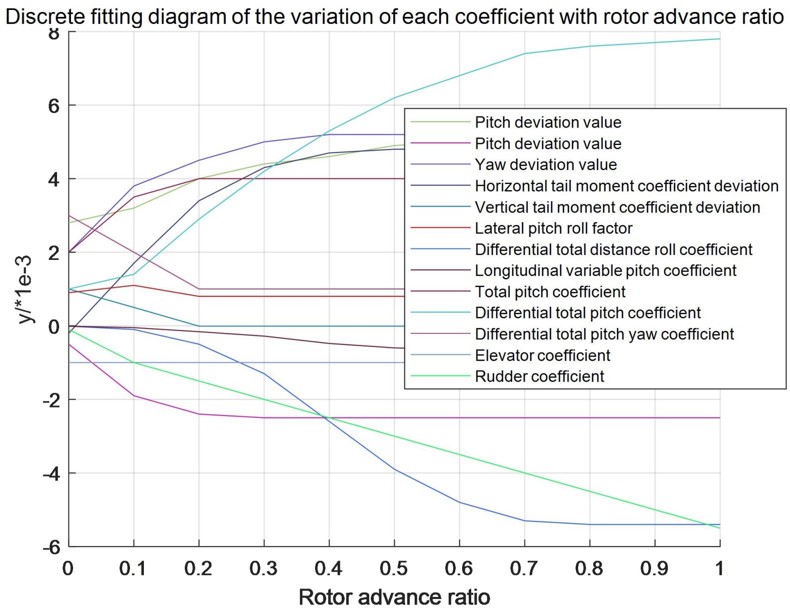


Figure 2 Discrete fitting diagram of the
variation law of each coefficient with rotor advance ratio

We independently fit the image of a rotor propulsion ratio greater than 0.4, and observe its trend. Draw discrete and continuous images of γ and variables just like the two pictures above. The first picture describes Continuous fitting diagram of the variation law of each coefficient with rotor advance ratio while the second one describe Discrete fitting diagram of the variation of each coefficient with rotor advance ratio. It will be readily found that the overall trend and shape of the two images are similar, so we use the average parameter to replace the instantaneous parameter, so as to give the solution method of $\vec{\sigma}$ and $\vec{\beta}$ in any state.

$$\frac{\sigma_i(\gamma,\vec{u})-\sigma_i([10\gamma]/10,\vec{u})}{10\gamma-[10\gamma]}=\sigma_i(\frac{([10\gamma]+1)}{10},\vec{u})-\sigma_i(\frac{[10\gamma]}{10},\vec{u}) \qquad (4-1-6)$$

$$\frac{\beta_i(\gamma,\vec{u})-\beta_i([10\gamma]/10,\vec{u})}{10\gamma-[10\gamma]}=\beta_i(\frac{([10\gamma]+1)}{10},\vec{u})-\beta_i(\frac{[10\gamma]}{10},\vec{u}) \qquad (4-1-7)$$

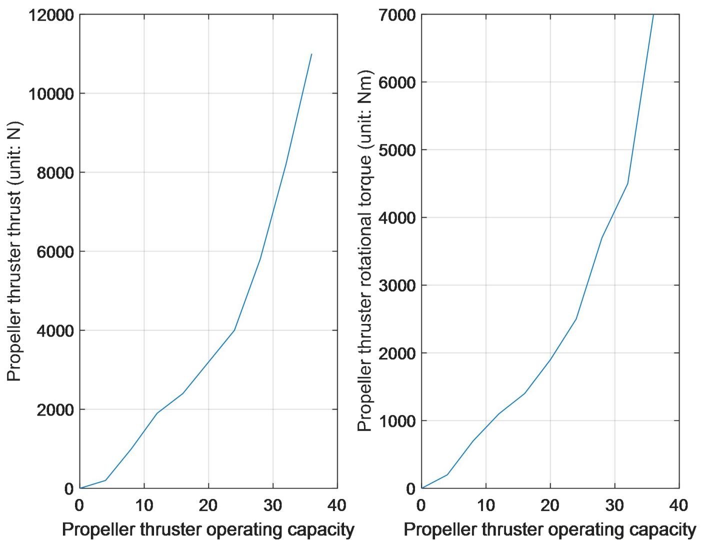


Figure 3 Propeller thruster thrust and rotational torque

With the increase in propeller thruster operating capacity, propeller thruster thrust and propeller thruster rotational torque are similar to a functional increase.

Through observing the model built above, we can conclude them into four parameters:Control parameters, Deviation parameters, State parameters, Model parameters.

And by observing the data in the appendix, we can find that both $\vec{\beta}$ and $\vec{\sigma}$ is connected with γ, which means we can represent $\vec{\sigma}$ and $\vec{\beta}$ in terms of γ.

$$\vec{\sigma} = \vec{\sigma}(\gamma) \qquad (4-1-8)$$

$$\vec{\beta} = \vec{\beta}(\gamma) \qquad (4-1-9)$$

4.2Modal of question 1

Objective function:

$$M_\theta = C_{c\theta}\rho S_P u + \frac{1}{2}C_{H\theta}\rho U^2 S_H x_H + 0 + F_P z_P \qquad (4-2-1)$$

$$M_\theta = I_\theta\ddot{\theta} \qquad (4-2-2)$$

In Equation $(4-2-1)$, M_θ indicates the pitch moment of the compound helicopter, $C_{c\theta}$ indicates Rotor roll moment coefficient, $C_{H\theta}$indicates the Horizontal tail moment coefficient, FP indicates the force of the thruster; in Equation $(4-2-2)$, I_θ indicates the moment of inertia of the roll angle, $\ddot{\theta}$ indicates the acceleration of the roll angle.

Parameter variables:

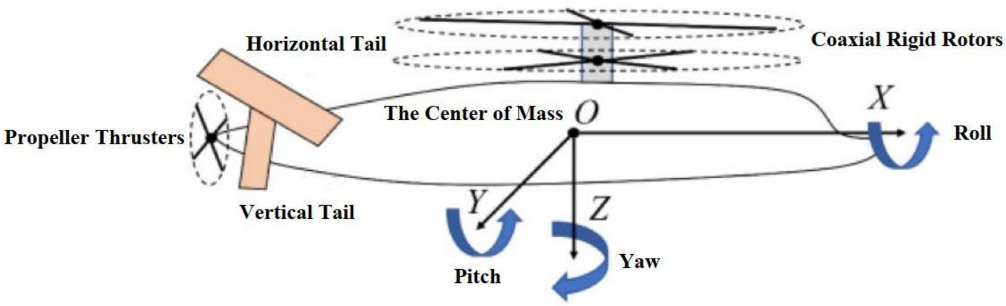
$$C_{c\theta} = \sigma_\theta + \beta_V u_e + \beta_{all} u_c + \beta_\theta u_{cd} \qquad (4-2-3)$$

$$C_H = \sigma_H + \beta_e u_{eh} \qquad (4-2-4)$$

In Equation$(4-2-3)$, $C_{c\theta}$ indicates the Rotor roll moment coefficient, σ_θ indicates pitch deviation value; CH indicates the Horizontal tail moment coefficient; in Equation$(4-2-4)$,CH indicates the Horizontal tail moment coefficient.

4.3Modal of question 2

The space rectangular coordinate system is established as shown in the figure:



Objective function:

$$\overrightarrow{M_C} = \rho S_c u(C_{c\phi}, C_{c\theta}, C_{c\Psi}) \qquad (4-3-1)$$

$$\overrightarrow{M_H} = \frac{1}{2}\rho v^2 S_H x_H (0, C_H, 0) \qquad (4-3-2)$$

$$\overrightarrow{M_V} = \frac{1}{2}\rho v^2 S_V C_V (z_V, 0, x_V) \qquad (4-3-3)$$

$$\overrightarrow{M_P} = (M_0, F_P z_P, F_P x_P) \qquad (4-3-4)$$

$$\overrightarrow{M} = \overrightarrow{M_C} + \overrightarrow{M_H} + \overrightarrow{M_V} + \overrightarrow{M_P} \qquad (4-3-5)$$

$$\overrightarrow{M} = (I_\phi \ddot{\phi} + I_\theta \ddot{\theta} + I_\Psi \ddot{\Psi}) \qquad (4-3-6)$$

In Equation$(4-3-1)$,$\overrightarrow{M_C}$ indicates roll moments, $(C_{c\phi}, C_{c\theta}, C_{c\Psi})$ indicates a set of vector parameters; In Equation$(4-3-2)$,$\overrightarrow{M_H}$ *indicates* pitch moments, $(0, C_H, 0)$ indicates a set of vector parameters ; In Equation$(4-3-3)$,$\overrightarrow{M_V}$ *indicates* yaw moments, $C_V(z_V, 0, x_V)$ indicates a set of vector parameters;In Equation$(4-3-4)$,$\overrightarrow{M_P}$ indicates the thruster moment, M_0 indicates torque

generated by the thruster itself, F_P indicates the force of the thruster;In Equation$(4-3-6)$, I_ϕ indicates the moment of inertia of roll angle, I_θ indicates the moment of inertia of pitch angle, I_Ψ indicates the moment of inertia of yaw angle, $\ddot{\phi}$ indicates the angular acceleration of the roll Angle, $\ddot{\theta}$ indicates the angular acceleration of the pitch Angle, $\ddot{\Psi}$ indicates the angular acceleration of the yaw Angle.In Equation$(4-3-2)$ and Equation$(4-3-3)$, to facilitate merging, we write the parameters as vectors, such as, $(0, C_H, 0)$and $C_V(z_V, 0, x_V)$.
parameter variable:

$$C_{c\phi} = \sigma_\phi + \beta_l u_a + \beta_\phi u_{cd} \qquad (4-3-7)$$

$$C_{c\theta} = \sigma_\theta + \beta_V u_e + \beta_{all} u_c + \beta_\theta u_{cd} \qquad (4-3-8)$$

$$C_{c\Psi} = \sigma_\Psi + \beta_\Psi u_{cd} \qquad (4-3-9)$$

$$C_v = \sigma_v + \beta_v u_{av} \qquad (4-3-10)$$

$$C_H = \sigma_H + \beta_e u_{eh} \qquad (4-3-11)$$

In Equation $(4-3-7)$,$C_{c\phi}$ indicates the Rotor roll moment coefficient; In Equation $(4-3-8)$, $C_{c\theta}$ indicates the Rotor pitch moment coefficient; In Equation $(4-3-9)$, $C_{c\Psi}$ indicates the Rotor yaw moment coefficient; In Equation $(4-3-10)$, C_v indicates the Vertical tail moment coefficient; In Equation$(4-3-11)$, C_H indicates the Horizontal tail moment coefficient.

4.4Modal of question 3

At low speed, the two variables u and v determine the moment of the aircraft, and then affect the flight state of the aircraft. When u_i takes different u, the piecewise function f(u) has different values.

k_{max} and k_{min} are the upper and lower limits of the evaluation of the tail and rotor respectively, which are related to the external factors such as the structure, reaction time and performance parameters of the aircraft.

Then two related parameters b_1 and b_2 are introduced, and the following equation relationship is obtained by solving $\frac{\partial^2 f}{\partial V^2}|_{v=\frac{v_u+v_c}{2}} = 0$.

$$b_1 = \frac{2\ln\left(\frac{k_{max}}{k_{min}} - 1\right)}{v_u + v_b} \qquad (4-4-1)$$

$$b_2 = \frac{2\ln\left(\frac{k_{min}}{k_{max}} - 1\right)}{v_u + v_b} \qquad (4-4-2)$$

It can be seen that equation $\int_0^{|u_i|} |u_i| f(u_i, v) du_i$ represents the u_i evaluation result. For practical and theoretical purposes, we just need to keep the attitude Angle as small as possible. Under the condition of definite solution of $\overrightarrow{M} = 0$ and $\overrightarrow{u_b} \le \vec{u} \le \overrightarrow{u_u}$, find the minimum value of the following objective function

$$\min\sum\int_0^{|u_i|} f(u_i,v)du_i \qquad (4-4-3)$$

The first new parameter component K_{3*7} in the equation is obtained by combining the vector parameters listed above, and it can be seen that most of the components of $\vec{u}$ are linearly related to K_{3*7}, while u_t is not linearly related to K all the time.
When $j = 5$, then

$$K_{ij} = K_{ij}(u_t) \qquad (4-4-4)$$

And when $j \neq 5$, then C_{3*1} and K_{3*7} are independent parameters, independent of u_t.
According to the title, we can list two constraints:

$$\overrightarrow{u_b} = (0,-25,-25,-25,0,-25,-25) \qquad (4-4-5)$$

$$\overrightarrow{u_u} = (30,25,25,25,36,25,25) \qquad (4-4-6)$$

After analysis and calculation, we can get the following expression:

$$K_{11} = K_{13} = K_{16} = K_{24} = K_{27} = K_{31} = K_{33} = K_{34} = K_{36} = 0 \quad (4-4-7)$$

4.5Modal of question 4

The fourth question is similar to the third, with the main difference being that $M \neq 0$, so we just need to keep the module of M as small as possible. Therefore, it is only necessary to take $M = 0$ out of the equation and turn it into another objective function, that is, multidimensional objective optimization. The constraints in this case are:

$$\begin{cases} \overrightarrow{u_b} \le \vec{u} \le \overrightarrow{u_u} \\ ma = F(u_t) \\ \vec{u}(0) = \vec{u} \end{cases} \qquad (4-5-1)$$

Introduce an initial state condition u_g. When the problem is transformed into a determined u_t, when t=0, the minimum value of M is the optimal solution of the problem, and the subsequent analysis and solution process is the same as that of the third question.

v is a function of v_t, which is $v = v(t)$, and then we differentiate both sides of the equation, $dv(t) = adt$. Because they say we want to keep the acceleration constant, so a is constant. And then based on that analysis, we can figure out

$$\min\sum\int_0^t\int_{|u_i(0)|}^{|u_i(t)|} |u_i(t)|\, f(u_i(t), v(t))du_i(t)dt$$

$$\min|\vec{M}| = |K\vec{u} + C|$$

When solving for u_{av}, use $\{u_i\} = \{u_{av}\}$;when solving for u_c and u_{eh}, we use $\{u_i\} = \{u_c, u_{eh}\}$.

5.Test the Models

5.1Test of question 1

5.1.1Solving progress

At the same time, we can know that by using these five parameters, it is easy to develop a model for the variation of attitude angles.

$$M_\theta = M_\theta(\vec{u},\vec{\sigma},\vec{\beta},\vec{\varepsilon},\vec{\alpha}) \quad (5-1-3)$$

After simplification, we get:

$$M_\theta = M_\theta(\vec{u},\gamma,\vec{\varepsilon},\vec{\alpha}) \quad (5-1-4)$$

Thus, when $\vec{u},\gamma,\vec{\varepsilon},\vec{\alpha}$ is determined, y is also determined. According to this conclusion, we can write such a differential equation:

$$M_\theta = I_\theta\ddot{\theta} \quad (5-1-5)$$

Under the following conditions:

$$\theta = 0 \quad (5-1-6)$$

$$\dot{\theta} = \theta \quad (5-1-7)$$

We can get such an answer as the follows:

$$\theta = \frac{1}{2}\frac{M_\theta}{I_\theta}t^2 \quad (5-1-8)$$

5.1.2Solution result

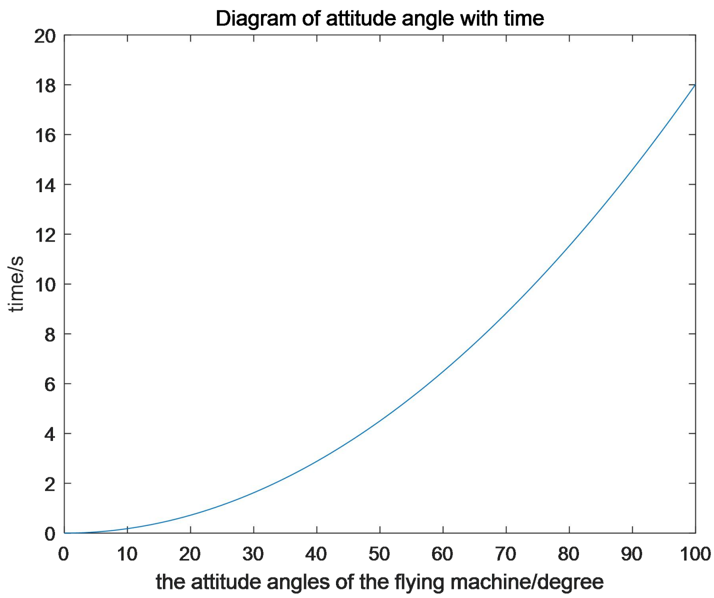


Figure 4 Diagram of attitude angle with time

Attitude Angle of the vehicle at different times(I)

time/s	**5**	**10**	**20**
$\boldsymbol{\theta}$**/rad**	0.0450537493601286	0.189214997449481	0.720859989761926

The magnitude of the pitching moment is

$$M_\theta = 72.085998976192585$$

5.2Test of question 2

5.2.1Solving progress

The second problem is a general generalization of the first problem, and we can solve this problem by using similar ideas.

Thus,according to the first problem, when $\vec{u}, \gamma, \vec{\varepsilon}, \vec{\alpha}$ is determined, y is also determined. According to this conclusion, we can write such a differential equation:

$$\vec{M} = (I_{\phi}\ddot{\phi}, I_{\theta}\ddot{\theta}, I_{\Psi}\ddot{\Psi}) \tag{5-2-1}$$

Under the following conditions:

$$(\phi, \theta, \Psi) = \vec{0} \tag{5-2-2}$$

$$(\dot{\phi}, \dot{\theta}, \dot{\Psi}) = (\phi, \theta, \Psi) \tag{5-2-3}$$

We can get such an answer as the follows:

$$(\phi I_{\phi}, \theta I_{\theta}, \Psi I_{\Psi}) = \frac{1}{2}\vec{M}t^2 \tag{5-2-4}$$

5.2.2Solution result

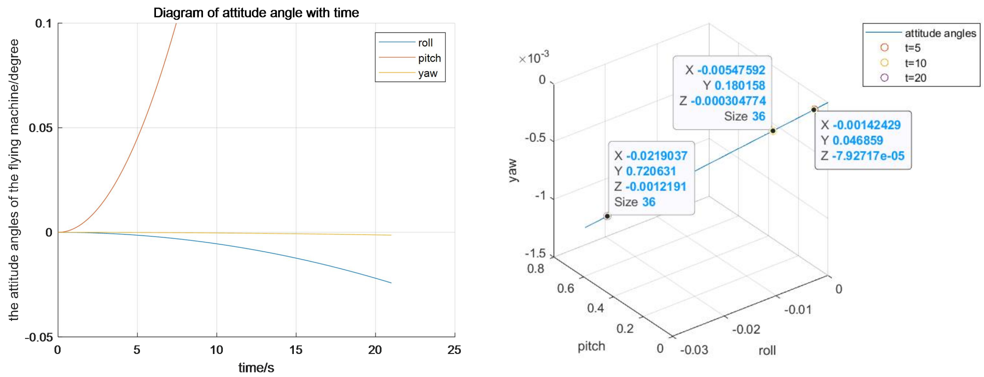


Figure 5 Diagram of attitude angle with time

Attitude Angle of the vehicle at different times(II)

time/s	5	10	20
ϕ/rad	-0.001371562613118	-0.005486250452473	-0.021945001809891
θ/rad	0.045053749360120	0.180214997440481	0.720859989761926
Ψ/rad	-0.000075675812299	-0.000302703249194	-0.001210812996777

5.3Test of question 3

5.3.1Solving progress

The problem requires solving the maneuvering characteristics at low speed and high speed, so as to design the maneuvering amplitude of each component. We introduce an S-type function to characterize which variables are adjusted at low and high speeds.

Draw the moment scatter diagram of each part of the aircraft in random state at low speed and high speed respectively. It can be analyzed that the aircraft propeller plays a leading role in regulating the torque at low speed. Therefore, we ignore the torque generated by the propeller in the second figure, and learn from the image that the torque of the tail is small compared with the rotor, so in the case of the torque generated by the propeller and rotor is much larger than the horizontal and vertical torque, it is possible to make such an assumption.

$$u_{av} = u_{eh} = 0 \qquad (5-3-1)$$

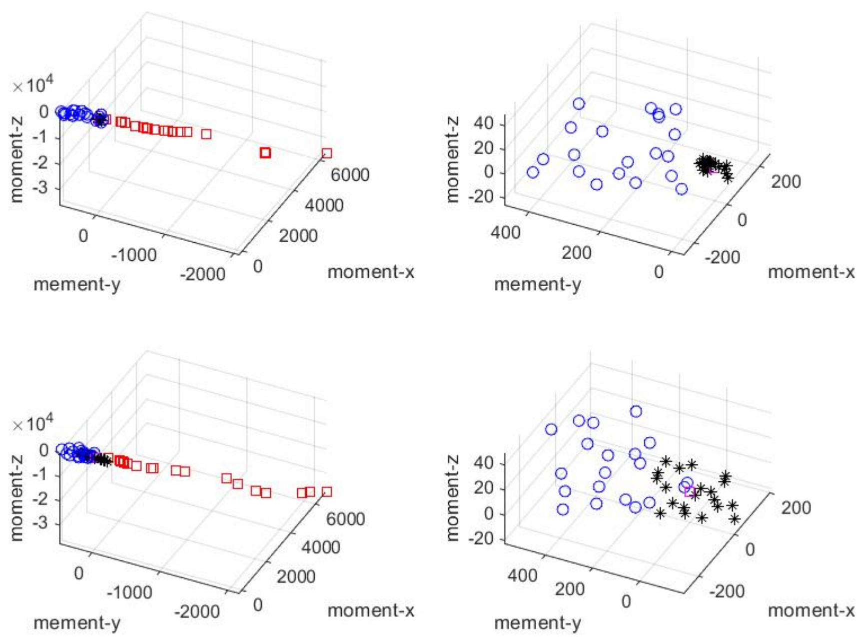


Figure 6 Three-dimensional torque scatter diagram

We draw the three-dimensional vector scatter plot composed of u_{cd}, u_t, u_a and u_c, u_t, u_e respectively, and the corresponding result can be obtained.

At low speed, the two variables u and v determine the moment of the aircraft, and then affect the flight state of the aircraft. When u_i takes different u, the piecewise function f(u) has different values.

Considering the flying state of helicopter at low speed, the three-dimensional scatter diagram of torque and gear is drawn by MATLAB. After image analysis, we can find that only the zero stop satisfies that the moment is zero, that is, the yaw angle and the roll angle are zero. While the other gear is greatly affected by the propulsive force, the torque changes little. Therefore, we take the optimal solution, that is, when the gear is 0, the aircraft can achieve level flight.

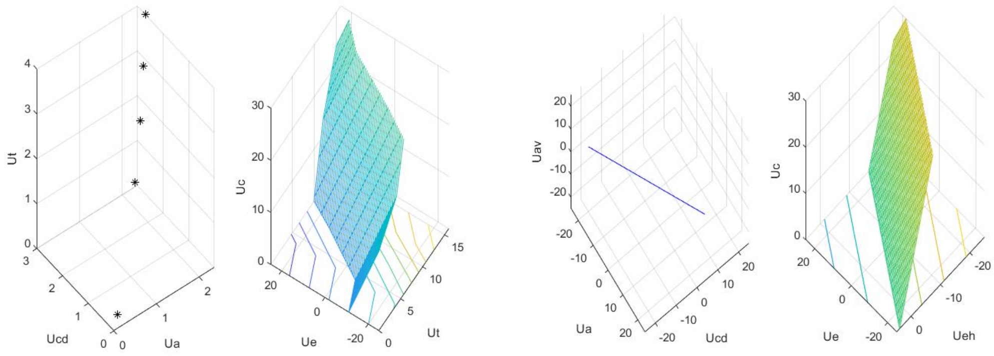


Figure 7 Scatter diagram of three dimensional control parameters

According to the analysis, when drawing with MATLAB for the second time, we ignored the influence of propulsion force and only considered the torque generated by the tail and rotor. At the same time, in the process of drawing, we found that the equation could not be solved from the 17th gear, so the gear range was reduced to 0 to 16 gear. From the analysis of the image, it can be concluded that when u_t=0, the equation always has a solution, and u_c is effective only in the feasible region of the grid we draw. To sum up, considering that in order to make the adjustment of u_c and u_e parameters as small as possible, we take the center of the edge part of the grid area with u_t=0 as the optimal solution.

k_{max} and k_{min} are the upper and lower limits of the evaluation of the tail and rotor respectively, which are related to the external factors such as the structure, reaction time and performance parameters of the aircraft.

Then two related parameters b_1 and b_2 are introduced, and the following equation relationship is obtained by solving $\frac{\partial^2 f}{\partial \mathrm{V}^2}|_{v=\frac{v_u+v_c}{2}} = 0$.

$$b_1 = \frac{2\ln\left(\frac{k_{max}}{k_{min}} - 1\right)}{v_u + v_b} \qquad (5-3-2)$$

$$b_2 = \frac{2\ln\left(\frac{k_{min}}{k_{max}} - 1\right)}{v_u + v_b} \qquad (5-3-3)$$

It can be seen that equation $\int_0^{|u_i|} |u_i| f(u_i, v) du_i$ represents the u_i evaluation result. Taking integration as the goal has a good effect on optimization[3].

For practical and theoretical purposes, we just need to keep the attitude Angle as small as possible. Under the condition of definite solution of $\overrightarrow{M} = 0$ and $\overrightarrow{u_b} \le \vec{u} \le \overrightarrow{u_u}$, find the minimum value of the following objective function

$$\min \sum \int_0^{|u_i|} f(u_i, v) du_i \qquad (5-3-4)$$

The first new parameter component K_{3*7} in the equation is obtained by combining the vector parameters listed above, and it can be seen that most of the components of $\vec{u}$ are linearly related to K_{3*7}, while u_t is not linearly related to K all the time.

When $j = 5$, then

$$K_{ij} = K_{ij}(u_t) \qquad (5-3-5)$$

And when $j \neq 5$, then C_{3*1} and K_{3*7} are independent parameters, independent of u_t.

According to the title, we can list two constraints:

$$\vec{u_b} = (0,-25,-25,-25,0,-25,-25) \qquad (5-3-6)$$

$$\vec{u_u} = (30,25,25,25,36,25,25) \qquad (5-3-7)$$

After analysis and calculation, we can get the following expression:

$$K_{11} = K_{13} = K_{16} = K_{24} = K_{27} = K_{31} = K_{33} = K_{34} = K_{36} = 0 \quad (5-3-8)$$

Observing the equation, it is not difficult to solve:

$$-K_{32}u_{cd} = K_{35}u_t + K_{37}u_{av} + C_3 \qquad (5-3-7)$$

$$-K_{14}u_a = K_{12}u_{cd} + K_{15}u_t + K_{11}u_{av} + C_1 \qquad (5-3-8)$$

$$-K_{21}u_c - K_{23}u_e = K_{22}u_{cd} + K_{25}u_t + K_{26}u_{eh} + C_3 \qquad (5-3-9)$$

Combined with the assumption above, then several variables in the above equation satisfy such a set of equations:

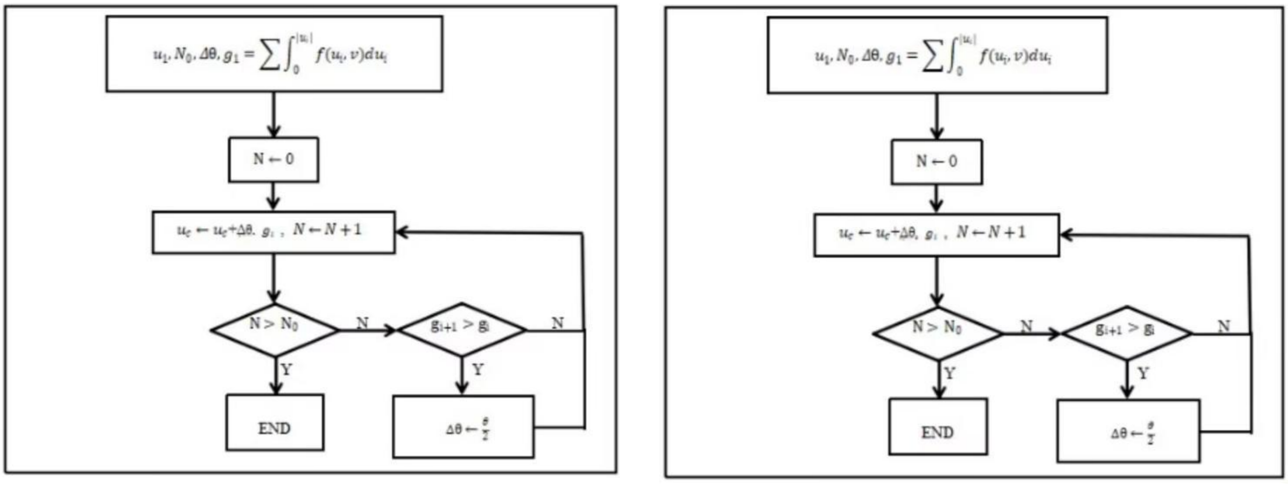
$$\begin{cases} u_{cd} = u_{cd}(u_t) \\ u_a = u_a(u_t) \\ u_c = u_c(u_t, u_e) \end{cases} \qquad (5-3-10)$$

According to the above analysis and problem, we can write the variation range of each component of $\vec{u}$ at low speed.

$$\begin{cases} 0 \le u_c \le 9.80 \\ u_{cd} = -2.053 \\ -20 \le u_e \le -11.90 \\ u_a = -1.920 \\ u_t = 0 \\ u_{eh} = 0 \\ u_{av} = 0 \end{cases} \qquad (5-3-11)$$

According to the above analysis and problem, we can write the variation range of each component of $\vec{u}$ at high speed.

$$\begin{cases} 0 \le u_c \le 28.17 \\ -24.82 \le u_{cd} \le 16.50 \\ -25 \le u_e \le 25 \\ -12.96 \le u_a \le 7.260 \\ u_t = 0 \\ -25 \le u_{eh} \le 3.75 \\ -25 \le u_{av} \le 15 \end{cases} \qquad (5-3-12)$$



The following is a flowchart of the problem solving process at low and high speed respectively.

And through the constraints and objective function, we can get the minimum value of the dynamic moment when each component takes what value.
In the flow chart, assign the value of $\{u_c, u_{eh}\}$ to $\{u_i\}$.

5.3.2Solution result

Under low speed conditions, the optimal solution can be obtained when

$$\vec{u} = (0, -2.053, -10.97, -1.920, 0, 0, 0).$$

Under high speed conditions, the optimal solution can be obtained when

$$\vec{u} = (0, 0, 0, -0.8143, 0, -6.289, -0.9729).$$

5.4Test of question 4

5.4.1Solving progress

In the acceleration process, a is only related to u_t, and because the control acceleration is required to be constant in the problem condition, $F(u_t)$ is certain. It is advisable to ignore the air resistance and take u_t as a fixed value. So, and there's an assumption in the condition that it might as well be that the propulsion is provided by the rear thruster, so let's ignore the air resistance and analyze this situation. The model combined with the second and third questions in this topic, its real variable parameters are actually only 3, namely u_{av}, u_{eh}, u_c. The solution after that is the same as the high-speed solution in the third position.

A time variable is introduced and the stable state of each instant is determined using the same solution as the third question.At the same time, the control difficulty $\min \sum \int_0^t \int_{|u_i(0)|}^{|u_i(t)|} |u_i(t)|\, f(u_i(t), v(t)) du_i(t) dt$ is considered as the constraint condition.

5.4.2Solution result

In the three images, the four related control parameters of the coaxial rotor, the control parameters of the vertical and horizontal tail, and the relationship between the control parameters of the propeller and the time are drawn respectively. It can be seen from the figure that the change trend of the control parameters with the increase of time.

In Figure 1, it is found that u_a and u_{cd} show a stepped-type steady growth, which is because the aircraft state appears unstable mutations in the process of partial shifting, while u_c and u_e show a parabolic growth, which can be observed to change over time. These two control parameters have an increasing impact on the aircraft state, and the impact change rate is increasing.

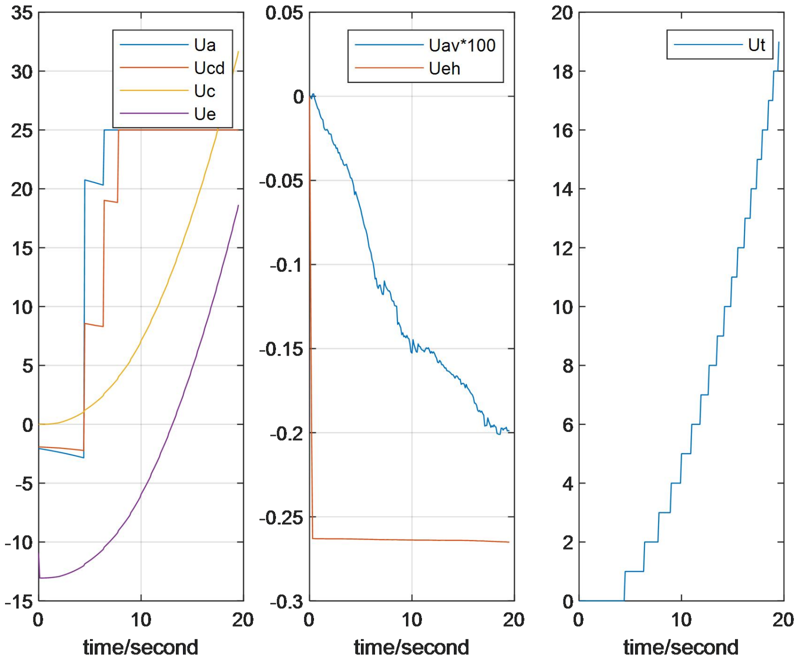


Figure 8 Relationship between control parameters and acceleration time

However, in Figure 2, u_{eh} has a large mutation near 0 seconds, because when the aircraft accelerates from zero, there is a large acceleration, which causes u_{eh} to have a large mutation. At the same time, it is also found that the u_{av} also has a decreasing trend, but for the convenience of presenting the image, we multiply the u_{av} by 100, so we can see that the control parameter of the u_{av} has only a small effect on the aircraft state.

The third diagram reflects the step-up variation of the thruster control parameters with respect to time as a function of first order, which also takes into account the change of gear. In the very front section, the helicopter has just started and the thruster has not been fully activated, so there is almost no change in u_t.

6. Sensitivity Analysis

6.1Analysis of question 1

In the test of the first question, we use a small disturbance to test, mainly to test whether the angular velocity of the aircraft still can not change too much after the disturbance, so as to prove that our solution is relatively reasonable. So we use the method of taking random numbers, and then carry out a random left and right disturbance on it, and then we find that its relative angular velocity changes by ten to the negative fourth power, and ten to the negative fourth power is a very small number, so we can get this result is relatively stable. In addition, the abscissa is the number of perturbations, and the ordinate is the Angle of each disturbance. In 100 perturbations, it is found that the angular velocity of pitch Angle is relatively stable, so our model is reasonable.

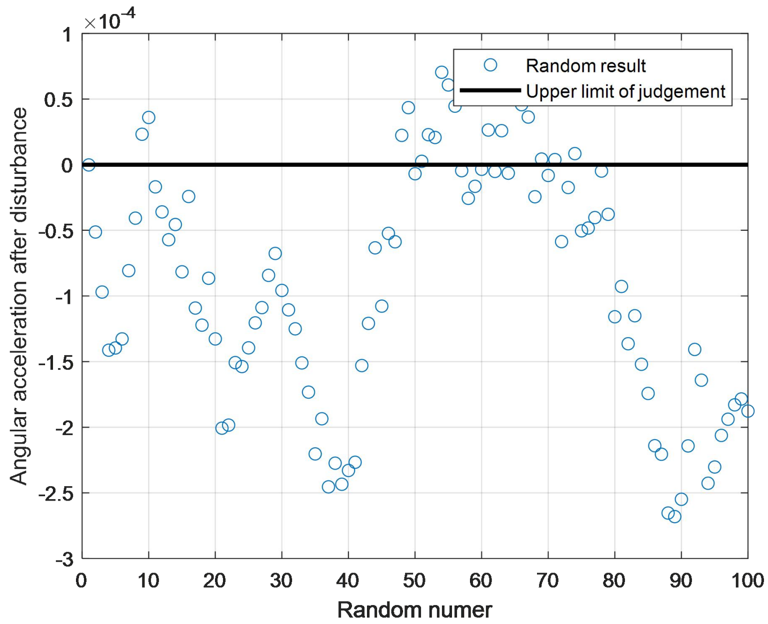


Figure 8 Sensitivity Analysis of question 1

6.2Analysis of question 2

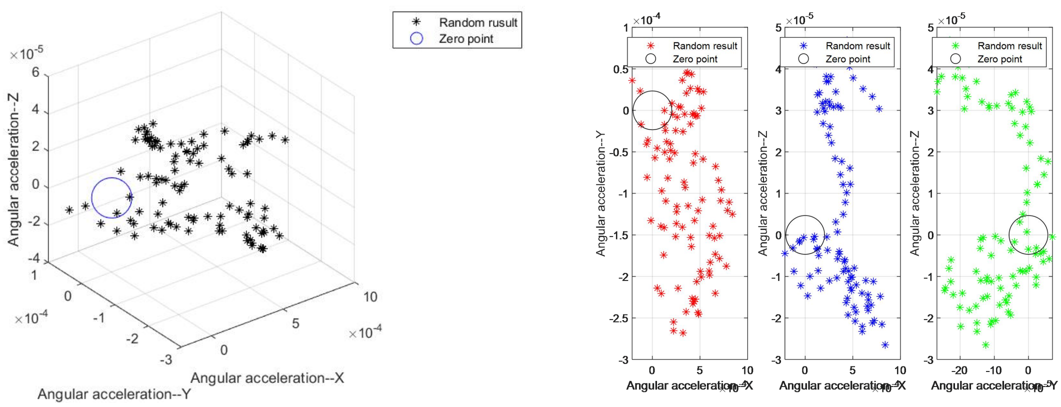


Figure 9 Sensitivity Analysis of question 2

For the test of the second question, because it is required to analyze and solve the problem in three directions, such as rolling angle, pitch angle and yaw angle, we give a random disturbance to the angular velocity of the aircraft in the three directions.

The first plot is a three-dimensional plot with X-Y-Z coordinates representing the angular velocity perturbations in each direction. In order to facilitate the viewing and understanding, we draw the three two-dimensional scatter diagrams at the bottom, representing the first three-dimensional three-dimensional map, which are respectively the projection in the X-Y direction, the projection in the X-Z direction, and the projection in the Y-Z direction.

To reflect the position distribution of each scatter point and the origin, we use a large circle to circle the origin. It can be seen from the scatter distribution in the figure that most of the scatter points are concentrated near the origin, and in the same figure, the scatter points are uniformly on the right or left, which indicates that the solution of our hypothesis is relatively reasonable.

Analyze the reasons why scatter points are more uniformly on the right or left. For the scatter plot of the X-Y image, the origin of most of the scatter points is to the right, because the weight of each control parameter is different in the course of considering the flight of the aircraft, so that it is also different in the size of the torque generated by it. The reason why the scatter is at the bottom right of the origin and not uniform is because the perturbation is a random perturbation of each parameter, and the weights of each parameter are different, so it will be skewed to the right. The X-Z and Y-Z images are similar to the X-Y analysis.

6.3Analysis of question 3

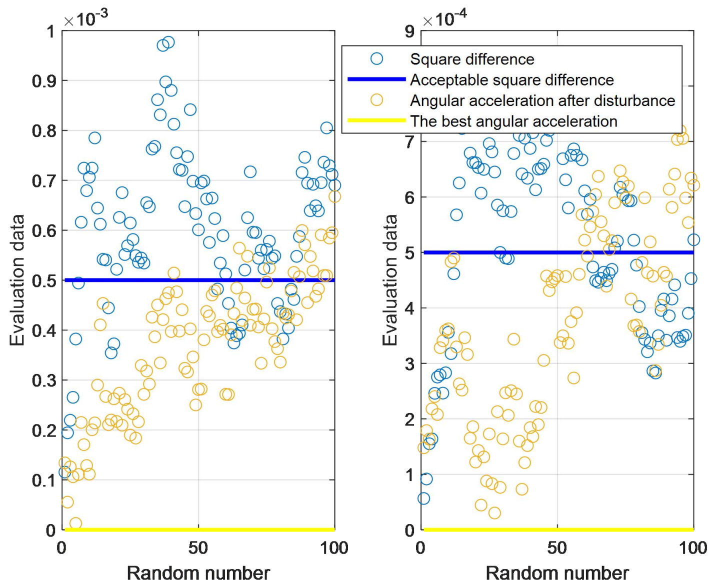


Figure 9 Sensitivity Analysis of question 3

For the test of the third question, we are also divided into low-speed state and high-speed state test. The first graph is the test at low speed, and the second graph is the test at high speed at low speed. The blue dots represent the acceptable square variance, which can be used for correctness tests and a stability test; The yellow dots represent the angular acceleration behind the disturbance. And we draw a blue line right in the middle of the graph, which is 0.5*10^-3, and we find that the variance is around here. Second, the yellow line in the figure represents both the origin and the original torque as a reference.

In the third question, after we obtained the corresponding results, we found that the fluctuation of the blue dots changed little after a small disturbance to the results, and almost all of them were less than 10^-3. Therefore, we can conclude that the variance of the angular velocity caused by the small disturbance is very small at low speed. And the fact that each variance is small proves that this result is stable and then this yellow dot this yellow dot refers to this well, when we perturb it, this new moment, minus the existing moment, is always greater than zero, which means that our optimal solution is reasonable, and this yellow dot is an optimal test. Therefore, the result of our analysis and calculation is more reasonable.

After analyzing the helicopter state at low speed, the same method is used to analyze the changes of the helicopter flight state at high speed when each control parameter is subjected to small disturbance. It can also be found that the fluctuation of the blue dots is small, and almost all of them are less than 9*10^-4, which also indicates that the variance of the angular velocity caused by small perturbations is very small in the high-speed state, and every variance is very small, which proves that the result is very stable, so our analysis and calculation results are more reasonable.

As for why the blue dots are more concentrated near the blue line at high speed, and the yellow dots are more concentrated at low speed. This is because the effect of small perturbations on the torque at high speed is greater, but the effect of small perturbations is smaller at low speed.

6.4Analysis of question 4

In the previous analysis, it can be seen that different parameters, due to different proportion effects, will change in the accelerated flight state of the helicopter due to gear shifting or sudden acceleration, and such changes are within a small acceptable range, so the model we established is relatively reasonable.

7.Strengths and Weakness

7.1 Strengths

1. Our model well determines the attitude angle of the helicopter in the fixed flight state, and can solve how to adjust the control parameters to make the helicopter fly stably in this state. At the same time, considering the difficulty of parameter change, the optimal solution in the stable flight is given.

2. For the flight at uniform acceleration, we can give a more reasonable optimal solution under the premise of considering the difficulty of changing the control parameters.

3. It is not difficult to see that the discretization processing makes our model also processable for non-uniform acceleration.

7.2 Weakness

1. Due to the constraint of time, we ignore the additional torque generated by air

resistance, which leads to some unreasonable results in the calculation of helicopter flight at high speed.

2. The discretization of the task in order to speed up the task in a short period of time leads to a certain error in the result. At the same time, the lack of adaptive iterative procedures leads to a large amount of computation[4].

8.Conclusion

This paper focuses on the modeling and optimal control of the composite helicopter. First of all, we summarized and analyzed the data given in the attachment, and obtained the coaxial rotor torque coefficient and the tail torque coefficient in the case of a certain propulsion ratio and control parameters. Then the torque equation of the composite helicopter under the condition of force balance is established successfully. Based on the above analysis results, we use the problem stem data to find that in a short time, the attitude Angle offset caused by external torque is proportional to the square of time.

According to the above moment equation, the attitude angle change, control parameter change and response difficulty in stable flight state are considered, and then the optimal attitude Angle model of stable flight is established. The feasible range of stable state is directly solved by matrix element analysis, and the optimal solution at low and high speed is obtained by linear optimization. The correctness, stability and optimality are verified by micro-perturbation test.

Considering the change of external state, the change of gear caused by acceleration, and the limitation of control parameter range, we establish the optimal attitude Angle model under accelerated flight. Subsequently, we improved the local search algorithm, realized the adaptive control of the step size by combining the dichotomy, reduced the error of the optimal solution, and realized the multi-dimensional optimization by introducing multi-dimensional vectors. Then, by discretizing the time and using the improved local search, the optimal solution of the control parameters with time is given. Then, according to the trend graph, the rationality of the trend of the optimal solution is analyzed.

References

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[3] Adrian M. STOICA;;Victor G. ADÎR.Integral LQR Control of a Star-Shaped Octorotor[J].INCAS Bulletin,2012.

[4] Ma Rui;Ding Li;Wu Hongtao.Dynamic Decoupling Control Optimization for a S mall-Scale Unmanned Helicopter[J].Journal of Robotics,2018.

Appendix

1.Correlation function

(1)draw

```
function [fitresult, gof] = createFit5(gama, bita, w)
[xData, yData, weights] = prepareCurveData( gama, bita, w );
% Set up fittype and options.
ft = fittype( 'smoothingspline' );
opts = fitoptions( 'Method', 'SmoothingSpline' );
opts.SmoothingParam = 0.999912158958922;
opts.Weights = weights;
% Fit model to data.
[fitresult, gof] = fit( xData, yData, ft, opts );
% Plot fit with data.
h=plot( fitresult );
hold on
plot(gama(1:4),bita(1:4),'k.');
% Label axes
xlabel( 'gama', 'Interpreter', 'none' );
ylabel( 'bita8', 'Interpreter', 'none' );
grid on
```

(2) Solve

```
function [si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita)
sigma1=[0.0002    0.00035 0.0004    0.0004 0.0004 0.0004 0.0004 0.0004 0.0004
0.0004 0.0004];
sigma2=[0.001     0.0014    0.0029    0.0042 0.0053 0.0062 0.0068 0.0074 0.0076
0.0077 0.0078];
sigma3=[0.0003    0.0002    0.0001    0.0001 0.0001 0.0001       0.0001 0.0001
0.0001 0.0001     0.0001];
sigma4=[-0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001
-0.0001 -0.0001 ];
sigma5=[-0.00001       -0.0001         -0.00015       -0.0002
-0.00025:-0.00005:-0.00055];
bita1=[0.00028    0.00032 0.0004    0.00044 0.00046 0.00049 0.00050 0.00050
0.00050 0.0005 0.00050];
bita2=[-0.00005 -0.00019       -0.00024       -0.00025 -0.00025 -0.00025 -0.00025
-0.00025 -0.00025 -0.00025 -0.00025];
bita3=[0.0002      0.00038 0.00045 0.0005 0.00052 0.00052 0.00052 0.00052 0.00052
0.00052 0.00052 ];
bita4=[-0.00002 0.00017 0.00034 0.00043 0.00047 0.00048 0.00048 0.00048 0.00048
0.00048 0.00048];
```

```
bita5=[0.0001	0.00005 -0.000001	-0.000001 -0.000001 -0.000001 -0.000001
-0.000001 -0.000001 -0.000001 -0.000001 ];
bita6=[0.00009	0.00011 0.00008 0.00008 0.00008 0.00008 0.00008 0.00008
0.00008 0.00008 0.00008];
bita7=[-0.0 00001	-0.00001	-0.00005	-0.00013	-0.00026	-0.00039
-0.00048	-0.00053 -0.00054 -0.00054	-0.00054 ];
bita8=[-0.000001	-0.000005	-0.000016	-0.000028	-0.000048 -0.000060
-0.000065 -0.000067 -0.000069 -0.000070 -0.000070];
si1=sigma1(floor(yita)+1)+(sigma1(floor(yita)+2)-sigma1(floor(yita)+1))*(yita-floor(
yita));
si2=sigma2(floor(yita)+1)+(sigma2(floor(yita)+2)-sigma2(floor(yita)+1))*(yita-floor(
yita));
si3=sigma3(floor(yita)+1)+(sigma3(floor(yita)+2)-sigma3(floor(yita)+1))*(yita-floor(
yita));
si4=sigma4(floor(yita)+1)+(sigma4(floor(yita)+2)-sigma4(floor(yita)+1))*(yita-floor(
yita));
si5=sigma5(floor(yita)+1)+(sigma5(floor(yita)+2)-sigma5(floor(yita)+1))*(yita-floor(
yita));
bi1=bita1(floor(yita)+1)+(bita1(floor(yita)+2)-bita1(floor(yita)+1))*(yita-floor(yita));
bi2=bita2(floor(yita)+1)+(bita2(floor(yita)+2)-bita2(floor(yita)+1))*(yita-floor(yita));
bi3=bita3(floor(yita)+1)+(bita3(floor(yita)+2)-bita3(floor(yita)+1))*(yita-floor(yita));
bi4=bita4(floor(yita)+1)+(bita4(floor(yita)+2)-bita4(floor(yita)+1))*(yita-floor(yita));
bi5=bita5(floor(yita)+1)+(bita5(floor(yita)+2)-bita5(floor(yita)+1))*(yita-floor(yita));
bi6=bita6(floor(yita)+1)+(bita6(floor(yita)+2)-bita6(floor(yita)+1))*(yita-floor(yita));
bi7=bita7(floor(yita)+1)+(bita7(floor(yita)+2)-bita7(floor(yita)+1))*(yita-floor(yita));
bi8=bita8(floor(yita)+1)+(bita8(floor(yita)+2)-bita8(floor(yita)+1))*(yita-floor(yita));
%
end
```

(3) Propeller

```
function [Fp,M0] = tuijin(ut)
power=[0 4	8	12	16	20	24	28	32	36];
Fpower=[0 200	1000	1900	2400	3200	4000	5800	8200
11000];
Mpower=[0 200	700 1100	1400	1900	2500	3700	4500
7000];
utt=ut;ut=ut/4;ut=floor(ut)+1;
Fp=Fpower(ut)+(Fpower(ut+1)-Fpower(ut))*(utt/4-(ut-1));
M0=Mpower(ut)+(Mpower(ut+1)-Mpower(ut))*(utt/4-(ut-1));
end
```

(4) Evaluation

```
function	f= pingjia(k,v)
v=norm(v,2);
```

```
kmax=1000000000;kmin=1;
vmax=110;vmin=85;vjun=(vmax+vmin)/2;
b1=log(kmax/kmin-1)/vjun;
if(k==1)
f=kmax/(1+(kmax/kmin-1)*exp(-b1*v));
else
f=kmax-kmax/(1+(kmax/kmin-1)*exp(-b1*v));
end
end
```

(5) Moment

```
function [M,Mc,Mh,Mv,Mp]=zhuangtailiju(ua,ucd,ue,uc,uav,ueh,ut,v)
ro=0.9096;u=180;
Sc=6^2*pi;Sh=1;Sv=0.5;xh=-3;xv=-3;xp=-3.5;zv=0.2;zp=-0.2;
yita=norm(v,2)/u*10;
[Fp,M0] = tuijin(ut);
[si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita);
Cc1=si1+bi1*ua+bi2*ucd;Cc2=si2+bi3*ue+bi4*uc+bi5*ucd;Cc3=si3+bi6*ucd;Ch=si
4+bi7*ueh;Cv=si5+bi8*uav;
Mc=ro*Sc*u*[Cc1,Cc2,Cc3];
Mh=1/2*ro*norm(v,2)^2*Sh*xh*[0 Ch 0];
Mv=1/2*ro*norm(v,2)^2*Sv*Cv*[zv 0 xv];
Mp=[M0 Fp*zp Fp*xp];
M=Mc+Mp+Mv+Mp;
end
```

2.Preparation

2960	-4. 24	0. 9134	70463	0. 695	9. 79754
2970	-4. 30	0. 9124	70374	0. 695	9. 79751
2980	-4. 37	0. 9115	70285	0. 694	9. 79748
2990	-4. 43	0. 9105	70195	0. 693	9. 79745
3000	-4. 50	0. 9096	70106	0. 692	9. 79742
海拔（米）	温度（℃）	空气密度（kg/m³）	气压值（pa）	大气压（atm）	重力加速度
3010	-4. 56	0. 9087	70017	0. 691	9. 79739
3020	-4. 63	0. 9077	69928	0. 690	9. 79736
3030	-4. 69	0. 9068	69839	0. 689	9. 79733
3040	-4. 76	0. 9059	69751	0. 688	9. 79730

（atmospheric data around3000 meters）

```
clear clc
gama=0:0.1:1;
sigma1=[0.0002 0.00035 0.0004 0.0004 0.0004 0.0004 0.0004 0.0004 0.0004
0.0004 0.0004];
sigma2=[0.001 0.0014 0.0029 0.0042 0.0053 0.0062 0.0068 0.0074 0.0076 0.0077
0.0078];
sigma3=[0.0003 0.0002 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001
0.0001];
sigma4=[-0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001
-0.0001 -0.0001 ];
```

```
sigma5=[-0.00001 -0.0001 -0.00015 -0.0002 -0.00025:-0.00005:-0.00055];
bita1=[0.00028 0.00032 0.0004 0.00044 0.00046 0.00049 0.00050 0.00050 0.00050
0.0005 0.00050];
bita2=[-0.00005 -0.00019 -0.00024 -0.00025 -0.00025 -0.00025 -0.00025 -0.00025
-0.00025 -0.00025 -0.00025];
bita3=[0.0002 0.00038 0.00045 0.0005 0.00052 0.00052 0.00052 0.00052 0.00052
0.00052 0.00052 ];
bita4=[-0.00002 0.00017 0.00034 0.00043 0.00047 0.00048 0.00048 0.00048
0.00048 0.00048 0.00048];
bita5=[0.0001 0.00005 -0.000001 -0.000001 -0.000001 -0.000001 -0.000001
-0.000001 -0.000001 -0.000001 -0.000001 ];
bita6=[0.00009 0.00011 0.00008 0.00008 0.00008 0.00008 0.00008 0.00008
0.00008 0.00008 0.00008];
bita7=[-0.000001 -0.00001 -0.00005 -0.00013 -0.00029 -0.00060 -0.00100
-0.00150 -0.00200 -0.00280 -0.00400 ];
bita8=[-0.000001 -0.000005 -0.000016 -0.000028 -0.000048 -0.000065 -0.000100
-0.000150 -0.000210 -0.000300 -0.000450];
w=[1 1 1 1 0.9:-0.1:0.3];
c=rand(13,3);
figure
createFit5(gama, bita1, w);
createFit5(gama, bita2, w);
createFit5(gama, bita3, w);
createFit5(gama, bita4, w);
createFit5(gama, bita5, w);
createFit5(gama, bita6, w);
createFit5(gama, bita7, w);
createFit5(gama, bita8, w);
createFit5(gama, sigma1, w);
createFit5(gama, sigma2/10, w);
createFit5(gama, sigma3,w);
createFit5(gama, sigma4,w);
createFit5(gama, sigma5, w);
axis([0,1,-0.5/1000,1/1000])
hold off
figure
legend
hold on
grid on
plot(gama, bita1,'color',c(1,:));
plot(gama, bita2,'color',c(2,:));
plot(gama, bita3,'color',c(3,:));
plot(gama, bita4,'color',c(4,:));
plot(gama, bita5,'color',c(5,:));
```

```
plot(gama, bita6,'color',c(6,:));
plot(gama, bita7,'color',c(7,:));
plot(gama, bita8,'color',c(8,:));
plot(gama, sigma1,'color',c(9,:));
plot(gama, sigma2/10,'color',c(10,:));
plot(gama, sigma3,'color',c(11,:));
plot(gama, sigma4,'color',c(12,:));
plot(gama, sigma5,'color',c(13,:));
axis([0,1,-0.5/1000,1/1000])
hold off
figure
legend
power=[0 4 8 12 16 20 24 28 32 36];
Fpower=[0 200 1000 1900 2400 3200 4000 5800 8200 11000];
Mpower=[0 200 700 1100 1400 1900 2500 3700 4500 7000];
subplot(1,2,1)
plot(power,Fpower)
grid on
subplot(1,2,2)
plot(power,Mpower)
grid on
```

3.Result of question 1 and question 2

```
clear clc
format long
v=[80 0.2];
uc=0;ucd=-2.1552;ue=-3.4817;ua=-2.0743;ut=0;ueh=-9.0772/(10^7);uav=4.1869/(1
0^7);
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ua,ucd,ue,uc,uav,ueh,ut,v);
figure
t=0:100;
theta=1/2*M(2)/I(2).*t.^2;
plot(t,theta)
xlabel('the attitude angles of the flying machine/degree')
ylabel('time/s')
title('Diagram of attitude angle with time')
t1=[t(6) t(11) t(21)];
theta=1/2*M(2)/I(2).*t1.^2
figure
t=0:0.1:21;
theta2=1/2*M(2)/I(2).*t.^2;
hold on
grid on
legend
```

```
theta1=1/2*M(1)/I(1).*t.^2;
plot(t,theta1)
plot(t,theta2)
ylabel('the attitude angles of the flying machine/degree')
ylim([-0.05 0.1])
xlabel('time/s')
title('Diagram of attitude angle with time')
theta3=1/2*M(3)/I(3).*t.^2;
plot(t,theta3)
hold off
grid on
legend(["roll","pitch","yaw"])
plot3(theta1,theta2,theta3)
xlabel('roll')
ylabel('pitch')
zlabel('yaw')
hold on
t=[5 10 20];
theta1=1/2*M(1)/I(1).*t.^2
theta2=1/2*M(2)/I(2).*t.^2
theta3=1/2*M(3)/I(3).*t.^2
ax = gca;
chart = ax.Children(1);
scatter3(theta1(1),theta2(1),theta3(1));
scatter3(theta1(2),theta2(2),theta3(2));
scatter3(theta1(3),theta2(3),theta3(3));
legend
legend(["attitude angles","t=5","t=10","t=20"])
grid on
ax2 = gca;
chart2 = ax2.Children(3);
chart2 = ax2.Children(1);
datatip(chart2,-0.02195,0.7209,-0.001211);
chart2 = ax2.Children(2);
datatip(chart2,-0.005486,0.1802,-0.0003027);
```

4. Analysis of question 3

```
clear clc
format long
figure( 'Name', 'Scatter plot of moment in the immediate state' );
v=[80 0.2];
mmc=[];mmhv=[];mmp=[];
for j=1:1:4
subplot(2,2,j)
```

```
if(j>2)
v=[179 0.2];
end
for i=1:20
ua=rand(1)*50-25;ucd=rand(1)*50-25;ue=rand(1)*50-25;uc=rand(1)*30;uav=rand(
1)*50-25;ueh=rand(1)*50-25;ut=rand(1)*36;
I=[8000 20000 25000];
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ua,ucd,ue,uc,uav,ueh,ut,v);
plot3(Mv(1)+Mh(1),Mv(2)+Mh(2),Mv(3)+Mh(3),'k*')
hold on
plot3(Mc(1),Mc(2),Mc(3),'bo')
if(j==1||j==3)
plot3(Mp(1),Mp(2),Mp(3),'rs')
end
grid on
end
Mhv=[ -0.079236964027614 0.873221457600000 1.188554460414213];%低速
plot3(Mhv(1),Mhv(2),Mhv(3),'ys')
xlabel('moment-x')
ylabel('mement-y')
zlabel('moment-z')
view([-64 51])
end
```

5. result of question 3

(1) Low speed

```
clear clc
format long
ro=0.9096;u=180;v=[80 2];
Sc=6^2*pi;Sh=1;Sv=0.5;xh=-3;xv=-3;xp=-3.5;zv=0.2;zp=-0.2;
yita=norm(v,2)/u*10;
[si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita);
K(1,:)=[0 ro*Sc*u*bi2 0 ro*Sc*u*bi1 0 1/2*ro*norm(v,2)^2*Sv*bi8*zv ];
K(2,:)=[ro*Sc*u*bi4 ro*Sc*u*bi5 ro*Sc*u*bi3 0 1/2*ro*norm(v,2)^2*Sh*xh*bi7
0];
K(3,:)=[0 ro*Sc*u*bi6 0 0 0 1/2*ro*norm(v,2)^2*Sv*bi8*xv];
I=[8000 20000 25000];
ueh=0;uav=0;
ucd0=[];ua0=[];
ue0=-25:1:25;uc0=[];ut0=0:1:35;
for ut00=0:1:35
ut=ut00;
```

```
[Fp,M0] = tuijin(ut);
C0=[M0+ro*Sc*u*si1+1/2*ro*norm(v,2)^2*Sv*si5*zv;Fp*zp+ro*Sc*u*si2+1/2*r
o*norm(v,2)^2*Sh*xh*si4;Fp*xp+ro*Sc*u*si3+1/2*ro*norm(v,2)^2*Sv*si5*xv];
ucd=1/(-K(3,2))*(C0(3,1)+uav*K(3,6));
ua=1/(-K(1,4))*(C0(1,1)+K(1,2)*ucd+uav*K(1,6));
ucd0=[ucd0 ucd];ua0=[ua0 ua];
uchang=[];
for ue=-25:1:25
uc=1/(-K(2,1))*(C0(2,1)+K(2,2)*ucd+K(2,3)*ue+ueh*K(2,5));
uchang=[uchang uc];
end
uc0=[uc0;uchang];
if(ut00==0)
C00=C0;
end
end
subplot(1,2,1)
plot3(log10(ua0(1:5)),log10(ucd0(1:5)),ut0(1:5),'k*')
xlabel('Ua')
ylabel('Ucd')
zlabel('Ut')
grid on
subplot(1,2,2)
grid on
[X,Y]=meshgrid(ut0(1:16),ue0);
meshc(X,Y,uc0(1:16,:)')
xlabel('Ut')
ylabel('Ue')
zlabel('Uc')
axis([0,16,-25,25,0,30])
shading flat
subplot(1,2,2)
view([-57.0 38.0])
```

(2) high speed

```
clear clc
format long
ro=0.9096;u=180;v=[179 2];
Sc=6^2*pi;Sh=1;Sv=0.5;xh=-3;xv=-3;xp=-3.5;zv=0.2;zp=-0.2;
yita=norm(v,2)/u*10;
[si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita);
K(1,:)=[0 ro*Sc*u*bi2 0 ro*Sc*u*bi1 0 1/2*ro*norm(v,2)^2*Sv*bi8*zv ];
K(2,:)=[ro*Sc*u*bi4 ro*Sc*u*bi5 ro*Sc*u*bi3 0 1/2*ro*norm(v,2)^2*Sh*xh*bi7
0];
```

```
K(3,:)=[0 ro*Sc*u*bi6 0 0 0 1/2*ro*norm(v,2)^2*Sv*bi8*xv];
I=[8000 20000 25000];
ut=0;[Fp,M0] = tuijin(ut);
C0=[M0+ro*Sc*u*si1+1/2*ro*norm(v,2)^2*Sv*si5*zv;Fp*zp+ro*Sc*u*si2+1/2*r
o*norm(v,2)^2*Sh*xh*si4;Fp*xp+ro*Sc*u*si3+1/2*ro*norm(v,2)^2*Sv*si5*xv];
uav0=-25:5:25;ucd0=[];ua0=[];
for uav=-25:5:25
ucd=1/(-K(3,2))*(C0(3,1)+uav*K(3,6));
ua=1/(-K(1,4))*(C0(1,1)+K(1,2)*ucd+uav*K(1,6));
ucd0=[ucd0 ucd];ua0=[ua0 ua];
end
subplot(1,2,1)
plot3(ua0,ucd0,uav0,'b-')
xlabel('Ua')
ylabel('Ucd')
zlabel('Uav')
axis([-25,25,-25,25,-25,25]);
grid on
subplot(1,2,2)
for j=1:1:11%ucd
ucd=ucd0(1,j);%ucd
ue0=-25:25;ueh0=-25:25;
[X0,Y0]=meshgrid(ue0,ueh0);
uc=1/(-K(2,1))*(C0(2,1)+K(2,2)*ucd+K(2,3)*X0+Y0*K(2,5));
meshc(X0,Y0,uc);
subplot(1,2,1)
view([54.3 53.4])
subplot(1,2,2)
view([-135.8 43.8])
subplot(1,2,2)
hold on
xlabel('Ue')
ylabel('Ueh')
zlabel('Uc')
axis([-25,25,-25,5,0,30]);
shading flat
end
```

6. Result of question 4

```
clear clc
format long
ro=0.9096;u=180;v=[80 2];
Sc=6^2*pi;Sh=1;Sv=0.5;xh=-3;xv=-3;xp=-3.5;zv=0.2;zp=-0.2;
yita=norm(v,2)/u*5;
```

```
Me=1000;a=5;
detat=0.1;
ucd=-2.053;ua=-1.920;uav=0;
ueh=0;uc=0;ue=-10.97;


yita=norm(v,2)/u*10;
[si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita);
[Fp,M0] = tuijin(ut);
C0=[M0+ro*Sc*u*si1+1/2*ro*norm(v,2)^2*Sv*si5*zv;Fp*zp+ro*Sc*u*si2+1/2*r
o*norm(v,2)^2*Sh*xh*si4;Fp*xp+ro*Sc*u*si3+1/2*ro*norm(v,2)^2*Sv*si5*xv];
Czan=C0(2,1);


uavt=[uav];ucdt=[ucd];uat=[ua];%one
ueht=[ueh];uct=[uc];uet=[ue];%two
for g=0.1:detat:19.5
ut=floor(1/20*g^2)/5;
v(1)=v(1)+a*detat;
yita=norm(v,2)/u*10;
[si1,si2 ,si3,si4,si5,bi1,bi2,bi3,bi4,bi5,bi6,bi7,bi8 ] = zhuanhan(yita);
K(1,:)=[0 ro*Sc*u*bi2 0 ro*Sc*u*bi1 0 1/2*ro*norm(v,2)^2*Sv*bi8*zv ];
K(2,:)=[ro*Sc*u*bi4 ro*Sc*u*bi5 ro*Sc*u*bi3 0 1/2*ro*norm(v,2)^2*Sh*xh*bi7
0];
K(3,:)=[0 ro*Sc*u*bi6 0 0 0 1/2*ro*norm(v,2)^2*Sv*bi8*xv];
[Fp,M0] = tuijin(ut);
C0=[M0+ro*Sc*u*si1+1/2*ro*norm(v,2)^2*Sv*si5*zv;Fp*zp+ro*Sc*u*si2+1/2*r
o*norm(v,2)^2*Sh*xh*si4;Fp*xp+ro*Sc*u*si3+1/2*ro*norm(v,2)^2*Sv*si5*xv];
M=K*[uc ucd ue ua ueh uav]'+C0;
M1zan=sqrt(M(1)^2+M(3)^2);
M2zan=M(2);


uavqian=uav;ucdqian=ucd;uaqian=ua;uehqian=ueh;ucqian=uc;ueqian=ue;
xishu1=[0 0];u11=[];
xishu2=[0 0];u22=[];
for j=1:1:2
data1=0.00001;
data2=0.00001;
for n=1:10000%%uav(ueh),ua， ucd(uc,ue)
uav=uav+(j*2-3)*data1;ueh=ueh+(j*2-3)*data2;%ueh uav new
ucd=1/(-K(3,2))*(C0(3,1)+uav*K(3,6));ua=1/(-K(1,4))*(C0(1,1)+K(1,2)*ucd+uav*
K(1,6));
uc=max(ucqian+(-1/(K(2,1)+K(2,3))*(K(2,5)*ueh+C0(2,1)-Czan)),0);ue=ueqian+(-
1/(K(2,1)+K(2,3))*(K(2,5)*ueh+C0(2,1)-Czan));
M=K*[uc ucd ue ua ueh uav]'+C0;
```

```
if(sqrt(M(1)^2+M(3)^2)>M1zan)
uav=uav-(j*2-3)*data1;
data1=data1/2;
end
if(M(2)>M2zan)
ueh=ueh-(j*2-3)*data2;
data2=data2/2;
end
M1zan=sqrt(M(1)^2+M(3)^2);M2zan=M(2);
end
u11=[u11;uav,ucd,ua];
u22=[u22;ueh,uc,ue];
xishu1(j)=abs(uav-uavqian)*pingjia(0,norm(v,2))+abs(ucd-ucdqian)*pingjia(1,nor
m(v,2))+abs(ua-uaqian)*pingjia(1,norm(v,2));
xishu2(j)=abs(ueh-uehqian)*pingjia(0,norm(v,2))+abs(uc-ucqian)*pingjia(1,norm(v,
2))+abs(ue-ueqian)*pingjia(1,norm(v,2));
end
if(xishu1(1)<xishu1(2))
j1=1;
else
j1=2;
end
if(xishu1(1)<xishu1(2))
j2=1;
else
j2=2;
end
Czan=C0(2,1);
uav=u11(j1,1);ucd=min(u11(j1,2),25);ua=min(u11(j1,3),25);
uavt=[uavt uav];ucdt=[ucdt ucd];uat=[ uat ua];
ueh=u22(j2,1);uc=u22(j2,2);ue=u22(j2,3);
ueht=[ueht ueh];uct=[uct uc];uet=[ uet ue];
end
t=0.1:detat:19.5;t=[0 t];
subplot(1,3,1)
plot(t,ucdt,t,uat,t,uct,t,uet)
xlabel('time/second')
legend
grid on
subplot(1,3,2)
plot(t,uavt*100,t,ueht)
xlabel('time/second')
legend
grid on
```

```
subplot(1,3,3)
plot(t,floor(1/20*t.*t))
xlabel('time/second')
legend
subplot(1,3,1)
legend({'data1','data2','Uc','Ue'})
subplot(1,3,1)
legend({'Ua','Ucd','Uc','Ue'})
subplot(1,3,2)
legend({'Uav*100','Ueh'})
subplot(1,3,3)
legend('Ut')
```

7. Verification of question 1 and question 2

```
clear clc
format long
I=[8000 20000 25000];
v=[80 0.2];
ua=-2.0743;ucd=-2.1552;ue=-3.4817;uc=0;uav=4.1869/(10^7);ueh=-9.0772/(10^7);
ut=0;
ukong=[ua ucd ue uc uav ueh];
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ua,ucd,ue,uc,uav,ueh,ut,v);
Mshang=M;
raodong=0.1;
rao=zeros(100,3);
for i=1:100
ukong=ukong+(rand(1,6)*2-1)*raodong;
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ukong(1),ukong(2),ukong(3),ukong(4),ukong(5),
ukong(6),ut,v);
rao(i,:)=(M-Mshang)./I;
end
plot(1:1:i,rao(:,2),'o')
hold on
plot([0,100],[0,0],'k-',"LineWidth",2)
legend
xlabel('Random numer')
ylabel('Angular acceleration after disturbance')
grid on
legend({'Random result','Upper limit of judgement'})
figure
plot3(rao(:,1),rao(:,2),rao(:,3),'k*')
legend
hold on
plot3(0,0,0,'bo',"MarkerSize",30)
```

```
  xlabel('Angular acceleration--X')
  ylabel('Angular acceleration--Y')
  zlabel('Angular acceleration--Z')
  grid on
  legend({'Random rusult','Zero point'})
  subplot(1,3,1)
  plot(rao(:,1),rao(:,2),'r*')
  legend
  xlabel('Angular acceleration--X')
  ylabel('Angular acceleration--Y')
  hold on
  plot(0,0,'ko',"MarkerSize",30)
  grid on
subplot(1,3,2)
  plot(rao(:,1),rao(:,3),'b*')
  legend
  xlabel('Angular acceleration--X')
  ylabel('Angular acceleration--Z')
  hold on
  plot(0,0,'ko',"MarkerSize",30)
  grid on
subplot(1,3,3)
  plot(rao(:,2),rao(:,3),'g*')
  legend
  xlabel('Angular acceleration--Y')
  ylabel('Angular acceleration--Z')
  hold
  plot(0,0,'ko',"MarkerSize",30)
  grid on
  subplot(1,3,1)
  legend({'Random result','Zero point'})
  subplot(1,3,2)
  legend({'Random result','Zero point'})
  subplot(1,3,3)
  legend({'Random result','Zero point'})
```

8. Verification of question 3

```
  clear clc
  format long
  v0=[80,2;179,2];
  for tt=1:2
  subplot(1,2,tt)
  I=[8000 20000 25000];
  v=v0(tt,:);
```

```
if(tt==1)
uc=0;ucd=-2.053;ue=-10.97;ua=-1.920;ut=0;ueh=0;uav=0;%low best
else
uc=0;ucd=0;ue=0;ua=-0.8143;ut=0;ueh=-6.289;uav=-0.9279;%低 high best
end
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ua,ucd,ue,uc,uav,ueh,ut,v(1,:));Mshang=M;
jsd=Mshang./I
ukong=[ua ucd ue uc uav ueh];
raodong=0.1;
rao=zeros(1,100);
for i=1:100
ukong=ukong+(rand(1,6)*2-1)*raodong;
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ukong(1),ukong(2),ukong(3),ukong(4),ukong(5),
ukong(6),ut,v);
rao(1,i)=norm((M-Mshang)./I,2);
end
plot([1:i],rao,'o')
hold on
plot([1 100],[1 1]/1000/2,'b-',"LineWidth",2)
grid on
ukong=[ua ucd ue uc uav ueh];
raodong=0.1;
rao=zeros(1,100);
for i=1:100
ukong=ukong+(rand(1,6)*2-1)*raodong;
[M,Mc,Mh,Mv,Mp]=zhuangtailiju(ukong(1),ukong(2),ukong(3),ukong(4),ukong(5),
ukong(6),ut,v);
rao(1,i)=sum(abs((M-Mshang))./I);
end
plot([1:i],rao,'o')
xlabel('Random number')
ylabel('Evaluation data')
hold on
plot([1 100],[0 0]/1000/2,'y-',"LineWidth",2)
grid on
end
subplot(1,2,2)
legend({'Square difference','Acceptable square difference','Angular acceleration
after disturbance','The best angular acceleration'})
```