Problem Chosen	2023	Team Control Number
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Integrated cost-effectiveness and washing efficiency and laundry strategy optimization

Summary

Scientific research includes not only the exploration of high-end technologies and new fields, but also the study of common life topics, such as scientific washing. Optimizing the washing process not only contributes to resource utilization and environmental protection, but also significantly improves economic efficiency and washing efficiency. Effective washing programs help to save costs, especially by analyzing the costs of different detergents and water, and can provide cost-effective solutions. Optimizing the washing process helps develop smarter, energy-efficient and environmentally friendly washing solutions, providing better services to consumers and businesses, and promoting rational resource utilization and environmental sustainability.

For Problem 1, the core of the problem is to find the optimal number of washes and amount of water used per wash to minimize the amount of dirt after washing, given a fixed initial amount of dirt and available water. For this multi-objective optimization problem, two assumptions are made: (1) the amount of water used for each wash follows a fixed distribution, and (2) the solubility represents the proportion of the amount of dirt dissolved after the previous wash by each wash. Based on the assumptions, the problem is further simplified to an optimization problem of minimizing the number of washes under the premise that the clothes are washed clean. The model construction and solution results show that the amount of dirt gradually decreases as the number of washes increases. Setting the objective of reducing the dirt to one-tenth of the initial amount, three washes can be achieved. The article analyzes the influence of the values of initial water volume W, initial dirt volume D_0 and A_1 on the optimal number of washing times and the amount of residual dirt by plotting.

For Problem 2, the limitation on the amount of water available is eliminated from Problem 1, i.e., the number of washes can be increased indefinitely. In this case, the limitation of the maximum number of washing times N is eliminated and the threshold is set to $D_k \leqslant 0.001D_0$. In order to investigate the effects of the initial solubility A_1 and the solubility decay parameter m on k as well as D_k, we use the simulated annealing algorithm to analyze the nonlinear relationship between the parameters and plot the optimal k as well as the variation of D_k under different values of A_1 and m in a 3d plot, and the optimal k values under certain values of optimal k. In addition, the paper also plots the sensitivity of the variation of A_1 and m to the variation of k for different intervals.

For Problem 3, we considered cost and efficiency in the washing task, set the relevant washing parameters by reviewing the relevant literature, and then improved the clustering algorithm to cluster the pollutant contents of different clothes, so as to ensure that the distribution of pollutant contents of clothes in the same cluster is as similar as possible, and that those in different clusters are as dissimilar as possible, and utilized the direct search method to configure the different categories of clothes with the most efficient The most efficient detergents are allocated to different categories of clothes using a direct search method. Two different strategies were analyzed in comparison: washing efficiency only, and weighted average washing efficiency combined with detergent cost. The final result is that the first and the third detergents can fit most of the categories of laundry with a total cost of 116.37 when only considering the washing efficiency, and the second and the third detergents are preferred when combining the efficiency and the effectiveness, and their cost is 48.36. Compared to the former, the cost is significantly lower, which shows that our strategy is effective.

For Problem 4, compared to Problem 3, in Problem 4 we face a new scenario of adding the mixing restriction of clothing materials, so we need to further optimize the algorithm of Problem 3. For clustering effectiveness, we first introduce the acceptance parameter, which represents the total number of clothes that can be mixed-washed for a certain material of clothes, and select random clothes in the material with the lowest acceptance as the initial point, and invoke the improved clustering algorithm with random sampling to cluster all the clothes to be washed, and then weight the average washing efficiency combined with the cost of detergent to assign the most suitable detergent to them, and finally output the different categories of clothes in the combinations as well as washing schemes, and calculates its total cost as 445.41, which shows that the benefits of our algorithm provide a highly efficient and effective washing scheme.

Keywords: cost-effectiveness, simulated annealing, stochastic search, clustering

Content

1. Introduction

1.1 Background

Washing and cleaning are common activities in people's daily lives. The stain-removing capabilities of laundry detergent or liquid laundry detergent derive from the surfactant chemistry in them, which remove dirt particles by increasing the permeability of water and utilizing an intermolecular electrostatic repulsion mechanism. The laundering process weakens the molecular forces that maintain the bonding between water molecules (causing the water to form droplets similar to those encapsulated in an elastic membrane), allowing the water molecules to penetrate between the surface of the object being cleaned and the dirt particles. The mechanical action of the washing machine or manual scrubbing can dislodge the dirt particles attached to the surface surrounded by surfactant molecules, which in turn adhere to the lipophilic part of the surfactant molecules. This results in the removal of dirt still suspended on the surface of the object during the rinsing phase.

In practice, whether it is a small-scale laundry at home or a large-scale washing operation in hotels and professional organizations, one of the issues to be considered is how we can achieve a clean and tidy laundry at a lower cost [1]. This seemingly simple life problem actually contains profound mathematical principles.

1.2 restatement of the problem

1.Given the amount of dirt on a garment D and the total amount of water available W. The solubility of dirt in water Ak varies with the number of washes k, satisfying a_1=0.8 and a_1=0.5*A_{k-1}. The objective is to find a washing scheme, including the number of washes k and the amount of water used each time W_k, that minimizes the remaining amount of dirt D_k, after the nth wash (and is not negative). It is also important to discuss how variations in solubility A_k, initial fouling volume D, and water availability W affect the washing effect.

2.Under the condition that each washing time is the same and the amount of water is unlimited, our goal is to design a cleaning schedule that ensures that the final dirt residue is no more than one-thousandth of the initial dirt volume, and to analyze how the solubility a_k and the initial dirt volume D_0 affect the optimal washing schedule.

3. The amount and type of dirt on 36 pieces of clothing is given and there are 10 different detergents available that have different solubilities and unit prices. Considering that the cost of water is 3.80 per ton, we need to come up with a cleaning solution that is both cost effective and has good cleaning results.

4. Based on the third question, develop a cost-effective and efficient cleaning program that takes into account the type and amount of dirt in garments made of different materials as well as the requirement that certain garments should not be

mixed and washed.

2. Problem analysis

2.1 Analysis of question one

Problem 1 is concerned with achieving the lowest amount of residual soiling and the smallest number of washes for a fixed amount of soiling and water usage. For this purpose, the following steps are carried out in this paper:

1. Two discussions on solubility A_k: This paper firstly discusses the relationship between the key variable in the washing process, i.e., the amount of water used in the kth wash, W_k, and the solubility A_k, and secondly, the paper explores the specific meaning of solubility, i.e., whether it refers to the initial dirt D_0 or to the residual dirt D_{k-1} after the previous wash.

2. Establishing assumptions and proving their rationality: Based on the above discussion, two assumptions are proposed in this paper and their rationality is verified. Assumption 1: the kth washing water consumption W_k obeys a fixed distribution; Assumption 2: the solubility a_k represents the proportion of dissolution to the amount of dirt D_{k-1} at the beginning of that washing.

3. Transformation of multi-objective optimization problem: By transforming a multi-objective optimization problem into a single-objective optimization problem, the article takes minimizing the number of washing times k under the premise of ensuring that the clothes are washed clean as the objective. By modeling and solving methods, the article analyzes the effects of the values of washing times k, initial water volume W, initial dirt amount D_0 and solubility A_1 on the optimal washing times k and residual dirt amount D_k.

4. Relationship analysis: In this paper, the effects of different values of initial water volume W, initial dirt volume D_0 and solubility A_1 on the optimal number of washes k and residual dirt volume D_k are analyzed by plotting. The results show that the initial water volume W affects k and D_k by influencing the maximum number of washing times N. Specifically, the larger W is, the larger k is and the smaller D_k is. In addition, the initial dirt volume D_0 is inversely proportional to the optimal number of washes k and directly proportional to the residual dirt volume D_k, and the solubility A_1 has the opposite effect to the initial dirt volume D_0.

2.3 Analysis of question two

Problem 2 removes the restriction on the amount of water available, i.e., the maximum number of washes N, based on Problem 1, allowing a theoretically infinite increase in the number of washes. In this regard, the following steps are carried out in this paper:

1. Remove the limit on the number of washes and set a new threshold: Remove the limit on the maximum number of washes N and set the threshold for the amount of

residual dirt to $D_k \leq 0.001D_0$.

2. Application of simulated annealing algorithm: In order to deeply investigate the effects of initial solubility A_1 and solubility decay parameter m on the number of washing times k and the amount of residual dirt D_k, this paper adopts the simulated annealing algorithm. The simulated annealing algorithm, as an effective method for analyzing complex nonlinear relationships, has advantages in exploring complex optimization problems [2].

3. 3D graphical presentation and sensitivity analysis: In this paper, the variation of the optimal number of washing times k and the residual dirt amount D_k at different values of A_1 and m are presented through 3D graphs, and the sensitivity of A_1 and m to the variation of the number of washing times k at different intervals is analyzed. This method presents the effects of parameter variations on the washing process through visualization, which helps to understand the relationship between parameters more intuitively and provides a reliable data base for further research.

2.4 Analysis of question three

Problem 3 focuses on how to achieve the goal of the washing task by integrating cost-effectiveness while ensuring washing efficiency. To solve this problem, the following steps are carried out in this paper:

1. Setting the washing parameters: by reviewing the relevant literature, the key parameters of the washing process were set, such as the amount of water used per wash, the total amount of clothes washed per wash, and the amount of detergent required per wash, which provided the basis for the subsequent steps[3].

2. Improved Clustering Algorithm: The contaminant content on the laundry is classified using an improved clustering algorithm, with the aim of making the contaminant content of the laundry as similar as possible within the same bucket and as different as possible between different buckets, a step which contributes to the efficiency and cost-effectiveness of the washing process[4].

3. Detergent Configuration and Cost Analysis: The most efficient detergents were configured for different categories of laundry through a direct search method. In addition, by comparing and analyzing the two strategies of washing efficiency only and weighted average washing efficiency combined with detergent cost, the optimal detergent selection and cost are derived when efficiency only is considered and when efficiency and effectiveness are combined, respectively.

2.5 Analysis of question four

Problem 4 adds the limitation of mixing and washing of clothing materials on the basis of Problem 3, which requires optimization of the original algorithm to adapt to the new challenge, for which the following steps are carried out in this paper:

1. Algorithm optimization and acceptance parameter introduction: Firstly, the "acceptance" parameter is introduced, which represents the total amount of other clothes that can be accepted by a particular material for mixing with it. On this basis,

the material with the lowest acceptance level is selected as the initial point of the clustering algorithm.

2.Clustering algorithm based on random search: in order to effectively consider the material's mixing and washing limitations, this paper applies the improved random sampling clustering algorithm to classify all the laundry to be washed, which takes the mixing and washing effect into full consideration while taking the material limitations into account[5].

3. Detergent allocation and cost calculation: allocate the most suitable detergent for each category of laundry on the basis of weighted average cleaning efficiency and detergent cost, and finally output the combination of different categories of laundry and washing solutions, calculate the total cost, and analyze the effect of the model.

3. Symbol and Assumptions

Symbol	Symbol meaning
T_k	kth wash true solubility
A_k	kth wash solubility
W_k	Laundry water use for the kth wash
D_k	Amount of residual dirt
N	Maximum number of washes
k	Number of washings
S_i	Solubility of the ith pollutant
d_i	*Content of the ith pollutant*

4. Model for Problem 1

Due to the open-ended nature of Problem 1, we need to make several assumptions in order to proceed with the analysis. The following assumptions are made regarding the meaning of the solubility of dirt in water A_k and the meaning of the initial amount of dirt mentioned in the explanation of the meaning of A_k. The following assumptions are made for the meaning of solubility of dirt in water and the meaning of initial amount of dirt mentioned in the explanation of the meaning.

4.1 For a discussion of the meaning of solubility A_k:

Assumption1：

$$T_k = A_k \cdot W_k / D_{k-1}$$

In the above equation, T_k denotes the true solubility of dirt during the k wash when the amount of laundry water used is W_k and the amount of residual stains is D_k. The meaning of A_k is explained in the title, stating that A_k denotes the proportion of dirt dissolved by an equal amount of detergent relative to the initial amount of dirt during the k wash. As can be seen from the equation, T_k is a value that increases linearly with W_k.

In this problem we can consider water as detergent when the amount of water used for washing is equal to the initial amount of dirt in this washing, i.e.:

$$W_K = D_k$$

this time：

$$T_k = A_k$$

Conforms to the interpretation of the meaning of A_k in the title。

However, when we increase W_k, it follows from the assumptions that the true solubility of dirt D_k during the kth wash will always be greater than 1, which means that as long as the amount of water used in the laundry is sufficiently high, then the clothes will be cleaned in the first wash. Let's illustrate this with an example:

Let the initial amount of dirt before washing be 1, and the proportion of dirt dissolved by an equal amount of detergent relative to the initial amount of dirt during the first wash be 0.8, i.e:

$$A_1 = 0.8, D_0 = 1$$

From Assumption 1, the first wash i.e., when k=1:

$$T_1 = A_1 \cdot W_1$$

① When W_1=1, T_k=0.8, which means that if 1 unit of water is used in the first wash, 0.8 units of dirt can be washed away.

② When W1=1.25, Tk=1, which means that if 1.25 units of water are used for the first wash, 1 unit of dirt can be washed away, i.e., the dirt is washed away completely.

From the above example, we can see that under assumption 1, since A_k is attenuated, we should put all the water into the first washing to achieve the maximum washing benefit regardless of the initial water volume, and we can clean the clothes in the first washing if the ratio of the initial water volume to the initial dirt volume is 1.25. Obviously, hypothesis 1 does not match our actual situation in reality, so we further propose hypothesis 2.

Assumption 2：

$$T_k = A_K$$

The above equation indicates that the true solubility of dirt is then equal to the proportion of dirt dissolved by an equal amount of detergent relative to the initial amount of dirt during the kth wash, and from the above equation, it can be seen that the true solubility of dirt for each wash is independent of the amount of water used for washing, Wk. Therefore, we can set Wk to a fixed value, so that it obeys a certain fixed distribution, and the minimum amount of water used for each wash will always be used with a fixed value, viz:

$$W_K \sim N(\mu, \sigma^2)$$

According to the life experience and relevant literature, this assumption is in line with the reality, such as the washing machine commonly used in the family, usually according to the fixed amount of water we set up in advance for water refilling, the scale of the laundry factory, usually also set up the water program to control the amount of water used under different washing times. Therefore, as long as the initial water availability W is fixed, the maximum number of washable clothes N is a definite value, regardless of whether W_k is a fixed uniform distribution or a decaying distribution. Based on the above, the problem we face becomes a multi-objective optimization problem to find the optimal number of washes and the amount of water to be used for each wash under the premise of ensuring that the clothes can be washed clean and the water consumption is limited. We hope to find an optimal balance between ensuring the cleanliness of the laundry and minimizing the number of washes and water consumption. We analyze the problem in detail below.

4.2 Discussion of the meaning of initial fouling

The meaning of A_k is explained in the title, stating that A_k denotes the proportion of dirt dissolved by an equal amount of detergent during the kth wash relative to the initial amount of dirt, and we make the following assumptions about the initial amount of dirt in this meaning for further discussion.

Assumption 1： This initial amount of fouling is the amount of fouling without washing, then:

$$D_K = D_{K-1} - A_K \cdot D_0$$

We further illustrate Assumption 1 with an example by setting the initial amount of fouling to be 1, i.e:

$$D_0 = 1$$

From assumption 1:

$$D_1 = D_0 - A_1 \cdot D_0 = 0.2$$

$$D_2 = D_1 - A_2 \cdot D_0 = -0.2$$

At this point, the dirt is completely cleaned, so regardless of the value of the initial amount of dirt D_0, the clothes can be cleaned after only two washes. Obviously this conflicts with common sense and subsequent questions, so we reject this assumption.

Assumption 2： This initial amount of dirt is the remaining amount of dirt after k-1 washes D_{k-1},then:

$$D_K = D_{K-1} - A_K \cdot D_{k-1}$$

Summarizing the discussion on the meaning of solubility A_k and the meaning of initial fouling amount, it can be seen that the assumption 1 of both discussions is counter-intuitive, therefore, in the following, we will analyze Problem 1 based on the assumption 2 of these two discussions.

4.3 Problem analysis

Based on the discussion above, the following analysis is based on the following two assumptions:

Assumption 1：

$$T_k = A_K, W_K \sim N(\mu, \sigma^2)$$

Assumption 2：

$$D_K = D_{K-1} - A_K \cdot D_{k-1}$$

In the above equation, Assumption 1 indicates that the true solubility of the kth wash is independent of the amount of water added, and that the amount of water used for each wash, W_k, is a fixed value or is set in accordance with a certain procedure to satisfy that the amount of water used for each wash obeys a fixed distribution, so that the amount of water used for each wash always meets the requirement of the maximum solubility, while minimizing the amount of water used. Assumption 2 indicates that the initial dirt amount in the meaning of A_k is the remaining dirt amount D_{k-1} after k-1 washes.

Based on the above assumptions, the amount of water available is W, the amount of water used per time Wk obeys a fixed distribution, and the maximum number of washings N is fixed, at this time, the problem we face becomes a multi-objective optimization problem to investigate the optimal number of washings and the amount of water used per washings under the premise of ensuring that the clothes can be cleaned and the amount of water used is limited. We hope to find an optimal balance between ensuring the cleanliness of the clothes and minimizing the number of washes and water consumption.

In the following, we build a model to solve the problem:

initial conditions：

$$D_0 = D_0, A_1 = 0.8$$

recursive equation：

$$D_K = D_{K-1} - A_K \cdot D_{k-1}$$

$$A_K = 0.5 A_{K-1}$$

The general expression for D_k with respect to the number of washes k is given by:

$$D_k = D_0 \cdot \prod_{i=1}^{k} (1 - A_1 \cdot 0.5^{i-1})$$

From the above equation, it can be seen that the expression is a decreasing function, and the amount of residual dirt decreases with the increase in the number of washes.

The final mathematical model is obtained as follows:

$$\min k$$

s.t

$$D_k = D_0 \cdot \prod_{i=1}^{k} (1 - A_1 \cdot 0.5^{i-1})$$

$$D_K < 0.001 D_0$$

$$0 \le k \le N, k \text{ is a whole number}$$

According to the given model, we need to find a minimum value within the range of the number of washing times k such that the concentration of stains in the washed clothes does not exceed one thousandth of the initial concentration during these k washing cycles. We solve the model by setting N to 100000, and the final solution finds that the optimal number of washes k is still equal to the maximum number of washable times N. In other words, no matter how the number of washes is adjusted, the finite amount of water will always fail to satisfy the requirement of one-thousandth of a percent of the cleanliness in the possible cycles. This conclusion suggests that the current system is unable to achieve the required cleaning standard under the given model and conditions.

In order to get the optimal benefit of the number of washes we output the variation of residual dirt amount D_k with k for the kth wash in the following graph:

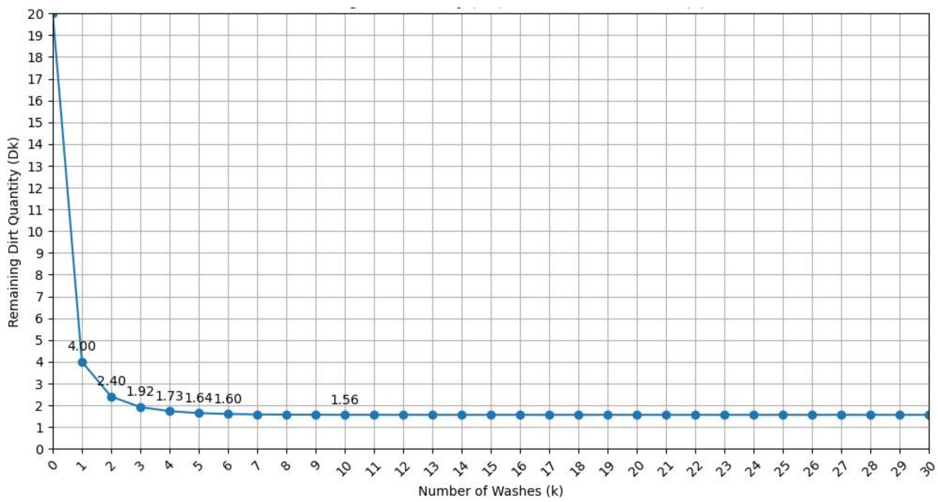

Figure 1 remaining dirty quantity Dk vs. Number of washes k

We set the initial amount of dirt before washing D_0=20,the initial solubility A_1=0.8,the maximum number of washable times N=50.In the above chart, we can observe that the remaining amount of dirt, D_k, is reduced from 20 to 1.6 after 6 washings, and the amount of dirt can be reduced to one-tenth of the original amount by performing another 3 washings. However, as the number of washes increases, the dirt reduction effect becomes smaller and smaller, indicating that with an initial dirt amount of 20, the dirt can already be significantly reduced by 3 washes, and that further increase in the number of washes has limited enhancement of the cleaning effect, so based on the logical analysis, 3 washes is a more feasible option in this case, which can achieve the desired cleaning in a shorter time.

In order to analyze the effect of various factors on D_k, we set k to 50, and based on the observation of the above function, the clothes are adequately cleaned when k takes the value of 50. On the premise that the clothes can be adequately cleaned, the effect of A1 and D_0 values on D_k is plotted as follows:

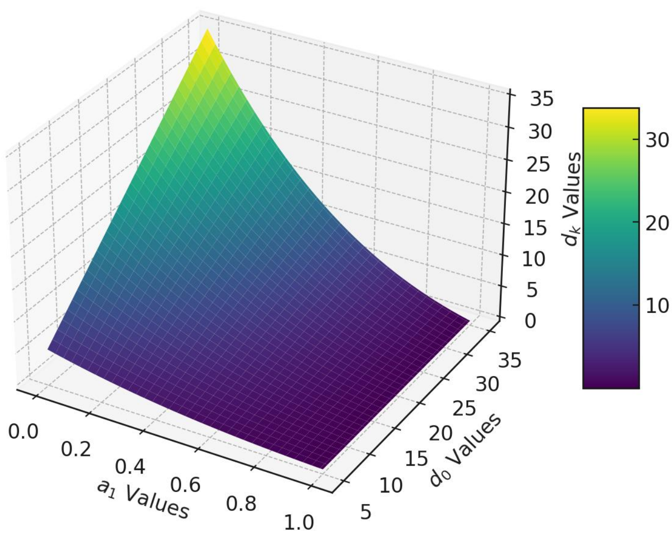

Figure 2 3D plot of D_k varying A_1 and D0 with k=50

Based on the above graph we found that:

(1) When A_1 is higher, more dirt can be dissolved per wash, so for the same number of washes k, a higher value of A_1 will result in a lower D_k, which means a more efficient wash and more complete dirt removal.

(2) When D_0 is higher, the total amount of dirt remaining after cleaning, D_k, will be more even if the dissolution rate is the same. This is because the initial amount of contaminants is large, and even if the ratio goes up to remove a lot, there is still more residue in absolute amount.

In the following, in order to further analyze the initial amount of dirt, the initial amount of water, the initial solubility on the optimal number of times k and D_k of cleaning, we divided into three points to analyze and discuss them in detail.

1.Initial fouling volume D0 influence

In the following, we further explore the effect of the initial amount of dirt on the efficiency curve of the number of cleaning, for the convenience of plotting, we fix A_1=0.5 and the amount of dirt takes the values of 5, 10, 15, 20, 25, 30, 35, and get the following figure:

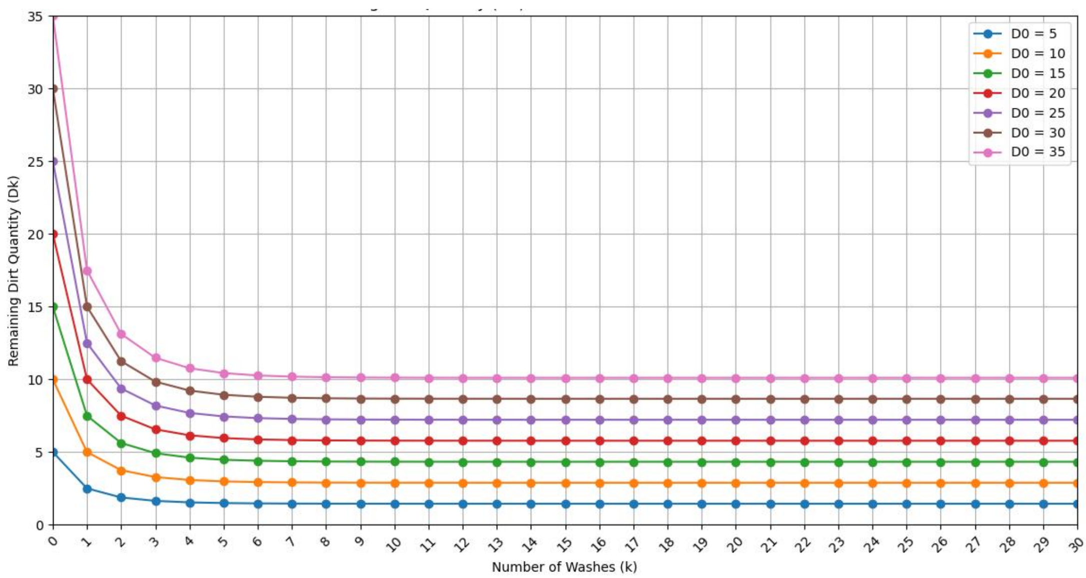

Figure 3 remaining dirt quantity D_k vs. Number of washes for D_0

As can be seen from the above figure, the smaller the initial amount of dirt D0, the faster the function converges, and if we reduce the dirt content to one-tenth of the original amount of dirt as a threshold, we find that for different values of D0, carrying out three cleanings can reach that threshold, and when we adjust A1 back to 0.8, we output the amount of different residual pollutants D3 after three cleanings and find that, although the three cleanings can all reach the original dirt one-tenth of the original dirt, but for the same number of cleanings, the higher the D0, the higher the amount of contaminant after the same number of cleanings.

We output the number of washes and dk that reach the threshold at different D0 as follows:

Table 1 k-values and Dk-values with different D0

D0	k_value	dk

5	3	0. 48
10	3	0. 96
15	3	1. 44
20	3	1. 92
25	3	2. 4
30	3	2. 88
35	3	3. 36

We find that the optimal number of washes to reach the threshold remains the same corresponding to different D_0s, and we can guess that the change in D_0 is independent of the optimal k (a proof will be given below), but that dk increases as D_0 increases.

2.W effect of initial water volume

It can be inferred that the initial amount of water affects the maximum number of washes N and thus the optimal number of washes, which in turn affects the final washing result. If the amount of water available increases, the maximum number of washes increases, and the amount of dirt in the final wash, D_k, decreases accordingly. Therefore, on this basis, we can conclude that increasing the amount of water available for washing is an effective way to improve the washing result.

3.Impact of A_k

due to:

$$a_k = m \cdot a_{k-1,} a_1 = 0.8, m = 0.5$$

Since therefore when we explore the impact of A_k, we can do so in terms of the different values of m taken by A_1 as well.

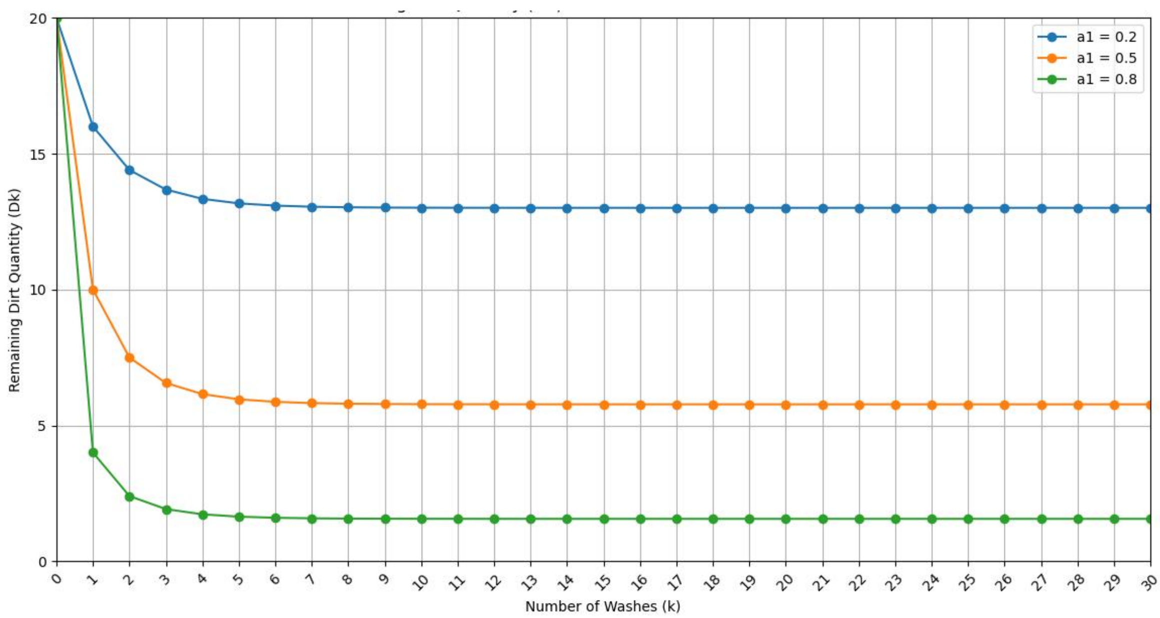

Figure 4 Remaining dirt quantity Dk vs. Number of washes for different A_1

According to the above figure, it can be observed that when the value of dirt removal efficiency A_1 is higher, the washing curve becomes steeper and the dirt reduction is faster. At the same time, increasing the value of A_1 leads to a decrease in the amount of dirt remaining after washing, D_k. In other words, the magnitude of initial solubility affects the efficiency of dirt dissolution and the degree of complete dissolution of dirt after washing. Therefore, we can conclude that the magnitude of the initial solubility has a significant effect on the efficiency of dirt removal and the cleaning effect after washing, specifically, increasing the initial solubility increases the efficiency of dirt dissolution and the degree of dirt dissolution after sufficient washing.

In order to further determine the minimum number of washings k required for D_K to reach the threshold D_k<=D_0/10 for different values of A_1, we calculate by using the formula: replace 0.8 with a1

$$d_0 \cdot \prod_{i=1}^{k}(1-0.8\cdot 0.5^{i-1}) \le \frac{d_0}{10}$$

Since d0 exists on both sides, we eliminate it, and this elimination operation illustrates a further validation of the above conjecture that the optimal number of washes, k, takes a value independent of D_0. The elimination of D_0 yields:

$$\prod_{i=1}^{k}(1-0.8\cdot 0.5^{i-1}) \le \frac{1}{10}$$

Calculating the above equation yields the following table:

Table 2 k-values with different A_1

A_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k	∞	∞	∞	∞	∞	∞	∞	∞	3	1	1

Because of the arithmetic problem, so in this paper we set above 1000 to infinity, we find that the optimal number of cleanings k rises exponentially as A_1 decreases.

5. Model for Problem 2

5.1 Problem analysis

On the basis of the first question $a_k = m \cdot a_{k-1,} a_1 = 0.8, m = 0.5$, we have obtained that the threshold of one in a thousand can be reached only for nearly infinite cycles. So we will next discuss the value of k that reaches the threshold for different values of A1,m.

In that analysis we still follow the assumptions of question one:

Assumption 1:

$$T_k = A_K, W_K \sim N(\mu, \sigma^2)$$

Assumption 2:

$$D_K = D_{K-1} - A_K \cdot D_{k-1}$$

5.2 Simulated annealing algorithm

5.2.1 Algorithm fitness analysis

We chose to use the simulated annealing algorithm to study the optimal number of times k for different values of A_1, m for the following reasons:

(1) Complexity of the problem

We are faced with an optimization problem where the objective is to minimize the number of cleanings for a specific fouling removal effect, and since the effects of parameters and pairs are nonlinear and quite complex, traditional optimization methods may have difficulty in finding an effective solution, especially when the parameter space is large, and the traditional methods may fall into the problem of searching with low efficiency. Therefore, in our study, we choose to use the simulated annealing algorithm, whose stochastic nature and the ability to regulate the temperature policy enable it to efficiently search for optimal solutions in large-scale parameter spaces.

(2) Search for global optimal solutions

The simulated annealing algorithm is able to effectively avoid falling into local optimal solutions and search for global optimal solutions by performing random search at high temperatures and by gradually decreasing the temperature in order to reduce the probability of accepting suboptimal solutions. For the problem we face, i.e., finding the optimal solution to minimize the number of cleaning in a large range of parameter space, this method is very suitable.

(3) Flexibility and Adaptability

The simulated annealing algorithm is a flexible optimization algorithm, whose core idea is to explore the solution space based on the strategy of random search and gradual cooling, and gradually reduce the probability of accepting suboptimal solutions. The algorithm is highly adjustable, and its parameters, such as initial temperature and cooling rate, can be adjusted according to the characteristics and structure of the problem, thus adapting to problems of different scales and complexity and being able to find the optimal solution.

5.2.2 Simulated annealing algorithm setup

(1) Parameter settings:

Initial temperature: set to a relatively high value (e.g., 10000) to allow the algorithm to explore a wider range of the solution space at an early stage.

Cooling rate: set to a moderate value (e.g., 0.99) to ensure that the algorithm has enough time to explore the solution space while converging gradually.

Range of m values for A1 sum: avoid endpoint values, usually between 0.01 and 0.99.

Maximum number of cycles L = 1000

(2) Algorithm flow

Initialization: an initial solution X0 is randomly generated, here X0 contains two variables A1 and m, representing the initial solubility and solubility decay coefficient, respectively.

Loop start: set the current solution as the best solution X_{best}=X0, initialize the temperature Tk, and set the number of iterations k=0.

Loop body: sub-L iterative search at temperature Tk:

Generate a new solution Xnew based on the current solution Xk, which is reached by slightly modifying the values of A1 and m.Calculate the fitness value of the new solution $f(X_{new})$, the new A1 and m correspond to the minimum number k of cleanings.

If $f(X_{new}) < f(X_k)$ then the new solution is accepted and the best solution is updated.

Otherwise,calculate the probability of accepting the new solution $P(t_k) = e(f(X_{new}) - f(X_k))/t_k$ .And determines whether to accept a new solution according to the common metropolis criterion.If $\text{random}(0.1) \ll P(t_k)$ ， then accept the new solution。

temperature drop:

lower the temperature $t_{k=1} = t_k \cdot \text{cooling rate}$ ， where cooling rate is the cooling rate, e.g. 0.99.Increase the number of iterations k

Termination condition: the algorithm is stopped when the temperature Tk drops below a certain threshold or reaches a preset maximum number of iterations.

Output: Output X_{best}, the optimal A_1 and m found in the given simulated annealing process, and the corresponding k values.

(1) Algorithm Flowchart

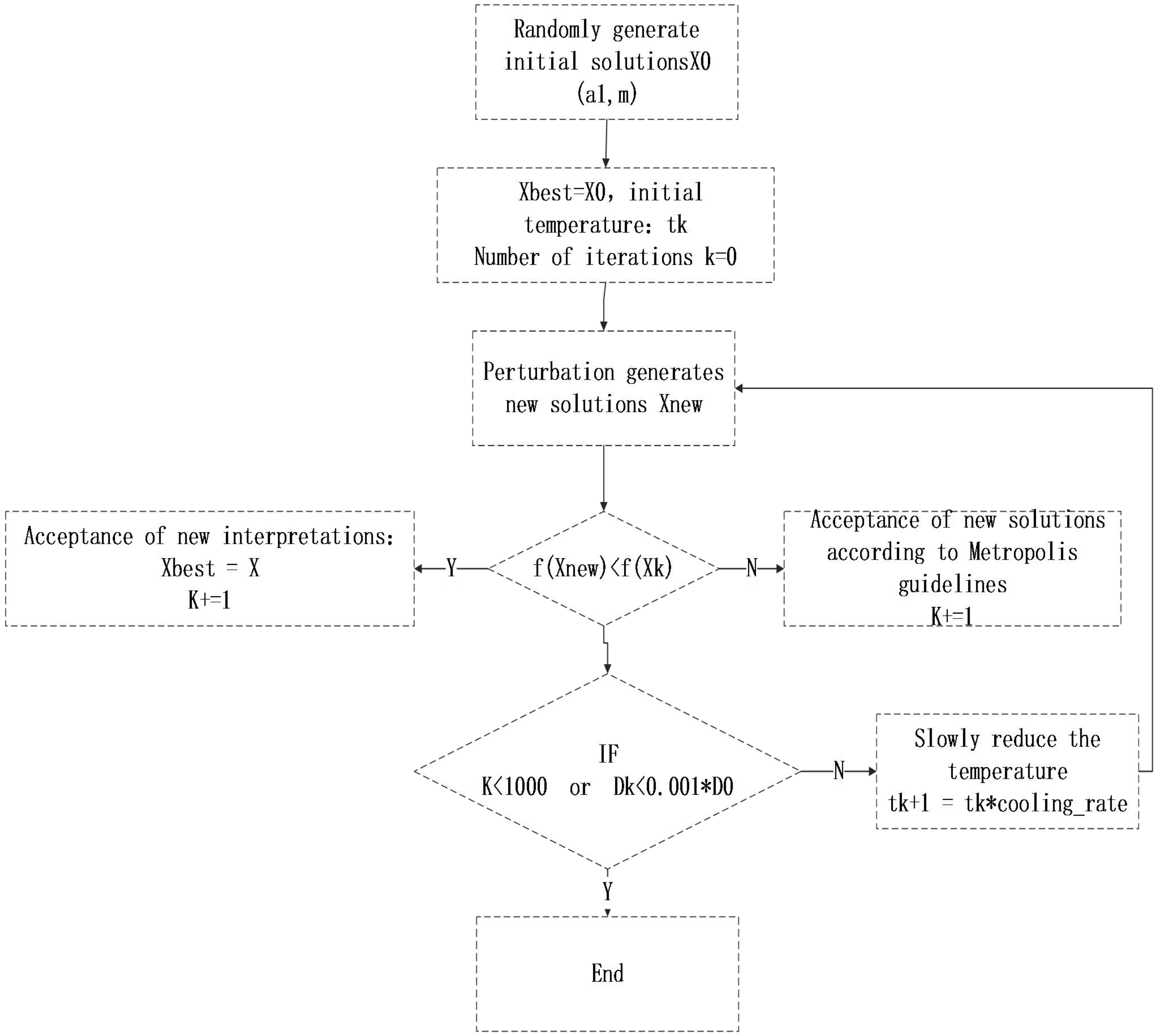

Figure 5 Flowchart of simulated annealing algorithm

（1） Simulated annealing algorithm pseudocode：

```
# Initialize parameters
a1, m = random_initial_values()    # Randomly generate initial values
for a1 and m
X0 = [a1, m]
Xbest = X0.copy()
tk = initial_temperature    # Initial temperature
k = 0    # Iteration count
L = number_of_iterations_at_each_temperature
cooling_rate = 0.99    #    set the cooling rate

# Main loop
while not termination_condition(tk, k):
for i in range(L):
# Generate new solution
Xnew = generate_new_solution(Xk)

# Calculate fitness
fitness_new = calculate_fitness(Xnew)
fitness_current = calculate_fitness(Xk)

# Check if new solution is accepted
if fitness_new < fitness_current:
Xk = Xnew
if fitness_new < calculate_fitness(Xbest):
Xbest = Xnew
else:
# Calculate acceptance probability
P = math.exp((fitness_new - fitness_current) / tk)
if random.uniform(0, 1) < P:
Xk = Xnew
# Decrease temperature
tk *= cooling_rate

# Update iteration count
k += 1
# Output the best solution
Print    Xbest
```

5.2.3 Analysis of results

We randomly selected 30 sets of different combinations of A_1, m and output their k values in the following table:

Table 3 Examples of optimal values of k for different values of A1, m

a1	m	k	a1	m	k	a1	m	k
0.14	0.38	1000	0.5	0.95	17	0.79	0.96	7
0.23	0.16	1000	0.51	0.67	1000	0.8	0.97	6
0.28	0.05	1000	0.54	0.93	19	0.81	0.84	19
0.29	0.8	1000	0.6	0.93	14	0.84	0.9	8
0.33	0.32	1000	0.6	0.91	19	0.84	0.97	6
0.4	0.96	23	0.64	0.44	1000	0.89	0.95	5
0.41	0.98	17	0.72	0.93	10	0.91	0.79	18
0.42	0.45	1000	0.76	0.86	18	0.95	0.9	5
0.48	0.55	1000	0.76	0.9	10	0.97	0.85	5
			0.78	0.85	20	0.98	0.91	4

As can be seen from the above table, k increases with the decrease of A_1 and m. In order to further explore the relationship, we plotted a three-dimensional diagram to show the results of k values calculated by the simulated annealing algorithm under different combinations. Due to the limitation of the computational performance, we limit the number of iterations to more than 1000, and obtain the results under different combinations as the following figure:

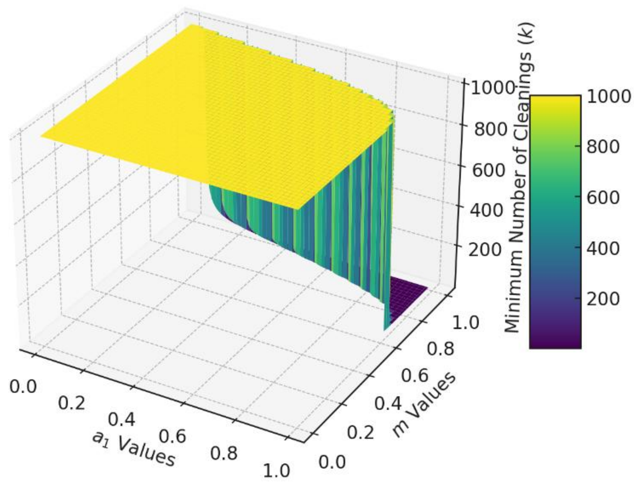

Figure 6 3D plot of minimum k for D_k<=0.001*D_0 with varying A_1 and m

As can be seen from the chart above:

(1) When A_1 is high, more dirt can be dissolved per wash, so fewer washes k are required.

(2) When m is high, the rate of decrease in solubility is slower, so that a high rate of fouling dissolution is maintained as the number of cleanings increases, which usually results in a lower k value.

By combining and analyzing the effect of A_1 values and m values on the minimum number of cleanings, k, in the three-dimensional plot, we can conclude that higher A_1 values and m value combinations usually lead to lower values, indicating higher cleaning efficiency. Conversely, a lower combination of a1 and m values may result in a higher value of k, indicating that more washes are required to achieve the same cleaning effect. Combining the effects of sum values can help us choose the best cleaning strategy to improve cleaning effectiveness and save cleaning time.

5.2.4 sensitivity analysis

In order to analyze the sensitivity of the change of the optimal k value to the change of a1 and m in order to reach the threshold one thousandth of the value, we fixed a1=0.9, m=0.9 (we set it a little higher to prevent it from reaching the threshold even after a sufficient number of iterations, so that the k takes on the value of L=1000), and the number of loops is restricted to within 1000 this (more than 1000 is denoted as 1000), and the number of cycles is restricted to within 1000 this (more than 1000 is denoted as 1000). The result is shown below:

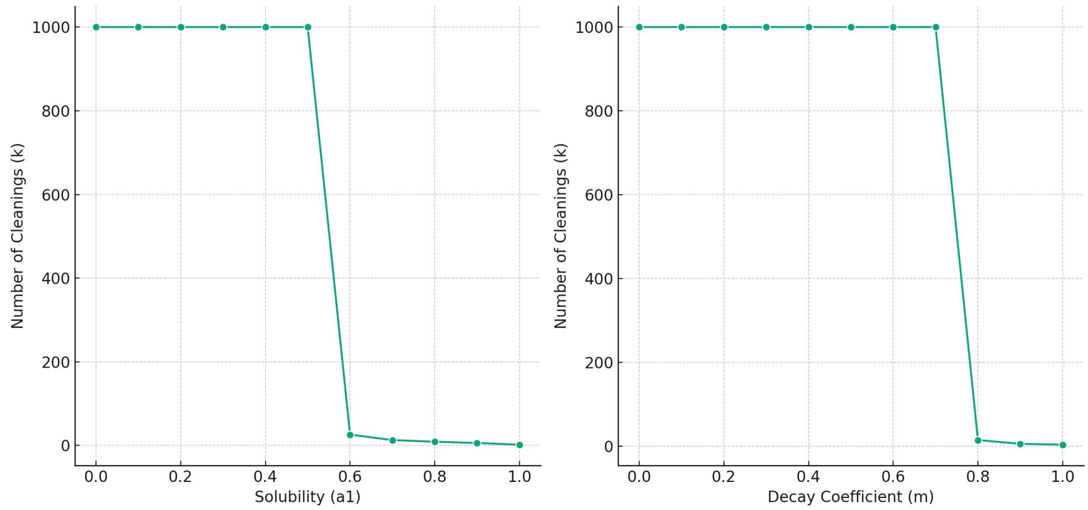

Figure 7 Sensitivity of k to A_1 and Sensitivity of k to m

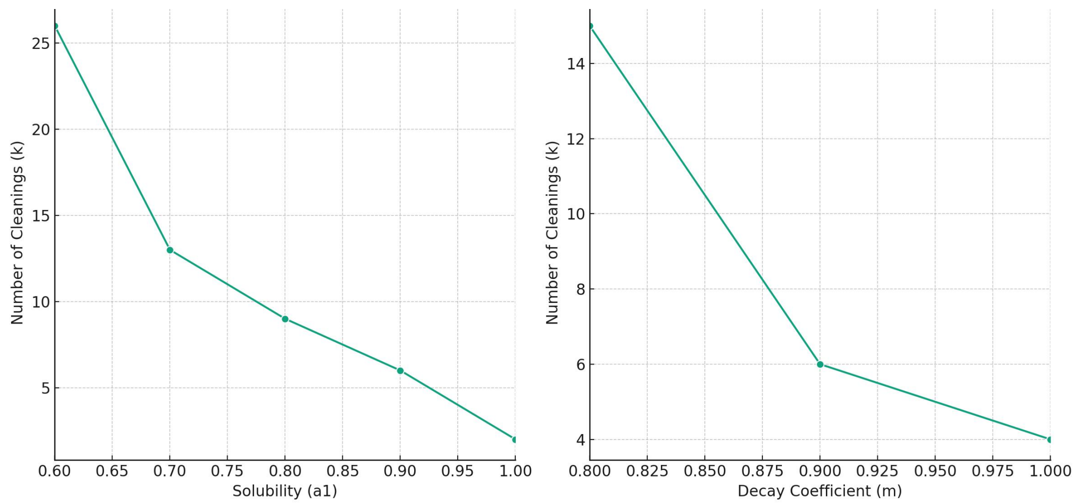

Figure 8 Sensitivity of k to A_1 and Sensitivity of k to m (partial)

It is easy to see from the above graph that both are on a downward trend, which means that the sensitivity to changes in k decreases as the value of both increases. This means that at lower values of both, a slight change can make a huge change in the optimal number of washes k.

6. Model for Problem 3

6.1 Problem analysis

In our problem analysis of the previous two questions, we assumed that the water consumption obeys some kind of fixed distribution, and transformed the limitation on water consumption into a limitation on the maximum number of washes, but since the question needs to take into account the price factor of water, we need to make assumptions on the water consumption per washing, and by reviewing the relevant literature, the water consumption per wash, for example, for the most common washing machines, is approximately between 100-120L. Each wash can wash 5-6 pieces of clothing, and the washing process consists of one wash and three rinses, in the washing process, an average of 21g of detergent will be used, which means an average of 3.5g of detergent per piece of clothing. Considering all these factors, we can define a complete laundry process as 100L of water discharge per wash, washing 6 pieces of clothes, adding 21g of detergent, which means an average of 3.5g/piece. Such a definition can help us better understand the amount of water and detergent used in the laundry process and provide a more accurate basis for subsequent analysis.

Based on the above assumptions, we can cluster the data in Annex 1 by calculating the minimum Euclidean distance, dividing all the data into six classes with six pieces of laundry in each class, which is exactly the amount of one wash. Subsequently, we can select the detergent with the most dissolution efficiency and match the best detergent for different categories of laundry. This classification scheme is in line with realistic scenarios because in daily washing, we seldom use separate detergents for a single piece of laundry and it is unscientific to wash one piece of laundry at a time. Therefore, combining laundry and configuring the best detergent for each combination and then washing 6 pieces of laundry at the same time can achieve a more efficient, practical, economical and environmentally friendly washing process, while minimizing the use of laundry detergent, reducing costs, increasing efficiency, and contributing to the protection and environment and resource conservation.

6.2 Clustering models and search strategies

6.2.1 Pollutant clustering

Aiming at the shortage of conventional clustering algorithms such as K-means in terms of sample size balance, we optimize on the conventional clustering algorithms and adopt a direct search strategy to solve the problem. The main steps are as follows:

(1) Randomly select a sample, i.e., a certain piece of clothing.

(2) Calculate the Euclidean distance between that garment and all other garments where the pollutant content is taken.

(3) Aggregate the five closest clothes to that garment into a cluster.

(4) Subsequently randomly select another clothing from the unselected clothing and repeat steps (2) and (3) above.

(5) Calculate the distance between the centroids of each cluster.

(6) Once again, clothes different from the first step are selected as the initial clustering points, and the above process is continued to calculate the distance between the new centers of each cluster.

(7) Continuously loop, and finally select one of the combination strategy that makes the largest cluster center point as the final combination result.

The above improved clustering algorithm is very flexible and can form clusters dynamically, while avoiding the situation that ordinary clustering may have inconsistent sample sizes in the categories.

After the above clustering, the clothing contained in different categories is obtained as shown in the table below:

Table 4 Laundry and Sorting Strategies for Clothing

Garment	Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8	condition	cluster
1	8	5	2	4	3	1	0	0	1	5
2	3	2	4	5	0	0	0	1	1	1
3	2	8	2	1	2	2	1	0	1	2
4	2	0	5	3	2	1	4	3	1	3
5	2	0	4	7	1	3	4	2	1	4
6	0	0	0	6	5	4	0	0	1	4
7	2	7	4	5	2	2	5	1	1	2
8	1	0	4	3	6	5	2	0	1	4
9	7	2	0	5	3	1	0	4	1	0
10	5	6	4	2	2	2	4	4	1	1
11	5	0	5	0	2	4	2	4	1	3
12	6	2	8	3	4	0	2	3	1	2
13	0	6	5	4	2	2	3	1	1	3
14	5	2	3	1	2	3	4	0	1	1
15	3	0	0	0	4	5	7	6	1	5
16	0	0	0	4	0	5	6	9	1	5
17	5	6	0	5	2	4	1	1	1	0
18	5	1	2	3	4	0	4	2	1	1
19	1	7	8	2	3	0	0	4	1	4
20	2	0	1	0	4	3	0	0	1	2
21	6	5	4	0	2	3	1	1	1	0
22	5	2	0	4	5	1	3	2	1	0

23	0	4	1	5	1	0	3	3	1	3
24	1	1	2	5	3	0	4	3	1	3
25	5	0	0	5	6	3	1	2	1	2
26	1	1	3	3	0	0	4	2	1	3
27	9	2	3	2	5	6	1	2	1	1
28	6	2	4	2	0	0	4	3	1	2
29	0	3	4	0	0	0	0	0	1	4
30	7	5	0	0	5	2	2	0	1	0
31	5	2	3	3	7	0	1	1	1	0
32	4	2	0	3	1	0	2	1	1	1
33	0	0	2	4	6	0	1	0	1	4
34	0	5	2	0	1	0	5	1	1	5
35	2	1	2	4	0	3	4	6	1	5
36	1	2	5	3	4	6	1	4	1	5

The total amount of different pollutant types in different categories of clothing is shown in the Table below:

Table 5 Totals for different categories of clothing corresponding to different pollutant types

Cluster	Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8
0	35	22	7	17	24	11	8	9
1	31	15	16	16	14	11	15	10
2	23	19	19	16	18	10	13	9
3	9	12	21	20	10	7	20	16
4	4	10	22	22	21	12	7	6
5	14	13	11	15	12	20	23	26

As we can see from the table above, the number of different pollutant species on different classifications is differentiated, and we have visualized them in order to show the differences more intuitively:

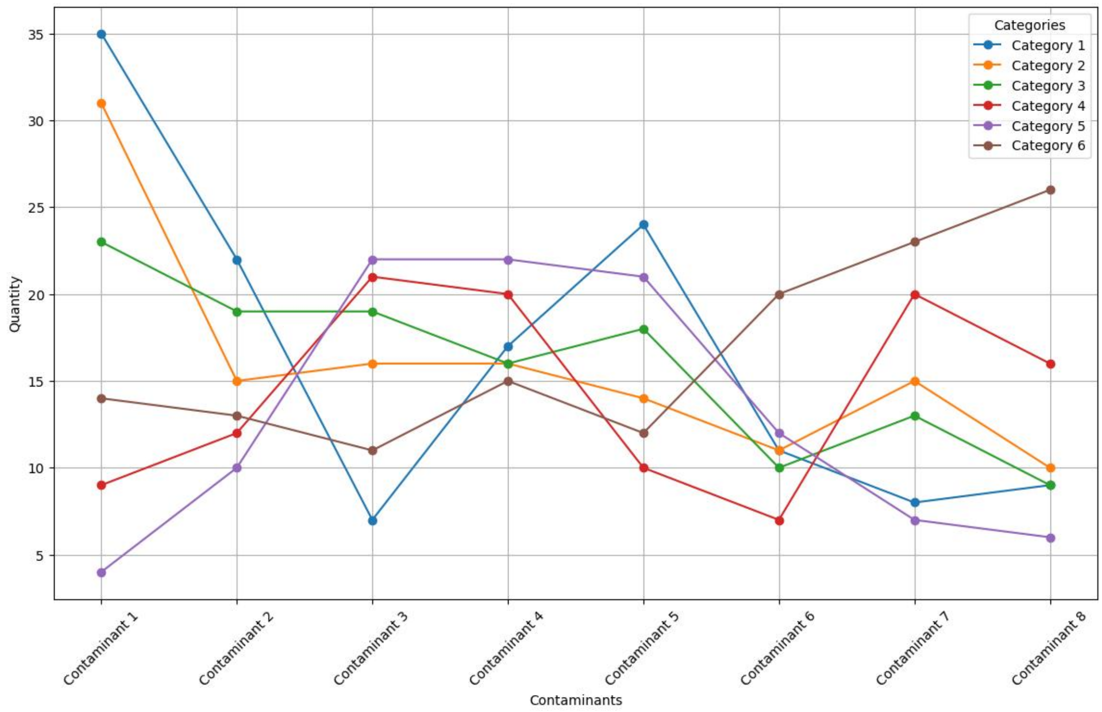

Figure 9 Contaminant levels in different categories

The graph shows the quantities of different pollutants corresponding to the six different categories, and we can clearly see that the differences in the distribution of the types and quantities of different categories of pollutants are very obvious. Based on the same data, we transformed it into the following heat map of the content of different categories of pollutants:

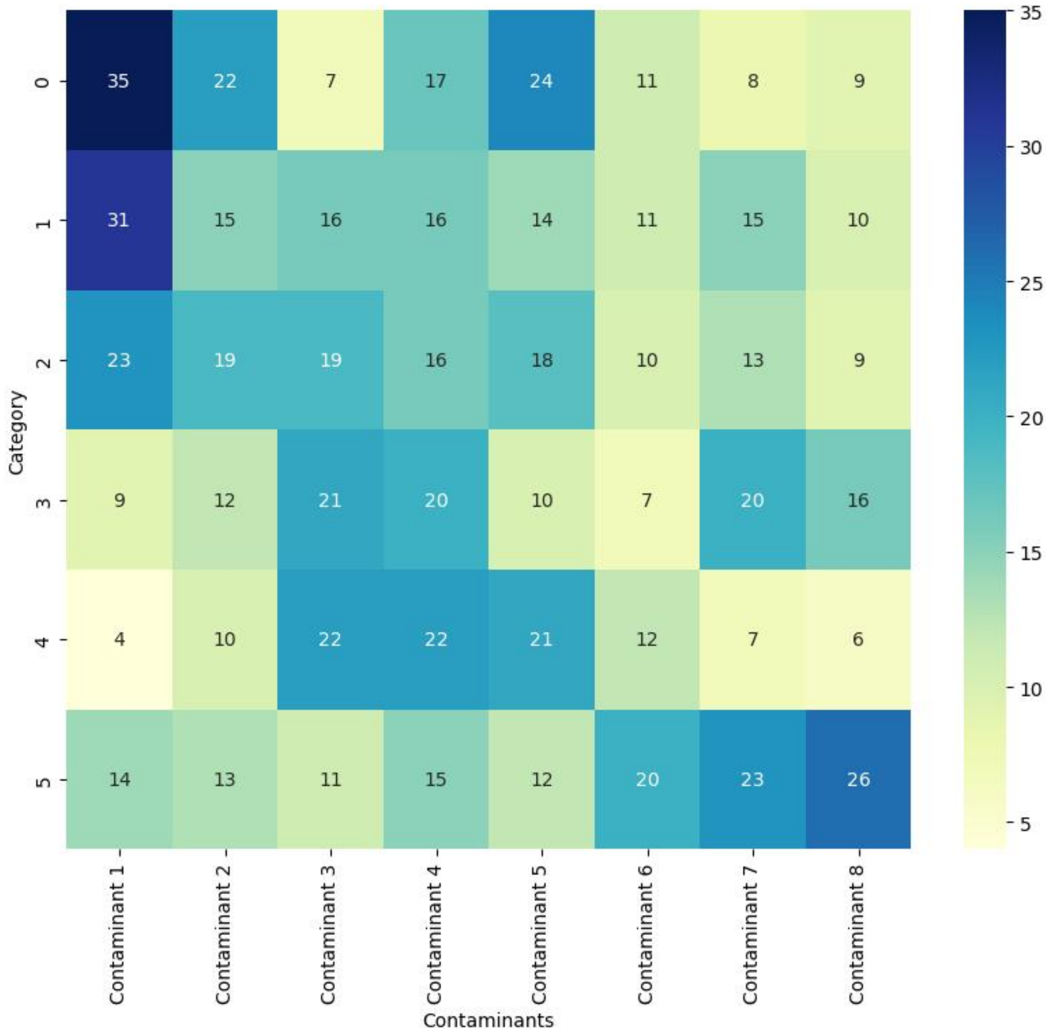

Figure 10 Heatmap of contaminant levels in different categories

As we can see from the heat map, for example, the categorization with code 0 has higher levels of the first few types of pollutants, while the categorization with code 5 shows the opposite performance, which indicates that our categorization is effective.

6.2.2 Strategic analysis

strategy I: Solution selection with the objective of maximizing the cleaning speed and minimizing the number of washings.

The objective function is:

$$\min k$$

Based on the above clustering results, as well as the above objective function, we only need to select the detergent that maximizes the cleaning efficiency of the clothes in the category for different categories, without considering the washing unit price for the time being. The cleaning efficiency can be expressed as: dissolution efficiency of the detergent * total amount of pollutants in the category.

For different categories of clothes, we can configure its most suitable detergent, the results of the configuration of the following table:

Table 6 strategy I Detergent Configuration Results

Contaminant 1	**Contaminant 2**	**Contaminant 3**	**Contaminant 4**	**Contaminant 5**	**Contaminant 6**	**Contaminant 7**	**Contaminant 8**	**Detergent**
35	22	7	17	24	11	8	9	1
31	15	16	16	14	11	15	10	1
23	19	19	16	18	10	13	9	1
9	12	21	20	10	7	20	16	1
4	10	22	22	21	12	7	6	1
14	13	11	15	12	20	23	26	3

From the above table we can observe that the first detergent is adapted to most categories of clothes. Based on the above results of detergent configuration for different categories of clothes, we wash the clothes according to the standard of D_k decaying to one thousandth of D_0 and output the washing results under strategy I as shown in the table below:

Table 7 strategy I Cleaning Effectiveness

cluster	**wash_number**	**water_cost**	**detergent_cost**
0	24	1.82	21.00
1	24	1.82	21.00
2	24	1.82	21.00
3	24	1.82	21.00
4	26	1.98	22.75
5	25	1.90	9.63

With about the same number of washes, the choice of detergent determines the difference in price. The total cost of strategy I was calculated to be $116.37. Based on the data in the Appendix, we can observe that the first detergent is more expensive, even though it is the most efficient choice for most of the laundry categories, however, there is a problem with the above optimization strategy in that the choice of detergent is based only on the principle of the highest marginal washing efficiency, in other words, it only considers maximizing the contaminant content of each dissolution and does not take into account the factor of the detergent's unit price. Therefore, next we will discuss the washing scheme that combines unit price and washing efficiency.

strategy II： Detergent allocation strategy that integrates cost-effectiveness and time-effectiveness

In order to consider the washing efficiency and the detergent unit price in a comprehensive way, we can use a weighted approach to get a comprehensive index. We define the solubility of this washing strategy as weighted average solubility efficiency/unit price + 0.3, and the formula for weighted solubility is as follows:

$$\frac{\sum_{i=1}^{8} S_i \cdot d_i}{\sum_{i=1}^{8} d_i}$$

In the above equation, S_i denotes the solubility of the ith pollutant, and d_i denotes the content of the ith pollutant. The reason we want to set the unit price +0.3 is that according to the data observation, the unit price is mostly less than 0.4, and the direct division will result in the influence of the unit price is obviously greater than the dissolution efficiency, in order to eliminate this influence, we carried out a simple normalization process. The final results obtained for the detergent configuration under strategy II are shown in the table below:

Table 8 Strategy II Detergent Configuration Results

Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8	Detergent
35	22	7	17	24	11	8	9	2
31	15	16	16	14	11	15	10	3
23	19	19	16	18	10	13	9	2
9	12	21	20	10	7	20	16	3
4	10	22	22	21	12	7	6	3
14	13	11	15	12	20	23	26	3

From the table above, we can see that the strategy that combines cleaning efficiency and detergent unit price in the selection of detergents (strategy II) significantly changes the configuration of detergents compared to the strategy that only considers cleaning efficiency (strategy I). Even though the first detergent has the best cleaning effect in most clothing categories, its high price makes it no longer the best choice after considering cost-effectiveness; therefore, in strategy II, for different categories of clothing, we select appropriate detergents to achieve the desired cleaning effect and economy. Below we output the contaminant content and cleaning cost of the corresponding detergents after cleaning different categories of clothes based on strategy II, as shown in the table below:

Table 9 strategy II Cleaning Effectiveness

cluster	wash_number	water_cost	detergent_cost
0	21	1.596	6.615
1	22	1.672	8.47
2	20	1.52	6.3
3	23	1.748	8.855
4	22	1.672	8.47
5	25	1.9	9.625

We summarized the total cost of cleaning and obtained that the total cost of cleaning for strategy II is 48.36, which is reduced to one-third of its cost compared to strategy I, indicating that the cleaning strategy II is successful and well considers the cost-effectiveness and cleaning efficiency.

7. Model for Problem 4

Problem 4 restricts the laundry combinations based on Problem 3 by stipulating that clothes of different materials cannot be washed at the same time, in order to satisfy this restriction we need to adapt the clustering algorithm, this improved clustering algorithm ensures that the final laundry combinations are compliant by adding the restriction on the material of the clothes.

7.1 Data preprocessing

Firstly, in the data pre-processing of Annex 3 and Annex 4, we organized and counted Annex 3 in order to derive the number of clothing of different materials, which provided the data basis for further analysis. The figure below illustrates the distribution of the number of clothing of different materials:

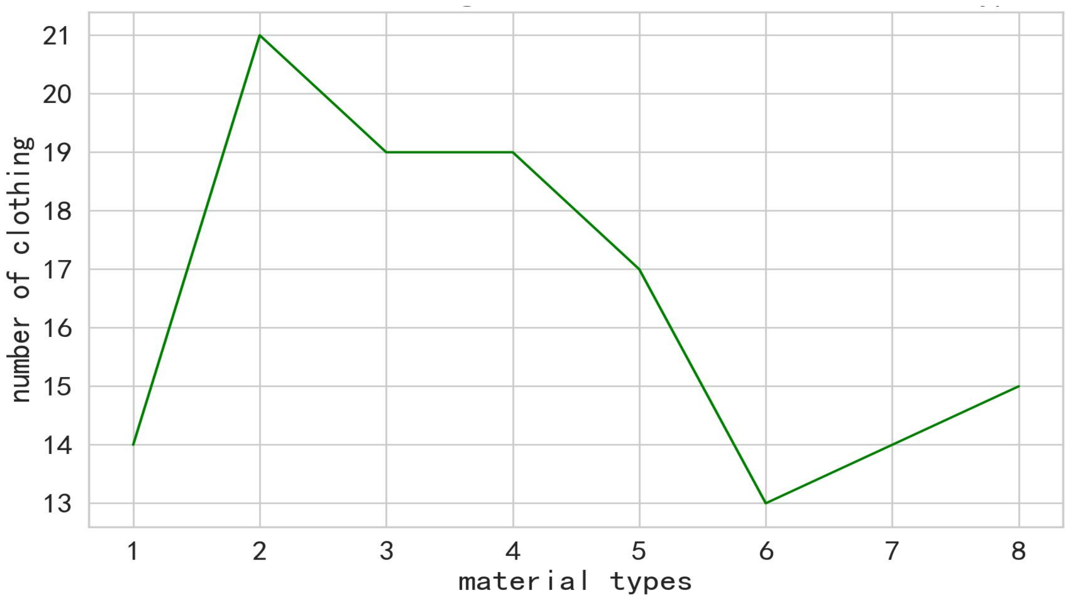

Figure 11 The number of clothing itens of different material types

For Annex 4, we have converted the question of whether clothes between different materials can be washed together into the following table:

Table 10 Mixed Laundry Relationship of Clothing Materials

Material	0	1	2	3	4	5	6	7
0	1	0	1	1	1	1	0	0
1	0	1	0	0	0	0	0	1
2	1	0	1	0	0	1	1	1
3	1	0	0	1	0	1	1	1
4	1	0	0	0	1	0	1	1
5	1	0	1	1	0	1	1	1

6	0	0	1	1	1	1	1	1
7	0	1	1	1	1	1	1	1

7.2 Model Adjustment

Given the limitations imposed by Problem 4 on mixed washing, we need to improve the model compared to Problem 3. The main adjustments are as follows:

(1) Select a piece of clothing at random and confirm its contaminant content and material.

(2) Calculate the Euclidean distance between the garment and the value of the contaminant content of the garment with which it can be mixed.

(3) Aggregate the 5 nearest garments to the garment into a cluster.

(4) The steps (2) and (3) are then repeated by randomly selecting additional clothing from the unselected clothing.

(5) Calculate the distance between the centroids of each cluster.

(6) Once again, select clothing that is different from the first step as the initial clustering point, and continue to perform the above process to calculate the distance between the new centers of each clustering.

(7) Continuously loops, and finally selects one of the combination strategies that maximizes the clustering centroids as the final combination result.

In addition to this, we also observed that due to the different number of different types of clothes and the different total number of clothes repelled by different materials of clothes, the use of the above search strategy with random sampling may lead to a situation where the number of clothes that can be allocated decreases at the later stage of the clustering, leading to a situation where the number of clothes in multiple buckets fails to reach six, a situation that is due to the conflict that exists between the clothes. Therefore, we plan to further improve the algorithm by prioritizing the allocation of clothes that have a greater degree of conflict with other clothes and solving the above problem by prioritizing the sampling of clothes types that are more repulsive, as follows:

(1) Use the total number of garments that can be mixed together as a measure of the receptivity of the garment material.

(2) Prioritize any of the clothing materials with the lowest acceptance.

(3) Continue with the above search strategy.

The chart below measures the amount of receptivity of different materials of clothing:

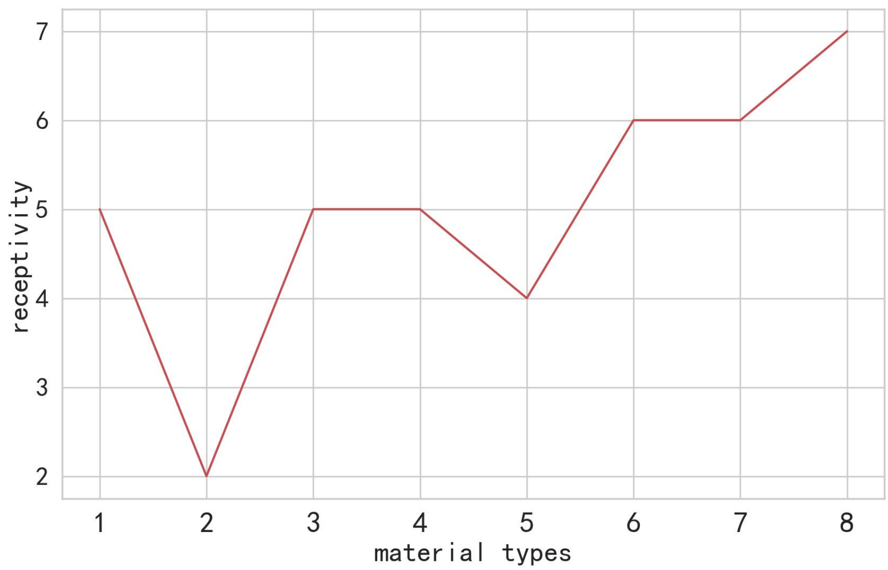

Figure 12 Total amount of clothes that can be mixed and washed with different materials

From the above figure, it can be seen that material 2 has the lowest acceptance, so we first choose any piece of clothing in material 2 as the starting point for the search algorithm. We consider both cost-effectiveness and washing efficiency, and the strategy II according to the above problem 3 has the following formula:

$$\frac{\sum_{i=1}^{8} S_i \cdot d_i}{\sum_{i=1}^{8} d_i}$$

Based on the clustering results of the final algorithm, the total amount of pollutants under different categories and the washing strategies for different categories are obtained as shown in the table below:

Table 11 Laundry Strategies

cluster	Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8	Detergent
0	31	24	30	14	19	18	12	20	3
1	21	11	5	24	37	25	28	15	3
2	48	34	2	14	19	18	26	16	2
3	17	18	31	17	24	19	17	17	3
4	17	32	11	19	12	19	12	18	3
5	10	24	15	15	27	19	18	27	3
6	11	28	9	7	18	15	18	22	3
7	20	34	9	13	9	13	19	26	3
8	27	25	13	26	29	12	21	6	2
9	13	13	16	33	16	29	36	15	3
10	16	4	29	28	26	19	21	20	3

11	19	17	16	18	16	21	17	34	3
12	12	19	22	26	17	26	24	25	3
13	11	6	7	8	18	15	9	7	3
14	30	23	12	16	21	36	13	24	3
15	33	29	18	26	7	6	12	12	2
16	15	20	15	27	25	13	13	27	3
17	15	19	15	12	12	17	17	28	3
18	20	14	21	9	14	16	15	18	3
19	19	5	4	1	2	8	6	3	3
20	11	18	26	21	23	16	21	15	3
21	29	26	12	10	26	27	32	19	3
22	19	16	23	28	18	30	17	12	3
23	3	2	1	4	2	7	7	0	3

The cleaning results obtained according to the above cleaning rules are shown in the table below:

Table 12 Breakdown of laundry costs

cluster	wash_number	water_cost	detergent_cost
0	23	1.748	8.855
1	23	1.748	8.855
2	21	1.596	6.615
3	23	1.748	8.855
4	24	1.824	9.24
5	24	1.824	9.24
6	24	1.824	9.24
7	24	1.824	9.24
8	20	1.52	6.3
9	23	1.748	8.855
10	23	1.748	8.855
11	25	1.9	9.625
12	24	1.824	9.24
13	23	1.748	8.855
14	24	1.824	9.24
15	21	1.596	6.615
16	24	1.824	9.24
17	25	1.9	9.625
18	24	1.824	9.24
19	23	1.748	8.855
20	23	1.748	8.855
21	23	1.748	8.855
22	23	1.748	8.855
23	22	1.672	8.47

As can be seen from the above table, a total of 132 pieces of clothes were washed under this strategy, the total cost of washing accumulated to $445.41, and the unit price of washing was $3.37/piece, compared to Problem 3, in which a total of 36 pieces of clothes were washed, the total cost of washing was 48.34, and the unit price of washing was $1.34/piece. The unit cost has increased slightly, but the strategy is to add the limit of mixing and washing on the basis of question 3, so the increase is also acceptable.

8. Strengths and Weakness

8.1 advantages

1. model strengths in Problem 1 and Problem 2:

a. Precise problem definition and assumption setting: In Problem 1, by precisely defining the key variables in the washing process, such as the amount of water used and the degree of solubility, and by setting up reasonable assumptions based on these definitions, a clear starting point for the problem is provided, and this clear problem framework and assumption setting are the basis for effective analysis.

b. Converting complex problems into tractable optimization models: In both problems, the real-world washing problem is converted into a mathematical optimization problem, allowing the problem to be solved by mathematical models and algorithms. This conversion makes the complex real-world problem more quantifiable and resolvable.

c. Use of efficient algorithms: the simulated annealing algorithm applied in Problem 2 is an efficient global optimization algorithm that is particularly suitable for solving problems with complex nonlinear relationships. The application of this algorithm improves the accuracy and efficiency of the analysis.

d. Flexible response to different constraints: although Problem 1 and Problem 2 are based on the same washing scenario, the constraints are different, and by flexibly adapting the model and algorithm to different constraints (e.g., removing the water quantity restriction in Problem 2), this approach shows a high degree of adaptability and practicality.

e. Multi-dimensional data analysis and visualization: Interpretation of data is made intuitive and easy to understand by presenting the results in the form of three-dimensional charts. This multi-dimensional presentation and analysis of data helps to understand the relationships between variables in greater depth.

f. Sensitivity analysis: By analyzing the effects of different parameters (e.g., initial water volume, initial dirt volume, and solubility) on the optimization results, it reveals the importance of these parameters and how they affect the final washing effect, increasing the practical value of the model.

1. model strengths in Problem 3 and Problem 4:

a. Combined cost and efficiency considerations: By balancing washing efficiency with cost-effectiveness, the method provides a cost-effective solution for the washing industry. This balance is particularly important as it is directly related to sustainability and economic efficiency.

b. Improved clustering algorithm: an improved random sampling clustering algorithm was used to improve the efficiency and accuracy of clustering, especially when differences in clothing materials were taken into account. And by using the clustering algorithm to categorize the contaminant content of the clothes, the method effectively ensures that similar clothes can be washed together, thus improving the efficiency and cost-effectiveness of washing.

c. The introduction of acceptance parameter: by introducing the acceptance parameter, the problem of mixed washing of clothes of different materials is effectively dealt with, and this new parameter enhances the practicality and adaptability of the model.

d. Combination of economy and efficiency: Both cleaning efficiency and cost are taken into account when dispensing detergents, providing a viable solution for achieving the optimal balance between economy and cleaning efficiency.

In summary, the advantages of these methods and models are that they combine accurate mathematical modeling, efficient algorithmic applications, flexible problem adaptability, and in-depth data analysis to provide powerful tools for solving real-world washing problems. These advantages not only improve the efficiency and accuracy of problem solving, but also provide reference and inspiration for similar optimization problems.

8.2 Weaknesses and their improvement

1. Shortcomings and improvements of the model and methodology in questions 1 and 2:

a. Simplifications and assumptions of the model: The models in questions 1 and 2 are based on specific assumptions, such as a particular way of calculating solubility or the possibility of an infinite increase in the number of washes. These assumptions may be oversimplified and not entirely realistic. Improvements could include the introduction of more complex models that better simulate the real-world washing process.

b. Limitations of the Simulated Annealing Algorithm: although the Simulated Annealing Algorithm is a powerful optimization tool, it may fall into local optimal solutions in some cases, especially in complex optimization problems. Consideration can be given to combining other optimization algorithms, such as genetic algorithms, in order to increase the likelihood of finding a globally optimal solution.

c. Practical feasibility of parameters: Parameters used in the problem (e.g., solubility, detergent efficacy) need to be assured of their feasibility and accuracy in practical applications. Practical testing and adjustment may help to improve the practical application value of the model.

1. Shortcomings and improvements of the model and methodology in questions 3 and 4:

a. Applicability of clustering algorithms: While clustering algorithms are effective in categorizing clothing, they may not be able to accurately handle extreme or special cases, such as very unique or rare types of contaminants. Improvements could include more refined classification criteria or the introduction of more advanced machine learning techniques.

b. Complexity of cost-benefit analysis: Cost-benefit analyses may be oversimplified and do not take into account all possible variables, such as fluctuations in water and electricity costs, changes in detergent prices, etc. Improvements include the introduction of more comprehensive cost models that take into account a wider range of economic factors.

Overall, the models and methods presented in this paper provide powerful tools for solving the scrubber optimization problem, but they may need to be further adapted and optimized in practical applications to ensure their effectiveness and usefulness in different environments and conditions.

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Appendix

The appendix contains all the code for the problem run, the result files, and the references.