# 微分方程数值解第二次大作业

杜鸿宇
2016141211049

**第一题**：用二维 ADI 方法计算热传导方程随时间的扩散

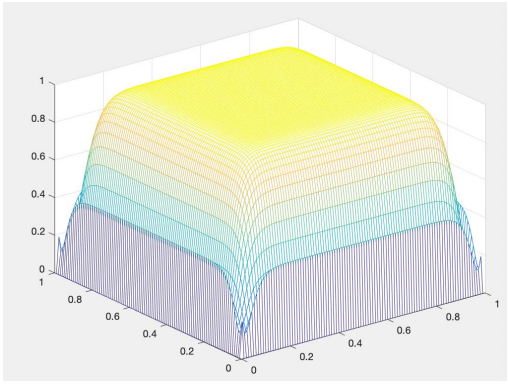
x,y 等分成 100 份，时间步长为 0.1。

**代码**：

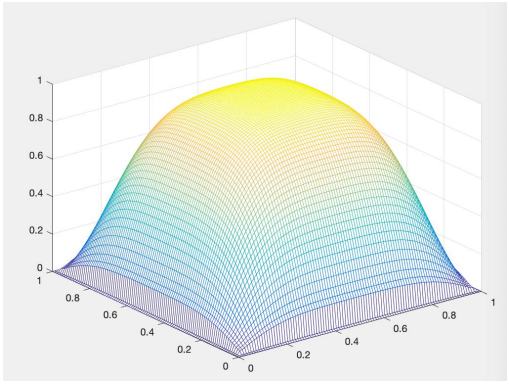
```
h=1/100;
x=0:h:1;
y=0:h:1;
A=[];
A(1,1:(1/h)+1)=0;
A((1/h)+1,1:(1/h)+1)=0;
A(1:(1/h)+1,1)=0;
A((1/h)+1:1:(1/h)+1)=0;
A(2:(1/h),2:(1/h))=1;
for t=0:0.01:1
    U1=[];
    U1(1,1:(1/h)+1)=0;
    U1((1/h)+1,1:(1/h)+1)=0;
    U1(1:(1/h)+1,1)=0;
    U1((1/h)+1:1:(1/h)+1)=0;
    M=diag((1+0.001/(h^2))*ones((1/h)-1,1))+diag((-0.5*0.001/(h^2))*ones((1/h)-2,1),1)
+diag((-0.5*0.001/(h^2))*ones((1/h)-2,1),-1);
    for i=2:1/h
        v=[];
        for j=2:1/h
            v(j-1)=(0.5*0.001/(h^2))*A(i+1,j)+(1-0.001/(h^2))*A(i,j)+(0.5*0.001/(h^2))*A(i
-1,j);
        end
        u=inv(M)*v';
        U1(i,2:1/h)=u;
    end
    U2=[];
    U2(1,1:(1/h)+1)=0;
    U2((1/h)+1,1:(1/h)+1)=0;
    U2(1:(1/h)+1,1)=0;
    U2((1/h)+1:1:(1/h)+1)=0;
    for j=2:1/h
        v=[];
        for i=2:1/h
            v(i-1)=(0.5*0.001/(h^2))*U1(i,j+1)+(1-0.001/(h^2))*U1(i,j)+(0.5*0.001/(h^2))
*U1(i,j-1);
```

```
        end
        u=inv(M)*v';
        U2(2:1/h,j)=u;
    end
    A=U2;
end
mesh(x,y,U2)
```

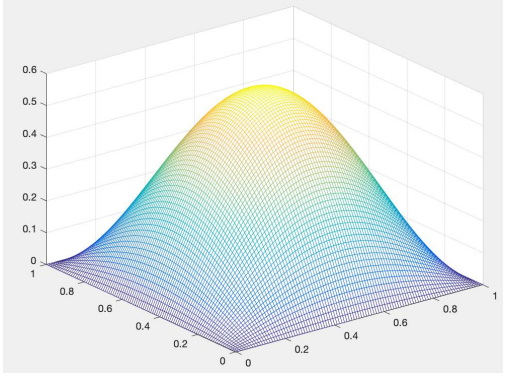
**图像：**



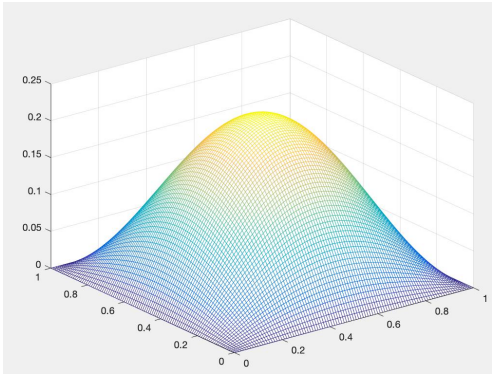
t=0.01



t=0.1



t=0.5



t=1

**第二题：**用二维 LOD 方法计算热传导方程随时间的扩散

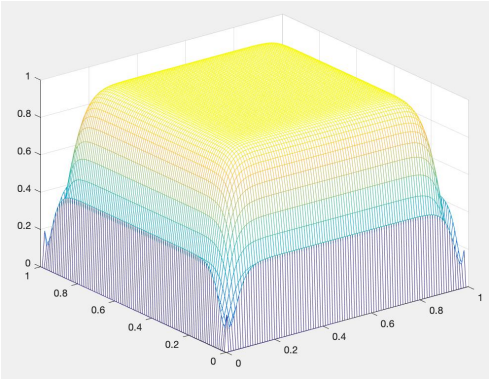
x,y 等分成 100 份，时间步长为 0.1。

代码：

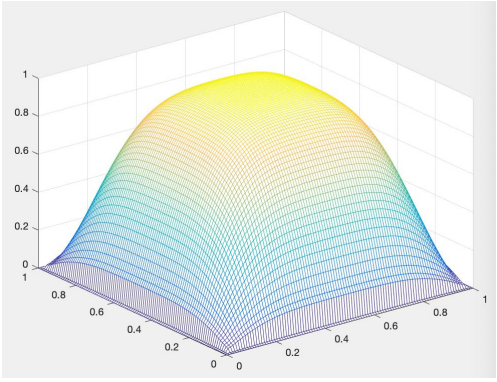
```
h=1/100;
x=0:h:1;
y=0:h:1;
A=[];
A(1,1:(1/h)+1)=0;
A((1/h)+1,1:(1/h)+1)=0;
A(1:(1/h)+1,1)=0;
A((1/h)+1:1:(1/h)+1)=0;
A(2:(1/h),2:(1/h))=1;
for t=0:0.01:1
    U1=[];
```

```
        U1(1,1:(1/h)+1)=0;
        U1((1/h)+1,1:(1/h)+1)=0;
        U1(1:(1/h)+1,1)=0;
        U1((1/h)+1:1:(1/h)+1)=0;
        M=diag((1+0.001/(h^2))*ones((1/h)-1,1))+diag((-0.5*0.001/(h^2))*ones((1/h)-2,1),1)
+diag((-0.5*0.001/(h^2))*ones((1/h)-2,1),-1);
        for j=2:1/h
            v=[];
            for i=2:1/h
                v(j-1)=(0.5*0.001/(h^2))*A(i,j+1)+(1-0.001/(h^2))*A(i,j)+(0.5*0.001/(h^2))*A(i,
j-1);
            end
            u=inv(M)*v';
            U1(2:1/h,j)=u;
        end
        U2=[];
        U2(1,1:(1/h)+1)=0;
        U2((1/h)+1,1:(1/h)+1)=0;
        U2(1:(1/h)+1,1)=0;
        U2((1/h)+1:1:(1/h)+1)=0;
        for i=2:1/h
            v=[];
            for j=2:1/h
                v(i-1)=(0.5*0.001/(h^2))*U1(i+1,j)+(1-0.001/(h^2))*U1(i,j)+(0.5*0.001/(h^2))
*U1(i-1,j);
            end
            u=inv(M)*v';
            U2(i,2:1/h)=u;
        end
        A=U2;
    end
    mesh(x,y,U2)
```

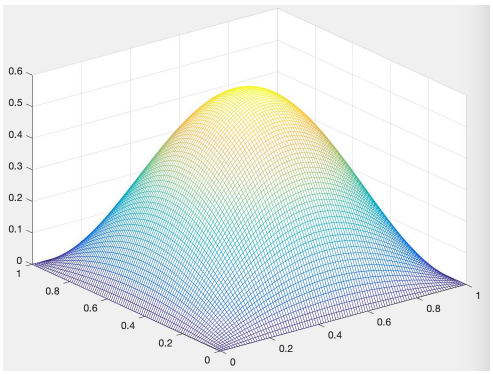
图像：



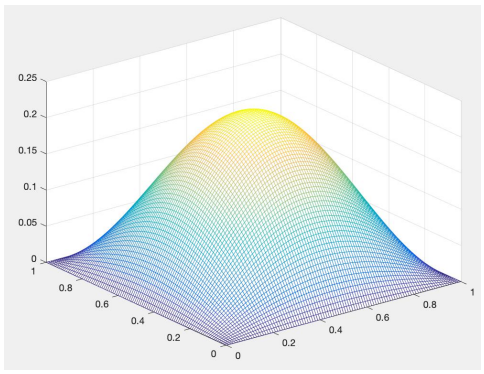
t=0.01



t=0.1



t=0.5



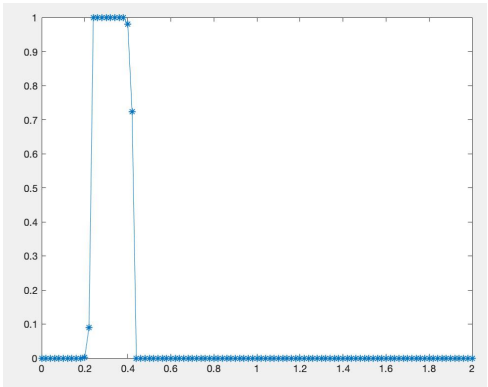
t=1

**第三题：**迎风格式书上例题：

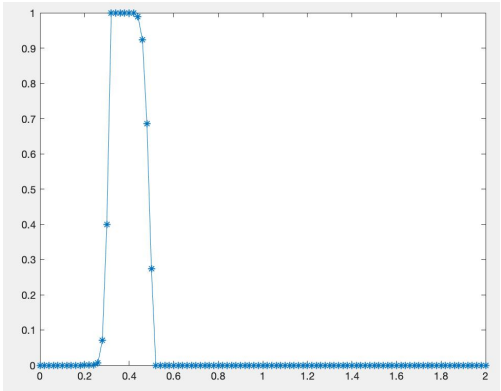
代码：

```
x=[0:0.02:2];
y=[];
y(1,1:10)=0;
y(1,11:21)=1;
y(1,21:101)=0;
g=[];
g(1)=0;
for t=0:0.02:0.1
    for i=2:101
        v=(1+x(i)^2)/(1+2*x(i)*t+2*(x(i)^2)+x(i)^4);
        g(i)=(1-v)*y(i)+v*y(i-1);
    end
    y=g;
end
plot(x,y,'-*')
```

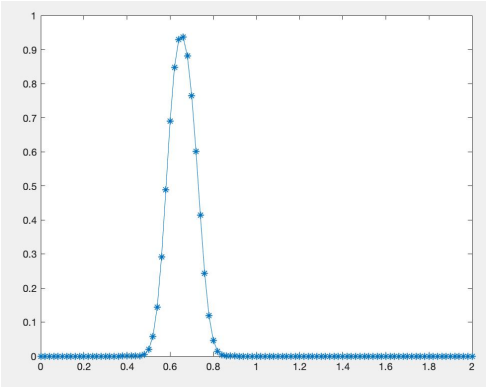
图像：



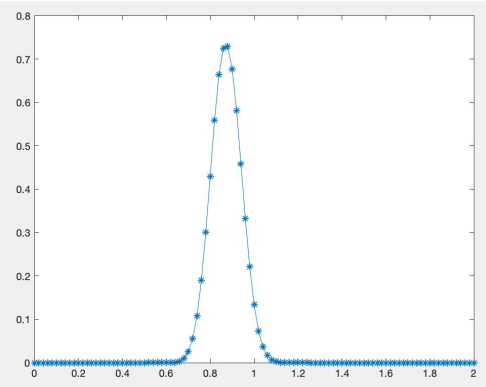
t=0.02



t=0.1



t=0.5



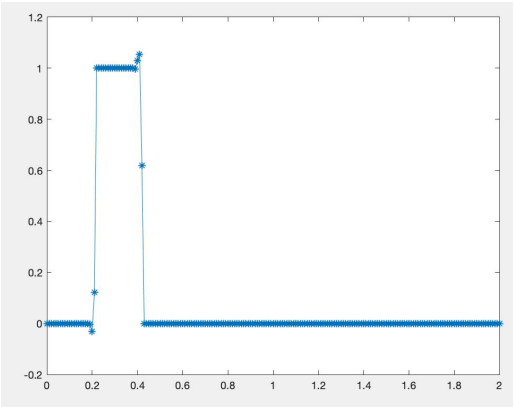
t=1

**第四题：**书上 Lax-Wendroff 格式例题：

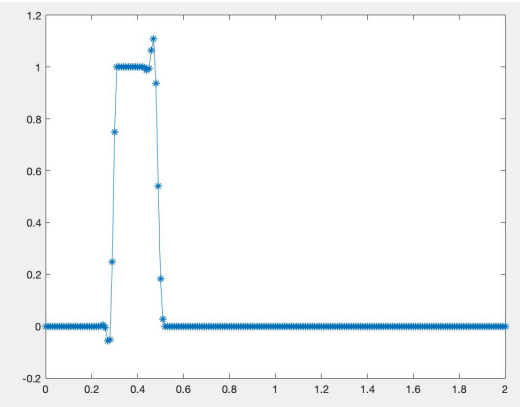
代码(0.01):

```
x=[0:0.01:2];
y=[];
y(1,1:20)=0;
y(1,21:41)=1;
y(1,42:201)=0;
g=[];
g(1)=0;
for t=0:0.01:1
    for i=2:200
        v=(1+x(i)^2)/(1+2*x(i)*t+2*(x(i)^2)+x(i)^4);
        g(i)=0.5*v*(1+v)*y(i-1)+(1-v*v)*y(i)-0.5*v*(1-v)*y(i+1);
    end
    y=g;
    y(201)=0;
end
plot(x,y,'-*')
```

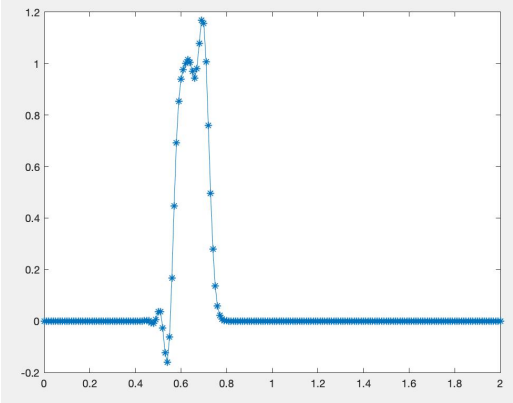
图像：



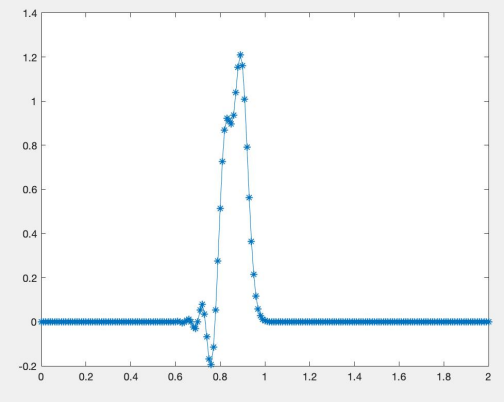
t=0.01



t=0.1



t=0.5

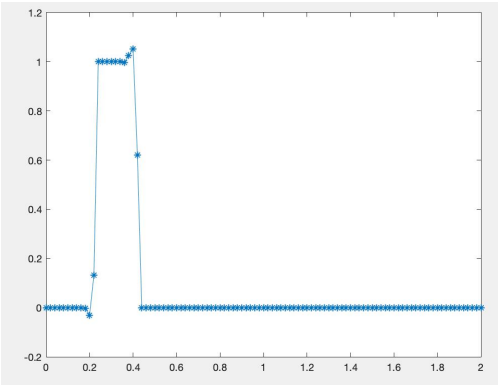


t=1

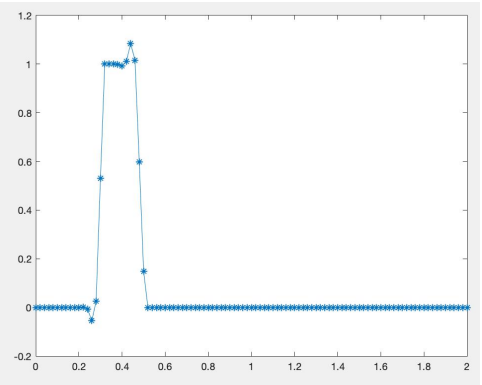
代码(0.02):

```
x=[0:0.02:2];
y=[];
y(1,1:10)=0;
y(1,11:21)=1;
y(1,21:101)=0;
g=[];
g(1)=0;
for t=0:0.02:1
    for i=2:100
        v=(1+x(i)^2)/(1+2*x(i)*t+2*(x(i)^2)+x(i)^4);
        g(i)=0.5*v*(1+v)*y(i-1)+(1-v*v)*y(i)-0.5*v*(1-v)*y(i+1);
    end
    y=g;
    y(101)=0;
end
plot(x,y,'-*')
```

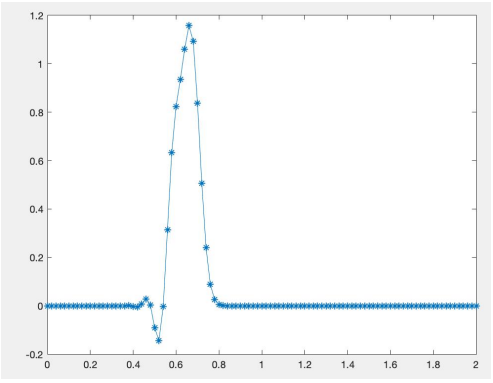
图像:



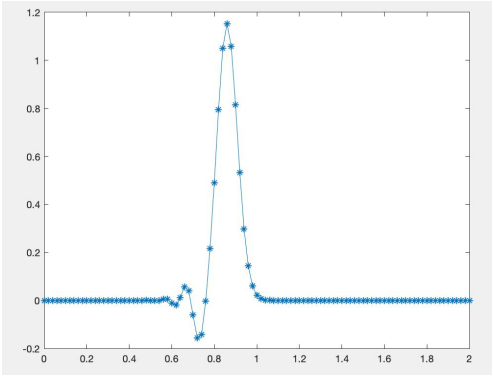
t=0.02



t=0.1



t=0.5



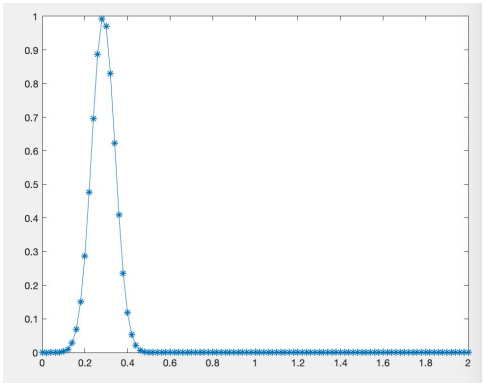
t=0.1

**第五题**：书上 Lax-Wendroff 格式第二个例题：

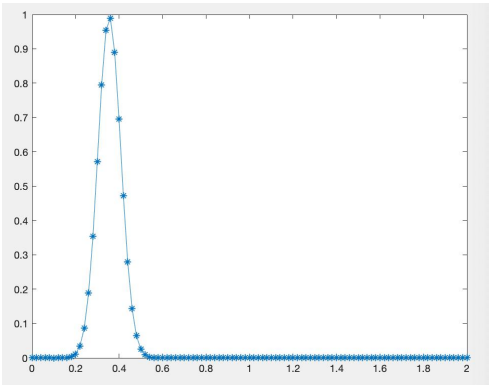
代码(0.02)：

```
x=[0:0.02:2];
y=[];
for j=1:101
    y(j)=exp(-10*((4*x(j)-1)^2));
end
g=[];
g(1)=0;
    for t=0:0.01:0.5
        for i=2:100
            v=(1+x(i)^2)/(1+2*x(i)*t+2*(x(i)^2)+x(i)^4);
            g(i)=0.5*v*(1+v)*y(i-1)+(1-v*v)*y(i)-0.5*v*(1-v)*y(i+1);
        end
     y=g;
     y(101)=exp(-10*((4*x(101)-1)^2));
end
plot(x,y,'-*')
```

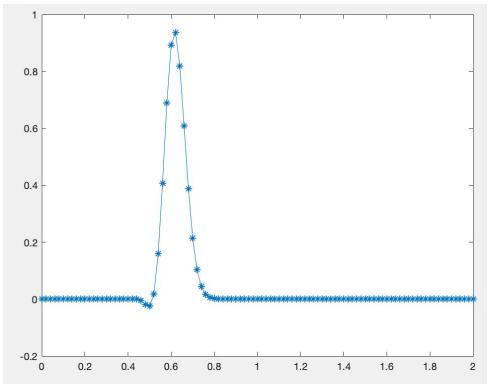
图像：



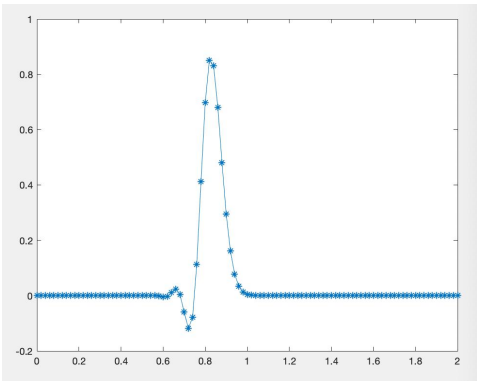
t=0.02



t=0.1



t=0.5

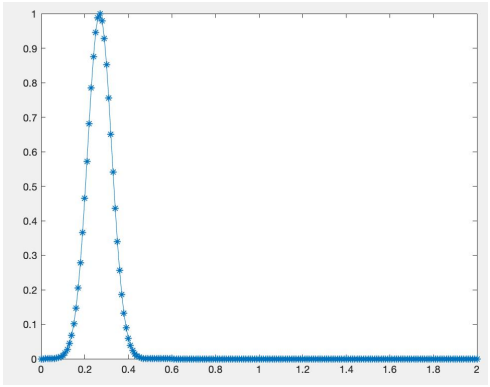


t=1

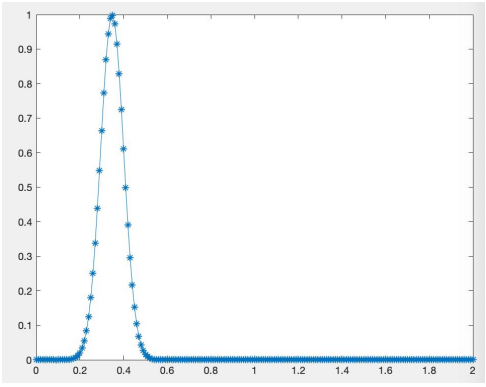
代码(0.01):

```
x=[0:0.01:2];
y=[];
for j=1:201
    y(j)=exp(-10*((4*x(j)-1)^2));
end
g=[];
g(1)=0;
for t=0:0.01:1
    for i=2:200
        v=(1+x(i)^2)/(1+2*x(i)*t+2*(x(i)^2)+x(i)^4);
        g(i)=0.5*v*(1+v)*y(i-1)+(1-v*v)*y(i)-0.5*v*(1-v)*y(i+1);
    end
    y=g;
    y(201)=exp(-10*((4*x(101)-1)^2));
end
plot(x,y,'-*')
```

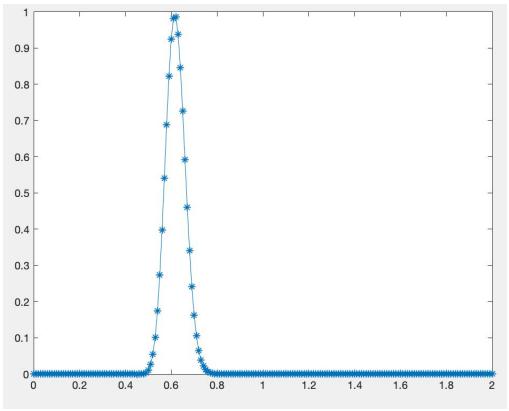
图像:



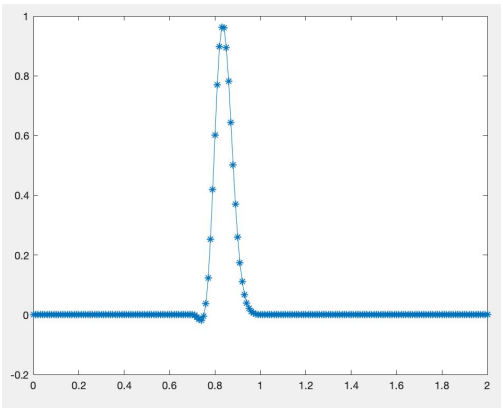
t=0.01



t=0.1


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

t=0.5


1
0.8
0.6
0.4
0.2
0
-0.2
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

t=1