Base-two primitive permutation groups

Hong Yi Huang

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University of BRISTOL

Bases

Let $G \leqslant \mathrm{Sym}(\Omega)$ be a **transitive** permutation group, where $|\Omega|$ is finite.

Definition

A **base** for G is a subset Δ of Ω such that $\bigcap_{\alpha \in \Delta} G_\alpha = 1$.

Base size $b(G)$: the minimal size of a base for G.

Examples

- $G = S_n$, $\Omega = \{1, \ldots, n\}$: $b(G) = n - 1$.
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- $G = D_{2n}$, $\Omega = \{1, \ldots, n\}$: $b(G) = 2$.

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p prime, $G = D_{2p}$ and $\Omega = \{1, \ldots, p\} \implies G$ primitive and $b(G) = 2$.

Some results

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- Progress where $G < L \wr P$ **(Burness & H, 2022+)**

Probabilistic methods

Consider

$$Q(G) = \frac{|\{(\alpha, \beta) \in \Omega^2 : G_\alpha \cap G_\beta \neq 1\}|}{|\Omega|^2},$$

the probability that a random pair in Ω is **not** a base for G.

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Saxl graphs

Assume $G \leqslant \mathrm{Sym}(\Omega)$ is transitive of degree n and $b(G) = 2$.

Burness & Giudici, 2020: **Saxl graph** $\Sigma(G)$:

$$\text{vertices } \Omega, \text{ with } \alpha \sim \beta \iff \{\alpha, \beta\} \text{ is a base.}$$

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  Hence, $\Sigma(G) \cong \mathbf{K}_{q^2-1} - (q+1)\mathbf{K}_{q-1}$ is **complete multipartite**.

Further example

- $G = \mathrm{PGL}_2(q)$ and $\Omega = \{$2-subsets of $\{$1-spaces in $\mathbb{F}_q^2\}\}$.

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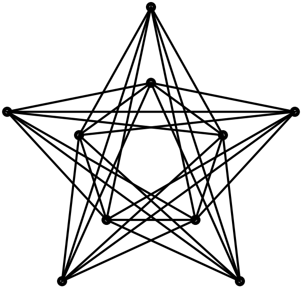
  For example, when $q = 4$ we have the complement of the Petersen.



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Example

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- Diagonal type groups

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