Simultaneous System Identification and Model Predictive Control with No Dynamic Regret

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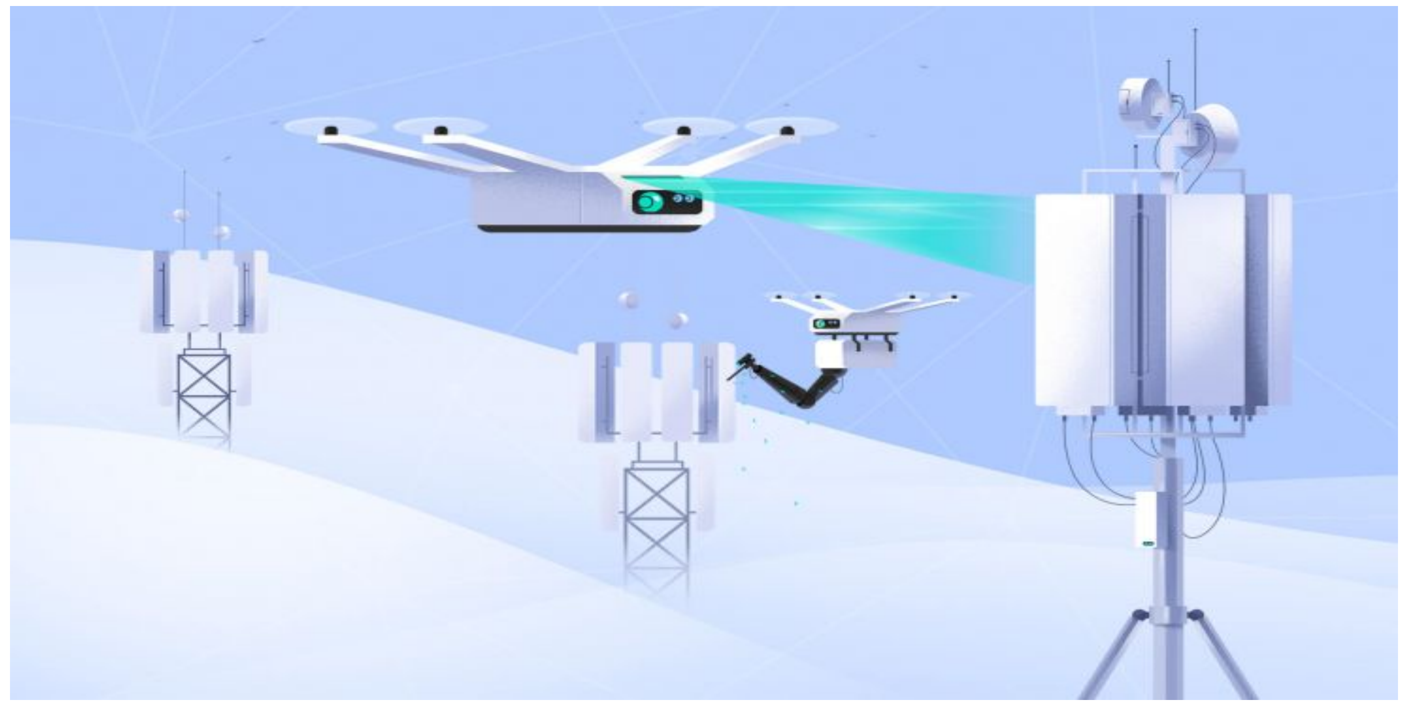
Agile and Precise Motion Control Under Uncertainty

Drone Delivery

Target Tracking

Inspection & Maintenance



Packages with unknown weights

Unknown aero-dynamic effects

Complication: Unknown dynamics or disturbances

Problem: How to achieve agile and precise tracking control under unknown dynamics and disturbances?[1,2,3]

[1] Ackerman, IEEE Spectrum '13
[2] Chen, Liu, Shen, IROS '16
[3] Seneviratne, Dammika, et al., Acta Imeko '18

Model Predictive Control Under Uncertainty

All above scenarios are control problems under uncertainty

Goal: Find control input to minimize loss under unknown system dynamics or disturbances:

$$\begin{aligned} \underset{u_t,\ \ldots,\ u_{t+N-1}}{min} \quad & \sum_{k=t}^{t+N-1} c_k\left(x_k, u_k\right) \\ subject\ to \quad & x_{k+1} = f\left(x_k\right) + g\left(x_k\right)u_k + h\left(z_k\right), \\ & u_k \in \mathcal{U}, \\ & k \in \{t, \ldots, t+N-1\}. \end{aligned}$$

convex

known system dynamics

unknown dynamics
or disturbances

control constraints

All above scenarios are control problems under uncertainty

Goal: Find control input to minimize loss under unknown system dynamics or disturbances:

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convex

known system dynamics

unknown dynamics
or disturbances

control constraints

Difficulty: $h\left(z_k\right)$ is unknown and adaptive to the state and the control

All above scenarios are control problems under uncertainty

Goal: Find control input to minimize loss under unknown system dynamics or disturbances:

$$\begin{aligned} \min_{u_t,\ \ldots,\ u_{t+N-1}} \quad & \sum_{k=t}^{t+N-1} c_k\left(x_k, u_k\right) \\ \text{subject to} \quad & x_{k+1} = f\left(x_k\right) + g\left(x_k\right) u_k + \hat{h}\left(z_k; \hat{\alpha}\right), \\ & u_k \in \mathcal{U}, \\ & k \in \{t, \ldots, t+N-1\}. \end{aligned}$$

convex

known system dynamics

estimated unknown dynamics/disturbances

control constraints

Assumptions

Assumption (Function Space[4])

$h : \mathbb{R}^{d_z} \to \mathbb{R}^{d_x}$ lies in a subspace of a Reproducing Kernel Hilbert Space *(RKHS)* $\mathcal{H}$ where the kernel K is considered known[5] and can be written via a feature map $\Phi : \mathbb{R}^{d_z} \times \Theta \to \mathbb{R}^{d_x \times d_1}$ as

$$K(z_1, z_2) = \int_\Theta \Phi(z_1, \theta) \Phi(z_2, \theta)^\top \mathrm{d}\nu(\theta),$$

where $d_1 \leq d_x$, ν is a known probability measure on a measurable space Θ.

Assumption (Operator-Valued Bochner's Theorem[6])

The measurable space Θ is a subset of $\mathbb{R}^{d_z+1}$ such that $\theta \in \Theta$ can be written as $\theta = (w, b)$, where $w \in \mathbb{R}^{d_z}$ and $b \in \mathbb{R}$. Also, the feature map can be factorized as $\Phi(z, \theta) = B(w)\phi\left(w^\top z + b\right)$, where $B : \mathbb{R}^{d_z} \to \mathbb{R}^{d_x \times d_1}$ and $\phi : \mathbb{R} \to [-1, 1]$ is a 1-Lipschitz function.

[4] Bach, Journal of Machine Learning Research '17

[5] Carmel, et al., Analysis and Applications '16

[6] Brault, et al., Asian Conference on Machine Learning '16

Assumptions

Definition (Value Function[7])

Given a state x and parameter $\hat{\alpha}$, the *value function* $V_t\left(x;\hat{\alpha}\right)$ is defined as the optimal value of

$$
\begin{aligned}
\underset{u_t,\ldots,\ u_{t+N-1}}{min} \quad & \sum_{k=t}^{t+N-1} c_k\left(x_k, u_k\right) \\
subject\ to \quad & x_{k+1} = f\left(x_k\right) + g\left(x_k\right) u_k + \hat{h}\left(z_k\right), \\
& x_t = x,\ u_k \in \mathcal{U}, \\
& k \in \{t, \ldots, t+N-1\}.
\end{aligned}
$$

[7] Grimm, et al., TAC '05

Assumptions

Assumption (Stability Condition)

There exist positive scalars $\underline{\lambda}$, $\overline{\lambda}$, and a continuous function $\sigma : \mathbb{R}^{d_x} \to \mathbb{R}_+$, such that

1. $c_t(x, u) \geq \underline{\lambda}\sigma(x), \forall x,\ u,\ t;$
2. $V_t(x; \hat{\alpha}) \leq \overline{\lambda}\sigma(x), \forall x,\ t;$
3. $\lim_{\|x\| \to \infty} \sigma(x) \to \infty.$

Assumption (Lipschitzness of Value Function)

Assume that $c_t(x, u)$ is locally Lipschitz in x and u, and $\hat{h}(\cdot)$ is locally Lipschitz in $\hat{\alpha}$. Then, the value function is Lipschitz in the initial condition x and the parameter $\hat{\alpha}$ over bounded domain sets.

Remark: The stability condition ensures that the system $x_{k+1} = f(x_k) + g(x_k)\ u_k + \hat{h}(z_k; \hat{\alpha})$ is globally asymptotic stable under MPC policy[7].

[7] Grimm, et al., TAC '05

Assumptions

Assumption (Input-to-State-Stability of MPC against Estimation Errors)

The MPC policy renders the estimated system dynamics $x_{k+1} = f(x_k) + g(x_k)\,u_k + \hat{h}(z_k)$ input-to-state-stable against the estimation error $e_t \triangleq \hat{h}(z_t) - h(z_t)$: there exists a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ, such that

$$\|x_t\| \leq \beta\left(\|x_{t_0}\|,\ t - t_0\right) + \gamma\left(e_{max}\right),\ \forall t \geq t_0.$$

where $e_{max} \triangleq \sup_{t_0 \leq \tau \leq t} \|e_\tau\|$,

Suboptimality Metric against Optimal Control Policies in Hindsight

Definition (Dynamic Regret)

Assume a total time horizon of operation T, and loss functions c_t, $t = 1, \ldots, T$. Then, *dynamic regret* is

$$\text{Regret}_T^D = \sum_{t=1}^{T} c_t\left(x_t, u_t, h(z_t)\right) - \sum_{t=1}^{T} c_t\left(x_t^\star, u_t^\star, h(z_t^\star)\right),$$

where $x_t^\star$ and $u_t^\star$ are the optimal trajectory and control input in hindsight, and the cost c_t depends on the unknown disturbance h explicit.

Remark:

- The regret is sublinear if $\lim_{T\to\infty} \frac{\text{Regret}_T^D}{T} \to 0$, which implies $c_t\left(x_t, u_t, h(z_t)\right) - c_t\left(x_t^\star, u_t^\star, h(z_t^\star)\right) \to 0$.
- h adapts (possibly differently) to the state and control sequences (x_1, u_1), $\ldots$, (x_T, u_T) and $(x_1^\star, u_1^\star)$, $\ldots$, $(x_T^\star, u_T^\star)$ since h is a function of the state and the control input.

Problem

At each $t = 1, \ldots, T$,

- estimate the unknown disturbance $\hat{h}\,(\cdot)$;
- identify a control input u_t using MPC.

The goal is to minimize dynamic regret:

$$\sum_{t=1}^{T} c_t\left(x_t, u_t, h(z_t)\right) - \sum_{t=1}^{T} c_t\left(x_t^\star, u_t^\star, h(z_t^\star)\right).$$

State of the Art

Offline Learning for Control[8]

- collects data offline and trains neural-networks or Gaussian-process models BUT
  - data-collection can be expensive and time-consuming
  - may not generalize to unseen environments

Robust Control[9]

- selects control inputs over a lookahead horizon BUT
  - conservative since assuming worst-case disturbances

[8] Sánchez-Sánchez et al., '18; Carron et al., RAL '19; Torrente et al., RAL '21; Shi et al., ICRA '19; O'Connell, et al., SR '22; …
[9] Goel et al., '20; Sabag et al., ACC '21; Goel et al., L4DC '21; Martin et al., L4DC '22; Didier et al., L-CSS '22; Zhou et al., CDC '23; …

State of the Art

Adaptive Control[10]

- estimates uncertainty and compensates control input with estimated uncertainty BUT
  - do not learn a model of uncertainty for predictive control

Non-Stochastic Control[11]

- updates control input online to adapt to observed uncertainty BUT
  - sensitive to tuning parameters
  - do not learn a model of uncertainty for predictive control

[10] Slotine, '91; Krstic, et al., '95; Ioannou et al., '96; Tal et al., TCST '20; Wu, et al., '23; Das et al., '24; Jia, et al., TRO ’23;…

[11] Agarwal et al., ICML '19; Hazan et al., ALT '20; Gradu et al., L4DC '23; Zhou et al., CDC '23; Zhou et al., RAL '23;...

Initialization:

- Gradient descent learning rate η, number of random Fourier features M, domain set $\mathcal{D}$, estimated parameter $\hat{\alpha}_{i,1} \in \mathcal{D}$;
- Randomly sample $\theta_i \sim \nu$ and formulate $\Phi\left(\cdot, \theta_i\right)$, where $i \in \{1, \ldots, M\}$;

At each iteration $t = 1, \ldots, T$:

1. Apply control input u_t using MPC with $\hat{h}(\cdot) \triangleq \frac{1}{M}\sum_{i=1}^{M} \Phi\left(\cdot, \theta_i\right) \hat{\alpha}_{i,t}$;
2. Observe state x_{t+1}, and calculate disturbance via $h\left(z_t\right) = x_{t+1} - f(x_t) - g(x_t)u_t$;
3. Formulate estimation loss $l_t\left(\hat{\alpha}_t\right) \triangleq \|h\left(z_t\right) - \frac{1}{M}\sum_{i=1}^{M} \Phi\left(z_t, \theta_i\right) \hat{\alpha}_{i,t}\|^2$;
4. Calculate gradient $\nabla_t \triangleq \nabla_{\hat{\alpha}_t} l_t\left(\hat{\alpha}_t\right)$;
5. Update $\hat{\alpha}'_{t+1} = \hat{\alpha}_t - \eta \nabla_t$;
6. Project $\hat{\alpha}'_{i,t+1}$ onto $\mathcal{D}$, *i.e.*, $\hat{\alpha}_{i,t+1} = \Pi_{\mathcal{D}}(\hat{\alpha}'_{i,t+1})$, for $i \in \{1, \ldots, M\}$.

Dynamic Regret Analysis

Theorem (No-Regret)

Our algorithm with $\eta = \mathcal{O}\left(1/\sqrt{T}\right)$ achieves $\text{Regret}_T^D \leq \mathcal{O}\left(T^{\frac{3}{4}}\right)$.

Remark:

- Our algorithm converges asymptotically to the optimial controller since $\lim_{T\to\infty} \frac{\text{Regret}_T^D}{T} \to 0$.

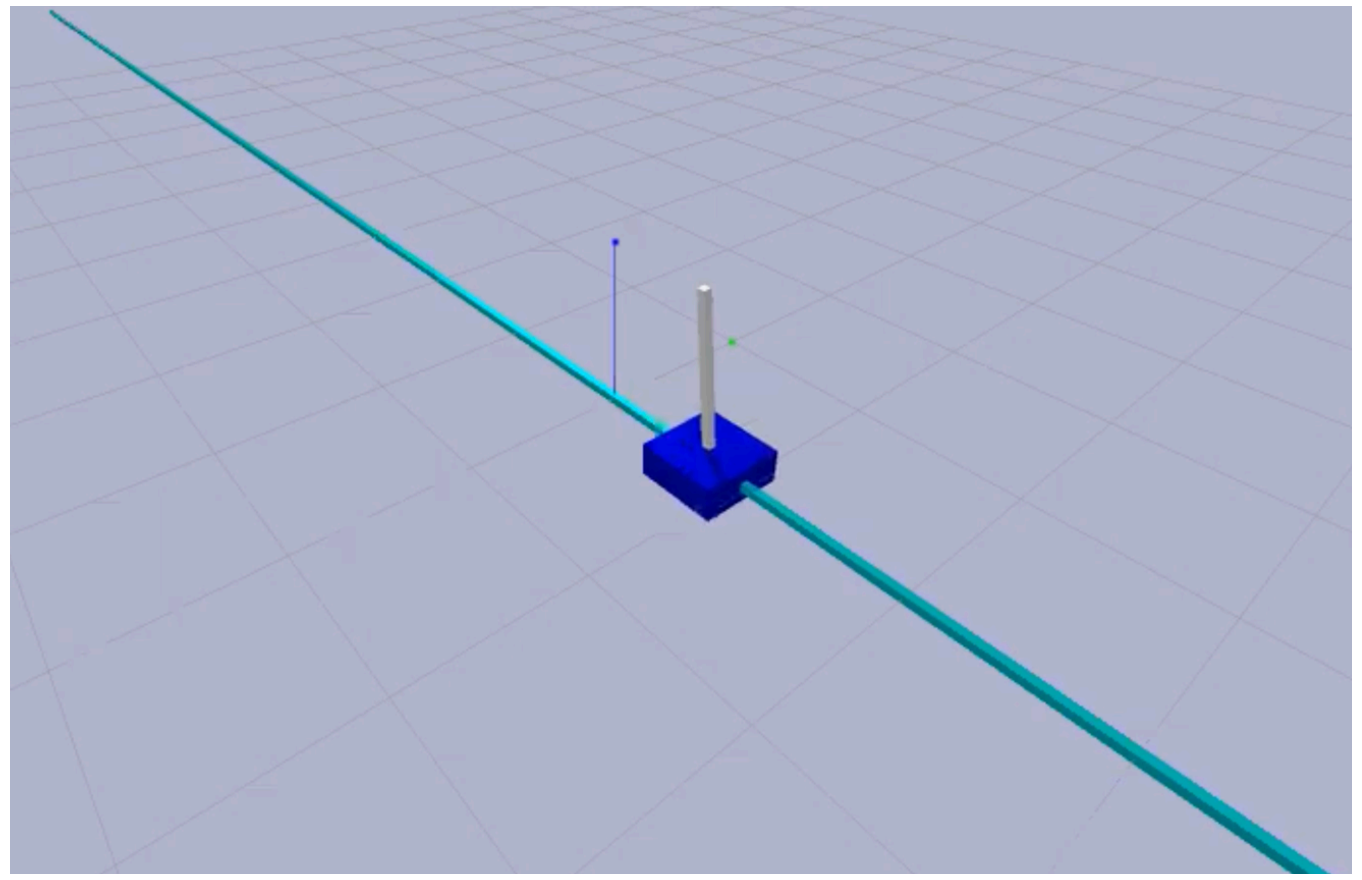
Simulations on Cart-Pole Stabilization with Inaccurate Model

Goal:

- Stabilize the cart-pole around the upright position of the pole

Setup:

- The model parameters (pole length & mass, cart mass) are inaccurate



Simulations on Cart-Pole Stabilization with Inaccurate Model

Goal:

- Stabilize the cart-pole around the upright position of the pole

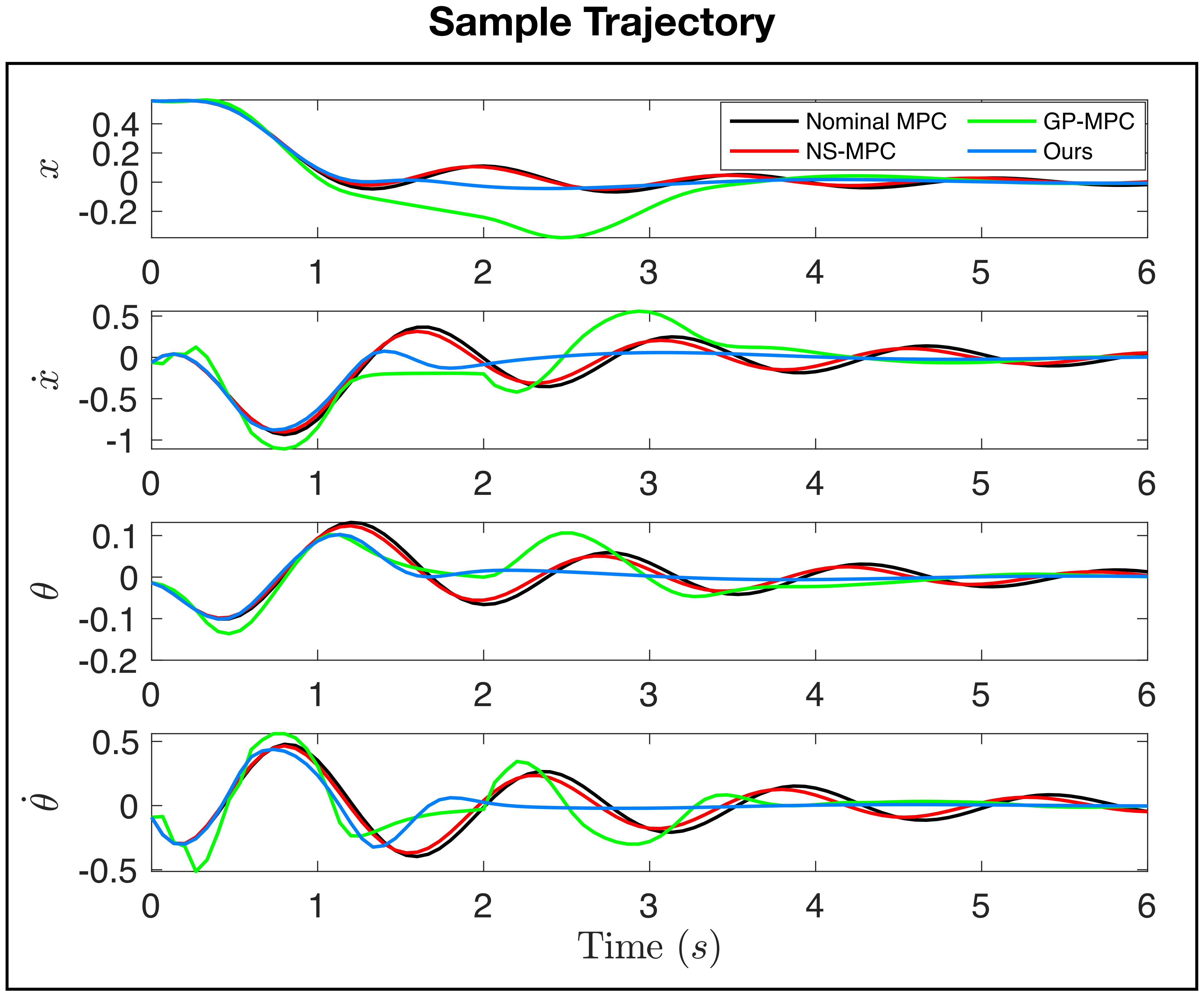
Setup:

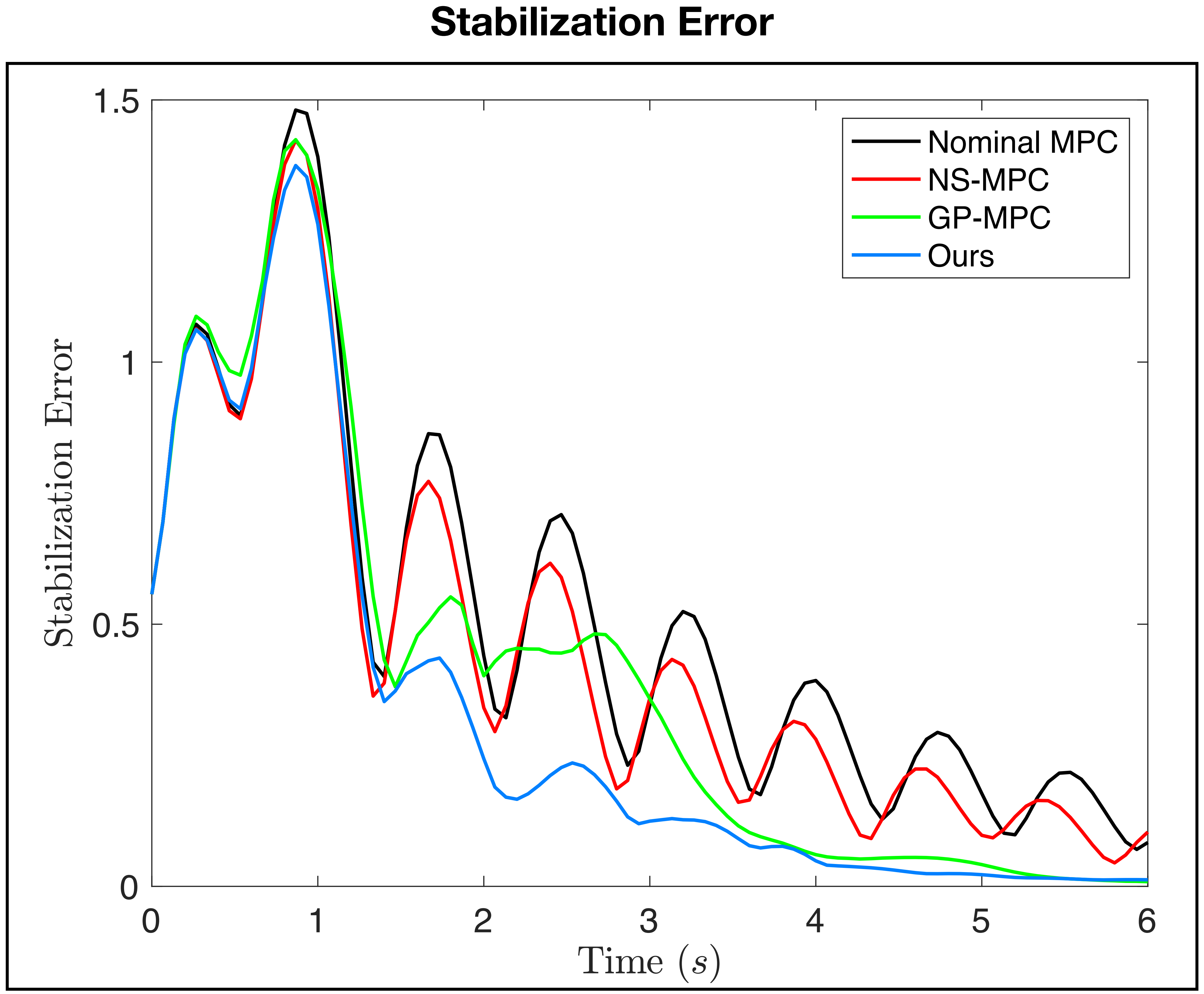
- The model parameters (pole length & mass, cart mass) are inaccurate

Compared algorithms: (i) Non-stochastic MPC[12] and (ii) Gaussian Process MPC[13]

[12] Zhou et al., RAL '23 [13] Hewing et al., TCST '19

Simulations on Cart-Pole Stabilization with Inaccurate Model





Results:

- Our method achieves fastest stabilization while other algorithms:
  - NS-MPC has marginal improvement over Nominal MPC
  - GP-MPC has larger deviation than our method

Simulations on Cart-Pole Stabilization with Inaccurate Model

TABLE I: **Computational Performance.** The red numbers correspond to the worse performance.

	MPC	NS-MPC	GP-MPC	Ours
Time (ms)	9.63 ± 3.34	9.94 ± 3.39	160.01 ± 26.96	16.28 ± 21.77

Results:

- Our method is computationally more efficient than GP-MPC

Simulations on Trajectory Tracking with Unknown Aerodynamic Effect

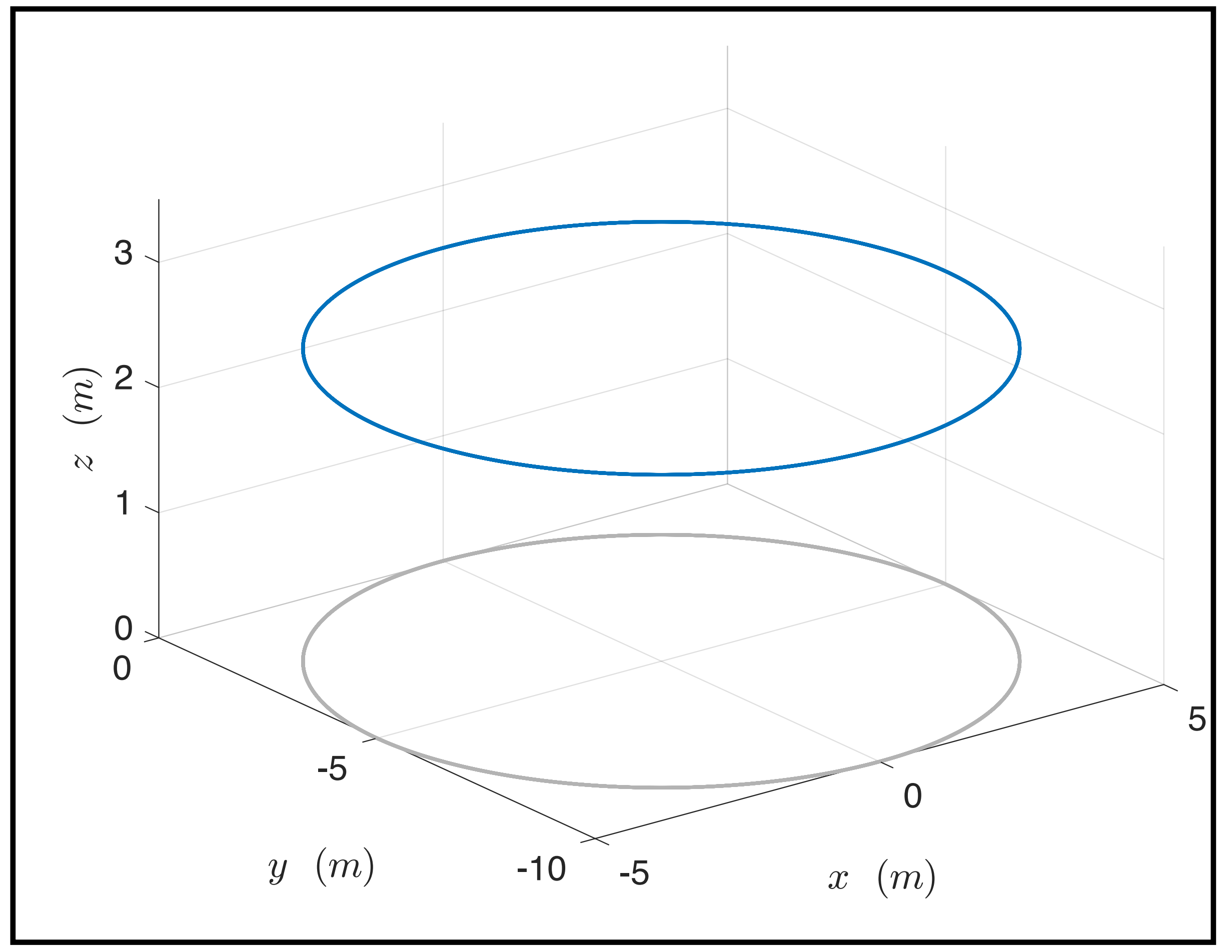
Goal:

- Track reference trajectories with a drone

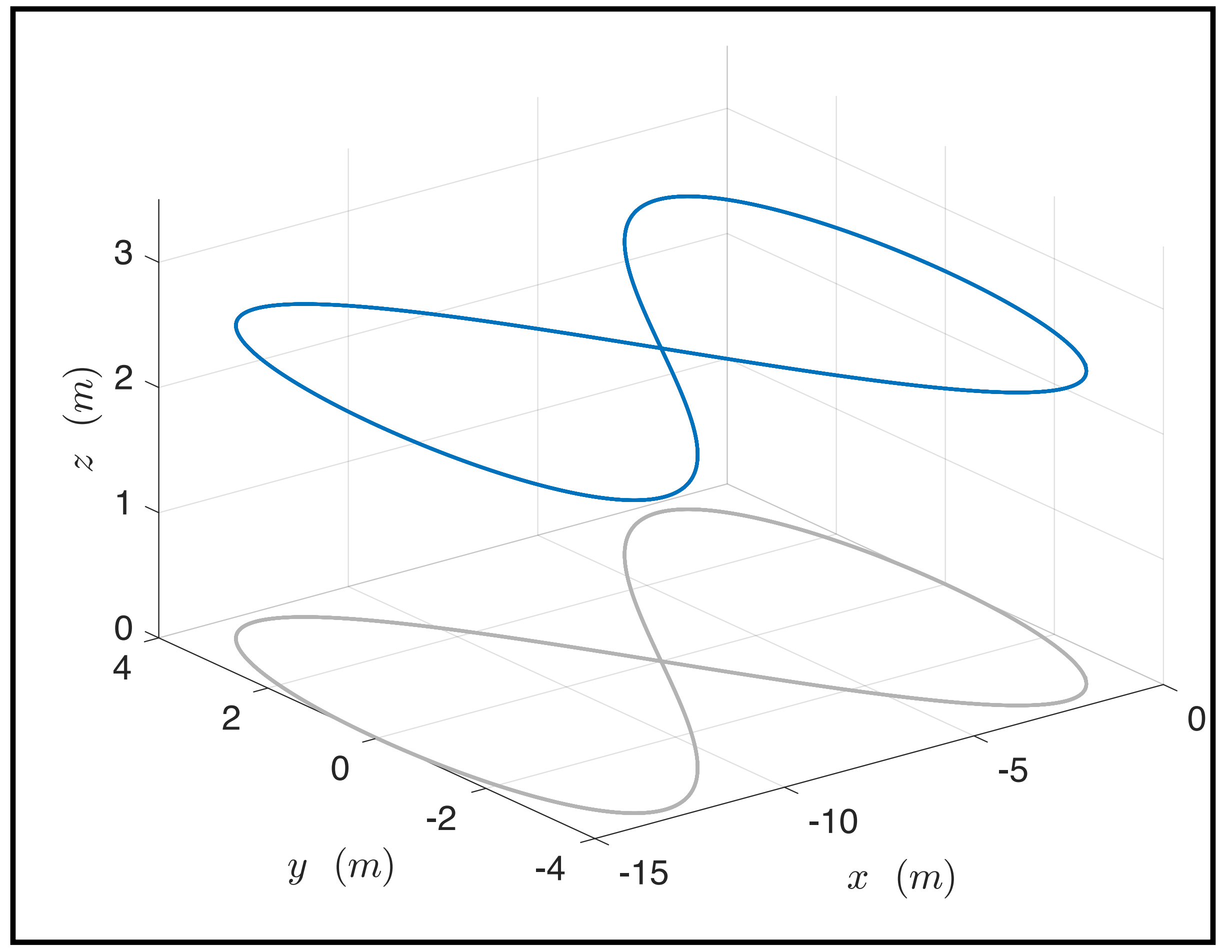
Setup:

- The system dynamics of the drone are corrupted with unknown drag effects

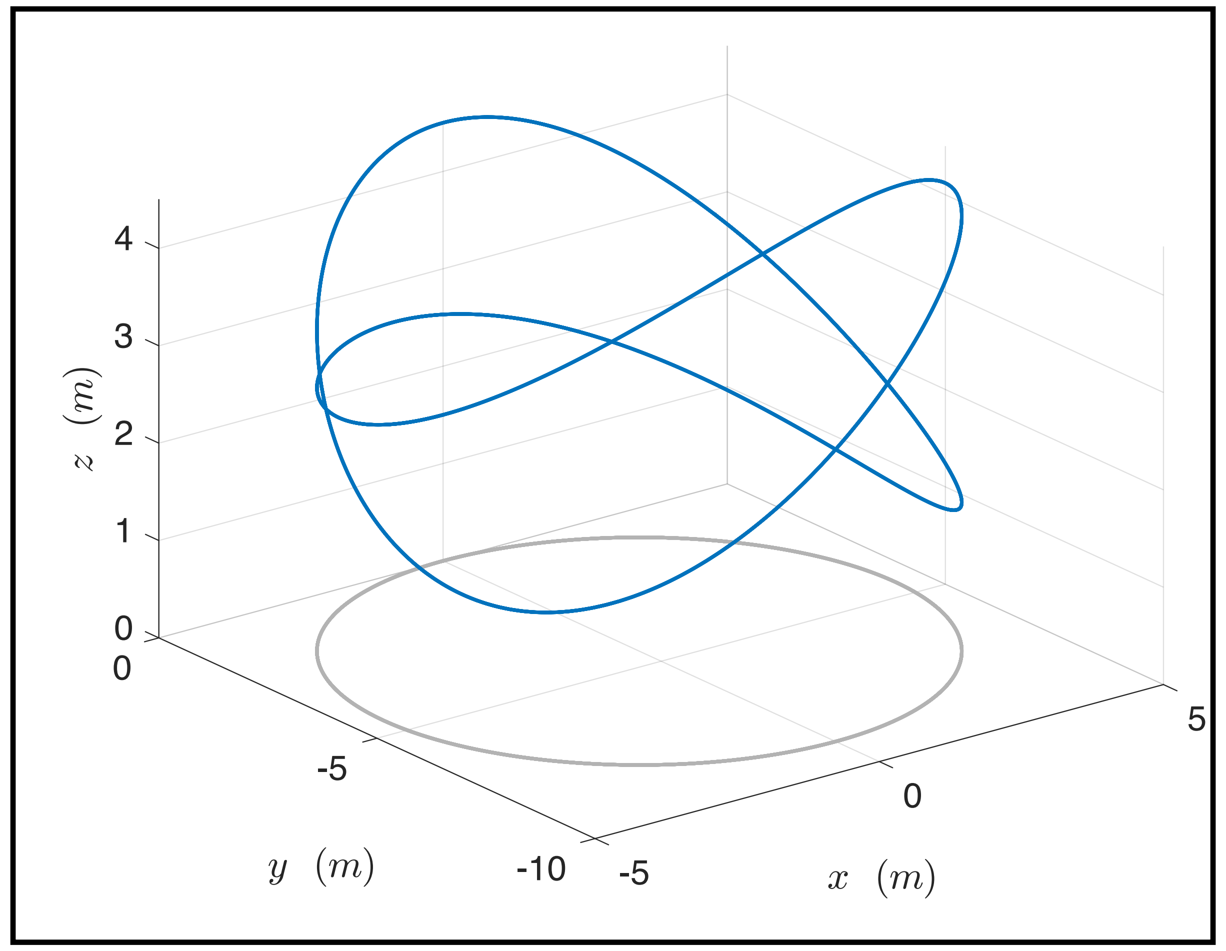
Circle



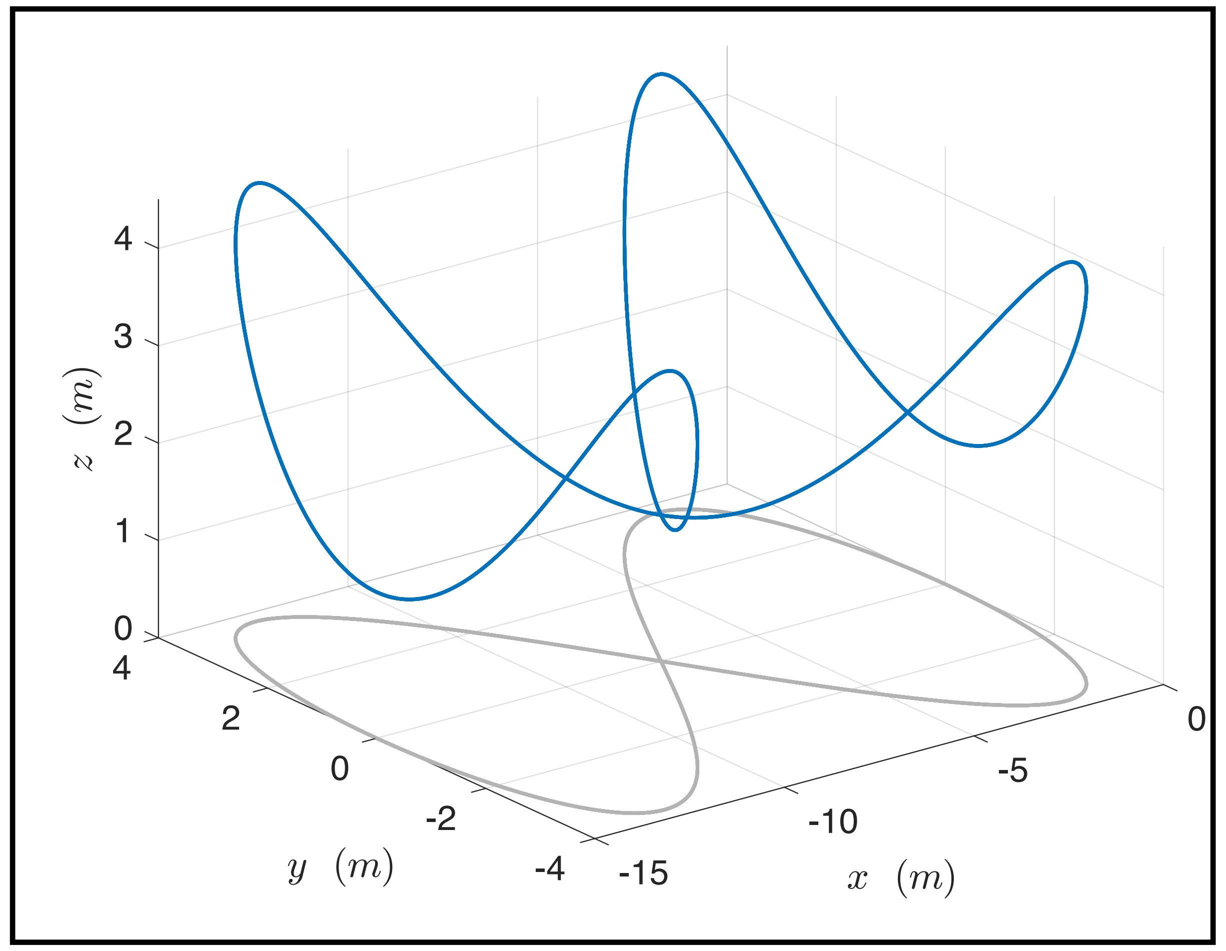
Lemniscate



Wrapped Circle



Wrapped Lemniscate



Simulations on Trajectory Tracking with Unknown Aerodynamic Effect

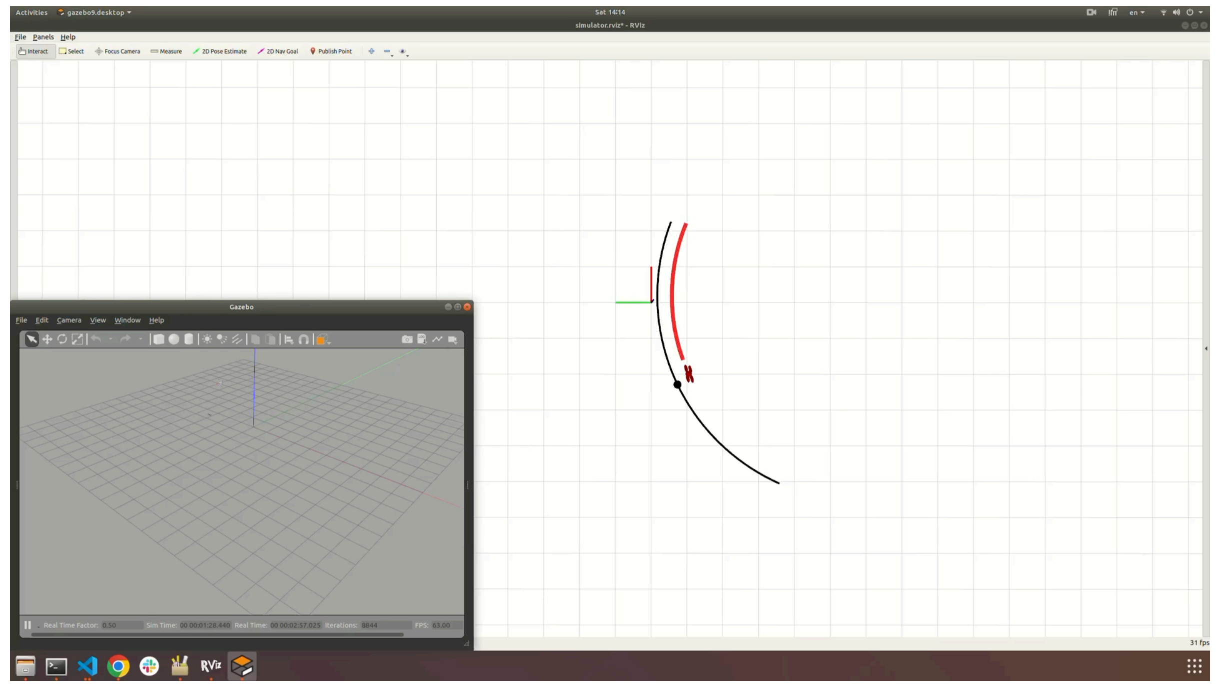
Goal:

- Track reference trajectories with a drone

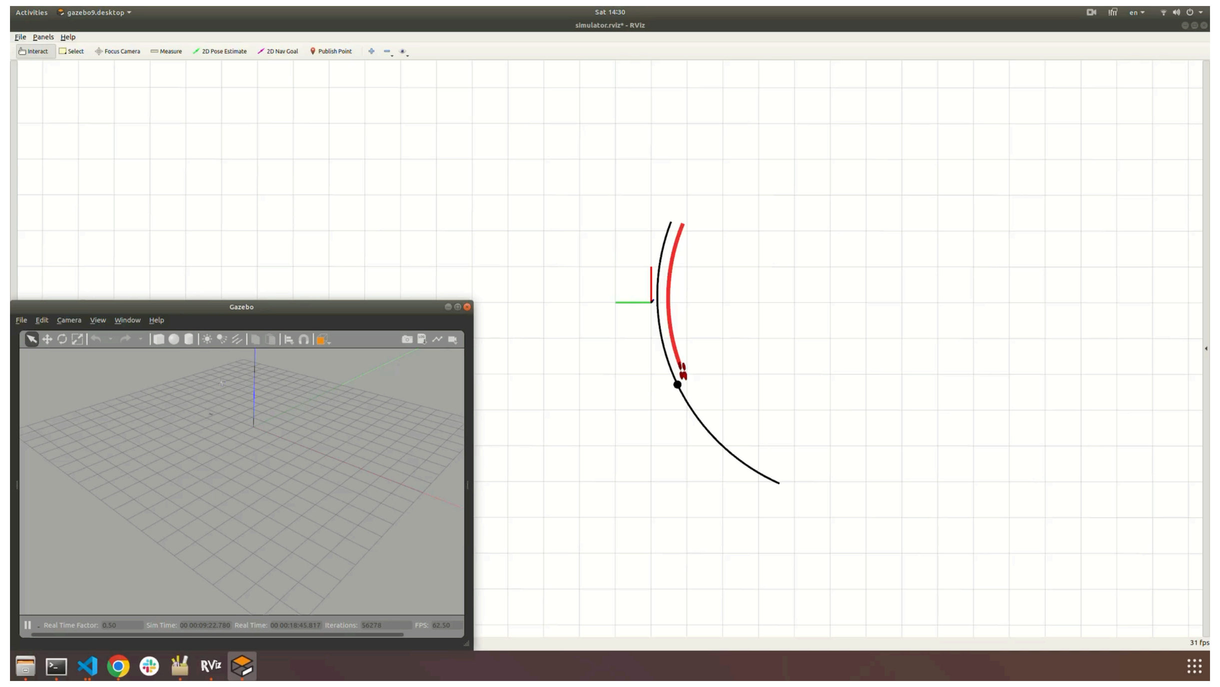
Setup:

- The system dynamics of the drone are corrupted with unknown drag effects

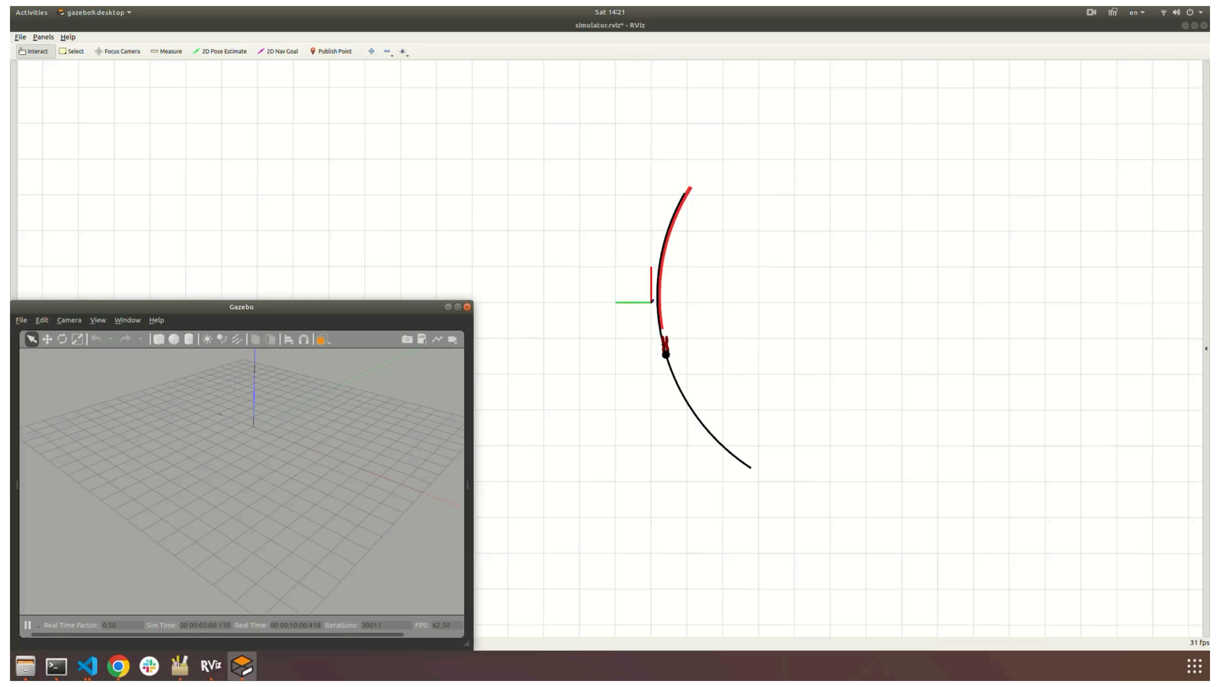
Compared algorithms: (i) Nominal MPC, (ii) Gaussian Process MPC[14], and (iii) Ours w/ INDI inner loop[15]



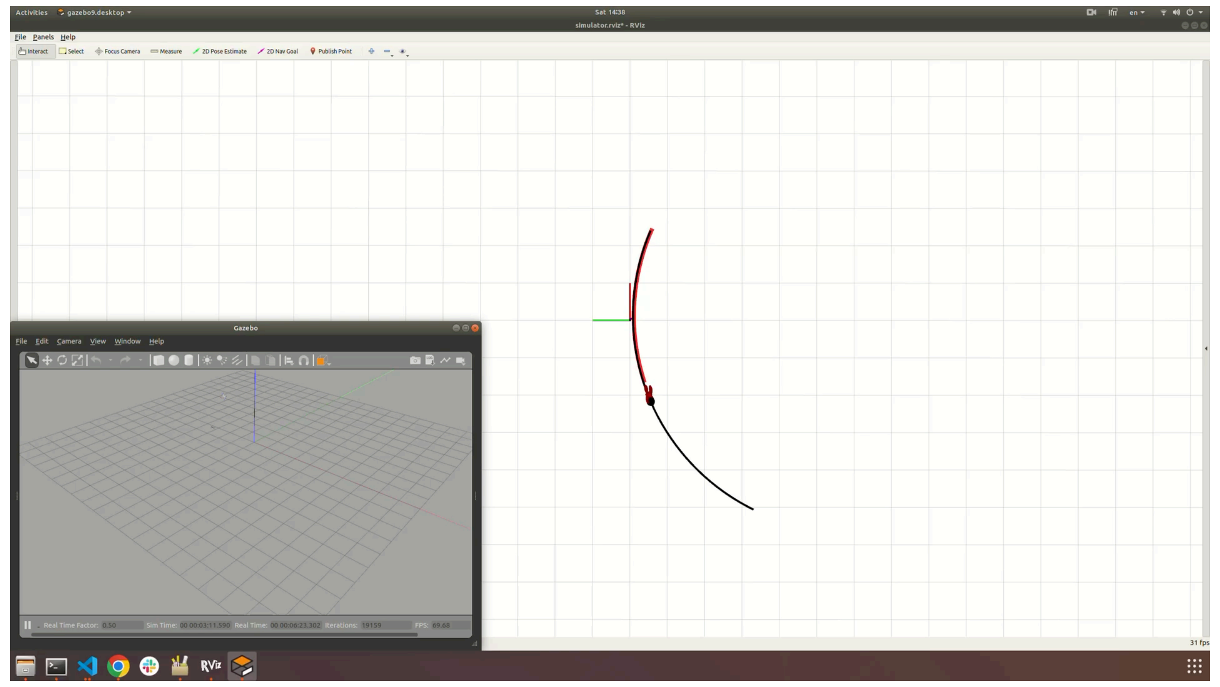
Nominal MPC



GP-MPC



Ours



Ours w/ INDI

[14] Z Torrente et al., RAL '21 [15] Tal et al., TCST '20

Simulations on Trajectory Tracking with Unknown Aerodynamic Effect

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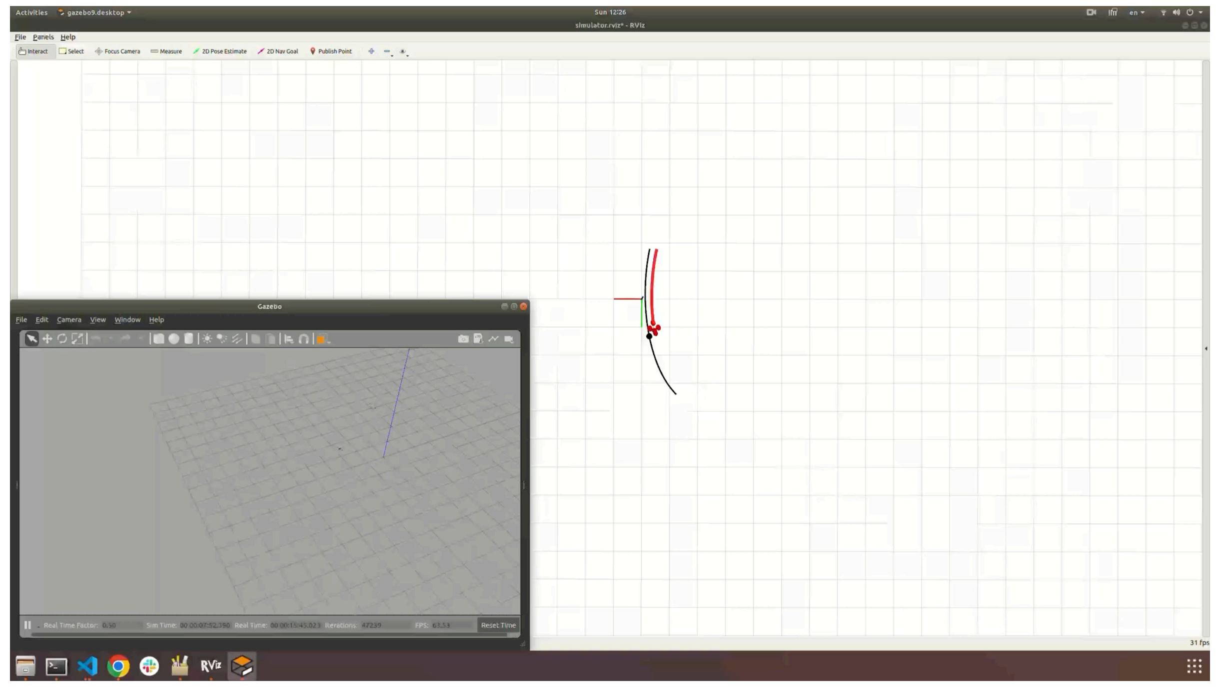
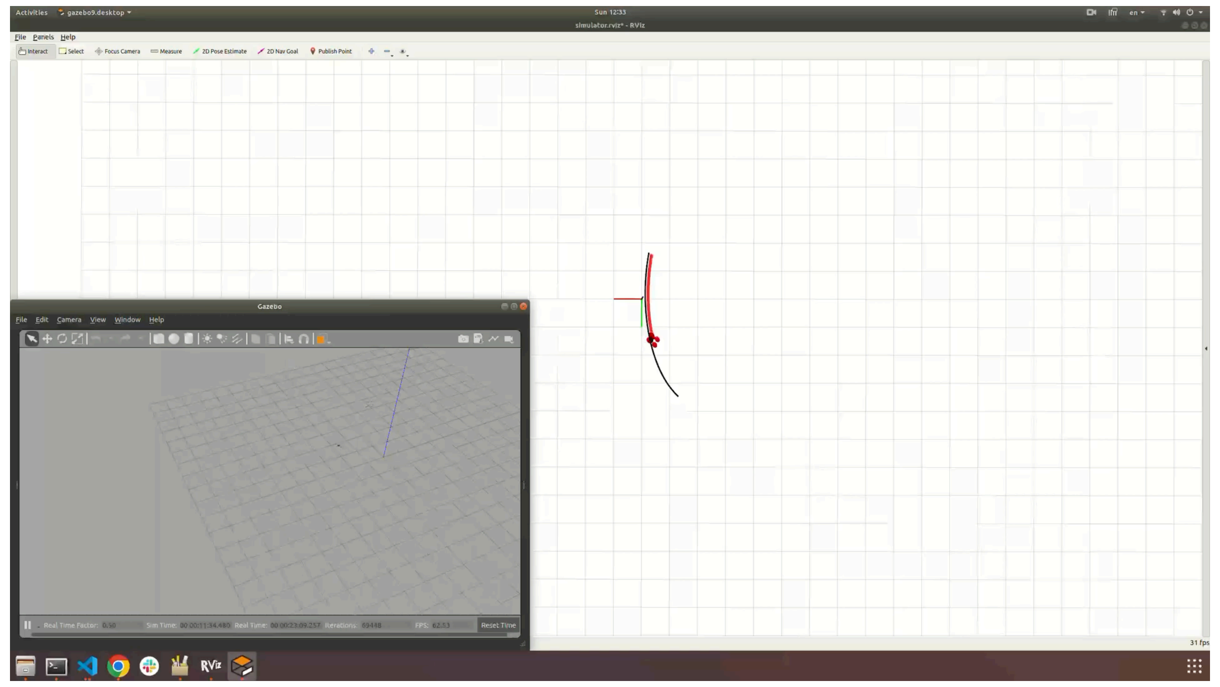
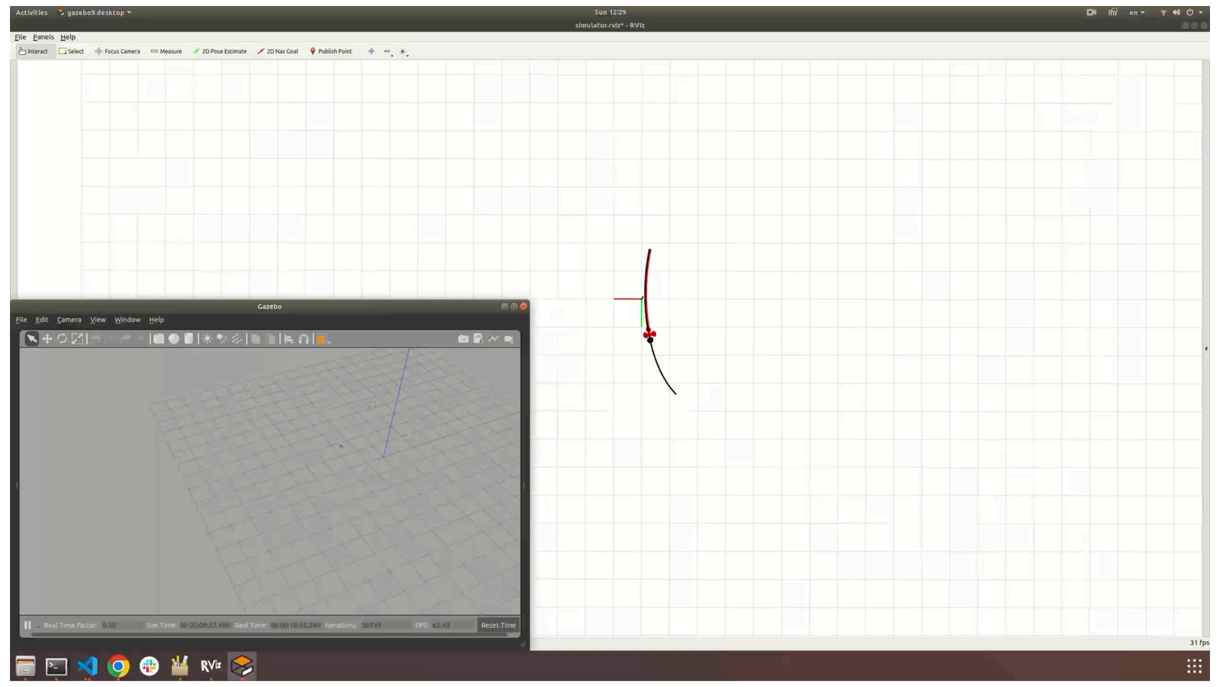


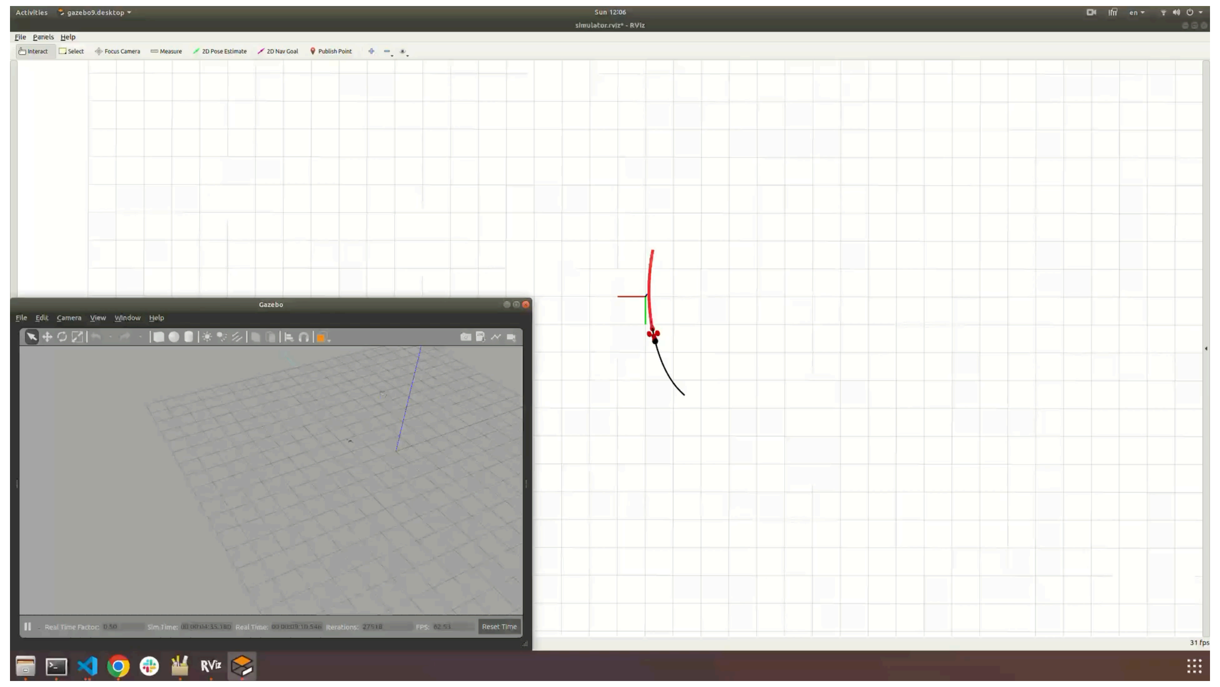
Nominal MPC



GP-MPC



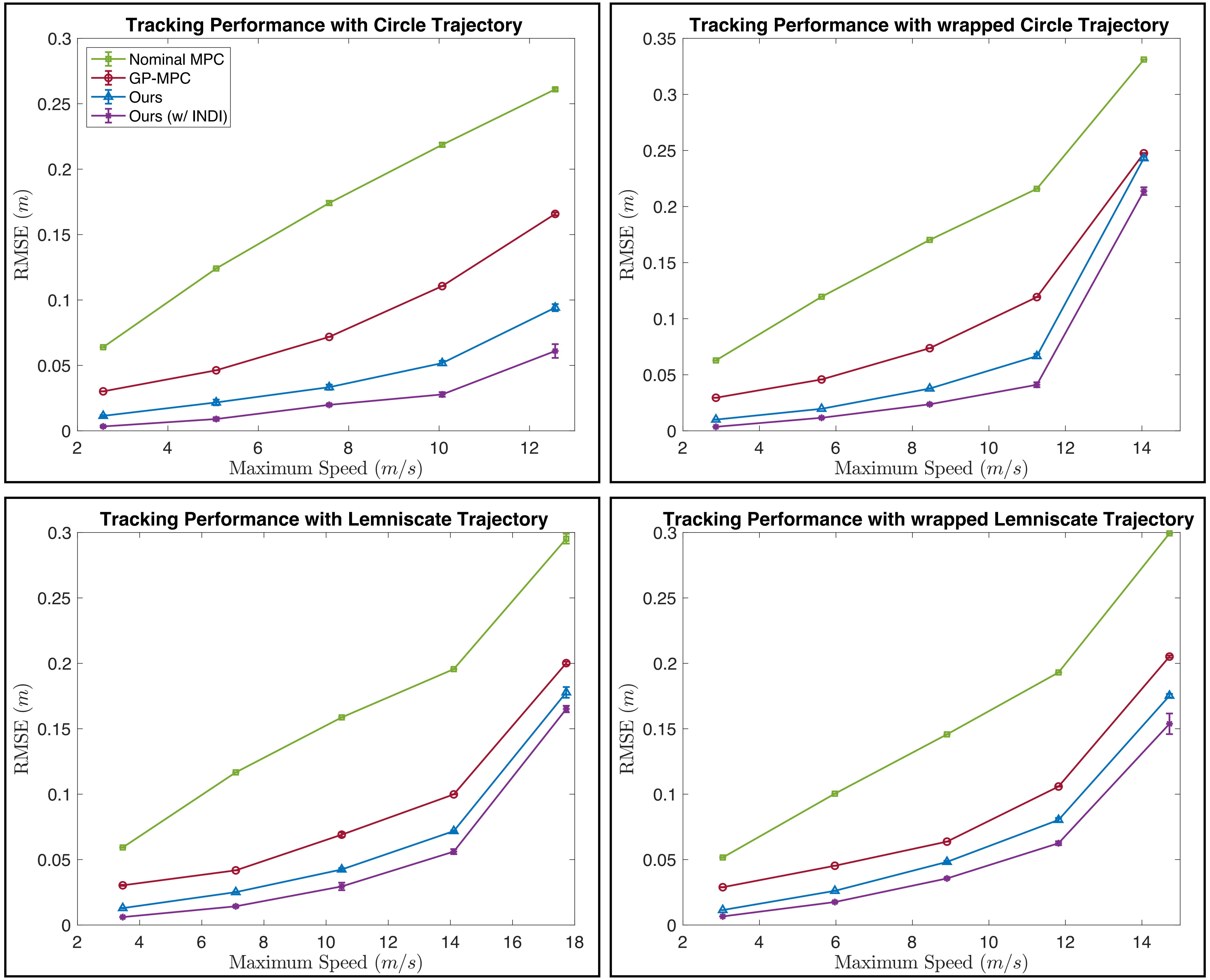
Ours



Ours w/ INDI

[14] Z Torrente et al., RAL '21 [15] Tal et al., TCST '20

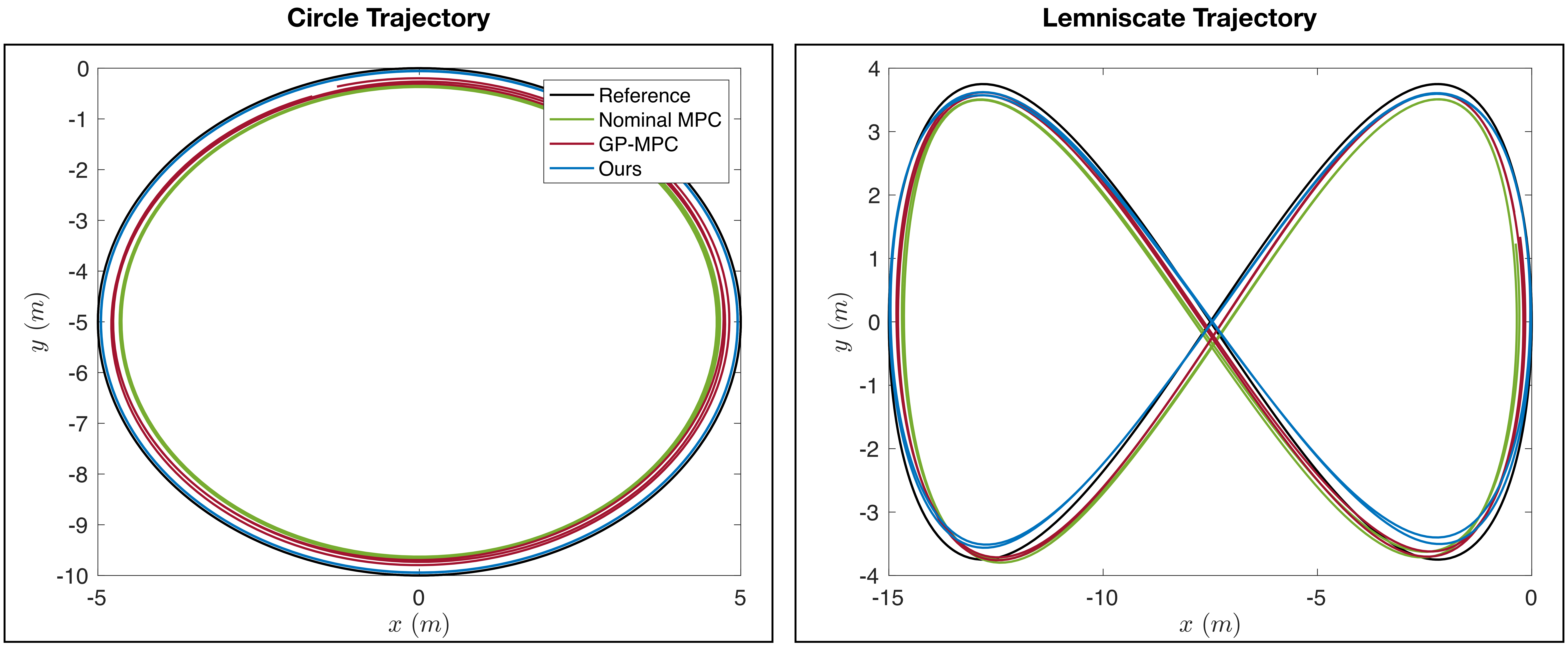
Simulations on Trajectory Tracking with Unknown Aerodynamic Effect



Results:

- Ours achieves lower tracking errors than GP
- Ours + INDI performs the best

Simulations on Trajectory Tracking with Unknown Aerodynamic Effect



Results:

- Ours achieves lower tracking errors than GP-MPC

Hardware on Trajectory Tracking with Unknown Aerodynamic Effect

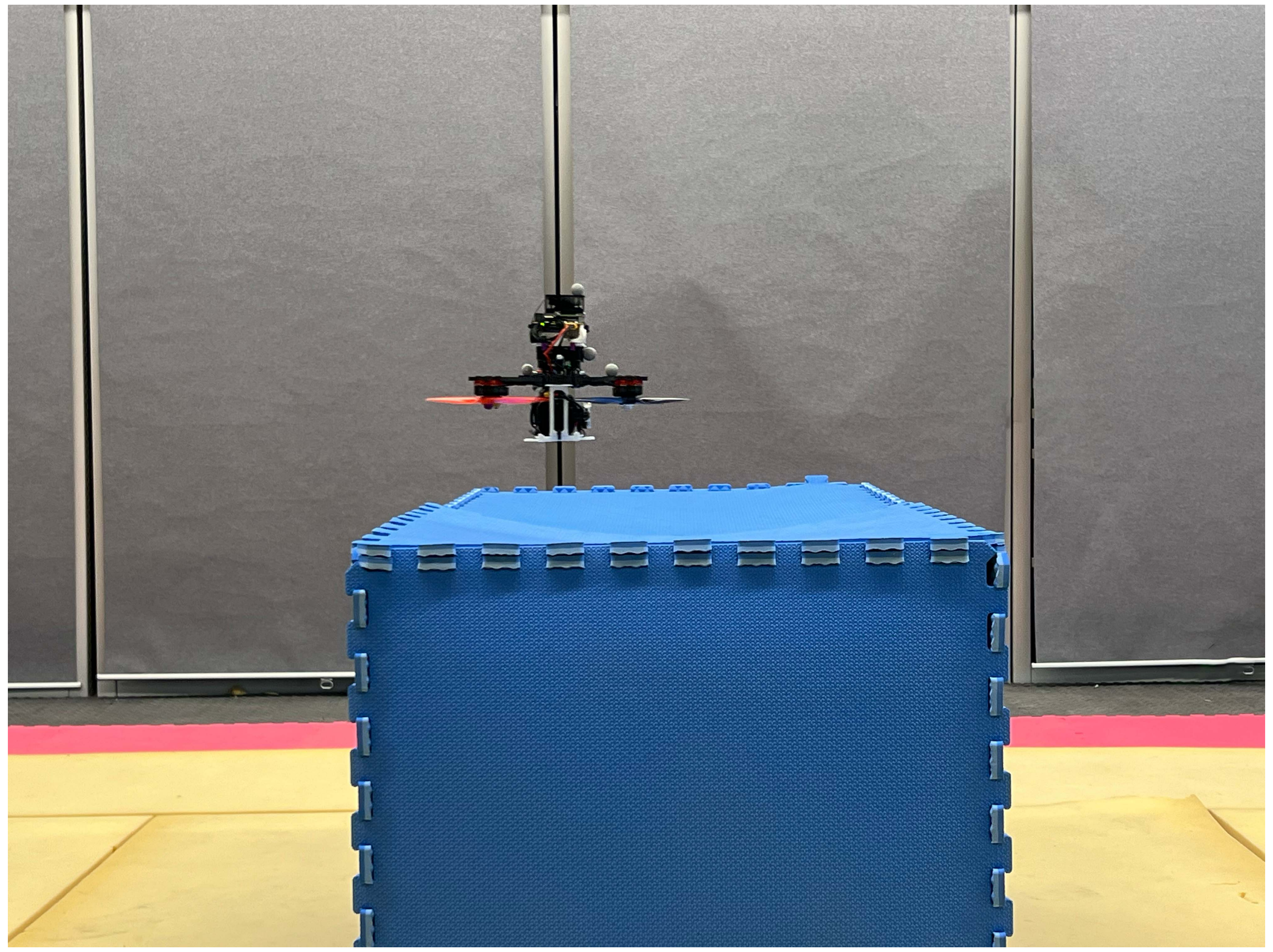
Goal:

- Track a circular trajectory with a drone

Setup:

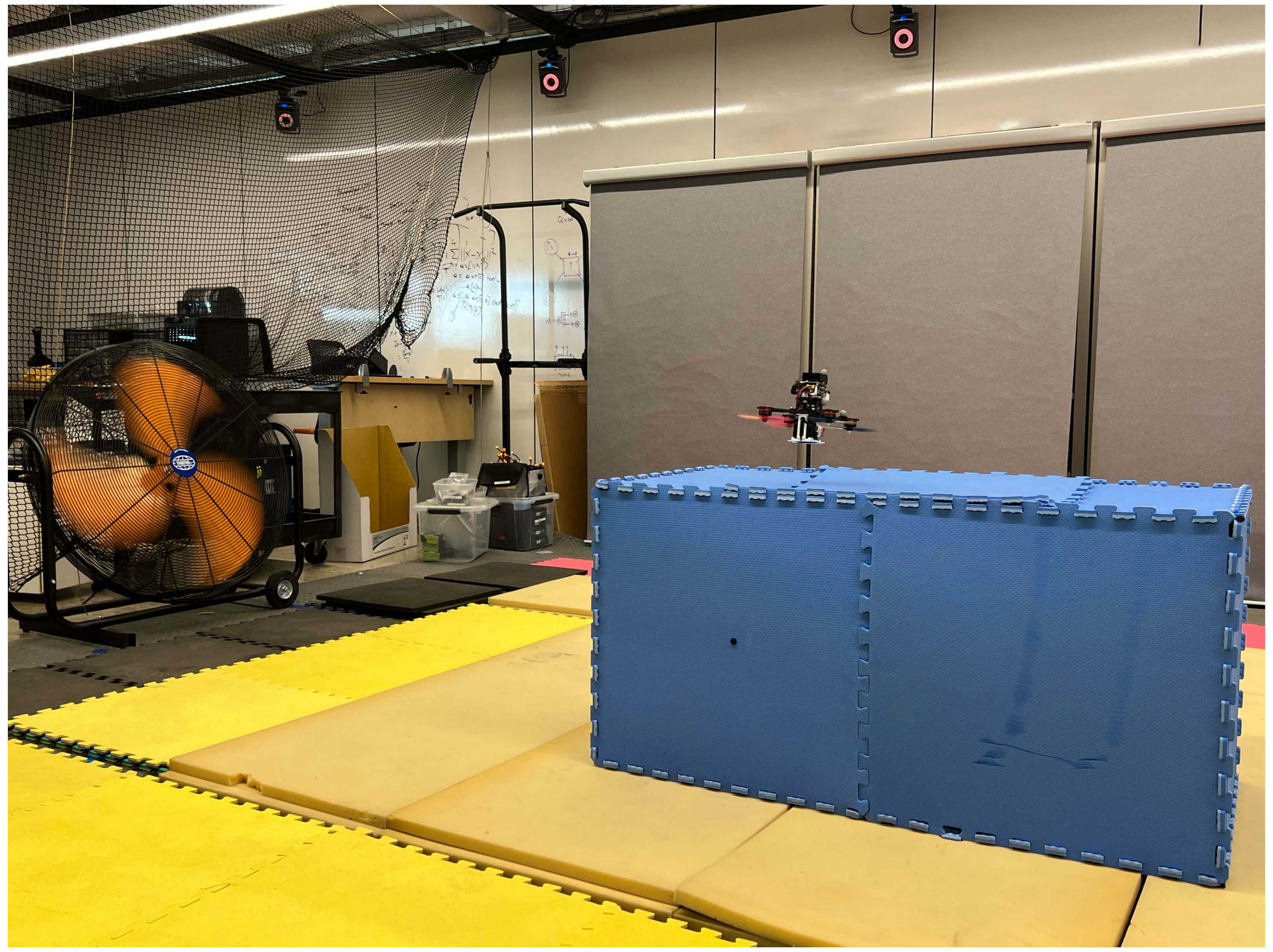
- The circular trajectory is $1m$ in diameter
- The speed is 0.8 m/s
- The drone suffers from:

(i) ground effect



(ii) wind disturbances

(iii) ground effect + wind disturbances



Hardware on Trajectory Tracking with Unknown Aerodynamic Effect

Goal:

- Track a circular trajectory with a drone

Setup:

- The circular trajectory is $1m$ in diameter
- The speed is 0.8 *m/s*
- The drone suffers from: (i) ground effect, (ii) wind disturbances, and (iii) ground effect + wind disturbances

Compared algorithms: (i) Nominal MPC and (ii) L1 adaptive MPC[16]

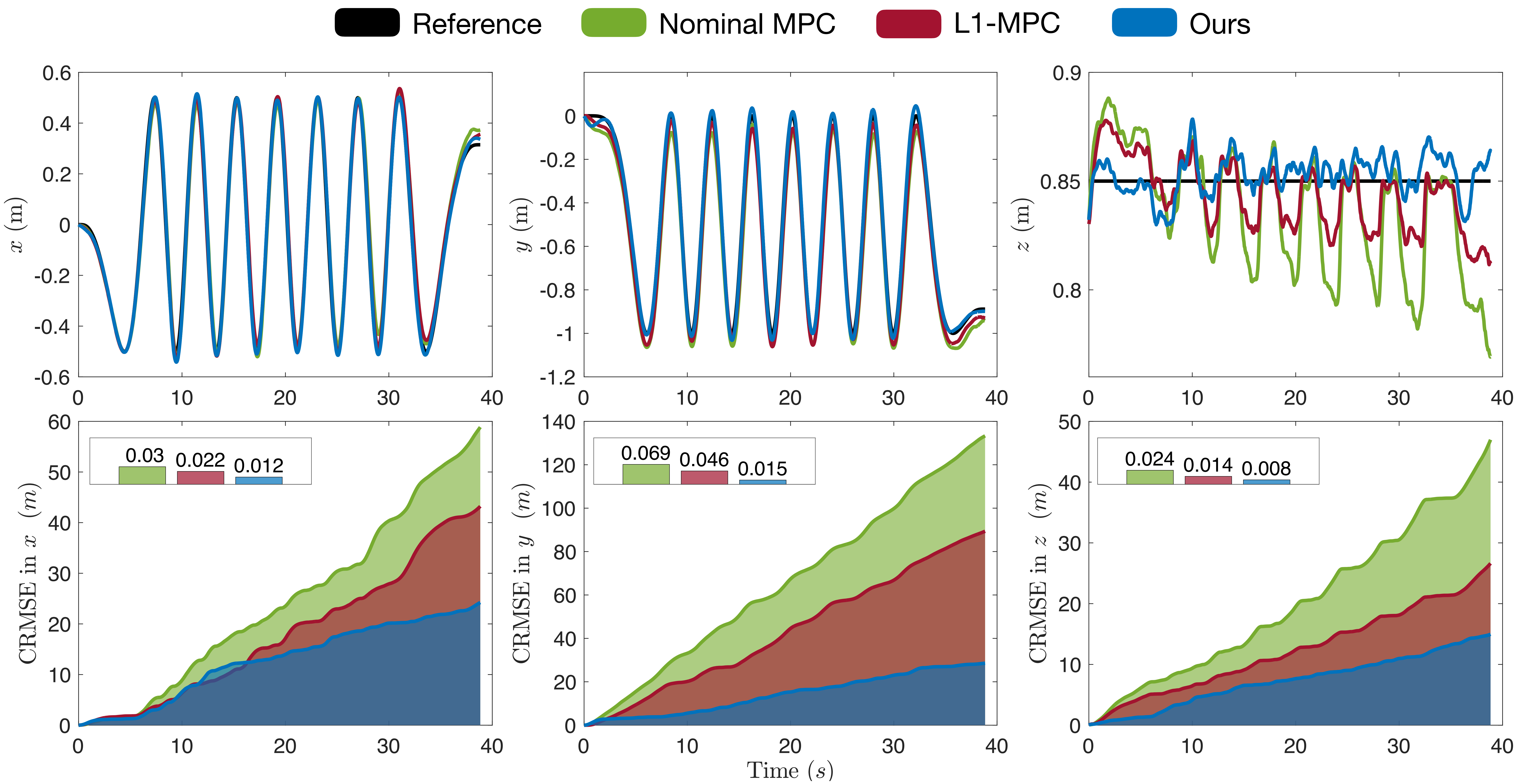
[16] Wu et al., arxiv '23

Hardware on Trajectory Tracking with Unknown Aerodynamic Effect

Results:

- Ours achieves lower tracking errors than L1-MPC
- Nominal MPC crashes under ground effect & wind disturbances

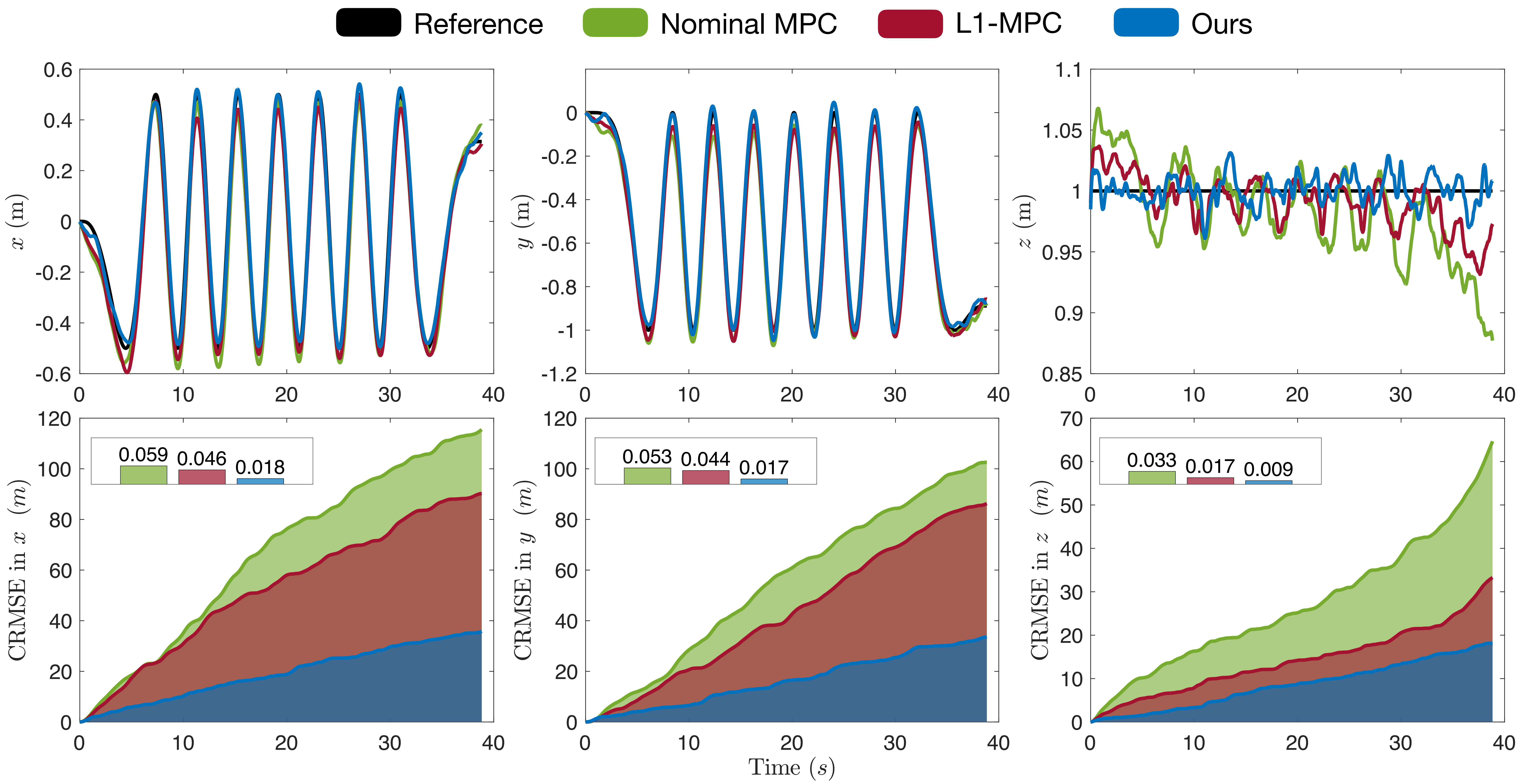
Sample Trajectory with Ground Effect



Results:

- Ours achieves lowest tracking errors
- L1-MPC do not learn the disturbance model and has large tracking error when the disturbances change abruptly
- Nominal MPC has poor *z*-direction tracking error due to ground effect and battery's voltage drop

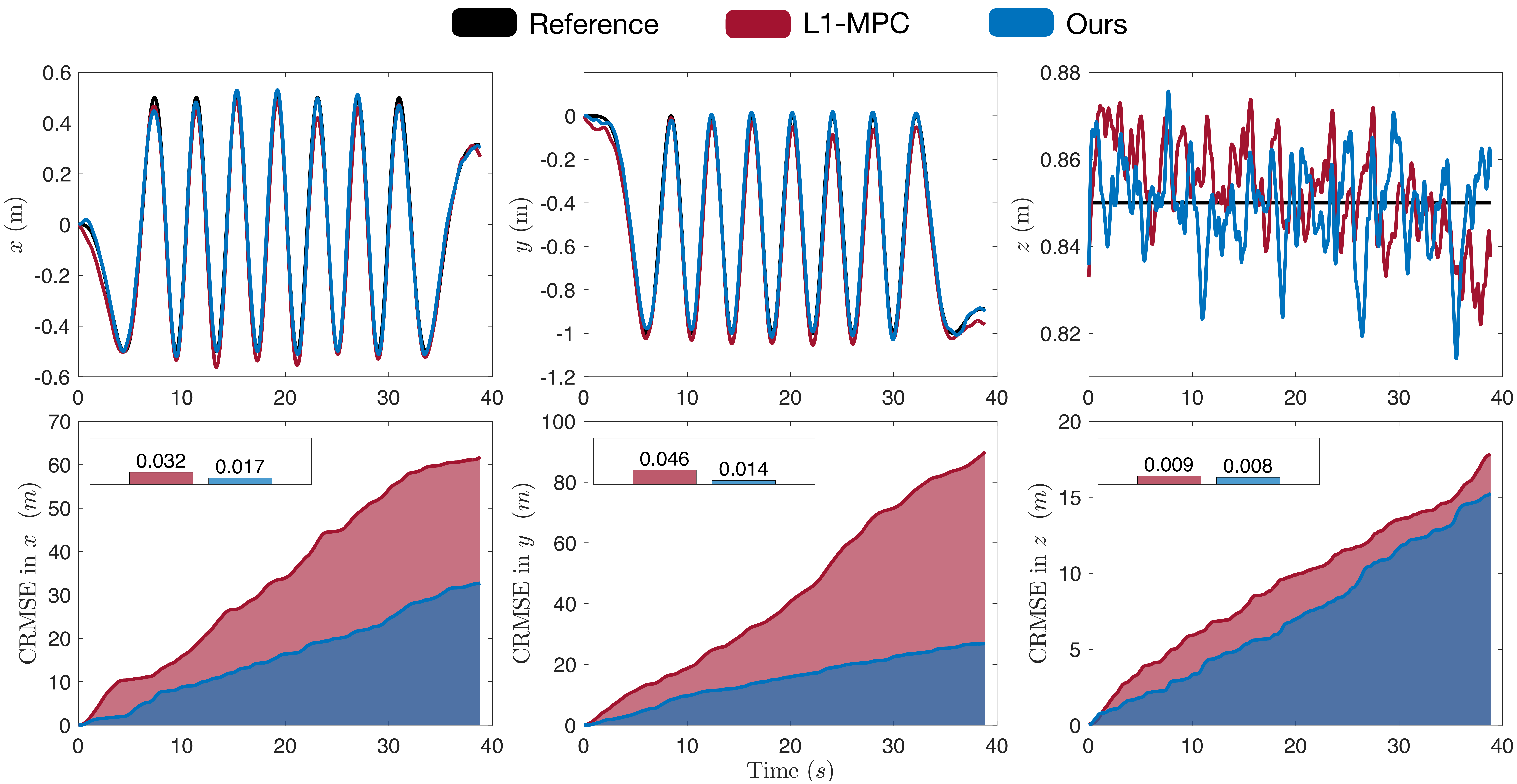
Sample Trajectory with Ground Effect



Results:

- Ours achieves lowest tracking errors
- Nominal MPC has poor z-direction tracking error due to wind and battery's voltage drop

Sample Trajectory with Ground Effect



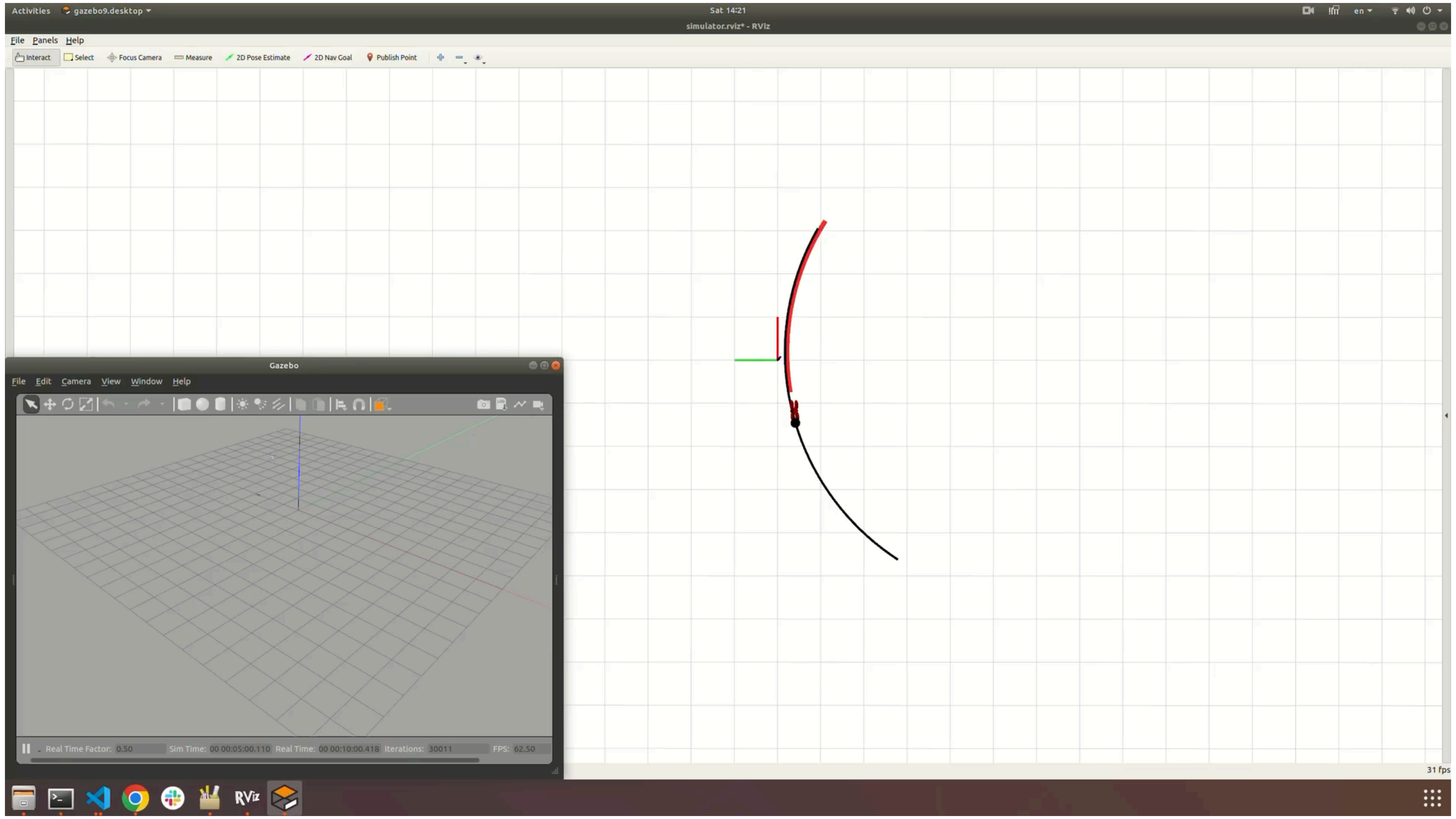
Results:

- Ours achieves lowest tracking errors
- Nominal MPC crashes under ground effect & wind disturbances

Summary and Extensions

Online control algorithm for partially unknown control-affine systems with:

- simultaneous system identification and model predictive control
- no-dynamic-regret performance guarantees



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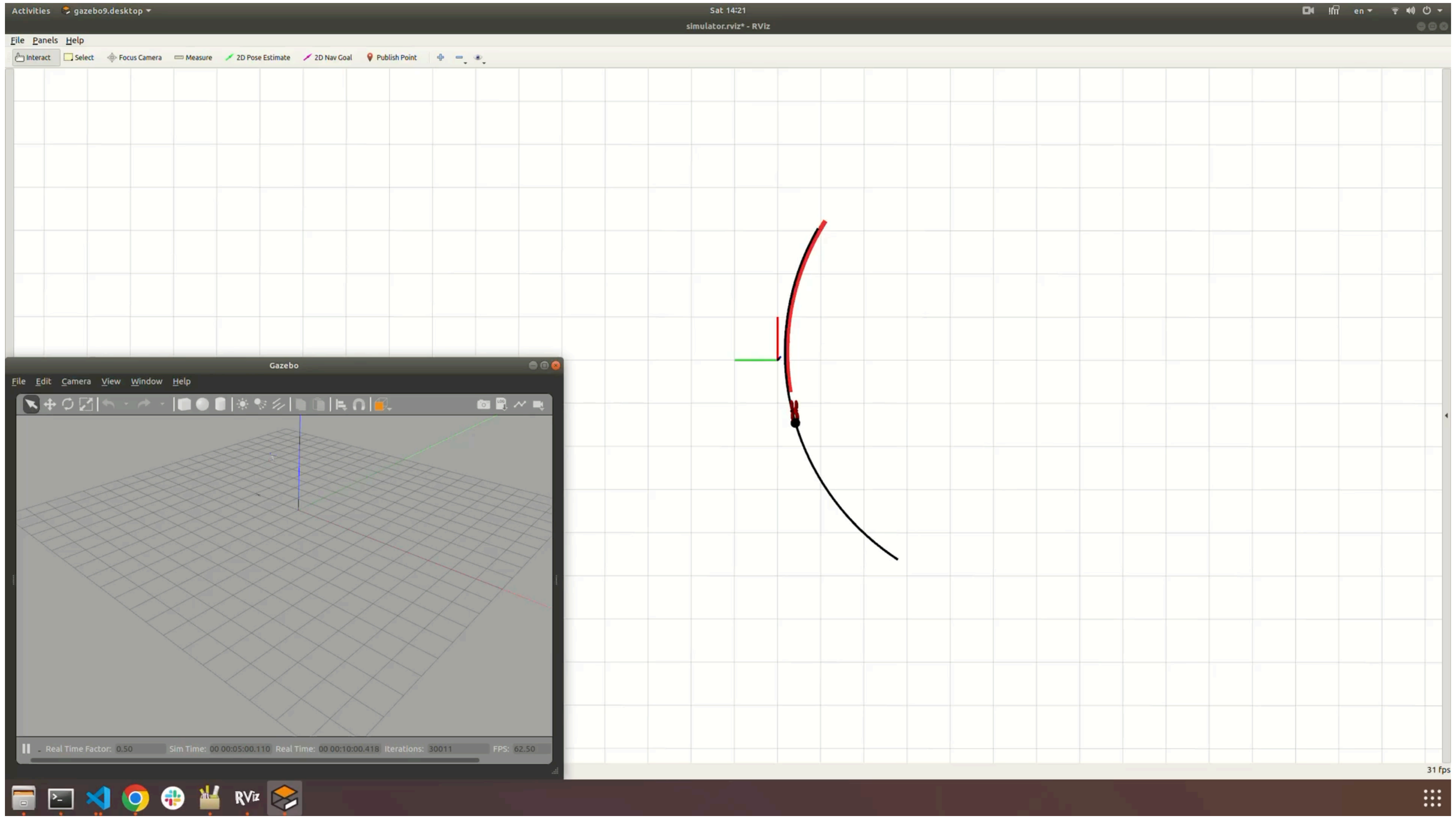

Extensions:

- active learning
- optimality of the regret bounds