Project description for the module of Advanced Model Based Design and Testing of Marine Control Systems (version 2)

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***Abstract*— This is a project description document formulating the final problem formulation of the project. The document is being updated throughout the course (Pay attention to the version of the document). In the final project we aim to build a controller the uses Machine Learning techniques to control an uncertain system. The structure of the controller is of the multi-controller kind and the benchmark example of the system is a multi-input multi-output plant with uncertainties.**

I. INTRODUCTION

In many practical applications of control theory, it is virtually impossible to obtain a highly accurate mathematical model of the physical process of interest. Moreover, all practical systems are subjected to uncertainty caused by different reasons such as unmodeled dynamics, uncertain and unknown system parameters, plant disturbances, measurement noise, changes in operating conditions, failure or degradation of components, or unexpected changes in system dynamics and etc. Consequently, a practicable controller, should be able to provide stability and performance in the presence of these uncertainties. This projects aims to build a step by step guide for building an adaptive multi-controller that benefits from machina learning techniques.

II. BENCHMARK EXAMPLE

In this section we describe the dynamics of our benchmark example which consists of system of motors and loads. A system of motors and loads connected to each other through flexible transmission as shown in Fig. 1;

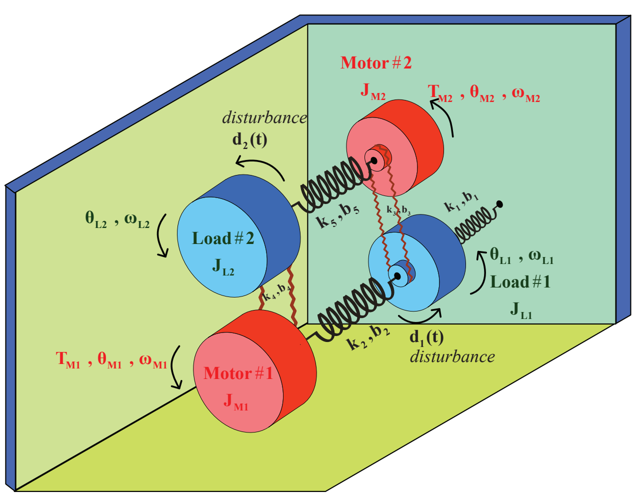

Fig. 1. The systems Motors and Loads with Elastic Transmission

The plant consists of two motors driving two loads through flexible couplings. Each load is connected to both motors.

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One load is connected to a wall through a torsional spring and torsional damper while the other is free. The output signals are the loads shaft angles the measurement of which are corrupted by measurement noises $v(t)$. The disturbance torques $d(t)$ affect only the loads. The disturbance torques $d(t)$ are independent stationary first-order (coloured) stochastic processes generated by driving a low-pass filter with continuous-time white noise of zero mean and unit intensity, i.e.,

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \frac{.2}{s+.2} & 0 \\ 0 & \frac{.2}{s+.2} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}.$$

The disturbance $w_1(t)$ and $w_2(t)$ are independent zero mean white noises with unit intensity.

Later in the project we will assume that the numerical parameters of elastic transmission are uncertain, we will also assume that there is, in the control channel, unmodelled time-delay τ whose maximum possible value is 50 ms. (These do not concern the first exercise on designing of the Kalamn filters)

A state-space representation of the plant, including the disturbance and noise inputs, is given by

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \tag{1a}$$
$$y(t) = Cx(t) + v(t), \tag{1b}$$

where $A, , B, G$ are presented in (8) where the state vector is selected as $x^T(t) = [\theta_{L1}\ \theta_{M1}\ \theta_{M2}\ \theta_{L2}\ \omega_{L1}\ \omega_{M1}\ \omega_{M2}\ \omega_{L2}\ d_1\ d_2]$ and $J_{M1} = J_{M2} = J_{L1} = J_{L2} = 1$ (Kgm^2), $k_1 = 0.15$ (N/rad), $k_3 = k_4 = 0.1$ (N/rad), $b_1 = b_2 = b_3 = b_4 = b_5 = 0.1$ (Ns/rad), and k_2 and k_5 are unknown parameters assumed to have values in the intervals $k_2 \in [0.75,\ 2.5]$ and $k_5 \in [0.9,\ 2.5]$. (for the first exercise, i.e. designing Kalamn filter assume that these are known and pick the center of the interval as their parametric value). The shaft angles of the loads θ_{L1} and θ_{L2} (in radians) are measured outputs and they are corrupted by independent zero mean white noises with intensity of 10^{-6}.

A. *Exercise one: Kalman Filter Design*

In order to design a KF, we follow the guidelines provided in the Kalman Filter self study guideline [1]. Before destining the discrete time KF, make yourself familiar with the simulation model.

It is very important to pay attention to the intensity values that are given in this exercise for $w(t)$ and $v(t)$. these values are expressed in courthouse time framework. In order to design a discrete time KF you need to find the equivalent density values in discreet-time framework which depends on sampling time that you have picked. In order to have a unified simulation in whole class, take sampling time as 1 millisecond. If you need some help about this, start exploring ”kalmd” command in Matlab.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2+k_3}{J_{L1}} & \frac{k_2}{J_{L1}} & \frac{k_3}{J_{L1}} & 0 & -\frac{b_1+b_2+b_3}{J_{L1}} & \frac{b_2}{J_{L1}} & \frac{b_3}{J_{L1}} & 0 & \frac{1}{J_{L1}} & 0 \\ \frac{k_2}{J_{M1}} & -\frac{k_2+k_4}{J_{M1}} & 0 & \frac{k_4}{J_{M1}} & \frac{b_2}{J_{M1}} & -\frac{b_2+b_4}{J_{M1}} & 0 & \frac{b_4}{J_{M1}} & 0 & 0 \\ \frac{k_3}{J_{M2}} & 0 & -\frac{k_3+k_5}{J_{M2}} & \frac{k_5}{J_{M2}} & \frac{b_3}{J_{M2}} & 0 & -\frac{b_3+b_5}{J_{M2}} & \frac{b_5}{J_{M2}} & 0 & 0 \\ 0 & \frac{k_4}{J_{L2}} & \frac{k_5}{J_{L2}} & -\frac{k_4+k_5}{J_{L2}} & 0 & \frac{b_4}{J_{L2}} & \frac{b_5}{J_{L2}} & -\frac{b_4+b_5}{J_{L2}} & 0 & \frac{1}{J_{L2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{J_{M1}} & 0 \\ 0 & \frac{1}{J_{M2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ .2 & 0 \\ 0 & .2 \end{bmatrix}. \quad (8)$$

B. Exercise two: Designing Dynamic Hypothetic Testing

In this section we assume that the parametric uncertainty is divided to three sub-sets ilustrated in Fig. 2

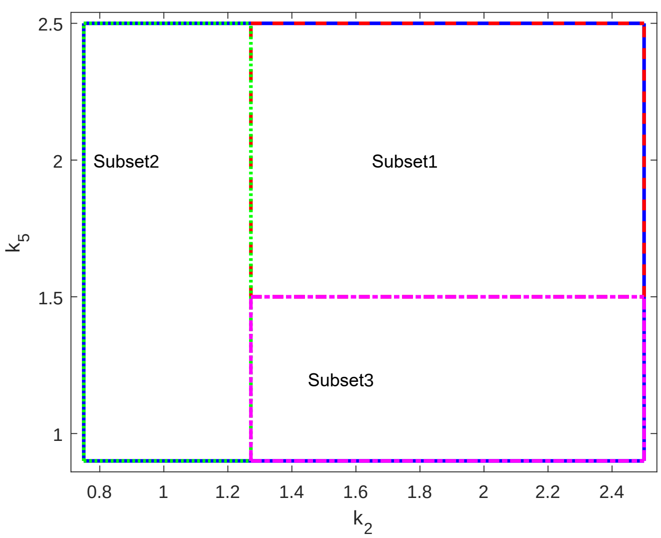

Fig. 2. The parametric uncertainty set is divided to three subsets

We will assume three hypotheses:

- *Hypothesis one*: the uncertain parameters are in subset 1. For hypothesis one we assume $k_2 = 2$ and $k_5 = 2$.
- *Hypothesis two*: the uncertain parameters are in subset 2. For hypothesis two we assume $k_2 = 1$ and $k_5 = 1.75$.
- *Hypothesis three*: the uncertain parameters are in subset 3. For hypothesis three we assume $k_2 = 2$ and $k_5 = 1.25$.

For each Hypothesis design a discrete time steady state Kalman filter (command kalmd in matlab).

Implement the dynamic hypothesis testing algorithm and assign a probability to each hypothesis.

$$h_i(t+1) = \frac{Pr\{y(t+1)|\mathcal{H} = \mathcal{H}_i\,,\, Z(t)\}}{\sum_{k=1}^{N} Pr\{y(t+1)\big|Z(t)\,,\, \mathcal{H} = \mathcal{H}_k\} h_k(t)} h_i(t). \quad (2)$$

Since, all the noises and disturbances were Gaussian, this can be simplified to

$$h_i(t+1) = \frac{\frac{e^{-\frac{1}{2}\tilde{y}^T_{\mathcal{H}_i}(t+1) S^{-1}_{\mathcal{H}_i} \tilde{y}_{\mathcal{H}_i}(t+1)}}{\sqrt{(2\pi)^2 |S_{\mathcal{H}_i}|}}}{\sum_{k=1}^{N} h_k(t) \frac{e^{-\frac{1}{2}\tilde{y}^T_{\mathcal{H}_k}(t+1) S^{-1}_{\mathcal{H}_k} \tilde{y}_{\mathcal{H}_k}(t+1)}}{\sqrt{(2\pi)^2 |S_{\mathcal{H}_k}|}}} h_i(t). \quad (3)$$

where $\tilde{y}_{\mathcal{H}_k}(t+1)$ represents the output error (residual) from k^{th} hypothesis (or k^{th} Kalman Filter) and $|S_{\mathcal{H}_k}|$ denotes determinant of the covariance matric of $y_{\mathcal{H}_k}$. Pay attention that for steady state Kalman filter $S_{\mathcal{H}_k}$ is a contant matrix can be calculated as $C\ P\ C^T + \Theta_d$, where P is steady-state error covariance (can be calculated from a Kalman Filter in Off-Line) and Θ_d discrete equivalent of continuous measurement noise covariance matrix (this is not exactly the intensity matrix of measurement noises. It basically needs to be discretized). Whenever needed, use sampling time of one millisecond.

References

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. New Jersey, USA: Prentice-Hall, 1979.