

微分方程数值解 2 第一次上机作业

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1. 题目和基本理论推导:

原题为: $\nabla \cdot (a \cdot \nabla u) + f = 0, \text{ in } \Omega;$
 $\Omega = (0, 1) \times (0, 1); u|_{\partial\Omega} = 0;$
 $u(x, y) = \sin(2\pi x) \cdot \sin(2\pi y)$
 $a(x, y) = (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$
 由于 ∇a 涉及到 $\frac{\partial a}{\partial x}$ 和 $\frac{\partial a}{\partial y}$
 ∇u 涉及到 $\frac{\partial u}{\partial x}$ 和 $\frac{\partial u}{\partial y}$
 因此我们先得到:

$$\frac{\partial a}{\partial x} = \frac{\partial ((x - \frac{1}{2})^2 + (y - \frac{1}{2})^2)}{\partial x} = 2x - 1$$

$$\frac{\partial a}{\partial y} = \frac{\partial ((x - \frac{1}{2})^2 + (y - \frac{1}{2})^2)}{\partial y} = 2y - 1$$

$$\frac{\partial u}{\partial x} = \frac{\partial (\sin(2\pi x) \cdot \sin(2\pi y))}{\partial x} = 2\pi \sin(2\pi y) \cdot \cos(2\pi x)$$

$$\frac{\partial u}{\partial y} = \frac{\partial (\sin(2\pi x) \cdot \sin(2\pi y))}{\partial y} = 2\pi \sin(2\pi x) \cdot \cos(2\pi y)$$
 又因为 $\nabla \cdot (a \cdot \nabla u) = 0$

$$\Rightarrow f = -\nabla \cdot (a \cdot \nabla u) = -\nabla a \cdot \nabla u - \nabla^2 u$$

$$= -(\frac{\partial a}{\partial x} + \frac{\partial a}{\partial y}) (\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}) - \nabla^2 u$$

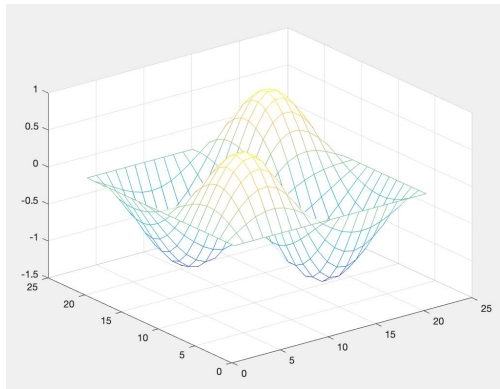
$$= -\frac{\partial a}{\partial x} \frac{\partial u}{\partial x} - \frac{\partial a}{\partial y} \frac{\partial u}{\partial y} - \nabla^2 u$$

$$= -\frac{\partial a}{\partial x} \frac{\partial u}{\partial x} - \frac{\partial a}{\partial y} \frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}$$
 经计算 \Rightarrow

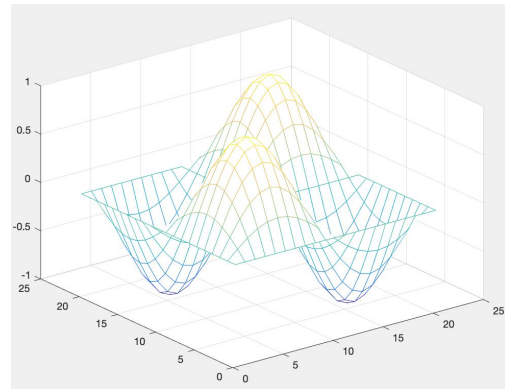
$$f = 2\pi \sin(2\pi x) \sin(2\pi y) + \sin(2\pi x) \sin(2\pi y) [(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2] 8\pi^2$$

$$- 4\pi y \sin(2\pi x) \cos(2\pi y) - 4\pi x \sin(2\pi y) \cos(2\pi x)$$

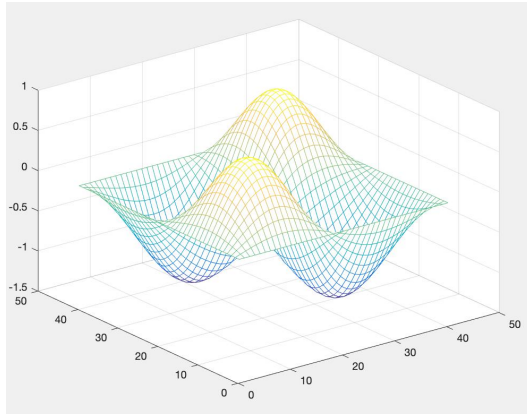
2. 图像:



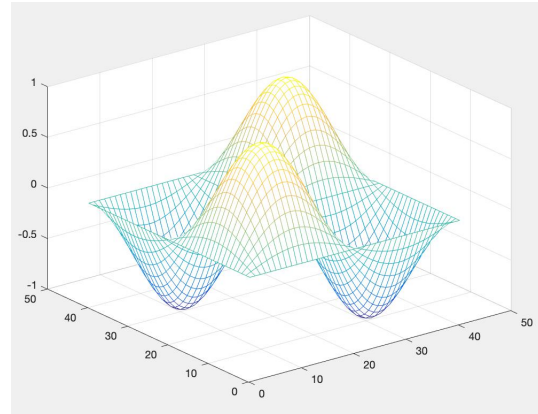
1/20 U



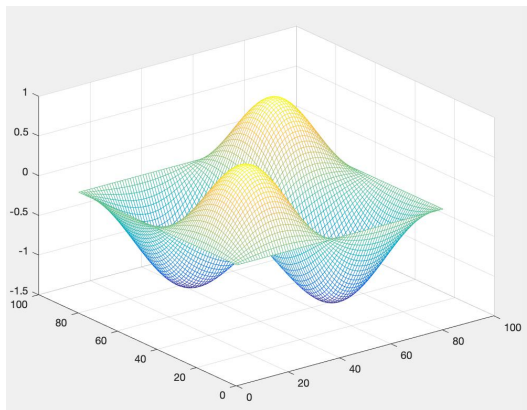
1/20 u



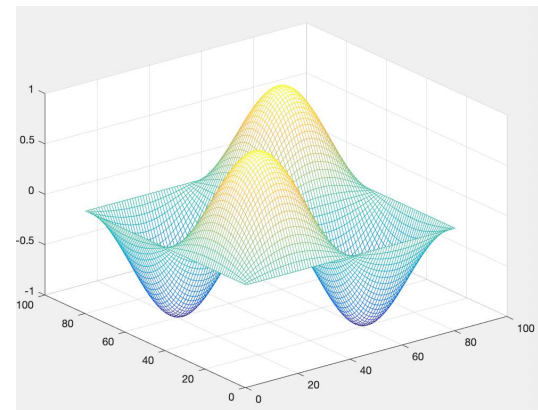
1/40 U



1/40 u



1/80 U



1/80 u

3.代码(U):

```
x1=0.0125;
y1=0.0125;
J=1/x1 ;
M=1/y1 ;
x=1:1:J+1;
y=1:1:M+1;
u=zeros(J+1,M+1);
a=zeros(2*J+1,2*M+1);
f=zeros(J+1,M+1);
e=eye(J+1,M+1);
for i=2:J
    for j=2:M
        f(i,j)=2*pi*sin(2*pi*(i-1)*x1+2*pi*(j-1)*y1)+8*pi*pi*sin(2*pi*(i-1)*x1)*sin(2*pi*(j-1)*y1)*[((i-1)*x1-1/2)^2+((j-1)*x1-1/2)^2]-4*pi*(i-1)*x1*cos(2*pi*(i-1)*x1)*sin(2*pi*(j-1)*y1)-4*pi*(i-1)*y1*sin(2*pi*(i-1)*x1)*cos(2*pi*(j-1)*y1);
    end
end
A=zeros((J-1)^2,(M-1)^2);
U=zeros(J+1);
for i=1:J-1
    for j=1:M-1
```

```

        A((i-1)*(M-1)+j,(i-1)*(J-1)+j)=((j+0.5)*y1-0.5)^2+((j-0.5)*y1-0.5)^2+((i+0.5)*x1-0.5)^2+((i-0.5)*x1-0.5)^2+2*(i*x1-0.5)^2+2*(j*y1-0.5)^2;
    if j~=M-1
        A((i-1)*(M-1)+j+1,(i-1)*(J-1)+j)=-((j+0.5)*y1-0.5)^2-(i*x1-0.5)^2;
    end
    if j~=1
        A((i-1)*(M-1)+j-1,(i-1)*(J-1)+j)=-((j-0.5)*y1-0.5)^2-(i*x1-0.5)^2;
    end
    if i~=J-1
        A((i-1)*(M-1)+j+(M-1),(i-1)*(J-1)+j)=-((i+0.5)*x1-0.5)^2-(j*y1-0.5)^2;
    end
    if i~=1
        A((i-1)*(M-1)+j-(M-1),(i-1)*(J-1)+j)=-((i-0.5)*x1-0.5)^2-(j*y1-0.5)^2;
    end
end
end
tt=f(2:J,2:M);
f=tt(:);
b=y1^2*f;
U0=A\b;
for k=1:J-1
    U(2:J,k+1)=U0((k-1)*(J-1)+1:k*(J-1));
end
mesh(U)

```

4.代码(u):

```

x1=0.05;
y1=0.05;
J=1/x1 ;
M=1/y1 ;
x=1:1:J+1;
y=1:1:M+1;
u=zeros(J+1,M+1);
for i=1:J+1
    for j=1:M+1
        u(i ,j)=sin(2*pi*(i-1)*x1)*sin(2*pi*(j-1)*y1);
    end
end
mesh(x,y,u)

```