Ensemble Learning

An ensemble method tries to make a strong learner by combining the predictions of many weak learners (**whose accuracy is a little bit higher than random learner**). Actually, for binary classification, we could prove that the majority vote makes correct decision if the weak classifies are **independent** to each other. It's called condorcet's rule. Two core elements have to be considered in ensemble learning: accuracy and diversity.

Ensemble learning is a method which could be used in both regression and classification and now a lot of ensemble algorithms are tree-based. There are two strategies to ensemble weak learners: boosting and bagging.

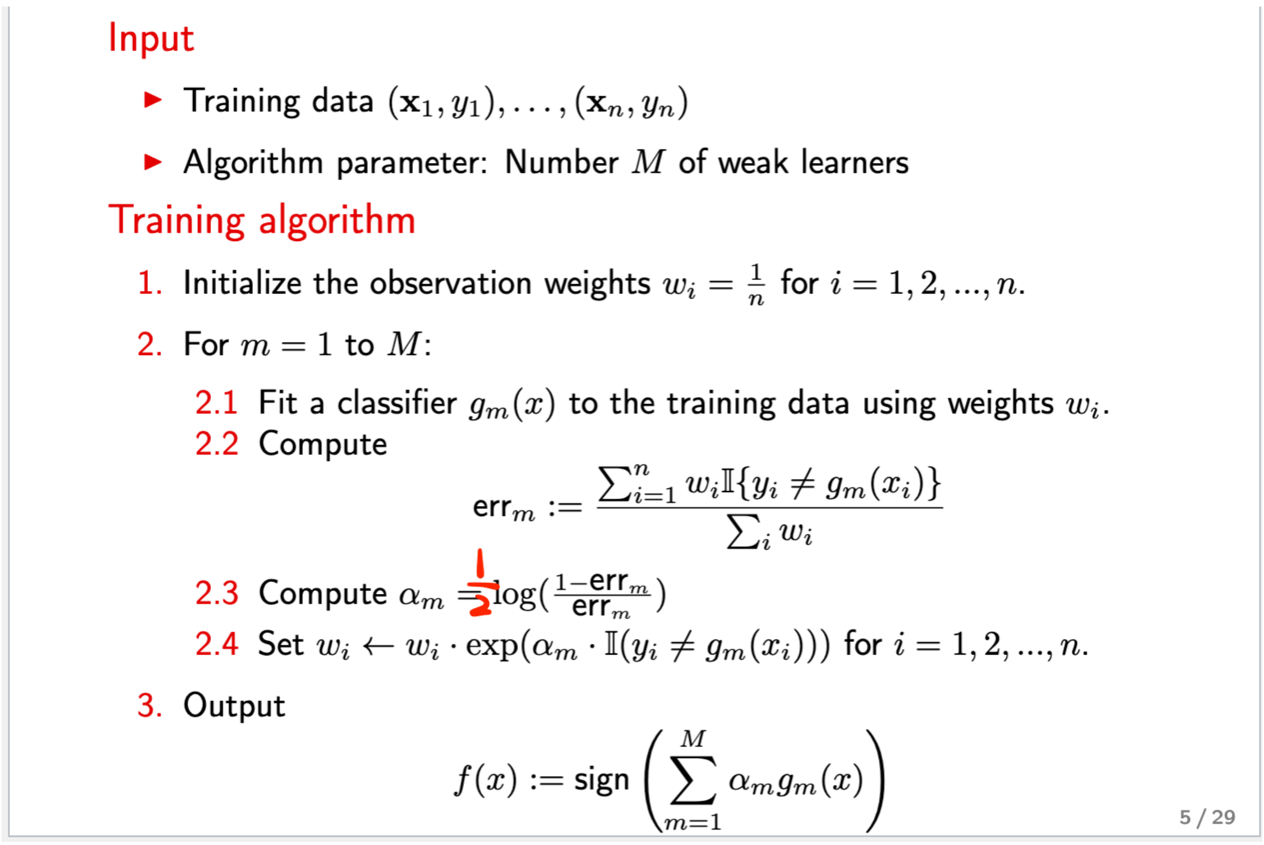
Boosting

Boosting is composed of:

- additive model $F(x) = \Sigma_{m=1}^{M} \alpha_m g_m(x)$, where $g_m(x)$ is a weak learner.
- reweighted samples after training each weak learner.

The most famous algorithm is Adaboost.

1. Algorithm - Adaboost



Input

- Training data $(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)$
- Algorithm parameter: Number M of weak learners

Training algorithm

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1. Initialize the observation weights $w_i = \frac{1}{n}$ for $i = 1, 2, ..., n$.
2. For $m = 1$ to $M$:
    2.1 Fit a classifier $g_m(x)$ to the training data using weights $w_i$.
    2.2 Compute
        $\text{err}_m := \frac{\sum_{i=1}^{n} w_i \mathbb{I}\{y_i \neq g_m(x_i)\}}{\sum_i w_i}$
    2.3 Compute $\alpha_m = \frac{1}{2}\log(\frac{1-\text{err}_m}{\text{err}_m})$
    2.4 Set $w_i \leftarrow w_i \cdot \exp(\alpha_m \cdot \mathbb{I}(y_i \neq g_m(x_i)))$ for $i = 1, 2, ..., n$.
3. Output
        $f(x) := \text{sign}\left(\sum_{m=1}^{M} \alpha_m g_m(x)\right)$
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Note: 1. standard adaboost could only be used in binary classification problem; 2. the weak learn whose error rate is larger than 0.5 would be discarded in iteration.

The idea of adaboost is to make the sample which is classified into a wrong class currently have a larger weight. In this case, if we want to minimize our loss, it's better to classify these wrong-classified samples correctly in later iterations.

2. Loss Function - Adaboost

The loss function used in Adaboost is exponential loss function:

$$L(y, f(x)) = e^{-yf(x)}$$

Actually we could derivate the updating policy for parameters in Adaboost from the perspective of minimizing loss function. The final policy is the same as the first part stated. The specific derivation could be found a chinese version in here.

3. Additionals

- How to use it in multi-classification problem?

  As we stated before, the standard adaboost could only be used in binary classification problem. What if we want to apply this algorithm for multi-classes problem? We could transform the standard algorithm in three ways:

  - choose weak learners that can handle multiple classification.
  - still use binary weak learners and combine the output and features into new sample set. Then train a multi-classifier based on the new sample set.
  - encode label by using m bits binary code and then train m weak learners. The final output is the label we predict.

Bagging

As we stated before, we want the weak learners are independent to each other so as to get a strong learner with high ability of generalization. In order to do this, bagging samples dataset randomly by **boostrap** (sample with replacement) in each iteration of training. The most famous algorithm is random forest.

1. Algorithm - Random Forest

- For t = 1, 2, 3, ... , T repeat
  - Randomly sample a subset of dataset
  - **Randomly sample a subset of features**
  - Train a **decision tree** $h_t(x)$on the subset
- Output: if it's for classification, we could use majority vote (*Note: if there are two classes with same votes, we could just pick one as the prediction randomly*) to predict while for regression, we could compute the average.

Random forest is an variation of bagging. It not only sample dataset randomly, but also make the features in each iteration to be different. This stragegy makes the weak learners in random forest more diverse/independent to each other.

2. Additionals

- Unlike adaboost, random forest could be used in both regression and classification (even multi-classification).

- Bagging is a stragety of ensemble learning which means the weak learners do not have to be decision trees. However, **for random forest, it is an bagging-idea-based algorithm with decision trees as weak learners**.
- Disadvantage: since each time we fit a decision tree to a boostrap sample, we will get a different tree. It would decrease the interpretability of our model.
- What is **out of bag estimate** in bagging?

  Since boostrap is a sample method with replacement, there always are some data which are not used in each training (*It could be shown that on average around 37% of the observations would not be drawned in each boostrap.*). Naturally, we could use these unsampled dataset to do cross validation so as to estimate the test error of a bagging estimate which is called out-of-bag estimate.
- What is **importance of feature** in random forest?

  We would still find the best splitting feature even though only a random subset of features would be considered in each iteration. This actually could help us evaluate the importance of each variable because for each predictor, we could add up the total amount by which the loss decreases every time we use the predictor in the decision tree. Finally, we would get the importance of feature by averaging over all weak learners.
- A trick of choosing the number of trees in random forest: In practise, we could use $\sqrt{p}$ to be the number of decision trees. But of course, we could also use cross validation to tune this parameter.