TMR4515 Marine Control Systems Specilisation Module: Advanced Model Based Design and Testing Project Report

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Abstract: In this project, we consider a multi-input multi-output benchmark with uncertainties. To control this plant, a controller that uses machine learning technique is used. Dynamic Hypothetic Testing is implemented, assuming three hypotheses. For each hypothesis, we design a discrete time steady state Kalman filter.

Keywords: Control Design, Kalman Filter, Dynamic Hypothetic Testing, Machine Learning.

1. INTRODUCTION

In practice, it is difficult to obtain a highly accurate mathematical model of the physical process of interest. These model uncertainties arise from, e.g. unmodeled dynamics, disturbances and measurement noise. Hence, the control system in practice should be robust and reliable under uncertain situation. In this project, Dynamic Hypothesis Testing is implemented by using a discrete time steady state Kalman filter.

2. BENCHMARK EXAMPLE

In this project, we consider the dynamic of a benchmark. The system of benchmark is shown in 1.

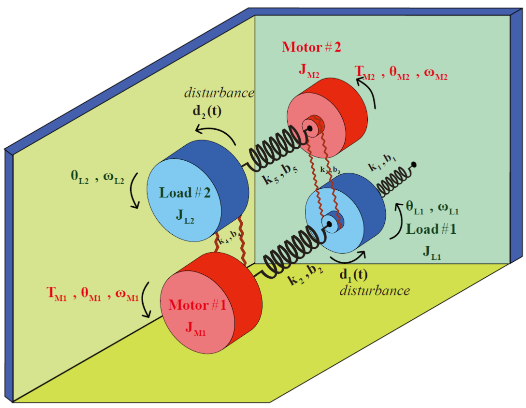

Fig. 1. the Simplified Benchmark Model

The plant has two motors driving two loads through flexible couplings. Each load is connected to both motors. In addition, $Load1$ is connected to wall through a torsional spring and torsional damper. The output signals are the loads shaft angles the measurement of these signals are corrupted by measurement noises $\mathbf{v}(t)$. The angular velocity of loads is affected by the disturbance torques $d(t)$, which are independent stationary first-order (coloured) stochastic processes generated by driving a low-pass filter with continuous-time white noise, defined as follow

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \dfrac{0.2}{s+0.2} & 0 \\ 0 & \dfrac{0.2}{s+0.2} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{1}$$

where $w_1(t)$ and $w_2(t)$ are white noises, which have zero mean and intensity of 1 and 11 respectively.

The state-space model of the plant, including disturbance and noise inputs, is defined as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{v}(t) \end{aligned} \tag{2}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}$ and $\mathbf{C}$ are given in equation 4 and 5, and $\mathbf{v}$ is measurement noise which is zero mean white noise with intensity of 10^{-6}.

The states of system are selected as

$$\mathbf{x}^T(t) = [\, \theta_{L1}\ \theta_{M1}\ \theta_{M2}\ \theta_{L2}\ \omega_{L1}\ \omega_{M1}\ \omega_{M2}\ \omega_{L2}\ d_1\ d_2 \,] \tag{3}$$

where θ_{L1}, θ_{L2}, θ_{M1}, θ_{M2} are rotating angle of $Load1$, $Load2$, $Motor1$ $Motor2$ and ω_{L1}, ω_{L2}, ω_{M1}, ω_{M2} are angular velocity of $Load1$, $Load2$, $Motor1$ $Motor2$ respectively.

3. KALMAN FILTER DESIGN

In exercise one, a Kalman filter is designed, the guideline of Kalman filter design is provided by Brian D. O. Anderson (1979).

3.1 Parameters

The parameters of state-space model is given in Table 1.

$$
\mathbf{A}=\begin{bmatrix}
0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0\\
-\frac{k_1+k_2+k_3}{J_{L1}}&\frac{k_2}{J_{L1}}&\frac{k_3}{J_{L1}}&0&-\frac{b_1+b_2+b_3}{J_{L1}}&\frac{b_2}{J_{L1}}&\frac{b_3}{J_{L1}}&0&\frac{1}{J_{L1}}&0\\
\frac{k_2}{J_{M1}}&-\frac{k_2+k_4}{J_{M1}}&0&\frac{k_4}{J_{M1}}&\frac{b_2}{J_{M1}}&-\frac{b_2+b_4}{J_{M1}}&0&\frac{b_4}{J_{M1}}&0&0\\
\frac{k_3}{J_{M2}}&0&-\frac{k_3+k_5}{J_{M2}}&\frac{k_5}{J_{M2}}&\frac{b_3}{J_{M2}}&0&-\frac{b_3+b_5}{J_{M2}}&\frac{b_5}{J_{M2}}&0&0\\
0&\frac{k_4}{J_{L2}}&\frac{k_5}{J_{L2}}&-\frac{k_4+k_5}{J_{L2}}&0&\frac{b_4}{J_{L2}}&\frac{b_5}{J_{L2}}&-\frac{b_4+b_5}{J_{L2}}&0&\frac{1}{J_{L2}}\\
0&0&0&0&0&0&0&0&-0.2&0\\
0&0&0&0&0&0&0&0&0&-0.2
\end{bmatrix} \tag{4}
$$

$$
\mathbf{B}=\begin{bmatrix}
0&0\\0&0\\0&0\\0&0\\0&0\\
\frac{1}{J_{M1}}&0\\
0&\frac{1}{J_{M2}}\\
0&0\\0&0\\0&0
\end{bmatrix}
\quad
\mathbf{G}=\begin{bmatrix}
0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0.2&0\\0&0.2
\end{bmatrix}
\quad
\mathbf{C}=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0
\end{bmatrix} \tag{5}
$$

Table 1. Parameters

Inertial moment ($\mathrm{Kg}\cdot\mathrm{m}^2$)		Damping (Ns/rad)		Spring (N/rad)	
J_{M1}	1	b_1	0.1	k_1	0.15
J_{M2}	1	b_2	0.1	k_2	1.625
J_{M3}	1	b_3	0.1	k_3	0.1
J_{M4}	1	b_4	0.1	k_4	0.1
		b_5	0.1	k_5	1.8

Parameters k_2 and k_5 are unknown and assumed to be in the intervals $k_2 \in [0.75, 2.5]$ and $k_5 \in [0.9, 2.5]$. In exercise one, we pick the center of intervals as their parametric value, which are $k_2 = 1.625$ and $k_5 = 1.8$.

3.2 Discrete-time Kalman Filter Design

The state-space model given in equation 2 is continuous. In order to design a discrete-time Kalman filter, we first need to discretize the state space model with $T_s = 0.001s$, given in below

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \mathbf{G}_k\mathbf{w}_k\\
\mathbf{y}_k &= \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k
\end{aligned} \tag{6}
$$

where

$$
\begin{aligned}
\mathbf{A}_k &= \mathbf{I} + T_s\mathbf{A}\\
\mathbf{B}_k &= \mathbf{A}^{-1}(\mathbf{A}_k - \mathbf{I})\mathbf{B}\\
\mathbf{G}_k &= \mathbf{A}^{-1}(\mathbf{A}_k - \mathbf{I})\mathbf{G}\\
\mathbf{C}_k &= \mathbf{C}
\end{aligned} \tag{7}
$$

In addition, the process and measurement noise covariance matrices is defined as

$$
\mathbf{Q}_k = \begin{bmatrix} 1 & 0\\ 0 & 11 \end{bmatrix}, \quad \mathbf{R}_k = \begin{bmatrix} 10^{-6} & 0\\ 0 & 10^{-6} \end{bmatrix} \tag{8}
$$

Since we adjust the expression of G_k, values for $w(t)$ and $v(t)$ in discrete time framework are independent on sampling time and equivalent to values in continuous time framework. With all the parameters and matrices defined, the Discrete-time Kalman Filter Algorithm is ready to be implemented. The algorithm is present below.

$$
\begin{aligned}
\mathbf{K}_k &= \bar{\mathbf{P}}_k\mathbf{C}_k^T[\mathbf{C}_k\bar{\mathbf{P}}_k\mathbf{C}_k^T + \mathbf{R}_k]^{-1}\\
\hat{\mathbf{x}}_k &= \bar{\mathbf{x}}_k + \mathbf{K}_k[\mathbf{y}_k - \mathbf{C}_k\bar{\mathbf{x}}_k]\\
\hat{\mathbf{P}}_k &= [\mathbf{I} - \mathbf{K}_k\mathbf{C}_k]\bar{\mathbf{P}}_k[\mathbf{I} - \mathbf{K}_k\mathbf{C}_k]^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T\\
\bar{\mathbf{x}}_{k+1} &= \mathbf{A}_k\hat{\mathbf{x}}_k + \mathbf{B}_k\mathbf{u}_k\\
\bar{\mathbf{P}}_{k+1} &= \mathbf{A}_k\hat{\mathbf{P}}_k\mathbf{A}_k^T + \mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^T
\end{aligned} \tag{9}
$$

3.3 Simulation Results

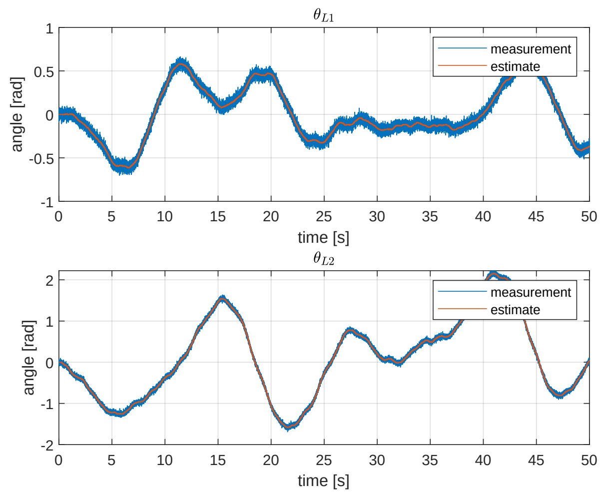

Fig. 2. Comparison between Measurement and Estimated result of Kalman Filter

The results of measurement and estimated outputs are shown in Figure 2. Kalman filter is able to estimate the outputs of the plant and filter out the high frequency noise. This is expected since the intensity of these noises is exactly known.

In the Discrete-time Kalman Filter Algorithm, the error covariance matrix $\mathbf{P}_k$ is time-varying. While we notice that the error between $\mathbf{P}_k$ and $\mathbf{P}_{k+1}$ will quickly become negligible. For example, after $t = 2s$, the order of magnitude of the error between $\mathbf{P}_k$ and $\mathbf{P}_{k+1}$ is -11. Hence, in the following exercise, the steady-state Kalman filter will be implemented.

4. DESIGNING DYNAMIC HYPOTHETIC TESTING

In exercise one, we neglect uncertainty on parameters k_2 and k_5 and use the mean value of intervals for each in the state-space model. In this task, k_2 and k_5 are assumed to be uncertain, and the parametric uncertainty is divided to three sub-sets, shown in Figure 3.

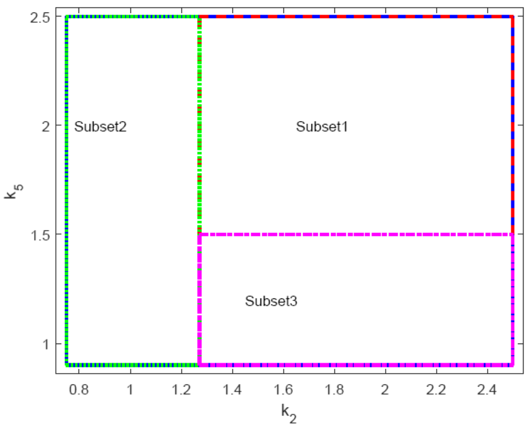

Fig. 3. The parametric uncertainty

4.1 Hypotheses

In this part, we assume three hypotheses, which are given in Table 2. For each hypothesis, a Kalman filter is designed the way in previous task.

Table 2. Three Hypotheses

Hypothesis	$\mathbf{k}_2$ (N/rad)	$\mathbf{k}_5$ (N/rad)
1	2	2
2	1	1.75
3	2	1.25

4.2 Dynamic Hypothesis Testing Algorithm

Each Kalman filter will give a estimate of measurement different from others. To decide which Kalman filter gives more accurate estimate, the dynamic hypothesis testing algorithm is implemented which assigns a probability to each hypothesis. The probability to each hypothesis is given as

$$h_i(t+1) = \frac{Pr\{y(t+1)|\mathcal{H} = \mathcal{H}_i, Z(t)\}}{\sum_{k=1}^{N} Pr\{y(t+1)|Z(t), \mathcal{H} = \mathcal{H}_k\}} h_i(t) \quad (10)$$

All the noise and disturbances are Gaussian, equation 10 can be simplified to

$$h_i(t+1) = \frac{\frac{e^{-\frac{1}{2}\tilde{y}^T_{\mathcal{H}_i}(t+1)\mathcal{S}_{\mathcal{H}_i}\tilde{y}_{\mathcal{H}_i}(t+1)}}{\sqrt{(2\pi)^2|\mathcal{S}_{\mathcal{H}_i}|}}}{\sum_{k=1}^{N} h_k(t)\frac{e^{-\frac{1}{2}\tilde{y}^T_{\mathcal{H}_k}(t+1)\mathcal{S}_{\mathcal{H}_k}\tilde{y}_{\mathcal{H}_k}(t+1)}}{\sqrt{(2\pi)^2|\mathcal{S}_{\mathcal{H}_k}|}}} h_i(t) \quad (11)$$

where $\mathcal{S}_{\mathcal{H}_k} = \mathbf{CPC}^T + \Theta_d$. $\mathbf{P}$ is steady-state error covariance and Θ_d is discrete equivalent of continuous measurement noise covariance matrix which is computed as $\Theta_d = \frac{\mathbf{R}_k}{T_s}$

Based on the dynamic hypothesis testing algorithm, we will choose the estimate corresponding to hypothesis with highest probability.

4.3 Simulation Result

We design a discrete time steady state Kalman filter for each hypothesis in Simulink. The Kalman filter and dynamic hypothesis testing algorithm Simulink diagram is shown in Figure 4. $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ are steady-state error covariance matrices which are obtained by simulating three Kalman filters separately without probability function.

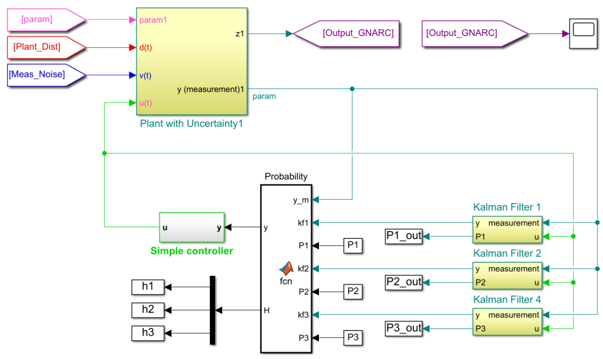

Fig. 4. Simulink diagram

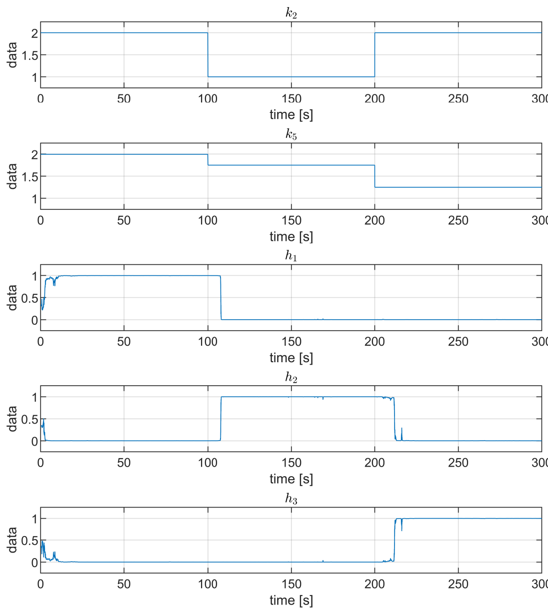

Fig. 5. The values of parameters k_2 and k_5, and the probabilities to three hypotheses with lower bound $ASTAN = 5 \times 10^{-5}$

The simulation result is present in Figure 5. The lower bound of probability is chosen as $ASTAN = 5 \times 10^{-5}$. When the parameter changes, the dynamic hypothesis testing algorithm will update the probability of three hypotheses and the probability of the correct hypothesis will converge to 1.

The lower bound of probability is decided by tuning. Results of different lower bound are present in Figure 6. If the lower bound is too small, it takes more time to switch to the roght hypothesis. To reduce delay in switching, a possible method is increasing the lower bound parameter $ASTAN$ in the testing algorithm. When this value increase to 10^{-3} or more, disturbance will appear on the probability testing result.

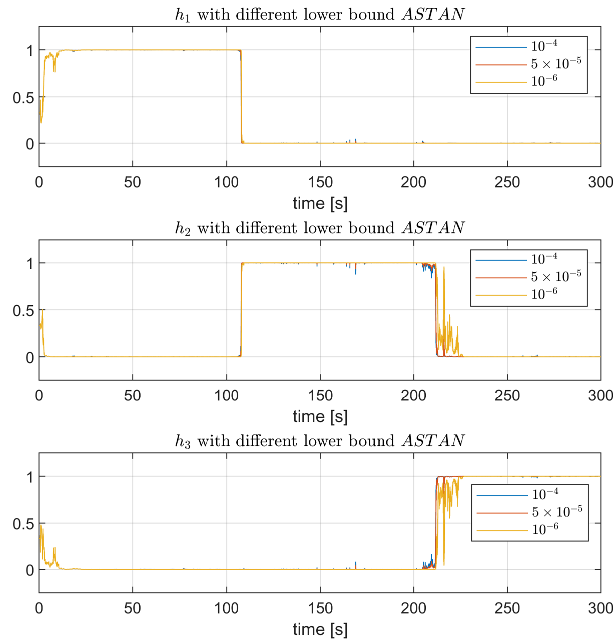

Fig. 6. The probabilities to three hypotheses with three different lower bound $ASTAN$

5. CONCLUSION

This project constructs a discrete Kalman Filter for a benchmark model to filter out measurement and processing noise in the system, where shaft angles of loads θ_{L1} and θ_{L2} are measurements. The discrete Kalman filter can estimate measured states from the plant properly and remove high frequency components.

Due to uncertainties in parameter k_2 and k_5, a dynamic hypothesis testing algorithm is added into the benchmark system, automatically selecting the Kalman filter that has the most probability. The result shows that the system can correctly switch its Kalman filter in response to the parameter changes of k_2 and k_5. A problem of this algorithm is lag out of Kalman filter switching. To reduce delay in switching, a possible method is increasing the lower bound parameter $ASTAN$ in the testing algorithm. This value cannot increase to 10^{-3} or more because an apparent disturbance will appear on the probability testing result.

REFERENCES

Brian D. O. Anderson, J.B.M. (1979). *Optimal Filtering*. PRENTICE-HALL, INC., Australia.