第二次数值实验

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目录

1.概念回顾：

Jacobi 迭代：

设原方程组为：

$$Ax = b;$$

其中 A 为非奇异的矩阵。

如果 A 的所有对角元 $a_{ii} \neq 0$，那么原问题可以改写为：

$$x = Bx + g \quad \text{或者} \quad (I - B)x = g;$$

那么我们可以得到：

$$B = I - diag(a_{ii}{}^{-1})A;$$

$$g = diag(a_{ii}^{-1})b;$$

因为 A 是非奇异的，所以 $I - B$ 也是非奇异的，设 $x^{(0)}$ 是任意的一个初始迭代向量，构造向量序列：

$$x^{(m)} = Bx^{(m-1)} + g;$$

若向量序列收敛则称之为 J 方法，否则为发散。若向量序列收敛于 x^*，则最终可以得到 $Ax^* = b$。

根据以上思想，我们得到原题中的解用 J 方法最终求得的结果为：

$$x_1 = -8.9893, x_2 = -9.4845, x_3 = 10.0510;$$

这是经过 1000 次迭代之后所得到的结果。

代码如下：

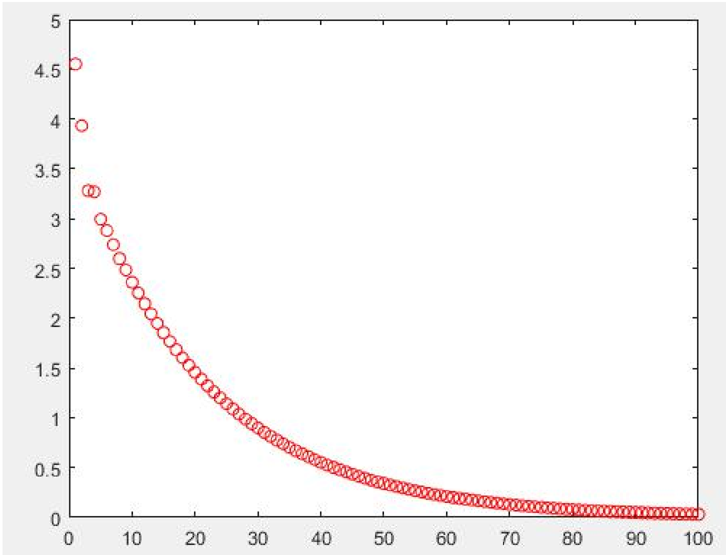
```
A = [4.63,-1.21,3.22;-3.07,5.48,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.48 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 1000
      xi = B*xi + g;
      i = i+1;
end
xi
```

Jacobi 迭代的图像绘制：

接着，我们将绘制在二范数意义下的 G-S 迭代的误差规律图，即为：

$$\| A(x) - b \|_2$$

通过前面的理论分析我们知道这个范数当 n 足够大的时候，一定趋近零，用 matlab 得到的图像如下：



代码为：
```
A = [4.63,-1.21,3.22;-3.07,5.48,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.48 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 100
   E = [i];
   xi = B*xi + g;
   F = norm((A*xi-b)',2);
   z(i)=F;
   i = i+1;
end
k=linspace(1,100,100);
plot(k,z,'ro');
```

2.G-S 迭代：

如同 J 方法我们将矩阵 A 分成三块，分别是：D（对角阵），$-L$（下三角阵），$-U$（上三角阵），则 J 方法可以表示成：

$$x^{(m)} = D^{-1}(L+U)x^{(m-1)} + D^{-1}b\text{；}$$

可以得到更加有效的迭代项式：

$$x^{(m)} = D^{-1}Lx^{(m)} + D^{-1}Ux^{(m-1)} + D^{-1}b\text{；}$$

等价地为：

$$(I - D^{-1}L)x^{(m)} = D^{-1}Ux^{(m-1)} + D^{-1}b\text{；}$$

于是得到：

$$x^{(m)} = (I - D^{-1}L)D^{-1}Ux^{(m-1)} + (I - D^{-1}L)D^{-1}b\text{；}$$

这个称为 G-S 迭代法，其中：

$$B_{GS} = (I - D^{-1}L)D^{-1}U = (D-L)^{-1}U\text{；}$$

于是得到 G-S 迭代的标准形式：

$$x^{(m)} = (D-L)^{-1}Ux^{(m-1)} + (D-L)^{-1}b\text{；}$$

本题在 G-S 迭代 1000 次情况下得到的答案为：

$$x_1 = -8.9893, x_2 = -9.4845, x_3 = 10.0510\text{；}$$

代码如下：

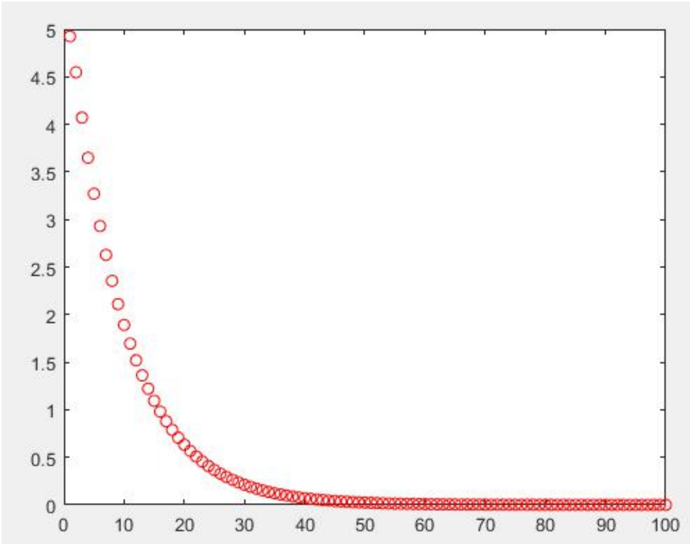
```
A = [4.63,-1.21,3.22;-3.07,5.48,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.48,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 1000
      xi = B*xi + g;
      i = i+1;
end
xi
```

G-S 迭代的图像绘制：

接着，我们将绘制在二范数意义下的 G-S 迭代的误差规律图，即为：

$$\| A(x)-b \|_2$$

通过前面的理论分析我们知道这个范数当 n 足够大的时候，一定区域零，用 matlab 得到的图像如下：



代码为：
```
A = [4.63,-1.21,3.22;-3.07,5.48,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.48,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 100
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
k=linspace(1,100,100);
plot(k,z,'ro');
```

严格对角占优与不可约对角占优：

为了解决$|a_{22}|$的绝对值最小问题，需要用到不可约对角占优。原问题中矩阵 A：

$$\begin{pmatrix} 4.63 & -1.21 & 3.22 \\ -3.07 & 5.48 & 2.11 \\ 1.26 & 3.11 & 4.57 \end{pmatrix};$$

因此，由定理若 A 是严格对角占优或不可约对角占优，则 J 方法和 GS 方法都是收敛的，证明如下：

由于 J 方法的迭代矩阵为 $B_J = I - diag(a_{ii}^{-1})A$，若 A 是严格对角占优矩阵，可知：

$\sum_{j=1,i\neq j}^{n} |a_{ij}| < |a_{ii}|, i = 1,\ldots\ldots,n$，因此可以得到 $\sum_{j=1,i\neq j}^{n} \left|\frac{a_{ij}}{a_{ii}}\right| < 1$，这意味着$\|B_J\|_\infty < 1$，因此得到 J 迭代和 GS 迭代是收敛的。若 A 是不可约对角占优矩阵，显然 $I - B_J$ 也是不可约对角占优矩阵。下面使用反证法证明。

假设 B_J 的某个特征值 λ 满足$|\lambda| \geq 1$，则由 $\det(\lambda I - B_J) = 0$ 得到 $\det(I - \frac{B_J}{\lambda})$,但这与 $(I - \frac{B_J}{\lambda})$ 是不可约对角占优矩阵矛盾，从而 $\rho(B_J) < 1$,J 方法收敛。

设矩阵 $(D-L)^{-1}U$ 的某个特征值 λ 满足$|\lambda| \geq 1$ 则有 $\det|\lambda I - (D-L)^{-1}U| = 0$ 推得 $\det(D - (L + \frac{1}{\lambda}U)) = 0$,这和 $D - (L + \frac{1}{\lambda}U)$ 是严格对角占优的矛盾，从而 $\rho(B_{GS}) < 1$,故 GS 迭代矩阵收敛。

因此当$|a_{22}|$不小于 5.18 时，J 和 GS 迭代矩阵是收敛的。

但是这只是一个必要条件，所以下确界可以更小，经过调试其值在 4.7~4.8 范围内可以达到精确值。所以以 1 精度来算其下确界为 4.8。

五个代表性的a_{22}：(前三个由于是可收敛的，所以迭代次数取得丝毫不差)

（1）$a_{22} = 6$

1. J 迭代情况下：

迭代次数	数值结果	$\|A(x)-b\|_2$	迭代矩阵的谱半径
183	x1=-5.3889,x2=-5.5232,x3=6.3626	2.1090e-06	0.9242

2. GS 迭代情况下：

迭代次数	数值结果	$\|A(x)-b\|_2$	迭代矩阵的谱半径
82	x1=-5.3889,x2=-5.5232,x3=6.3626	2.7854e-06	0.8372

（2） $a_{22}=5.5$

1. J 迭代情况下：

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
262	x1=-8.7579,x2=-9.2299,x3=9.8140	8.4945e-06	0.9515

2.GS 迭代情况下：

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
117	x1=-8.7579,x2=-9.2299,x3=9.8140	1.1177e-05	0.8938

（3） $a_{22}=5.18$

1. J 迭代的情况下：

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
473	x1=-15.0741,x2=-16.1791,x3=16.2845	3.5214e-06	0.9710

2. S 迭代的情况下：

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
210	x1=-15.0741,x2=-16.1791,x3=16.2845	4.7709e-06	0.9357

（4） $a_{22}=4$

1. J 迭代的情况下：

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
51	x1=-114.445,x2=-141.876,x3=121.903	84.2544	1.0625

2. GS 迭代的情况下

迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
51	x1=-7.1825,x2=-9.3474,x3=8.3425	5.4134e+03	1.1477

（5） $a_{22}=3$

1. J 迭代的情况下：

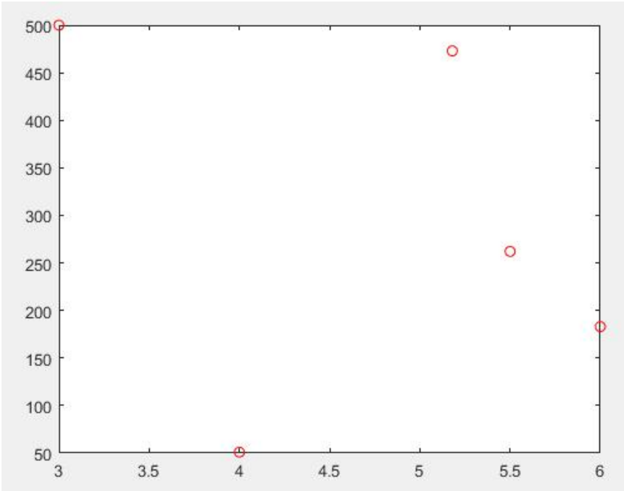
迭代次数	数值结果	$\|\| A(x)-b \|\|_2$	迭代矩阵的谱半径
500	x1=NaN ,x2=NaN, x3=NaN	NaN	1.1796

2. GS 迭代的情况下：

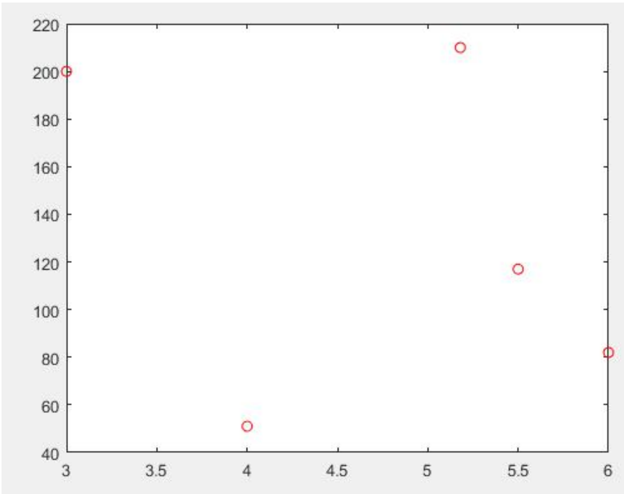
迭代次数	数值结果	$\| A(x)-b \|_2$	迭代矩阵的谱半径
200	x1=NaN ,x2=NaN, x3=NaN	NaN	1.4569

图像：

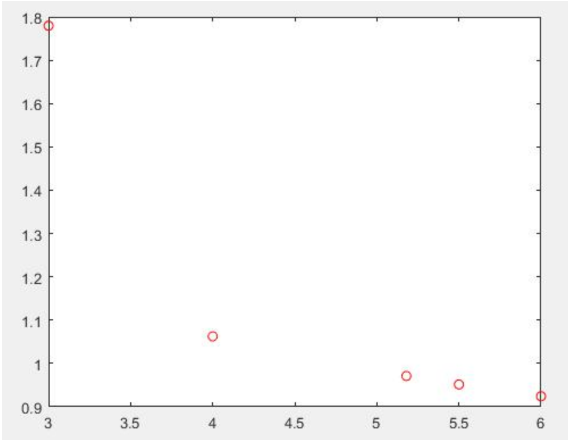
a_{22} 与迭代次数（J）：



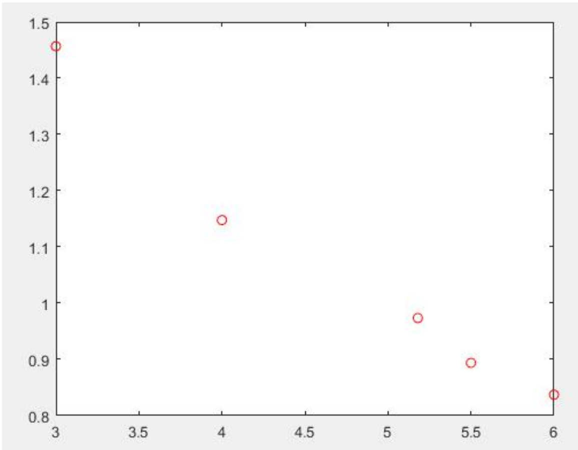
a_{22} 与迭代次数（G-S）：



a_{22} 与迭代矩阵谱半径（J）：



a_{22} 与迭代矩阵谱半径（G-S）：



代码：

$a_{22} = 6$：

J 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./6 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 183
```

```
        xi = B*xi + g;
        i = i+1;
    end
    xi
```

J 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./6 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 183
   E = [i];
   xi = B*xi + g;
   F = norm((A*xi-b)',2);
   z(i)=F;
   i = i+1;
end
F
```

J 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./6 1./4.57];
D = diag(v);
B = I - D*A;
vrho(B)
```

GS 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,6,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
```

```
i = 1;
while i <= 82
      xi = B*xi + g;
      i = i+1;
end
xi
```

GS 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,6,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 82
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
F
```

GS 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,6,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,6,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
vrho(B)
```

$a_{22} = 5.5$：

J 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
```

```
I = eye(3);
v = [1./4.63 1./5.5 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 262
       xi = B*xi + g;
       i = i+1;
end
xi
```

J 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.5 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 262
   E = [i];
   xi = B*xi + g;
   F = norm((A*xi-b)',2);
   z(i)=F;
   i = i+1;
end
F
```

J 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.5 1./4.57];
D = diag(v);
B = I - D*A;
vrho(B)
```

GS 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.5,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 117
      xi = B*xi + g;
      i = i+1;
end
xi
```

GS 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.5,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 117
   E = [i];
   xi = B*xi + g;
   F = norm((A*xi-b)',2);
   z(i)=F;
   i = i+1;
end
F
```

GS 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,5.5,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.5,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
```

```
K = inv(D+L);
B = K*U;
vrho(B)
```

$a_{22} = 5.18$：

J 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.18 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 473
       xi = B*xi + g;
       i = i+1;
end
xi
```

J 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.18 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 473
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
F
```

J 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
```

```
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./5.18 1./4.57];
D = diag(v);
B = I - D*A;
vrho(B)
```

GS 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.18,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 210
      xi = B*xi + g;
      i = i+1;
end
xi
```

GS 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.18,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 210
   E = [i];
   xi = B*xi + g;
   F = norm((A*xi-b)',2);
   z(i)=F;
   i = i+1;
```

```
        end
    F
```

GS 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,5.18,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,5.18,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
vrho(B)
```

$a_{22} = 4$：

J 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./4 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 51
        xi = B*xi + g;
        i = i+1;
end
xi
```

J 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./4 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
```

```
while i <= 51
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
F
```

J 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./4 1./4.57];
D = diag(v);
B = I - D*A;
vrho(B)
```

GS 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,4,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 51
      xi = B*xi + g;
      i = i+1;
end
xi
```

GS 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,4,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
```

```
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 51
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
```

F

GS 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,4,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,4,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
vrho(B)
```

$a_{22}=3$：

J 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./3 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 500
        xi = B*xi + g;
        i = i+1;
end
xi
```

J 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./3 1./4.57];
D = diag(v);
B = I - D*A;
g = D*b;
xi = [0,0,0]';
i = 1;
while i <= 500
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
F
```

J 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
I = eye(3);
v = [1./4.63 1./3 1./4.57];
D = diag(v);
B = I - D*A;
vrho(B)
```

GS 迭代数值结果：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,3,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 200
      xi = B*xi + g;
      i = i+1;
```

```
        end
        xi
```

GS 迭代 2 范数：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,3,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
g = K*b;
xi = [0,0,0]';
i = 1;
while i <= 200
    E = [i];
    xi = B*xi + g;
    F = norm((A*xi-b)',2);
    z(i)=F;
    i = i+1;
end
F
```

GS 迭代谱半径：

```
A = [4.63,-1.21,3.22;-3.07,3,2.11;1.26,3.11,4.57];
b1 = [2.22,-3.17,5.11];
b = b1';
D = [4.63,0,0;0,3,0;0,0,4.57];
L = [0,0,0;-3.07,0,0;1.26,3.11,0];
U = [0,1.21,-3.22;0,0,-2.11;0,0,0];
K = inv(D+L);
B = K*U;
vrho(B)
```

a_{22} 与迭代次数（J）：

```
A = [3 4 5.18 5.5 6];
B = [500 51 473 262 183];
plot(A,B,'ro')
```

a_{22} 与迭代次数（G-S）：

```
A = [3 4 5.18 5.5 6];
B = [200 51 210 117 82];
```

```
plot(A,B,'ro')
```

a_{22} 与迭代矩阵谱半径（J）：

```
A = [3 4 5.18 5.5 6];
B = [1.7796 1.0625 0.9710 0.9515 0.9242];
plot(A,B,'ro')
```

a_{22} 与迭代矩阵谱半径（G-S）：

```
A = [3 4 5.18 5.5 6];
B = [1.4569 1.1477 0.97357 0.8938 0.8372];
plot(A,B,'ro')
```