Retailer Initiated Inventory-Based Financing

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Abstract

In this paper we study an innovative financing scheme in which a large retailer provides inventory-based financing (IBF) to a small retailer selling through her own brick-and-mortar channel. In anticipation of a peak selling period, the small retailer could repeatedly pledge her on-hand inventory in exchange for a loan amount, which is in turn used to procure more inventory, i.e., stockpiling to fulfil a stochastic customer demand. Following sales proceeds, the small retailer buys back the pledged inventory to the extent possible, and defaults on any leftovers which will be liquidated by the big retailer via his own platform. We analyze the effectiveness of such financing scheme through a Stackelberg game, and explore its impact on operational decisions and contract design. In particular, we derive the optimal joint inventory ordering and pledging decisions for the small retailer during the stockpiling phase; we further characterize the optimal loan interest rate for the big retailer. Using datasets obtained from the financial technology arm of a major retailer, we provide empirical evidence that small retailers exhibit stock-piling behavior when using IBF. When further verify the predictions from the theoretical results through reduced-form analysis.

1 Introduction

One of the challenges faced by small and medium sized enterprises (SMEs) is working capital finance. Firms of large size or with established business usually have fixed assets, such as plants, machinery, equipment, etc., that could be used as collateral to secure loans from financial institutions. This is often not the case for fast growing, entrepreneurial firms, especially in the retail sector. In the absence of fixed assets, small retailers are constantly challenged to obtain bank loans at competitive interest rate. This in turn generates operational inefficiencies since the lack of working capital finance could severely constrain firm's inventory orders, among other operations.

One way to alleviate such problem is inventory-based financing (IBF). As a form of asset-based lending, inventory-based financing is increasingly adopted by high-growth retailers (Buzacott and Zhang, 2004). To facilitate IBF, a lender (usually a bank) performs due diligence of the borrower (a small retailer), and provides loans with maximum amount linked to the inventory value of the borrowing firm. The loan limit and interest rate are chosen depending on the potential sales proceeds and the salvage value of unsold inventory in the event of bankruptcy. The use of IBF opens doors for small, high-growth retailers with limited alternatives of working capital financing.

However, the interest rate of this traditional bank-initiated IBF is still too high in practice, thus prohibiting its mass adoption by smaller retailers who buy and sell in price-competitive markets. In addition, the maximum loan amount is often time limited in supporting the optimal inventory procurement policy in face of a growing market. The high interest rate and low loan limit are results of the risk management of the bank. Most IBF contracts do not entitle banks the ownership of inventory before the loan is due; in the event of bankruptcy, the bank usually has limited capability to re-sell the leftover inventory, realizing a very low salvage value. Due to these frictions, IBF often reaches its limit in providing competitive financing to a mass pool of retailers.

On the other hand, big retailers, especially online platforms, enjoy great advantages in devising and providing IBF to small retailers. Compared with banks, big retailers have more experience and data on the underlined asset as they sell similar products via their own online portals; unlike bank-initiated IBF, retailer-initiated IBF grants the lender the ownership of the secured inventory by obliging the borrower to stock in the lender's warehouse; in the event of loan default, the big retailer could sell the pledged inventory through its own channel without suffering from low salvage value. The above-mentioned advantages fundamentally change the risk management model and potentially lower the interest rate while increasing the loan limit. This makes such big retailer more suited in providing IBF to small retailers.

Our interest in retailer-initiated IBF was motivated by the practice employed by JD Finance, which is the financing leg of JD.com. JD.com is China's largest online business-to-consumer retailer offering more than one million product selections from thousands of brands. Over the years, JD has established a portfolio of supply chain finance products, ranging from reverse factoring (Tunca and Zhu, 2017), micro-lending, to more recent inventory-based financing. The latter mainly focus on the small retailers selling outside of JD.com but becoming part of JD's ecosystem through inventory-based loans. During the one year history of IBF, JD has issued over 1000 loans to small retailers.

In this paper, we study this innovative financing scheme initiated by large retailer such as JD. In the scheme initiated by JD, the small retailer will pledge her inventory in exchange for loan which is priced by JD. The loan will be used to purchase extra products in order to fulfil future demand. The small retailer will later buy back the inventory pledged to the extent possible, depending on the sales of her products. If not all products are bought back by the small retailer, JD takes the remaining inventory and sells through his own channel.

We are particularly interested in the effectiveness of such scheme and its performance in comparison to bank loan and other forms of financing schemes that help small retailers such as trade credit. We both theoretically and empirically explore the benefit of IBF in the rest of the paper.

2 Literature Review

Our work studies inventory based financing scheme. As such, it is closely related to previous literature on asset-based lending. **?**) demonstrate that when a bank is a profit maximizer, the collateral value of inventory is a function of the bank's belief XXXXX. Using a two-period game-theoretic model, Iancu et al. (2017) study the effectiveness of various IBF contracts (covenants) in mitigating agency issues stemming from operating flexibility. XXX

Our paper is also related to previous literature on retailer initialed supply chain financing schemes. Specifically, XXXX.

Our work is at the interface of operations and financial decisions, exploring financing in the supply chain when there are payment delays contractually agreed between the buyer and the supplier. As such, it is related to the trade credit literature, which studies such payment delays in the channel often employed to provide financial flexibility to buyers with capital limitations. Trade credit has been extensively studied in a stream of finance literature, exploring a broad range of topics including supplier size, product differentiation, and insurance and default premiums on trade credit contract terms (see Smith 1987, Petersen and Rajan 1997, Cunat 2007, Giannetti et al. 2011, Klapper et al. 2011, and Murfin and Njoroge 2015 among others). In the operations management literature, Xu and Birge (2004) provide one of the early studies that captures the decision of a capital constrained buyer. They show that firm value can be significantly improved by integrating financial and operational decisions. Dada and Hu (2008) show that in a Stackelberg game setting, a capital constrained newsvendor would borrow from bank but order an amount that is less than what would be optimal. Zhou and Groenevelt (2008) investigate the case when a supplier provides subsidies to a budget constrained retailer. Caldentey and Chen (2009), Kouvelis and Zhao (2011, 2012) and Jing et al. (2012) examine the interplay between a supplier, a budget constrained retailer and a bank, demonstrating that when bank loans are competitively priced, retailers will prefer supplier financing to bank financing under trade credit, but when the bank has market power in setting the interest rate, either form of financing can be preferable depending on the market parameters.

Yang and Birge (2016) show that even when bank financing and supplier financing can be used jointly, supplier financing is still preferred to bank financing. In addition, using firm-level data, they find that the financing pattern predicted by their model is used by a wide range of firms. regarding the firm's demand distribution. Luo and Shang (2018) explore the interaction between the inventory policy and trade-credit in multi-period setting, demonstrating that a simple myopic inventory policy based on a target stock level and the firm's working capital is optimal.

Our paper is centered on the financial constraints of buyers who usually cannot obtain trade credit. In current practice, there are few ways, if any, a independent large retailer can help smaller retailers to reduce their interest rates.

Tanrisever et al. (2012) study how reverse factoring creates value for each party in the supply chain and how the value is affected by the spread in exogenous financing costs determined by the bank, the working capital policy, the payment period extension and the risk free rate. They also explore the impact of reverse factoring on operational decisions using make-to-order and make-to-stock models. Van der Vliet et al. (2015) study the effect of payment extension in reverse factoring using simulation-based optimization, finding that length of the payment period can get reflected on the supplier's financing cost in a nonlinear way. Rui and Lai (2015) explore deferred payments to suppliers as a way to incentivize suppliers to invest in improving their product quality. They show that deferred payments can improve investments and compare its effectiveness to product inspections by the buyer. Wu et al. (2014) explore buyer-backed supplier finance through a centralized two-stage stochastic programming model, finding that the buyer's guarantee in financing is necessary if the demand is large, supplier's capital is inadequate or the market finance interest rate is high. They also find that in this single decision-maker setting, the buyer can improve her payoff by guaranteeing the supplier's loan. In our paper, we study buyer intermediated financing in a three-way decentralized game between the supplier, the buyer and the bank with supplier defects and endogenous buyer determined interest rate and wholesale price. We compare the performance of two different financial schemes in this strategic setting with asymmetric information, as employed in practice by companies such as JD. We further identify the conditions, under which in equilibrium the buyer intermediated financing scheme can improve the buyer's payoff or reduce it. Further, filling a void in the literature, we apply the theoretical findings to data from JD to estimate unobservable parameters, test the theory, and empirically measure efficiency.

3 Model

We consider a scenario where a lender (e.g., a big retailer) $\mathscr{L}$ initiates an inventory based financing scheme to lend to a borrower (e.g., a budget-constrained small retailer) $\mathscr{B}$[1]. Specifically, the lender sets the interest rate of the loan, and the borrower in turn determines the amount of inventory to be pledged to achieve the optimal inventory level. In what follows, we first introduce a single period model to demonstrate the interplay between the borrower and the lender under the IBF scheme. We then extend the single period model by allowing the borrower to repeatedly pledge inventory over a time horizon.

3.1 Single Period

We start with the case in which the borrower has a one-shot opportunity to pledge her on-hand inventory as collateral to the lender in exchange for finance. We model the interaction between the borrower and the lender as a sequential-move Stackleberg with the following chronology. At $T = 0$, the lender sets the interest rate r. Then at $T = 1$, the borrower decides z, the quantity to be pledged, after observing r. Meanwhile, we denote the valuation the lender sets for each pledged product as v, which is assumed to be exogenous throughout the paper[2]. We also assume that $v > c$ where c is the production cost. In this way, the borrower would receive vz in cash when z units are pledged to the lender, and the borrower would in turn acquire vz/c units of the product, all purchased on loan. The demand realizes at $T = 2$. The borrower satisfies the demand with her on-hand inventory at unit price p and repays her loan to the lender to the extent possible. In case of partial payment, the lender then salvages the rest of product at unit price s.

Notably, the borrower will not be interested in holding any cash, as doing so requires her to pledge more quantity than needed, and the cost of the loan is always higher than the risk-free rate r_f. As a result, the borrower would only pledge the exact amount needed to reach the desired

[1]We will be using borrower (lender) and small retailer (big retailer) interchangeably in the remainder of the paper

[2]In reality, the supply chain finance department communicate with the procurement department to get a quote on the salvage value of the product, the valuation of the product is then set as the salvage value plus some preset markup.

inventory level. Hence, the borrower's objective function can be written as

$$\Pi_{\mathscr{B}} = \max_{z \le q} \mathbb{E}\left[p \min\left(D, q + \frac{v}{c}z\right) - vRz\right]^+ ., \tag{1}$$

where R is defined as $1 + r$. Note that the borrower could reclaim the pledged inventory from the lender instantly to further satisfy customer demand. Hence the *total available stock* in the selling period is $q + vz/c$, i.e., the initial stock plus the additional inventory purchased through the inventory-based financing.

The lender's problem is to determine R to maximize his profit $\Pi_{\mathscr{L}}$, defined as the repayment from the borrower plus the revenue from salvaging the left-over inventory minus the cost of capital. Given the borrower's initial inventory q, the lender's objective function can be written as

$$\Pi_{\mathscr{L}} = \max_{R} \mathbb{E}\left[\min(pD, vRz) + s\left(z - \frac{p}{vR}D\right)^+\right] - vR_f z, \tag{2}$$

where z is the amount of inventory pledged by the borrower as in Equation (1) and $R_f = 1 + r_f$. Note also that since $p > c(1+R)$, the equivalent sales from the borrower's total available inventory, $p(Q + vz/c)$, is always greater than the total loan repayment vRz. Therefore, the actual repayment the lender receives is reduced to $\min(pD, vRz)$.

We solve the Stackelberg game by first analyzing the borrower's optimization problem. We assume that all cost parameters and demand distribution are public information. In addition, we make the following key assumptions on the distribution of demand D. Denoting the p.d.f., c.d.f. and the complementary c.d.f. of D as $f(\cdot)$, $F(\cdot)$ and $\overline{F}(\cdot) = 1 - F(\cdot)$, respectively, we have

Assumption 1 *For any $m > 0$ and $\alpha \in (0, 1)$, we assume the distribution for demand D satisfies the following requirements.*

1. *The equation $\overline{F}(m + x) - \alpha\overline{F}(\alpha x) = 0$ has at most a single root in $[0, \infty)$.*
2. *Function $(x + m)\overline{F}(x)$ is unimodal in $[0, \infty)$.*
3. $\lim_{x\to\infty} f(m + x)/f(\alpha x) = 0$.

Assumption 1 imposes several restrictions on the demand distribution, yet it is in fact satisfied by a wide range of distributions, such as the truncated normal distribution and the Gamma distribution with shape parameter $\alpha > 1$. Equipped with these conditions, we now analyze borrower's optimal pledging decision.

Proposition 1 *Suppose the distribution of demand D satisfies Assumption 1. Denote $S^* = F^{-1}(1 - Rc/p)$. Then the equation*

$$\overline{F}(q + \frac{v}{c}z)p - \overline{F}(\frac{vR}{p}z)cR = 0$$

has a unique solution $z^*(q)$ when $q < S^*$ and the borrower's optimal pledge quantity $z^O(q)$ can be characterized as

$$z^O(q) = \begin{Bmatrix} q, & \textit{if } q \leq \frac{c}{c+v}S^* \\ \min\{z^*(q), q\}, & \textit{if } \frac{c}{c+v}S^* \leq q \leq S^* \\ 0. & \textit{if } S^* < q \end{Bmatrix}. \tag{3}$$

Proposition 1 implies that, when q is sufficiently small, i.e., when $q \leq \frac{c}{c+v}S^*$, the borrower should pledge all her on-hand inventory in order to satisfy the demand as much as possible. Moreover, when q is large enough, i.e., when $q > S^*$, the borrower would no longer need to expand her inventory since it can meet the demand D appropriately. However, when q is moderate, $z^*(q)$ does not possess an explicit-form solution under general demand. Below we construct a heuristic policy by solving a simplified model of the original Equation (1).

Heuristic Formulation. By allowing negative ending cash position of the borrower, we define the simplified model as

$$\Pi_{\mathscr{B}}^H = \max_z \mathbb{E}\left[p \min\left(D, q + \frac{v}{c}z\right) - vRz\right]. \tag{4}$$

Let z^H be the heuristic solution of the original problem. We derive the heuristic policy from solving (4) in the following proposition.

Proposition 2 *The optimal solution of the profit function in (4) is given by*

$$z^H(q) = \begin{Bmatrix} q, & \textit{if } q \leq \frac{c}{c+v}S^* \\ \frac{c}{v}(S^* - q), & \textit{if } \frac{c}{c+v}S^* < q \leq S^* \\ 0, & \textit{if } S^* < q \end{Bmatrix}, \tag{5}$$

where $S^* = F^{-1}\left(1 - \frac{Rc}{p}\right)$.

We use the optimal policy of the simplified problem in (4) as the heuristic policy of the original problem in (1). This policy structure demonstrates the two distinct functions of inventory on the borrower's side: *loan-pledging capacity* and *demand-fulfilling stock*. Below we further illustrate these functions with respect to borrower's starting inventory.

Case 1. When the borrower starts with sufficiently low inventory level, i.e., $q \leq \frac{c}{c+v}S^*$, it is optimal to pledge all inventory to maximize her post-replenishment stock level. In this case, on-hand inventory mainly serves as a capacity constraint on the available loan amount to secure future stock. This constraint is binding due to relatively low inventory level compared with expected demand.

Case 2. When the borrower's initial inventory level is in the middle range, i.e., $\frac{c}{c+v}S^* < q \leq S^*$, it is optimal to pledge only part of her inventory for replenishment so that the total available stock reaches the base-stock level S^*. In this case, on-hand inventory provides capacity for loan pledging although the constraint is not binding. This is because the borrower starts with sufficient inventory thus does not need to pledge everything to reach the base-stock level.

Case 3. When the borrower's initial inventory level is higher than the base-stock level S^*, it is optimal not to pledge any stock. In this case, all her inventory serves to fulfill demand.

As shown in Proposition 1, the optimal policy of the original problem holds the piece-wise linear structure when initial inventory Q is sufficiently small or relatively large. The linear structure in the middle range only holds in the heuristic policy. We further conduct a numerical study to test the effectiveness of the heuristic policy. We can theoretically bound the gap between the optimal solution z^O and the heuristic solution z^H as follows.

Proposition 3 *If $p \geq vR$, then the gap between the heuristic solution and the optimal solution of the borrower's problem can be bounded as*

$$z^O - z^H \leq \frac{c}{v}\left[F^{-1}\left(\frac{p^2 - R^2c^2}{p}\right) - F^{-1}\left(\frac{p - Rc}{p}\right)\right].$$

(Weiming: discuss the tightness of the bound and tie to the distribution properties) Meanwhile, observing the borrower's initial inventory q and foreseeing her best response $z^O(q)$, the lender decides on the interest rate R to maximize his profit $\Pi_{\mathscr{L}}$. Nevertheless, the exact form of R is involved as the optimal pledging quantity $z^O(q)$ does not have an explicit form. We thus present the possible region for the optimal interest rate R^*.

Proposition 4 *The optimal interest rate R^* is bounded in the following region:*

$$R^* \in \left[\frac{p}{c}\bar{F}\left(\frac{c+v}{c}q\right), \frac{p}{c}\bar{F}(q)\right].$$

Proposition 4 implies that $R^* \approx p/c$ when the inventory level of the borrower is sufficiently low.

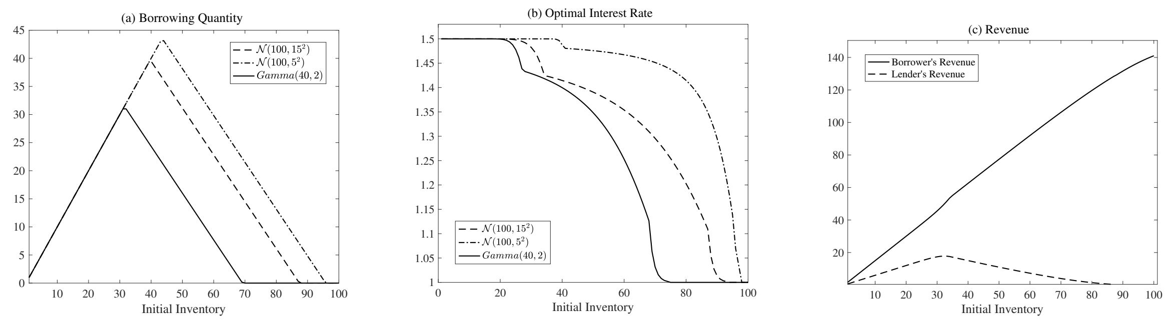

Figure 1: **Equilibrium outcomes in the single period scenario.** Panel (a) illustrates the optimal borrowing quantity of the borrower under different initial inventory and demand distribution facing a fixed interest rate $R = 1.2$. Panel (b) illustrates the optimal interest rate set by the lender when observing the borrower's initial inventory under different demand distributions. Panel (c) shows the best revenue gained by the lender and the borrower in the game when the demand distribution is $\mathcal{N}(100, 15^2)$. For all the panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1$ and $R_f = 1$.

In this case, the borrower can barely benefit from using IBF. For moderate or large q, however, the bound of R^* can be loose and not as informative. We thus present in Figure 1 the impact of borrower's initial inventory on borrower and lender's equilibrium outcomes. To start, Panel (a) of Figure 1 serves as a visual representation of Proposition 1. Facing a fixed interest rate, it is optimal for the borrower to pledge everything when the initial inventory level is low. As the initial inventory increases, the borrower pledges less and eventually does not need to resort to IBF to fulfill the demand. Panel (b) examines how initial inventory affects lender's optimal interest rate. We observe that when the initial inventory level is low, it is optimal for the lender to set a rate that is close to p/c. The interest rate then decreases in a nonlinear fashion as the initial inventory increases, highlighting the non-triviality of the equilibrium outcome. Finally, we present in Panel (c) the revenue changes caused by the inventory level. A higher inventory level leads to a higher revenue for the borrower, as the borrower will not only be able to satisfy a growing percentage of the demand, but that the financing costs associated with acquiring additional inventory decreases as well due to a fallen interest rate. For the lender, however, too little or too much inventory level both translate into reduced revenue, as the borrower either has limited inventory to pledge or the borrower is almost self-sufficient. Either way, the lender's revenue suffers from a dampened magnitude of loan principle or a reduced interest rate or both.

3.2 Multiple Periods

The IBF scheme employed by JD Finance states that the borrower can resort to IBF as long as 1) she has on-hand inventory to pledge to the lender and 2) the amount of the outstanding loan does not exceed the credit limit assigned by the lender. Notably, the fact that these two prerequisites do not prohibit the borrower from pledging inventory that is purchased on loan could give rise to stockpiling behavior. That is, a borrower with low initial inventory could exploit the IBF scheme to repeat the pledge-purchase cycle multiple times without repaying the outstanding loan in the meantime, until she reaches the desired inventory level. In this section, we study the equilibrium outcomes when the borrower engages in such stockpiling behavior. Later in Section 4, we will empirically demonstrate that such stockpiling behavior is in fact common among borrowers, especially before major holidays when they prepare for the surge in demand.

We now extend the single period model to a multiple period one, in which the borrower stockpiles by repeatedly pledging her on-hand inventory that she ordered from previous period using loan from the IBF scheme. In this case, the borrower now faces a contract horizon of N periods before a single opportunity to serve a stochastic demand D. The lender offers a fixed interest rate over the contract horizon, within which the borrower decides on a planning horizon of $T \leq N$ periods to prepare her operations, i.e., inventory level, for the final sale. In particular, the borrower repeatedly pledges part of her inventory to the lender, obtains cash credit in return, and further replenishes inventory with lead time $L = 1$. The detailed sequence of events is described as follows.

1. At the beginning of the contract horizon, the lender observes the borrower's initial inventory level q and determines the interest rate r for the entire horizon.

2. In each period $t = 0, 1, 2, ..., T-1$, the following events happen in sequence: (1) the borrower observes the updated on-hand inventory, the total pledged quantity, and the purchasing budget; (2) a pledge quantity decision is made and the corresponding inventory is transferred to the lender in exchange for a new loan amount; (3) an inventory order decision is made subject to the total purchasing budget; (4) the newly ordered inventory is delivered at the end of the period.

3. At time $t = T$, the demand occurs and is satisfied from borrower's on-hand inventory. The corresponding sales proceeds is used to pay the cumulative interest and reclaim (buy back) the pledged inventory. The reclaimed inventory could be further used to fulfill the demand. The process of reclaiming inventory occurs with zero lead time and could be repeated during the selling period. At the end of period T, unmet demand is lost without penalty and the salvage value for any remaining inventory is normalized to zero. The borrower should pay back the loan principle to the extent possible. The leftover inventory that is not reclaimed

is at the dispose of the lender, which he could salvage at a discount price via his own retail channel.

To characterize the game dynamics, we define the state and decision variables at the beginning of period t to be:

$$\begin{aligned} q_t &= \text{on-hand inventory level in Event (1);} \\ z_t &= \text{pledged quantity in Event (2);} \\ w_t &= \text{purchasing budget level in Event (2);} \end{aligned}$$

Given interest rate r, the borrower chooses the planning horizon $T \leq N$ and subsequently solves a joint inventory pledging and replenishment problem so as to maximize her expected profit $\Pi_{\mathscr{B}}$. In particular, the borrower decides on the pledge quantity z_t and the ordering amount u_t for each $t = 0, 1, 2, ..., T-1$. Without loss of generality, we assume at $t = 0$, the borrower starts with initial states $(q_0, x_0, w_0) = (q, 0, 0)$.

To start with, we first consider the problem where the planning horizon T are given. Specifically, the borrower faces a dynamic program defined as below.

$$B_t^{(T)}(q_t, x_t, w_t) = \max_{z_t \leq q_t,\ u_t \leq (vz_t + w_t)/c} B_{t+1}^{(T)}(q_{t+1}, x_{t+1}, w_{t+1}) \tag{6}$$

$$B_T^{(T)}(q_T, x_T, w_T) = \mathbb{E}\left[p \min(D, x_T + q_T) + w_T - vx_T - rv\sum_{t=1}^{T} x_t\right]^+, \tag{7}$$

where the state dynamics are the following.

$$\begin{aligned} q_{t+1} &= q_t + u_t - z_t, \\ x_{t+1} &= x_t + z_t, \\ w_{t+1} &= w_t + vz_t - cu_t. \end{aligned}$$

In Equation (6), each time the borrower makes pledging and purchasing decision, the pledged quantity may not exceed her current on-hand inventory level and the payment must satisfy the budget constraint. In Equation (7), the final objective function consists of the profit earned by having $x_T + q_T$ available stock and the total cost of the loan, which is defined by the capital vx_T and the interest $rv\sum_{t=1}^{T} x_t$. The borrower then must decide on the optimal planning horizon T^*, in which case the profit $\Pi_{\mathscr{B}}$ can be written as

$$\Pi_{\mathscr{B}} = B_0^{(T^*)}(q,0,0) = \max_T B_0^{(T)}(q,0,0). \tag{8}$$

Despite the simple structure, computing the objective order quantity Q directly from formulation (8) is challenging. Therefore, we first study the problem where, given a fixed objective inventory level $Q = x_T + q_T$, the borrower decides the optimal planning horizon T^* and the corresponding x_t, z_t that minimize the maximize the total revenue while achieving Q. In particular, for a fixed inventory level $Q = x_T + q_T$, the borrower's total revenue can be written as

$$\begin{aligned}\Pi_{\mathscr{B}}(Q) &= \max_T B_0^{(T)}(q,0,0) \\ &= \mathbb{E}\left[p\min(D,Q) + w_T - c(Q-q) - rvA(Q)\right]^+ .\end{aligned}$$

where $A(Q) = \sum_{t=1}^{T^*} x_t$ is the cumulative inventory throughout the planning horizon, which, given a fixed interest rate, in proportional to the total interest paid by the borrower. For a given inventory target Q, we can characterize the small retailer's optimal borrowing behavior as follows.

Proposition 5 *Given total inventory Q, the optimal planning horizon is*

$$T^* = \left\lfloor \frac{\log\left(\frac{Q}{q}(\theta-1)+1\right)}{\log\theta} \right\rfloor, \tag{9}$$

where $\theta = v/c$. And the optimal joint pledging and ordering decisions can be characterized as follows

$$\begin{aligned}
z_0^* &= \frac{1}{\theta^T}\left[Q - \frac{\theta^T - 1}{\theta - 1}q\right], \\
z_1^* &= q_1 = q + (\theta-1)z_0^*, \\
z_2^* &= q_2 = \theta q_1, \\
&\dots \\
z_{T^*-1}^* &= q_{T^*-1} = \theta^{T^*-1}q_1, \\
u_t^* &= \theta z_t^*. \ \forall t
\end{aligned}$$

Proposition (5) characterizes the optimal pledging quantity when the borrower is to reach a fixed inventory goal Q. It turns out that the optimal pledging strategy follows a simple structure: at time period $t \geq 1$, the borrower should pledge all on hand inventory; at $t = 0$, the borrower should pledge z_0^*, so that the borrower reaches the objective inventory level at time T, i.e., $x_T + q_T = Q$, after pledging all on hand inventory every period from $t = 1$ to $t = T - 1$. In this way, the total interest of the loan is minimized. Equipped with Proposition 5, we can now derive the optimal inventory level Q^O which we formally state in the following theorem.

Theorem 1 *Suppose the distribution of demand D satisfies Assumption 1. Then there exists a unique $Q^O \geq q$ that maximizes $\Pi_{\mathscr{B}}(Q)$ for a given interest rate r. Denote $S^* = F^{-1}(1 - \frac{Rc}{p})$, then we have*

$$Q^O(q) = \left\{ \begin{array}{ll} Q^*, & \text{if } q \leq S^* \\ 0, & \text{if } q > S^* \end{array} \right\},$$

where Q^* is the unique solution to the equation

$$\overline{F}(Q) - \overline{F}(\alpha_T Q - \beta_T)\alpha_T = 0,$$

and

$$\alpha_T = \frac{c}{p} + \frac{rv(\theta^T - 1)}{p\theta^T(\theta - 1)}, \qquad \beta_T = \frac{rvq}{p}\left(\frac{T}{\theta - 1} - \frac{\theta^T - 1}{\theta^T(\theta - 1)^2}\right) + \frac{cq}{p}$$

and T is defined in (9) for any Q.

Theorem 1 indicates that Q^* is critically determined by T^*. This is because a longer optimal planning horizon can lead to an inflated financing cost and bring down the order quantity. Nevertheless, T^* is affected by the optimal quantity Q^* at the same time according to Equation (9). To obtain a pair of Q^* and T^* that are consistent with each other, we utilize the uniqueness of the optimal quantity and provide in an iterative approach in Algorithm 1 to jointly solve for Q^* and T^*.

As for the lender, a total $M(Q) = rvA(Q) + c(Q - q)$ amount of loan is due in the final period, which consists of the cumulative interest and the original capital. We suppose the lender will split the due amount evenly to each single inventory. As a result, the lender's objective function can be written as

$$\Pi_{\mathscr{L}} = \max_R \mathbb{E}\left[\min(pD, M(Q)) + sQ\left(1 - \frac{pD}{M(Q)}\right)^+\right] - vr_f A(Q) - c(Q - q). \tag{10}$$

Algorithm 1: Derivation of the Optimal Inventory Level

input : q, p, c, v, r

intialize $T = 0$, $\tilde{T}^* = 1$

while $T \neq \tilde{T}^*$ **do**

  $T = T + 1$

  Solve FOC

$$\overline{F}(Q) - \overline{F}(\alpha_T Q - \beta_T)\alpha_T = 0$$

  to get the best response price Q_T^*

  Solve

$$\tilde{T}^* = \left\lfloor \log\left(\frac{Q_T^*}{q}(\theta - 1) + 1\right) / \log\theta \right\rfloor$$

end while

output: $T^* = \tilde{T}^*$,$Q^* = Q_T^*$

Similar to the single period scenario, the lender sets the interest rate R to maximize revenue. As the optimal interest rate and the optimal pledging quantity do not have explicit expressions, we resort to Figure 2 to demonstrate the lender's and borrower's equilibrium behaviors. Specifically, Panel (a) of Figure 2 shows the optimal pledging behavior according to Proposition 5. Depending on the initial inventory q, Panel (a) shows that the borrower may choose to start pledging at four and six periods ahead of the demand realization stage. Also, after the initial adjustment period, the borrower pledge an increasing quantity in the following periods at a rate of $v/c = 120\%$. Panel (b) demonstrates the relationship between the optimal planning horizon and the objective inventory Q^O. We observe that the borrower can postpone the pledging decision as q increases, which translates a reduced financing cost and further leads to a higher objective inventory Q^O. Panel (c) presents the optimal interest rate and the optimal cumulative quantity $A(Q^O)$ under different q. Due to the discrete nature of the planning period, α_T and β_T changes whenever the optimal planning horizon changes from T to $T + 1$. As a result, both the optimal interest rate and the corresponding order quantity exhibit several kinks as q increases. Notably, different from the single-period case in which the borrower's cumulative pledge quantity is bounded by the initial inventory q, we observe that in Panel (c) that the borrower actually pledges a significant amount when q is small. This is made possible thanks to the flexibility offered by the IBF scheme, which allows borrowers to repeatedly pledge their on-hand inventory over multiple periods to eventually stockpile way more than what they initially possess. Additionally, it is critical to point out that the interest rate jumps every time the planning horizon is reduced by one period, and the most significant climb occurs when the borrower has sufficient inventory to switch from repeated pledging to one-shot pledging. That

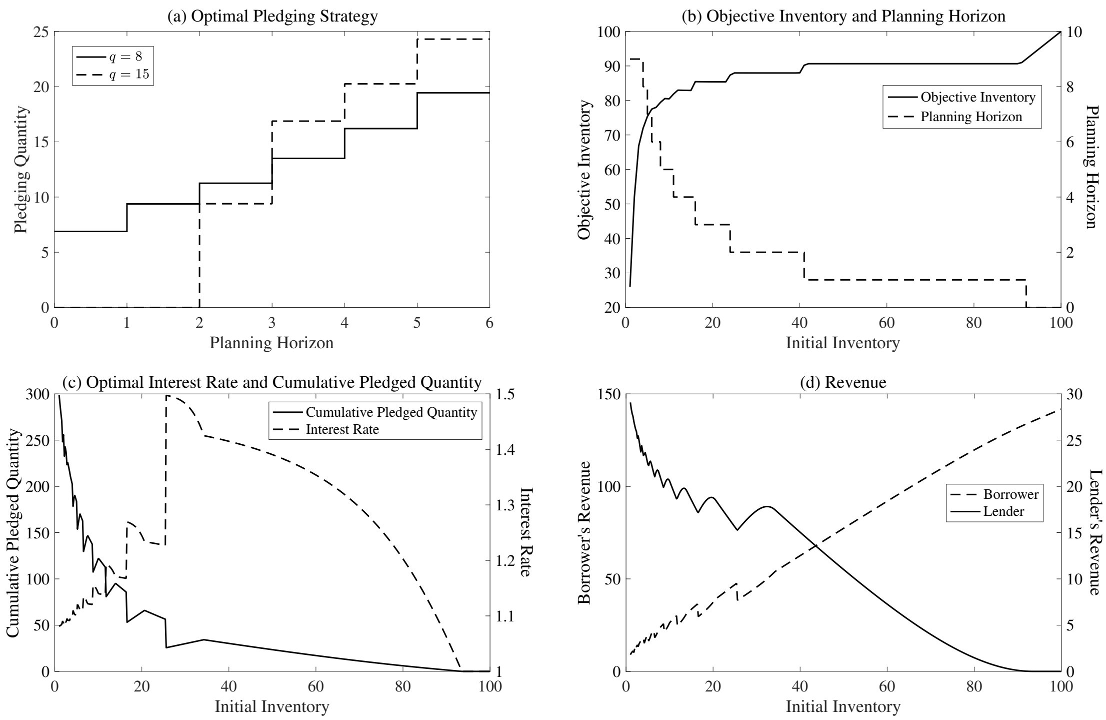

Figure 2: **Equilibrium outcomes in the multi-period scenario.** Panel (a) demonstrates the optimal borrowing quantity z_t in each time period of the borrower. The objective inventory is set to be 100 and the initial inventory is chosen as 8 and 15 respectively. Panel (b) illustrates the optimal objective inventory Q^O and the borrowing period T for the borrower facing a fixed interest rate $R = 1.1$. Panel (c) shows the optimal interest rate set by the lender and the corresponding optimal cumulative quantity $A(Q^O)$ pledged by the borrower. Panel (d) illustrates the revenue gained by the borrower and the lender. For all the panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1$ and $R_f = 1$.

is, when the initial inventory level is sufficiently high, the optimal interest rate becomes identical to that in the case of single period model. Finally, Panel (d) illustrate the revenue of the borrower and the lender under different q. Other than the dips around the kinks which are caused by jump in planning horizon, the borrower enjoys a higher revenue overall when q increases as the total financing costs decrease, which in turn dampens the lender's revenue.

3.3 Supply Chain Efficiency under IBF

In this section, we study the welfare implications of IBF on each stakeholder and the whole supply chain. Specifically, we compare the surplus of each party under the following scenarios: 1) the small retailer has no access to external financing at all; 2) the small retailer has access to IBF but is prohibited from pledging inventory purchased on loan; 3) the small retailer has access to IBF and is allowed to stockpile and 4) the small retailer has free and unlimited access to loan. Denoting these four cases as *Base Case*, *Single IBF*, *Multi IBF* and *First Best*, respectively, we present the surplus

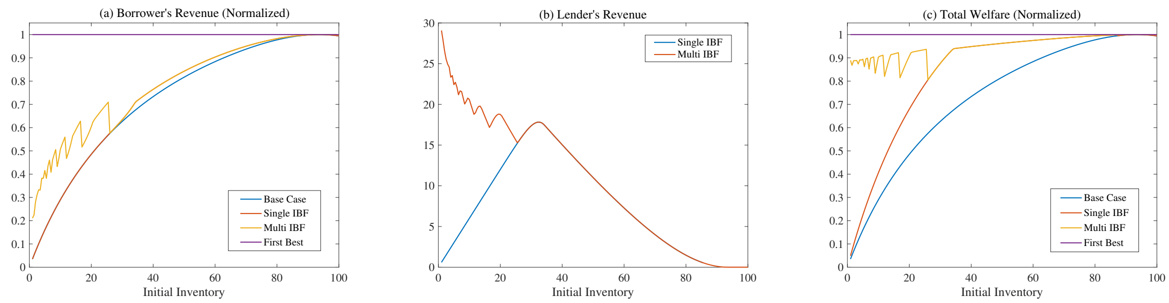

Figure 3: **Surplus comparisons for financing through IBF** Panel (a) shows the normalized borrower surplus, panel (b) shows the lender profits and panel (c) illustrates the normalized supply chain welfare. For all the panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1,\ R_f = 1$ and $D \sim \mathcal{N}(100, 15^2)$.

comparisons under these scenarios in Figure 3.

In Panel (a) of Figure 3, we observe that the borrower enjoys the highest revenue when stockpiling through IBF. Nevertheless, when the borrower's initial inventory level is low, the revenue difference between the *Single IBF* and *Multi IBF* scenarios are not immensely large. This can be explained as follows. According to Panel (b) in Figure 2, a low initial inventory implies a long planning horizon. In such case, the incurred interest expense over the planning horizon becomes substantial, making the borrower set a lower objective inventory level. Meanwhile, we also notice that, in the absence of stockpiling, the value of using IBF at low inventory level is minimal. This is because, knowing the small retailer would likely to pledge all her inventory, the lender sets the interest rate high enough to make the borrower indifferent between *Single IBF* and the *Base Case*. This is why, according to Panel (b), the lender is able to extract most of the surplus when the borrow has low initial inventory. Notably, Panel (b) indicates that the lender generates significantly higher revenue when the borrower stockpiles, which provides justification for the current contract terms that allows the borrower to pledge products that are purchased on loan. Finally, Panel (c) illustrates that the current IBF scheme can significantly improve the supply chain efficiency, achieving more than 90% of the total welfare compared to the first best scenario.

4 Empirical Analysis

In this section, we set out to seek for empirical evidence to verify the theoretical results derived from section 3. The datasets we use to test our theory are obtained from JD Finance, who has been implementing the IBF scheme to retailers since June 2016. In what follows, we first provide an overview to our datasets. We then describe our empirical strategy and present the results.

4.1 Data Description

We use two datasets for our empirical analysis. The first dataset contains detailed information on all 934 loans JD Finance issued from June 20th, 2016 to April 10th, 2018. Specifically, we observe for each loan the credit limit of the borrower, the loan principle, the interest rate, the date of issue, the corresponding due date. In addition, the dataset documents the actual payment date and the corresponding payment amount. At the same time, we see whether a loan defaults, and in case of default, the default amount and the liquidation value. The second dataset contains product level information. That is, We observe the product category and what SKUs are pledged in each loan, and, for each SKU, the quantity pledged as well as the product valuation. The summary statistics is presented in Table 1.

Table 1: Descriptive Statistics for the Loan Data

	Mean	Std.Dev	Min	Max
Loan Amount (Million Yuan)	0.55	0.89	1.50×10^{-3}	9.06
Interest Rate ($\times10^{-2}$)	14.34	1.82	7.00	17.00
Loan Repayment Period (Days)	91.83	4.47	89.00	122.00
Credit Limit of Retailer (Million Yuan)	16.17	21.12	1.00	67.00

Our datasets are primarily drawn from the loan contract, which include detailed information on loan related terms such as dates of loan origination and repayment, loan amount, interest rate, and the quantity and valuation of the pledged products. Thus, our empirical study focuses primarily on the loan borrowing behaviors and their impact on lender's interest rate.[3]

Proposition 5 implies that the borrower may stockpile by repeatedly pledging the inventory purchased through IBF in the previous periods in order to satisfy large future demand. Thus, we leverage the loan data to provide empirical evidence on the existence and the magnitude of stockpiling behavior. We also investigate when borrowers tend to engage in stockpiling behavior. Finally, we investigate how lender's interest rate decision is affected by the stockpiling activities.

[3]Notably, Theorem 1 predicts the exact quantity a borrower would pledge. Nevertheless, we do not have sufficient data to test if our theoretical results are accurate, as the equilibrium outcome hinges upon a number of variables exclusively known to the borrower (e.g., the production cost, the price and the demand forecast of the product). According to our interview with JD Finance's manager, such information is either imprecise as the product is sold through multiple channels, or is shielded from the lender, despite the dominant position and the strong bargaining power the lender has.

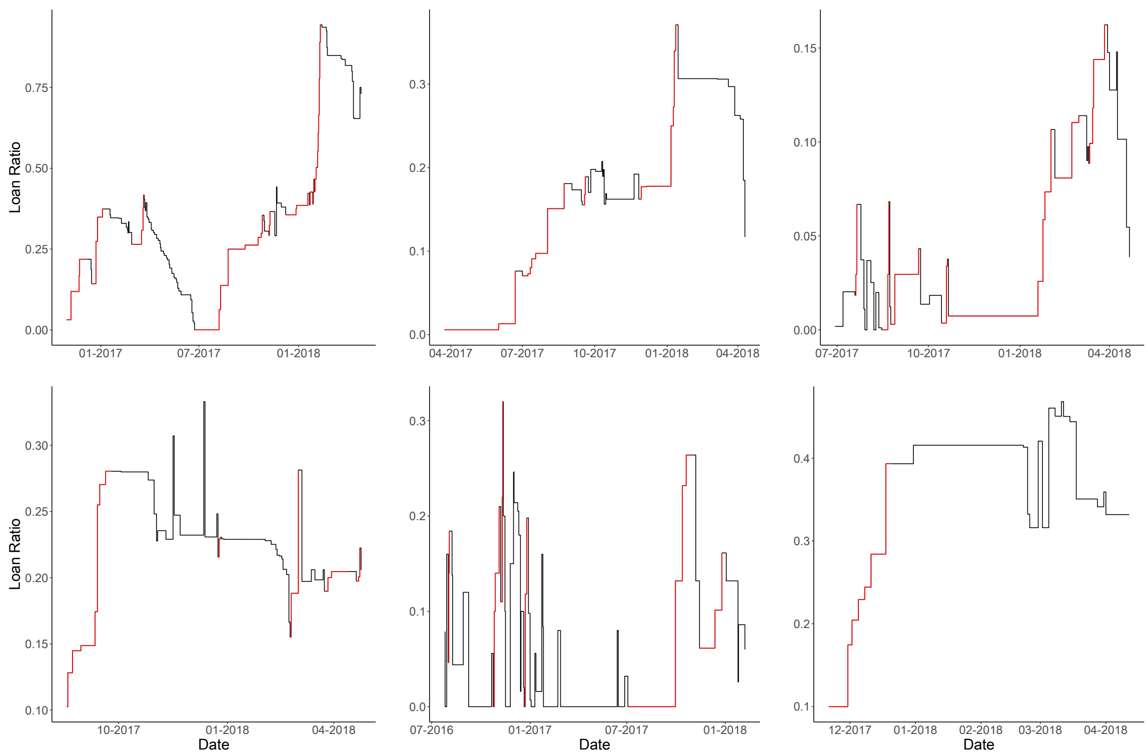

Figure 4: **Trajectory of the Outstanding Loans** These plots trace the loan ratio (outstanding loan amount divided by the credit limit) of six independent retailers. The identified stockpiling behaviors are colored in red.

4.2 Model-Free Evidence for Stockpiling Behavior

In this section, we present evidence that the borrowers indeed stockpile under the IBF scheme. By definition, to stockpile is to order consecutively in preparation for the upcoming high demand. We thus search for stockpiling behavior by identifying borrowing patterns that satisfy the following definition.

Definition 1 (Stockpiling) *The user of the IBF scheme is considered stockpiling if she borrows multiple loans consecutively without repaying to the lender in the meantime.*

Proposition 5 demonstrates that when faced with high future demand, the borrower has to repeat the pledge-purchase cycle multiple times to reach the objective inventory level. Thus, Definition 1 aims to distinguish the consecutive borrowing behavior of stockpiling from the one-shot borrowing pattern, i.e., borrowing followed by immediate repayment. Following this definition, we have identified 87 stockpiling incidences in our dataset, which correspond to 25.4% of the total loans incidences and 26.5% of the total loan amount. To showcase the general borrowing pattern under

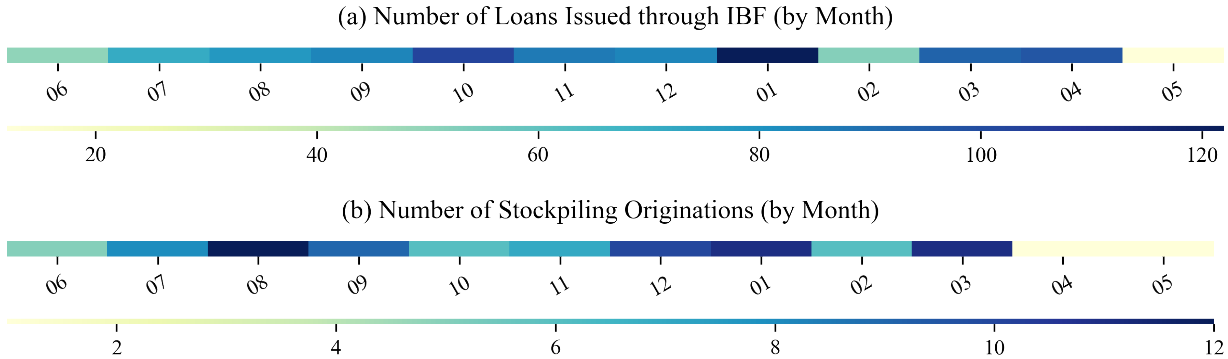

Figure 5: **Temporal distribution of borrowing and stockpiling behavior.** Panel (a) illustrates the number of loan issued across 12 months. Panel (b) summarizes the number of stockpiling originations within 12 months.

IBF, we draw on the loan data from JD Finance and plot in Figure 4 the typical outstanding loan trajectories from six borrowers. In particular, we color the stockpiling behaviors in red. These small retailers borrow 2.7 times on average when they stockpile. In some cases, however, the borrower may exploit IBF and borrow as many as nine consecutive loans.

Additionally, we study when borrowers resort to IBF and engage in stockpiling behaviors. To this end, we map the loan origination dates on the timeline in Figure 5. Panel (a) presents the heat map of all the loan origination dates under the IBF scheme (which are identical to the dates when borrowers actually receive the loan). We observe that the usage of IBF peaks at October and January, which are the months proceeding the Singles Day, the largest online shopping event in China taking place on November 11th, and the Chinese New Year, which is typically in February. On both holidays, retailers usually expect a flood of orders. This explains retailers' aggressive usage of IBF in these two months: to have the inventory properly built up by the holidays given the lead time, these small retailers have to pledge their inventory to place new order one month ahead, so that the products will arrive on time. Meanwhile, we plot exclusively the initiations of stockpiling in Panel (b). Interestingly, there is a noticeable shift of the loan borrowing date when we compare Panel (b) to Panel (a). When small retailers engage in stockpiling instead of one-shot borrowing, most of them would start pledging two or three months ahead of these holidays from as early as August and December. This is consistent with Proposition 5. That is, when the upcoming demand and the resulting objective inventory level is much larger than the current on-hand inventory, the borrowers would start pledging T^* periods before the demand realizes.

4.3 Implications of Stockpiling on Interest Rate

Given the prevalence of stockpiling behavior, it is crucial for the lender to discern if a loan application is part of a stockpiling plan in order to set a proper interest rate. Assuming that all the loans are one-time would make the lender set an interest rate that result in the small retailer setting a sub-optimal objective inventory level, which jeopardizes the lender's revenue as well as the supply chain efficiency. In reality, it can be difficult for the lender to determine if a loan is associated with stockpiling, as it requires the lender to actively communicate with the borrower and also to make predictions based on historical borrowing patterns. To investigate if JD Finance is aware of the stockpiling behaviors while setting the interest rate, we resort to Panel (c) of Figure 2, which visualizes the optimal interest rate under different inventory levels and carries three testable hypotheses:

- (H1) The optimal interest rates for one-shot loans are higher than that of stockpiling loans, as long as the initial inventory levels for borrowers who pledge one-shot are not too high.
- (H2) For loans associated with stockpiling activities, the optimal interest rate increases as the planning horizon shortens (i.e., the borrower has higher initial inventory level).
- (H3) For one-time loans, the optimal interest rate decreases as borrower's initial inventory level increases.

To begin with, we formally test (H1) through the following regression:

$$r_{it} = \beta_0 + \beta_1 \mathbb{1}_{\{stockpile_i=1\}} + \gamma \; CONTROL_{it} + \epsilon_{it} \,, \tag{11}$$

where r_{it} is the interest rate of loan i borrowed in month t, $\mathbb{1}_{\{stockpile_i=1\}}$ is the indicating function specifying if loan i is borrowed to stockpile, and $CONTROL_{it}$ includes the industry fixed effects and credit limit fixed effects. We report the regression results in specification (1) in Table 2. Notably, $\beta_1 = -0.0029$ and is significant, indicating that JD Finance in fact customizes the interest rates for loans associated with stockpiling activities. Meanwhile, Panel (c) of Figure 2 shows that the optimal interest rate increases as the borrower's initial inventory decreases for one-time loan. Therefore, removing the right tail of the interest rate curve should make interest rate gap between the one-time and multi-period loans more pronounced. Following this logic, we sort the all the one-shot loans each small retailer has borrowed in terms of amount, remove loans in the bottom 50% percentile, and rerun Regression (11) with the rest of the data as a robustness check. Specification

Table 2: Impact of Stockpile on Interest Rate

	(1)	(2)	(3)	(4)
	Interest Rate	Interest Rate (50% cut)	Interest Rate (stockpile)	Interest Rate (single period)
(Intercept)	0.1462^{***}	0.1490^{***}	0.1493^{***}	0.1379^{***}
	(0.0020)	(0.0021)	(0.0020)	(0.0044)
Stockpile	-0.0029^{**}	-0.0043^{***}		
	(0.0012)	(0.0012)		
Period			-0.0006^{*}	
			(0.0003)	
Loan Amount Percentile				0.0071^{*}
				(0.0034)
Industry Segment FE.	YES	YES	YES	YES
Credit Limit FE.	YES	YES	YES	YES
Adjusted R^2	0.2732	0.3101	0.5111	0.1431
Number of Observations	731	568	391	305

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

(2) shows that β_1 indeed has a larger magnitude and a higher level of significance.

Next, (H2) states that the optimal interest rate should increase as the planning horizon shortens. This is manifested in Panel (c) of Figure 2, in which all the interest rate experiences a sudden surge whenever the planning horizon decreases by one period. To formally test (H2), we rerun regression (11), with the stockpile indicating function $\mathbb{1}_{\{stockpile_i=1\}}$ replaced by the planning horizon, which is measured as the number of consecutive loans borrowed observed in the data. The result is reported in specification (3) in Table 2. We do observe that as the borrowing period increases, the interest rate becomes slightly lower. Nevertheless, the magnitude of the effect is small as -0.0006. In an extreme scenario, with everything else controlled, the interest difference between a borrower who plans to stockpile nine times and one who stockpiles two periods is merely 0.42%, which is negligible compared to the average interest rate which is 14.34%. This result indicates that there is room for JD Finance to improve its revenue by tailoring the interest rate for borrowers with different stockpile plans, provided that there is perfect information sharing between the lender and the small retailers.

Finally, according to (H3), we should expect the optimal interest rate to decrease as borrower's initial inventory level increases. Nevertheless, as we do not directly observe each borrower's initial inventory level q in our data, we use $Pecentile_{ij}$, the percentile of loan i when benchmarked against all the one-shot loans borrowed by small retailer j, as the proxy we come up with for q. The idea is, assuming the demand each small retailer faces when applying for one-shot loans to be stationary, higher borrowing amount would imply a lower level of initial inventory. Again, we substitute the stockpile indicating function $\mathbb{1}_{\{stockpile_i=1\}}$ with $Pecentile_{ij}$ and rerun regression 11. In specification (4), we observe the loan interest rate is indeed positively correlated with $Pecentile_{ij}$, indicating that (H3) is also supported.

5 Discussion

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Appendix for

Retailer Initiated Inventory-Based Financing

A Proof for Proposition 1

Proof. The borrower's objective function is

$$\begin{aligned}
\Pi_{\mathscr{B}}(z) &= \mathbb{E}\left[p\min\left(D, q+\frac{v}{c}z\right) - vRz\right]^+ \\
&= \int_0^{q+\frac{v}{c}z} (px - vRz)^+ f(x)dx + \int_{q+\frac{v}{c}z}^{\infty} (pq + p\frac{v}{c}z - vRz)^+ f(x)dx \\
&= \int_{\frac{vR}{p}z}^{q+\frac{v}{c}z} (px - vRz)f(x)dx + (pq + \frac{v}{c}pz - vRz)\overline{F}(q+\frac{v}{c}z) \\
&= \int_{\frac{vR}{p}z}^{q+\frac{v}{c}z} p\overline{F}(x)dx.
\end{aligned}$$

By taking its derivative, we have

$$\frac{\partial \Pi_{\mathscr{B}}(z)}{\partial z} = \overline{F}(q+\frac{v}{c}z)\frac{pv}{c} - \overline{F}(\frac{vR}{p}z)Rv.$$

Case 1: $q \le c/(c+v)S^*$. Then

$$\frac{\partial \Pi_{\mathscr{B}}(z)}{\partial z} \ge \overline{F}(q+\frac{v}{c}z)\frac{pv}{c} - Rv \ge \overline{F}(q+\frac{v}{c}q)\frac{pv}{c} - Rv \ge \overline{F}(S^*)\frac{pv}{c} - Rv = 0, \tag{A.1}$$

where we use the fact that $\overline{F}$ is monotone decreasing with $\overline{F}(0) = 1$. This means $\Pi_{\mathscr{B}}(z)$ is monotone increasing in $[0, q]$, hence we can have in this case $z^O(q) = q$.

Case 2: $c/(c+v)S^* < q < S^*$. Then we have

$$\left.\frac{\partial \Pi_{\mathscr{B}}}{\partial z}\right|_{z=0} = \overline{F}(q)\frac{pv}{c} - Rv > 0.$$

On the other hand, from the third requirement in Assumption 1, there exists a threshold M such that $\frac{\partial \Pi_{\mathscr{B}}}{\partial z} < 0$ when $z > M$. This along with the first assumption in Assumption 1 prove that there exists a unique solution $z^*(q) > 0$ to equation (A.1). And $\Pi_{\mathscr{B}}(z)$ increases in $[0, z^*(q)]$ while decreases in $[z^*(q), \infty]$. Hence the optimal solution would be $z^O(q) = \min\{q, z^*(q)\}$.

Case 3: $q \ge S^*$. Then we have $\frac{\partial \Pi_{\mathscr{B}}}{\partial z}\big|_{z=0} \le 0$. Note that under our Assumption 1, the derivative

$\frac{\partial \Pi_{\mathscr{B}}}{\partial z}$ would approach zero from the negative side as $z \to \infty$. Hence $\frac{\partial \Pi_{\mathscr{B}}}{\partial z} < 0$ and we would have $z^O(q) = 0$.

B Proof for Proposition 2

Proof. The borrower's objective function in this case can be written as

$$\begin{aligned}
\Pi_{\mathscr{B}}^H(z) &= \mathbb{E}\left[p \min\left(D, q + \frac{v}{c}z\right) - vRz\right] \\
&= \int_0^{q+\frac{v}{c}z} (px - vRz) f(x)dx + (pq + p\frac{v}{c}z - vRz)\overline{F}(q + \frac{v}{c}z) \\
&= -vRz + \int_0^{q+\frac{v}{c}z} p\overline{F}(x)dx.
\end{aligned}$$

Hence its derivative with respect to z is

$$\frac{\partial \Pi_{\mathscr{B}}^H(z)}{\partial z} = -vR + \frac{vp}{c}\overline{F}(q + \frac{v}{c}z),$$

which is monotone decreasing with respect to z.

Case 1: $q \le \frac{c}{c+v}Q^*$. Under this case, we have $\overline{F}(q + \frac{v}{c}z) \ge \overline{F}(\frac{c+v}{c}q) \ge \overline{F}(Q^*) = \frac{cR}{p}$, which indicates that $\frac{\partial \Pi_{\mathscr{B}}^H(z)}{\partial z} \ge 0$ when $z \in [0, q]$. Hence $\Pi_{\mathscr{B}}^H(z)$ achieves its maximum at $z = q$.

Case 2: $\frac{c}{c+v}Q^* < q < Q^*$. Then $\frac{\partial \Pi_{\mathscr{B}}^H(z)}{\partial z}$ has a single root $z^* = \frac{c}{v}(Q^* - q) \in [0, q]$. So $\Pi_{\mathscr{B}}^H(z)$ is increasing in $[0, z^*]$ and decreasing in $[z^*, q]$. Hence $\Pi_{\mathscr{B}}^H(z)$ achieves its maximum at $z = z^*$.

Case 3: $q \ge Q^*$. Then we can have $\overline{F}(q + \frac{v}{c}z) \le \overline{F}(Q^*) = \frac{cR}{p}$, which means $\frac{\partial \Pi_{\mathscr{B}}^H(z)}{\partial z} \le 0$ in $[0, q]$. Hence $\Pi_{\mathscr{B}}^H(z)$ achieves its maximum at $z = 0$.

C Proof for Proposition 3

Proof. Obviously, we only need to prove the case when $\frac{c}{c+v}Q^* < q \le Q^*$. Suppose the solution z^* satisfies

$$F(q + \frac{v}{c}z^*) = \frac{p - Rc}{p} + \frac{Rc}{p}F\left(\frac{vRz^*}{p}\right).$$

Because we assume $p \ge vR$, we can further have $F\left(\frac{vRz^*}{p}\right) \le F(z^*) \le F(q) \le F(Q^*) = 1 - Rc/p$. Hence

$$F(q + \frac{v}{c}z^*) \le \frac{p - Rc}{p} + \frac{Rc}{p}(1 - \frac{Rc}{p}) = \frac{p^2 - R^2c^2}{p}.$$

Note that the heuristic policy z^H satisfies $F(q + \frac{v}{c} z^H) = \frac{p - Rc}{p}$. So we can conclude that

$$
\begin{aligned}
z^* - z^H &= \frac{c}{v}\left[(q + \frac{v}{c} z^*) - (q + \frac{v}{c} z^H)\right] \\
&\leq \frac{c}{v}\left[F^{-1}\left(\frac{p^2 - R^2 c^2}{p}\right) - F^{-1}\left(\frac{p - Rc}{p}\right)\right].
\end{aligned}
$$

And our proposition follows from the fact that $z^O - z^H = \min\{z^*, q\} - z^H \leq z^* - z^H$.

D Proof for Theorem 4

Proof. The lender's objective function can be written as

$$
\begin{aligned}
\Pi_{\mathscr{L}}(R, v) &= \mathbb{E}\left[\min(pD, vRz) + s\left(z - \frac{p}{vR} D\right)^+\right] - vR_f z, \\
&= \int_0^{\frac{vRz}{p}} \left(px + s(z - \frac{px}{vR})\right) f(x)dx + \int_{\frac{vRz}{p}}^{\infty} vRz f(x)dx - vR_f z \\
&= \int_0^{\frac{vRz}{p}} \left(p - \frac{sp}{vR}\right) x f(x)dx + (s - vR)zF(\frac{vRz}{p}) + v(R - R_f)z \\
&= -\left(1 - \frac{s}{vR}\right) p \int_0^{\frac{vRz}{p}} F(x)dx + v(R - R_f)z
\end{aligned}
$$

When $R > \frac{p}{c}\overline{F}(q)$, we have $z^* = 0$ by Proposition 1, which means $\Pi_{\mathscr{L}}(R, v) = 0$.

When $R \leq \frac{p}{c}\overline{F}(\frac{c+v}{c} q)$, we have $z^* = q$ by Proposition 1. So we have $\frac{\partial z^*}{\partial R} = 0$ and we can calculate $\frac{\partial \Pi_{\mathscr{L}}}{\partial R}$ as

$$
\frac{\partial \Pi_{\mathscr{L}}(R, v)}{\partial R} = -(vz - \frac{sz}{R})F(\frac{vRz}{p}) + vz \geq \frac{sz}{R} F(\frac{vRz}{p}) > 0.
$$

Hence $\Pi_{\mathscr{L}}$ is monotone increasing in R and this case cannot be optimal.

Combining the above two statement, we can have the optimal R^* has the bound

$$
R^* \in \left[\frac{p}{c}\bar{F}\left(\frac{c+v}{c} q\right), \frac{p}{c}\bar{F}(q)\right].
$$

E Proof for Proposition 5

Proof. We first prove that if $w_0 = 0$, then $w_t^* = 0$ for every $t = 0, 1, \ldots, T$. This is because if the retailer's initial cash position is $w_0 = 0$, then holding positive cash means the retailer must be pledging some extra quantity than buying the exact amount of u_t at some time point. However, since the retailer can pledge any amount of quantity at any time point, it is better for him/her to pledge just the amount needed and save the rest as the product, in this way he/she wouldn't be charged the interest for the extra quantity pledged. In fact, for every pledging strategy $\{u_t, z_t\}$ with w_t as its cash position, we construct a corresponding policy $\{u_t', z_t'\}$ that satisfies $z_t' = cu_t/v$ and $u_t' = u_t$. By induction, it is easy to see that $vq_t' = vq_t + w_t$ for every $t = 0, 1, \ldots, T$, hence this policy is feasible. The total inventory in these two cases will be the same because $x_T + q_T = q + \sum_{t=0}^{T-1} u_t = x_T' + q_T'$. However, we get $w_t' = 0$ and $x_t' \leq x_t$ for $t = 0, 1, 2, \ldots, T$, which means the retailer will pay less interest to the platform, indicating the revenue will be higher under the policy $\{u_t', z_t'\}$.

By using the statement above, we can further write the state dynamics as

$$q_{t+1} = q_t + (\theta - 1)z_t \tag{A.2}$$

$$x_{t+1} = x_t + z_t, \tag{A.3}$$

where $\theta = \frac{v}{c}$. For a given T and Q, the objective function would be

$$B_0^T(q, 0, 0) = \mathbb{E}\left[p\min(D, Q) - vx_T - rv\Sigma_{t=1}^T x_t\right]^+ = \mathbb{E}\left[p\min(D, Q) - c(Q - q) - rvA(Q)\right]^+.$$

Hence maximizing $B_0^T(q, 0, 0)$ is equivalent to minimizing $A(Q) = \sum_{t=1}^T x_t = \sum_{t=0}^{T-1}(T - t)z_t$. So the dynamic programming problem can be written into a linear programming problem as

$$\begin{aligned} \min_{z_i} \ & \sum_{t=0}^{T-1}(T - t)z_t \\ s.t.\ & x_T + q_T = Q \\ & 0 \leq z_t \leq q_t \qquad t = 0, 1, 2, \ldots, T - 1. \end{aligned}$$

Note that from the states dynamics A.2 and A.3, we have $x_T + q_T = q + \sum_{t=0}^{T-1} z_t$ and $q_t =$

$q + (\theta - 1)\sum_{i=0}^{t-1} z_i$. So we can further write the problem as

$$\begin{aligned} \min_{z_i} \ & \sum_{t=0}^{T-1}(T-t)z_t \\ s.t. \ & q + \sum_{t=0}^{T-1} z_t = Q \\ & z_0 \le q \\ & z_t \le q + (\theta - 1)\sum_{i=0}^{t-1} z_i \qquad t = 1, 2, \ldots, T-1. \\ & z_t \ge 0 \qquad t = 0, 1, 2, \ldots, T-1 \end{aligned} \tag{A.4}$$

The Lagrangian of this problem is

$$L(\boldsymbol{z}, \mu, \boldsymbol{\lambda}, \boldsymbol{\kappa}) = \sum_{t=0}^{T-1}(T-t)z_t + \mu\left(q + \sum_{t=0}^{T-1} z_t - Q\right) + \lambda_0(z_0 - q) + \sum_{t=1}^{T-1}\lambda_t\left(z_t - q - (\theta-1)\sum_{i=0}^{t-1} z_i\right) - \sum_{t=0}^{T-1}\kappa_t z_t.$$

The first order condition would be

$$\frac{\partial L}{\partial z_t} = T - t + \mu + \lambda_t - (\theta - 1)\sum_{i=t+1}^{T-1}\lambda_t - \kappa_t = 0.$$

Taking subtraction of the t-th and $(t+1)$-th equation, we have

$$1 + \lambda_t - \theta\lambda_{t+1} = \kappa_t - \kappa_{t+1}. \tag{A.5}$$

If $\lambda_t > 0$, then by complementary slackness, we have $z_t = q + (\theta - 1)\sum_{i=0}^{t-1} z_i > 0$. Again by complementary slackness of κ_t, we have $\kappa_t = 0$. So in order to satisfy equation A.5, we must have $\lambda_{t+1} > 0$. By induction, we can have $\lambda_i > 0$ for every $i \ge t$. Hence there exists $0 \le T_0 \le T-1$ such that $\lambda_t = 0$ when $t \le T_0$ and $\lambda_t > 0$ when $t > T_0$.

For every $t \le T_0 - 1$, we have $\lambda_t = \lambda_{t+1} = 0$, so the left side of equation A.5 is 1, which indicates that $\kappa_t > 0$ and $z_t = 0$. For $t > T_0$, since $\lambda_t > 0$, we have $z_t = q + (\theta-1)\sum_{i=0}^{t-1} z_i$ by complementary slackness. As the result, we conclude that the optimal solution of linear programming A.4 has the following structure:

$$
\begin{aligned}
z_t &= 0, \qquad t = 0, 1, \ldots, T_0 - 1 \\
z_{T_0+1} &= q + (\theta - 1) z_{T_0} \\
z_t &= \theta z_{t-1}, \qquad t = T_0 + 2, T_0 + 2, \ldots, T - 1.
\end{aligned}
$$

So from the constraint

$$
\sum_{t=0}^{T-1} z_t = \sum_{t=T_0+1}^{T} \theta^{t-T_0-1} z_{T_0+1} + z_{T_0} = \frac{\theta^{T-T_0} - 1}{\theta - 1}(q + (\theta - 1) z_{T_0}) + z_{T_0} = Q - q,
$$

we can have

$$
z_{T_0} = \frac{1}{\theta^{T-T_0}} \left[Q - \frac{\theta^{T-T_0} - 1}{\theta - 1} q \right].
$$

From the constraint $0 \leq z_{T_0} \leq q$, we can have

$$
T - T_0 = \left\lfloor \frac{\log\left(\frac{Q}{q}(\theta - 1) + 1\right)}{\log \theta} \right\rfloor := T^*,
$$

which proves the statement in the theorem. And we can calculate $A(Q)$ as

$$
A(Q) = \sum_{t=0}^{T-1} (T - t) z_t = \frac{q(\theta^{T^*} - 1)}{(\theta - 1)^2} - \frac{qT^*}{\theta - 1} + z_0 \frac{\theta^{T^*} - 1}{\theta - 1}.
$$

F Proof for Theorem 1

Denote $M(Q) = c(Q - q) + rvA(Q)$, then the borrower's objective function can be written as:

$$
\begin{aligned}
\Pi_{\mathscr{B}}(Q) &= \mathbb{E}\left[p \min(D, Q) - c(Q - q) - rvA(Q)\right]^+ \\
&= \mathbb{E}\left[p \min(D, Q) - M(Q)\right]^+ \\
&= \int_0^Q (px - M(Q))^+ f(x) dx + (pQ - M(Q))^+ \overline{F}(Q) \\
&= \begin{cases} 0 & pQ < M(Q) \\ p \int_{M(Q)/p}^{Q} \overline{F}(x) dx & pQ \geq M(Q) \end{cases}
\end{aligned}
$$

Without loss of generality, we will only consider the region where $Q \geq M(Q)/p$ in the following text. By direct calculation, we can see that $M(Q)$ is linear in Q. In fact, $M(Q)$ can be written as:

$$
\begin{aligned}
M(Q) &= \left[c + rv\frac{\theta^T - 1}{\theta^T(\theta-1)}\right] Q - \left[rv\left(\frac{T}{\theta-1} - \frac{\theta^T - 1}{\theta^T(\theta-1)^2}\right) + c\right] q \\
&:= \alpha_T p Q - \beta_T p.
\end{aligned}
$$

As a result, the derivative of $\Pi_{\mathscr{B}}(Q)$ is

$$
\frac{\partial \Pi_{\mathscr{B}}(Q)}{\partial Q} = p\left(\overline{F}(Q) - \overline{F}\left(\alpha_T Q - \beta_T\right)\alpha_T\right). \tag{A.6}
$$

By basic calculus, we can see that $\alpha_T > 0, \beta_T > 0$ and α_T, β_T are monotone increasing with respect to T. (Here the assumption $\theta = v/c > 1$ is used.) This further implies that α_T and β_T are monotone increasing with respect to Q. We next prove that function (A.6) has at most a single root. In fact, by Assumption 1, we have function (A.6) has at most a single root for any fixed T. We now prove by contradiction. Suppose it has two roots $Q_1 < Q_2$. Then because the planning horizon T is increasing in Q, we must have $T_1 < T_2$ for these two pledging periods respectively. This further indicates that $\alpha_{T_1} < \alpha_{T_2}$ and $\beta_{T_1} < \beta_{T_2}$. We next show that from $\alpha_{T_1} < \alpha_{T_2}$ and $\beta_{T_1} < \beta_{T_2}$, we would get $Q_1 > Q_2$, which is a contradiction.

Denote

$$
\begin{aligned}
G_1(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_1} Q - \beta_{T_1}\right)\alpha_{T_1}, \\
G_2(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_2} Q - \beta_{T_2}\right)\alpha_{T_2}, \\
G_3(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_2} Q - \beta_{T_1}\right)\alpha_{T_2}.
\end{aligned}
$$

We already have Q_1, Q_2 is the solution to $G_1(Q), G_2(Q)$. It is easy to see that $G_3(Q_2) > G_2(Q_2) = 0$. Note that G_3 is approaching zero from the negative side as $Q \to \infty$, hence $G_3(Q)$ would also have a root, which we denote as Q_3. We next show that $Q_2 < Q_3 < Q_1$.

In fact, from Assumption 1, the equation

$$
\overline{F}(\alpha_{T_1} Q - \beta_{T_1})\alpha_{T_1} - \overline{F}(\alpha_{T_2} Q - \beta_{T_1})\alpha_{T_2} = 0 \tag{A.7}
$$

also has at most a single solution. If there is a solution Q_0 to (A.7), then because we have $x\overline{F}(x-m)$

is unimodal under Assumption 1, we would get

$$\overline{F}(\alpha_{T_1}Q_0 - \beta_{T_1})\alpha_{T_1} = \overline{F}(\alpha_{T_2}Q_0 - \beta_{T_1})\alpha_{T_2} \geq \overline{F}(Q_0 - \beta_{T_1}) \geq \overline{F}(Q_0).$$

Hence, $G_1(Q_0) < 0$. And we would get $G_1(Q) > G_3(Q)$ when $Q < Q_0$ and $G_1(Q) \geq G_3(Q)$ when $Q \geq Q_0$. If there is no solution to (A.7), we would naturally get $G_1(Q) > G_3(Q)$. As a result, we have $Q_3 < Q_1$ either way. On the other hand, by the monotonicity of $\overline{F}$, we have $0 = G_2(Q_2) < G_3(Q_2)$. This implies $Q_2 < Q_3$. So we achieve $Q_2 < Q_3 < Q_1$ and this is a contradiction to the fact that $Q_1 < Q_2$.

We now have proven that (A.6) has at most a single root. Similar to the case in the single period, we have $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}$ would approach zero from the negative side as $Q \to \infty$ by the third assumption in Assumption 1. Hence, $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}$ having a solution in $[q, \infty)$ is equivalent to $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}\Big|_{Q=q} \geq 0$, which is further equivalent to $q \leq S^*$, where $S^* = F^{-1}(1 - \frac{Rc}{p})$. As a result, when $q \leq S^*$, $\Pi_{\mathscr{B}}(Q)$ is increasing in $[q, Q^*]$ and decreasing in (Q^*, ∞) where Q^* is the root of function (A.6). And when $q > S^*$, we have $\Pi_{\mathscr{B}}(Q)$ is decreasing in $[q, \infty)$. This gives us the statement in the theorem.