Retailer Initiated Inventory-Based Financing

Hongyu Chen[1] and Weiming Zhu[2]

[1]Peking University

[2]IESE Business School, University of Navarra

Abstract

We study an innovative financing scheme in which a large retailer provides inventory-based financing (IBF) to a small retailer selling through her own brick-and-mortar channel. In anticipation of a peak selling period, the small retailer can repeatedly pledge her on-hand inventory in exchange for a loan amount, which is in turn used to procure more inventory, i.e., stockpiling to fulfill stochastic customer demand. Following sales proceeds, the small retailer buys back the pledged inventory to the extent possible and defaults on any leftovers, which will be liquidated by the large retailer via his own platform. We analyze the contract design and the effectiveness of such a financing scheme through a game-theoretical model. In particular, we derive the optimal joint inventory ordering and pledging decisions for the small retailer during the stockpiling phase; we further characterize the optimal loan interest rate for the large retailer. Using datasets obtained from the financial technology arm of a leading online retailer, we provide empirical evidence that small retailers stockpile through IBF, especially prior to shopping seasons and holidays. Furthermore, through double machine learning, we estimate that small retailers shorten the planing horizon by 18% of their lead time during the stockpiling phase or reduce the loan amount by 15% for single-period loans when facing a 1% increase in the interest rate.

1 Introduction

One of the challenges faced by small- and medium-sized enterprises (SMEs) is working capital finance. Firms of large size or with established business usually have fixed assets, such as plants, machinery, equipment, etc., that could be used as collateral to secure loans from financial institutions. This is often not the case for fast-growing, entrepreneurial firms, especially in the retail sector. In the absence of fixed assets, small retailers are constantly challenged to obtain bank loans at competitive interest rates. Alternatively, small retailers may resort to in-kind finance, such as trade credit, yet such financing schemes can be expensive and are offered only if the supply chain partner is not budget constrained. This scenario in turn generates operational inefficiencies since the lack of working capital finance could severely constrain a firm's inventory orders, among other operations.

One way to alleviate this problem is inventory-based financing (IBF). A form of asset-based lending, inventory-based financing is increasingly adopted by high-growth retailers (Buzacott and Zhang 2004). To facilitate IBF, a lender (usually a bank) performs due diligence of the borrower (a small retailer) and provides loans with the maximum amount linked to the inventory value of the borrowing firm. The loan limit and interest rate are chosen based on the potential sales proceeds and the salvage value of unsold inventory in the event of bankruptcy. The use of IBF opens doors for small, high-growth retailers with limited alternatives to working capital financing.

However, the interest rates of traditional bank-initiated IBF are still too high in practice, thereby prohibiting mass adoption by smaller retailers who purchase and sell in price-competitive markets. In addition, the maximum loan amount is often time limited in supporting the optimal inventory procurement policy in the face of a growing market. The high interest rate and low loan limit are results of the risk management of the bank. Most IBF contracts do not entitle banks the ownership of inventory before the loan is due; in the event of bankruptcy, the bank usually has limited capability to re-sell the leftover inventory, realizing a very low salvage value. Due to these frictions, IBF often reaches its limit in providing competitive financing to a mass pool of retailers.

On the other hand, large retailers, especially online platforms, enjoy considerable advantages in devising and providing IBF to small retailers. Compared with banks, large retailers have more experience and data on the underlined asset, as they sell similar products via their own online portals; unlike bank-initiated IBF, retailer-initiated IBF grants the lender ownership of the secured inventory by obliging the borrower to stock in the lender's warehouse. In the event of loan default, the large retailer could sell the pledged inventory through its own channel without suffering from low salvage value or insufficient demand. The above-mentioned advantages fundamentally change the risk management model and potentially lower the interest rate while increasing the loan limit, making such large retailers more suited to providing IBF to small retailers.

Our interest in retailer-initiated IBF was motivated by the practice employed by JD Finance. JD Finance is the financing arm of JD.com, the largest online business-to-consumer retailer in China, offering more than 35 million product selections from tens of thousands of brands[1]. Leveraging the abundance of product- and market-level data and its access to cheap loans (Huang et al. 2022), JD Finance has established a portfolio of supply chain finance products, ranging from reverse factoring (Tunca and Zhu 2018), micro-lending, to more recent inventory-based financing. The latter focuses on small retailers selling outside of JD.com but becoming part of JD's ecosystem through inventory-based loans. By 2018, one year after the launch of IBF,

[1] https://marketingtochina.com/the-secret-of-the-massive-sales-on-jd-com/

JD had issued over 1000 loans to small retailers.

In the inventory-based financing scheme initiated by JD Finance, the small retailer pledges her inventory in exchange for a loan that is priced by JD. The loan is then used to purchase extra products to fulfill future demand. The small retailer later buys back the inventory pledged to the extent possible, depending on the sales of the products. If not all products are bought back by the small retailer, JD takes the remaining inventory and sells through its own channel.

In this paper, we study this innovative financing scheme initiated by large retailers such as JD.com. To this end, we model the retailer-initialed IBF (abbreviated as IBF henceforth) using a Stackelberg game in which the small retailer has a single opportunity to pledge her inventory[2]. We characterize the small retailer's optimal pledging strategy and the large retailer's optimal interest rate. When granted a single opportunity to pledge, inventory serves two distinct functions for the borrower: *loan-pledging capacity* and *demand-fulfilling stock*. In such case, we demonstrate through numerical analysis that IBF improves the overall supply chain efficiency and is especially effective when the small retailer's initial inventory is not excessively low.

One notable feature of JD's IBF scheme is that small retailers are able to pledge inventory that is purchased on loan. As a result, a small retailer with low initial inventory but high credit limit could use the IBF scheme to repeat the pledge-purchase cycle multiple times without repaying the outstanding loan in the meantime, until she reaches the desired inventory level. To model the borrower's stockpiling behavior, we extend the single-period model to multiple periods. In this case, the borrower decides on a planning horizon of T periods. In each period, she pledges part or all of her on-hand inventory, obtains loans and replenishes inventory with a one-period lead time. Given an objective inventory level, we derive the optimal planning horizon and showcase that the borrower's optimal pledging strategy follows a simple structure: the borrower should pledge all on-hand inventory in every period except for the first one; in the first period, the borrower should pledge the amount that makes her reach the objective inventory level at the end of the planning horizon with the pledge-everything policy in the following periods. Importantly, we showcase through numerical study that stockpiling not only leads to increased profit for the large retailer but also eases the loan-pledging capacity constraint that initial inventory imposes and allows a borrower with low initial inventory to reach a high inventory level.

Fu et al. (2021) predicts that borrowers would stockpile under IBF when their on-hand inventory is significantly lower than the upcoming demand; our work arrives at the same conclusion. To verify our theoretical results, we use datasets obtained from JD Finance to study small retailers' borrowing patterns. We observe that stockpiling, defined as consecutive borrowing without repayment in the meantime, is common among borrowers. Spiking before major shopping seasons and holidays, loans tied to stockpiling account for more than 25% of total borrowing in terms of both amount and frequency. We further study if the impact of interest rates on small retailer's borrowing behavior aligns with our theoretical predictions. To mitigate

[2]Henceforth, we denote the small retailer by female pronouns and the large retailer by male pronouns

the bias caused by unobserved confounding variables and their nonlinear effects, we apply double machine learning (Chernozhukov et al. 2018) to estimate the effect of interest rate. We show that a 1% increase in the interest rate shortens the planning horizon by 18% of the lead time for stockpiling loans. For one-shot loans, a 1% increase in the interest rate can lower the borrowing amount by as much as 15%.

Contribution Our paper makes three contributions to the literature. First, we prescribe to small retailers the optimal pledging strategy to achieve an objective inventory level under IBF. Second, to the best of our knowledge, our work is the first study that empirically verifies small retailers' stockpiling behavior under IBF. We show that stockpiling behavior is in fact common among borrowers and tends to cluster before major shopping seasons and holidays. Finally, we provide empirical evidence of the impact of the large retailer's interest on the small retailer's borrowing strategy. As the smaller retailer's initial inventory is usually unobservable to the large retailer but can confound the result through a nonlinear relationship, we propose an empirical strategy in which we approximate the initial inventory through a number of time- and firm-specific variables and apply double machine learning to mitigate the bias in the estimated effect of the interest rate.

The remainder of this paper is organized as follows. We review the related literature in Section 2. In Section 3, we present our model framework and numerical studies. We then empirically test our theoretical predictions using datasets from JD Finance in Section 4. Finally, we conclude the paper in Section 5.

2 Literature Review

Our work studies an inventory-based financing scheme and is thus closely related to previous literature on bank-initiated asset-based lending (ABL), which is also called inventory-based lending and is widely used by budget-constrained retailers in the United States Foley et al. (2012). Buzacott and Zhang (2004) investigate ABL first through a multiperiod deterministic model in which a cash-constrained firm makes production and inventory decisions under ABL. Buzacott and Zhang (2004) formulate a single-period stochastic inventory model in which the firm makes inventory stocking and borrowing decisions to maximize its profit under ABL. Alan and Gaur (2018) extend Buzacott and Zhang (2004) by endogenizing the borrower's equity decision and incorporating information asymmetry between the bank and the borrower into the model. Alan and Gaur (2018) characterize the financial and operational decisions in equilibrium under ABL and show that ABL could reduce the information asymmetry between the bank and borrower via screening. Using country-level data in China, Hsu and Wu (2019) theoretically and empirically demonstrate that ABL can be used to take advantage of the financial arbitrage opportunities in financial markets with frictions. The two studies that are most closely to our work are Iancu et al. (2017) and Fu et al. (2021). Iancu et al. (2017) formulate a two-period game-theoretical model in which a firm can manage inventory through replenishments and partial liquidations. They demonstrate that when financed through debt, such flexibility in managing inventory can lead to agency issues. Nevertheless, such inefficiencies can be alleviated by borrowing base covenants in the debt agreement. Fu et al. (2021) study IBF through a novel dynamic inventory model in which the firm makes salvage, loan and ordering decisions. They demonstrate that the borrower's optimal decisions can be simplified by examining the state-dependent order-up-to level and the salvage-down-to level and that the firm may strategically build its inventory in exchange for a higher loan amount to meet future demand. Our work differs from the previous literature in two key aspects. First, we investigate both the borrower's optimal pledging decision and the lender's optimal interest rate, which depend on the borrower's initial inventory position. Notably, the interest rate in our setting is set by the lender to maximize his profit instead of being competitively priced by the market. This is because very few online retailers, if any other than JD Finance, provide IBF in China, making the market conditions far from perfect competition. Second, filling a void in the literature, we draw on data from JD Finance to provide empirical evidence on borrowers' stockpiling behavior under retailer-initiated IBF and examine the implication of the interest rate on the small retailer's borrowing patterns, drawing on our theoretical results.

Our paper is also connected to the literature on buyer-initiated supply chain financing schemes, i.e., reverse factoring. Tanrisever et al. (2012), Wu et al. (2014), Rui and Lai (2015), Van der Vliet et al. (2015) and Kouvelis and Xu (2021) study how reverse factoring creates value for each party in the supply chain and how the value is affected by factors such as the payment period extension and the spread in exogenous financing costs. Drawing on data from JD's reverse factoring scheme, Tunca and Zhu (2018) theoretically and empirically demonstrate that buyer intermediation lowers interest rates and wholesale prices, increases order fill rates, and boosts supplier borrowing. In our setting, the IBF scheme is also initiated by a large retailer. Nevertheless, the key difference is that the large retailer and the budget-constrained retailer are not in a vertical collaboration within a supply chain. For this reason, the small retailer must pledge her inventory as collateral to acquire the loan needed for purchasing additional products. As the loan amount the small retailer is able to obtain is limited by her on-hand inventory, the small retailer may need to repeat the pledge-purchase cycle multiple times to reach the desired inventory level. Thus, the focus of our paper is primarily to examine, both theoretically and empirically, the small retailer's optimal pledging strategy and the implications of interest rate on small retailer's borrowing patterns.

In our paper, the retailer-initiated IBF targets budget-constrained small retailers, which fall under the umbrella of supply chain finance solutions that provide financial flexibility to retailers with capital limitations. Among these solutions, the most well-studied financing scheme is trade credit. Trade credit, as a form of in-kind finance, allows a financially constrained buyer to purchase goods or service from a seller without paying upfront. Caldentey and Chen (2009), Kouvelis and Zhao (2011, 2012), Jing et al. (2012), Peura et al. (2017), Yang and Birge (2018) and Devalkar and Krishnan (2019) examine the interplay between a supplier, a budget-constrained retailer and a bank, demonstrating that retailers prefer trade credit to bank financing when bank loans are competitively priced. However, when the interest rate is set to maximize the bank's profit, the retailer's financing decision would depend on the market conditions and competition. In our work, the small retailer does not receive goods from the upstream party in the supply chain through trade credit, but pledges her on-hand inventory as collateral in exchange for finance from a third party online retailer.

3 Model

We consider a scenario where a lender (e.g., a large retailer) $\mathscr{L}$ initiates an inventory-based financing scheme to lend to a borrower (e.g., a budget-constrained small retailer) $\mathscr{B}$. We use borrower (lender) and small retailer (large retailer) interchangeably throughout this paper. Specifically, the lender sets the interest rate of the loan, and the borrower in turn determines the amount of inventory to be pledged to achieve the optimal inventory level. In what follows, we first introduce a single-period model to demonstrate the interplay between the borrower and the lender under the IBF scheme. We then extend the single-period model by allowing the borrower to repeatedly pledge inventory over a time horizon.

3.1 Single Period

We start with the case in which the borrower has a one-shot opportunity to pledge her on-hand inventory as collateral to the lender in exchange for finance. We model the interaction between the borrower and the lender as a sequential-move Stackleberg game with the following chronology. At $T = 0$, the lender sets the interest rate r. Then, at $T = 1$, the borrower decides z, the quantity to be pledged, after observing r. Meanwhile, we denote the valuation the lender sets for each pledged product as v, which is assumed to be exogenous throughout the paper[3]. We also assume that $v > c$, where c is the production cost. In this way, the borrower would receive vz in cash when z units are pledged to the lender, and the borrower would in turn acquire vz/c units of the product, all purchased on loan. The demand is realized at $T = 2$. The borrower satisfies the demand with her on-hand inventory at unit price p and repays her loan to the lender to the extent possible. In case of partial payment, the lender salvages the remaining product at unit price s.

Notably, the borrower will not be interested in holding cash, as doing so requires her to pledge more quantity than needed, and the cost of the loan is always higher than the risk-free rate r_f. As a result, the borrower would only pledge the exact amount needed to reach the desired inventory level. Hence, the borrower's objective function can be written as

$$\Pi_{\mathscr{B}} = \max_{z \leq q} \mathbb{E}\left[p \min\left(D, q + \frac{v}{c}z\right) - vRz\right]^{+}., \tag{1}$$

where R is defined as $1 + r$. Note that the borrower could reclaim the pledged inventory from the lender instantly to further satisfy customer demand. Hence, the borrower's *objective inventory level*, defined as the total usable stock in the selling period, is $q + vz/c$, i.e., the initial stock plus the additional inventory

[3] Through our interview with the manager of JD Finance, in the current practice, the supply chain finance department communicates with the procurement department to obtain a quote on the salvage value of the product; the valuation of the product is then set as the salvage value plus a preset markup.

purchased through inventory-based financing.

The lender's problem is to determine R to maximize his profit $\Pi_{\mathscr{L}}$, defined as the repayment from the borrower plus the revenue from salvaging the left-over inventory minus the cost of capital. Given the borrower's initial inventory q, the lender's objective function can be written as

$$\Pi_{\mathscr{L}} = \max_{R} \mathbb{E}\left[\min(pD, vRz) + s\left(z - \frac{p}{vR}D\right)^{+}\right] - vR_f z, \tag{2}$$

where z is the amount of inventory pledged by the borrower, as in Equation (1), and $R_f = 1 + r_f$. Note also that since $p > c(1+R)$, the equivalent sales from the borrower's total available inventory, $p(Q + vz/c)$, are always greater than the total loan repayment vRz. Therefore, the actual repayment the lender receives is reduced to $\min(pD, vRz)$.

We solve the Stackelberg game by first analyzing the borrower's optimization problem. We assume that all cost parameters and the demand distribution are public information. In addition, we make the following key assumptions on the distribution of demand D. Denoting the p.d.f., c.d.f. and the complementary c.d.f. of D as $f(\cdot)$, $F(\cdot)$ and $\overline{F}(\cdot) = 1 - F(\cdot)$, respectively, we have

Assumption 1 *For any $m > 0$ and $\alpha \in (0,1)$, we assume the distribution for demand D satisfies the following requirements.*

1. *The equation $\overline{F}(m+x) - \alpha\overline{F}(\alpha x) = 0$ has at most a single root in $[0,\infty)$.*
2. *Function $(x+m)\overline{F}(x)$ is unimodal in $[0,\infty)$.*
3. $\lim_{x\to\infty} f(m+x)/f(\alpha x) = 0$.

Assumption 1 imposes several restrictions on the demand distribution, yet it is satisfied by a wide range of distributions, such as the truncated normal distribution and the Gamma distribution with shape parameter $\alpha > 1$. Equipped with these conditions, we now analyze the borrower's optimal pledging decision.

Proposition 1 *Suppose the distribution of demand D satisfies Assumption 1. Denote $S^* = F^{-1}(1 - Rc/p)$. Then, the equation*

$$\overline{F}(q + \frac{v}{c}z)p - \overline{F}(\frac{vR}{p}z)cR = 0 \tag{3}$$

has a unique solution $z^*(q)$ when $q < S^*$ and the borrower's optimal pledge quantity $z^O(q)$ can be characterized as

$$z^O(q) = \left\{ \begin{array}{ll} q, & \textit{if } q \le \frac{c}{c+v}S^* \\ \min\{z^*(q), q\}, & \textit{if } \frac{c}{c+v}S^* \le q \le S^* \\ 0. & \textit{if } S^* < q \end{array} \right\}. \tag{4}$$

Proposition 1 implies that when q is sufficiently small, i.e., when $q \le \frac{c}{c+v}S^*$, the borrower should pledge all her on-hand inventory to satisfy as much demand as possible. Moreover, when q is sufficiently large, i.e., when $q > S^*$, the borrower would no longer need to expand her inventory since it can meet the demand D. However, when q is moderate, $z^*(q)$ does not possess an explicit-form solution under general demand. In any case, the objective inventory the borrower aims to achieve, defined as Q^O, can be formally expressed as $Q^O = q + vz^O(q)/c$.

Approximate Formulation. Now, to further understand the borrower's optimal borrowing scheme and to provide a practical solution for borrowers, we simplify the optimal pledging strategy by solving an approximate model of the original Equation (1). By allowing a negative ending cash position of the borrower, we define the simplified model as

$$\Pi_{\mathscr{B}}^H = \max_z \mathbb{E}\left[p \min\left(D, q + \frac{v}{c}z\right) - vRz\right]. \tag{5}$$

Let z^H be the heuristic solution of the original problem. We derive the heuristic policy by solving (5) in the following proposition.

Proposition 2 *The optimal solution of the profit function in (5) is given by*

$$z^H(q) = \left\{ \begin{array}{ll} q, & \text{if } q \leq \frac{c}{c+v}S^* \\ \frac{c}{v}(S^* - q), & \text{if } \frac{c}{c+v}S^* < q \leq S^* \\ 0, & \text{if } S^* < q \end{array} \right\}. \tag{6}$$

The heuristic policy has a simpler structure when the borrower starts with a moderate inventory level. Such a policy structure demonstrates the two distinct functions of inventory on the borrower's side: *loan-pledging capacity* and *demand-fulfilling stock*. Below, we further illustrate these functions with respect to the borrower's starting inventory.

Case 1. When the borrower starts with sufficiently low inventory level, i.e., $q \leq \frac{c}{c+v}S^*$, it is optimal to pledge all inventory to maximize her post-replenishment stock level. In this case, on-hand inventory mainly serves as a capacity constraint on the available loan amount to secure future stock. This constraint is binding due to relatively low inventory level compared with expected demand.

Case 2. When the borrower's initial inventory level is in the middle range, i.e., $\frac{c}{c+v}S^* < q \leq S^*$, it is optimal to pledge only part of her inventory for replenishment so that the total available stock reaches the base-stock level S^*. In this case, on-hand inventory provides capacity for loan pledging although the constraint is not binding. This is because the borrower starts with sufficient inventory and thus does not need to pledge everything to reach the base-stock level.

Case 3. When the borrower's initial inventory level is higher than the base-stock level S^*, it is optimal to not pledge any stock. In this case, all her inventory serves to fulfill demand.

As shown in Proposition 1, the optimal policy of the original problem holds a piece-wise linear structure when initial inventory Q is sufficiently small or relatively large. The linear structure in the middle range

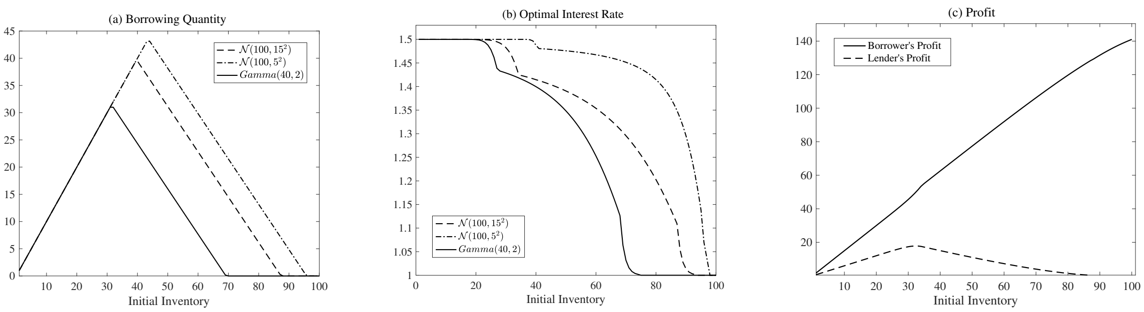

Figure 1: **Equilibrium outcomes in the single-period scenario.** Panel (a) illustrates the optimal borrowing quantity of the borrower under different initial inventory and demand distribution scenarios facing a fixed interest rate $R = 1.2$. Panel (b) illustrates the optimal interest rate set by the lender when observing the borrower's initial inventory under different demand distributions. Panel (c) shows the lender's and borrower's optimal profit when the demand distribution is $\mathcal{N}(100, 15^2)$. For all the panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1$ and $R_f = 1$.

holds only in the heuristic policy. We can theoretically bound the gap between the optimal solution z^O and the heuristic solution z^H as follows.

Proposition 3 *If $p \geq vR$, then the gap between the heuristic solution and the optimal solution of the borrower's problem can be bounded as*

$$z^O - z^H \leq \frac{c}{v}\left[F^{-1}\left(\frac{p^2 - R^2c^2}{p^2}\right) - F^{-1}\left(\frac{p - Rc}{p}\right)\right].$$

We further conduct a numerical study to test the closeness of the heuristic policy to the optimal solution. As demonstrated through Figure B.1 in the Appendix, this bound is tight when our demand distribution is thin-tailed. Intuitively, the difference between $F^{-1}\left(\frac{p^2-R^2c^2}{p^2}\right)$ and $F^{-1}\left(\frac{p-Rc}{p}\right)$ is small since most of the density is concentrated around the value $(p - Rc)/p$.

Meanwhile, observing the borrower's initial inventory q and foreseeing her best response $z^O(q)$, the lender decides on the interest rate R to maximize his profit $\Pi_{\mathscr{L}}$. Nevertheless, the exact form of R is involved, as the optimal pledging quantity $z^O(q)$ does not have an explicit form. We thus present the interval in which the optimal interest rate R^* resides in the following proposition.

Proposition 4 *The optimal interest rate R^* is bounded in the following region:*

$$R^* \in \left[\frac{p}{c}\bar{F}\left(\frac{c+v}{c}q\right), \frac{p}{c}\bar{F}(q)\right].$$

Proposition 4 implies that $R^* \approx p/c$ when the inventory level of the borrower is sufficiently low. For moderate or large q, however, the bound of R^* can be loose and not as informative. Thus, in Figure 1, we present the impact of the borrower's initial inventory on the borrower's and lender's equilibrium outcomes. To start, Panel (a) of Figure 1 serves as a visual representation of Proposition 1. Facing a fixed interest rate, it is optimal for the borrower to pledge everything when the initial inventory level is low. As the initial inventory increases, the borrower pledges less and eventually does not need to resort to IBF to fulfill demand. Panel (b) examines how initial inventory affects the lender's optimal interest rate. When the initial inventory level is low, it is optimal for the lender to set a rate that is close to p/c because when the borrower's initial inventory is extremely low compared to the upcoming demand, any additional unit she obtains can be sold almost surely. Therefore, she is willing to accept an interest rate that is equal to the marginal benefit of acquiring an additional product. The interest rate then decreases in a nonlinear fashion as the initial inventory increases. The reduced interest rate reflects a decreasing marginal benefit the borrower obtains from acquiring an additional unit of product as her initial inventory level increases and stems from the dual roles inventory plays in the IBF scheme, i.e., loan-pledging capacity and demand-fulfilling stock. The dynamics between the initial inventory and interest rate resonate with the *strategic inventory* concept proposed by Martínez-de Albéniz and Simchi-Levi (2013), which states that a buyer may strategically overstock to negotiate a lower price with a supplier. Finally, in Panel (c), we present the profit changes caused by the inventory level. A higher inventory level leads to a higher profit for the borrower, as the borrower will be able to satisfy a growing percentage of the demand and the financing costs associated with acquiring additional inventory decrease due to the lower interest rate. For the lender, however, an insufficient or excessive inventory level translates into reduced profit, as the borrower either has limited inventory to pledge or the borrower is almost self-sufficient. Either way, the lender's profit suffers from a dampened magnitude of loan principle or a reduced interest rate or both.

3.2 Multiple Periods

The IBF scheme employed by JD Finance states that the borrower can resort to IBF as long as 1) she has on-hand inventory to pledge to the lender and 2) the amount of the outstanding loan does not exceed the credit limit assigned by the lender. Notably, the fact that these two prerequisites do not prohibit the borrower from pledging inventory that is purchased on loan could give rise to stockpiling behavior. That is, a borrower with low initial inventory could exploit the IBF scheme to repeat the pledge-purchase cycle multiple times without repaying the outstanding loan in the meantime, until she reaches the desired inventory level. In this section, we study the equilibrium outcomes when the borrower engages in such stockpiling behavior. Later, in Section 4, we empirically demonstrate that such stockpiling behavior is in fact common among borrowers, especially before major holidays in preparation for the surge in demand.

We now extend the single-period model to multiple periods, where the borrower stockpiles by repeatedly pledging her on-hand inventory ordered in the previous period using a loan from the IBF scheme. In this case, the borrower now faces a contract horizon of N periods before a single opportunity to serve a stochastic demand D.[4] The lender offers a fixed interest rate over the contract horizon, within which the borrower

[4] As we demonstrate later in the empirical section, we observe most multiperiod loans before major shopping seasons during which promotions are common. We thus assume that customers would hold until the shopping season starts and that the demand before the major holiday is negligible.

decides on a planning horizon of $T \leq N$ periods to prepare her operations, i.e., inventory level, for the final sale. In particular, the borrower repeatedly pledges part of her inventory to the lender, obtains cash credit in return, and further replenishes inventory with lead time $L = 1$. The detailed sequence of events is as follows.

1. At the beginning of the contract horizon, the lender observes the borrower's initial inventory level q and determines the interest rate r for the entire horizon.

2. In each period $t = 0, 1, 2, ..., T-1$, the following events occur in sequence: (1) the borrower observes the updated on-hand inventory, the total pledged quantity, and the purchasing budget; (2) a pledge quantity decision is made and the corresponding inventory is transferred to the lender in exchange for a new loan amount; (3) an inventory order decision is made subject to the total purchasing budget; (4) the newly ordered inventory is delivered at the end of the period.

3. At time $t = T$, the demand occurs and is satisfied from the borrower's on-hand inventory. The corresponding sales proceeds are used to pay the cumulative interest and reclaim (buy back) the pledged inventory. The reclaimed inventory could be further used to fulfill the demand. The process of reclaiming inventory occurs with zero lead time and can be repeated during the selling period. At the end of period T, unmet demand is lost without penalty and the salvage value for any remaining inventory is normalized to zero. The borrower then pays back the loan principle to the extent possible. The leftover inventory that is not reclaimed is handled by the lender, who could salvage at a discount price via his own retail channel.

To characterize the game dynamics, we define the state and decision variables at the beginning of period t to be:

$$\begin{aligned} q_t &= \text{on-hand inventory level in Event (1);} \\ z_t &= \text{pledged quantity in Event (2);} \\ w_t &= \text{purchasing budget level in Event (2);} \end{aligned}$$

Given interest rate r, the borrower chooses the planning horizon $T \leq N$ and subsequently solves a joint inventory pledging and replenishment problem to maximize her expected profit $\Pi_{\mathscr{B}}$. Specifically, the borrower decides on the pledge quantity z_t and the ordering amount u_t for each $t = 0, 1, 2, ..., T-1$. Without loss of generality, we assume that at $t = 0$, the borrower starts with initial states $(q_0, x_0, w_0) = (q, 0, 0)$.

To begin, we first consider the problem where the planning horizon T is given. Specifically, the borrower faces a dynamic program defined as follows.

$$B_t^{(T)}(q_t, x_t, w_t) = \max_{z_t \leq q_t,\ u_t \leq (vz_t + w_t)/c} B_{t+1}^{(T)}(q_{t+1}, x_{t+1}, w_{t+1}) \tag{7}$$

$$B_T^{(T)}(q_T, x_T, w_T) = \mathbb{E}\left[p \min(D, x_T + q_T) + w_T - vx_T - rv\sum_{t=1}^{T} x_t\right]^+, \tag{8}$$

where the state dynamics are the following.

$$\begin{aligned} q_{t+1} &= q_t + u_t - z_t, \\ x_{t+1} &= x_t + z_t, \\ w_{t+1} &= w_t + v z_t - c u_t. \end{aligned}$$

In Equation (7), each time the borrower makes a pledging and purchasing decision, the pledged quantity may not exceed her current on-hand inventory level and the payment must satisfy the budget constraint. In Equation (8), the final objective function consists of the profit earned by having $x_T + q_T$ available stock and the total cost of the loan, which is defined by the capital vx_T and the interest $rv\sum_{t=1}^{T} x_t$. The borrower then must decide on the optimal planning horizon T^*, in which case the profit $\Pi_{\mathscr{B}}$ can be written as

$$\Pi_{\mathscr{B}} = B_0^{(T^*)}(q, 0, 0) = \max_T B_0^{(T)}(q, 0, 0). \tag{9}$$

Despite the simple structure, computing the objective order quantity Q directly from formulation (9) is challenging. Therefore, we first study the problem where, given a fixed objective inventory level $Q = x_T + q_T$, the borrower decides the optimal planning horizon T^* and the corresponding x_t, z_t that maximize the total profit while achieving Q. In particular, for a fixed inventory level $Q = x_T + q_T$, the borrower's total profit can be written as

$$\begin{aligned} \Pi_{\mathscr{B}}(Q) &= \max_T B_0^{(T)}(q, 0, 0) \\ &= \mathbb{E}\left[p \min(D, Q) + w_T - c(Q - q) - rvA(Q)\right]^+ . \end{aligned}$$

where $A(Q) = \sum_{t=1}^{T^*} x_t$ is the cumulative inventory throughout the planning horizon, which given a fixed interest rate, is proportional to the total interest paid by the borrower. For a given inventory target Q, we can characterize the small retailer's optimal borrowing behavior as follows.

Proposition 5 *Given total inventory Q, the optimal planning horizon is*

$$T^* = \left\lfloor \frac{\log\left(\frac{Q}{q}(\theta - 1) + 1\right)}{\log \theta} \right\rfloor, \tag{10}$$

where $\theta = v/c$. Moreover, the optimal joint pledging and ordering decisions can be characterized as follows

$$
\begin{aligned}
z_0^* &= \frac{1}{\theta^T}\left[Q - \frac{\theta^T - 1}{\theta - 1} q\right], \\
z_1^* &= q_1 = q + (\theta - 1) z_0^*, \\
z_2^* &= q_2 = \theta q_1, \\
&\ldots \\
z_{T^*-1}^* &= q_{T^*-1} = \theta^{T^*-1} q_1, \\
u_t^* &= \theta z_t^*. \ \forall t
\end{aligned}
$$

Proposition (5) characterizes the optimal pledging quantity when the borrower aims to reach a fixed inventory goal Q. The optimal pledging strategy follows a simple structure: at time period $t \geq 1$, the borrower should pledge all on-hand inventory; at $t = 0$, the borrower should pledge z_0^* such that the borrower reaches the objective inventory level at time T^*, i.e., $x_T + q_T = Q$, after pledging all on-hand inventory in every period from $t = 1$ to $t = T^* - 1$. In this way, the total interest of the loan is minimized. Equipped with Proposition 5, we can now derive the optimal inventory level Q^O, which we formally state in the following theorem.

Theorem 1 *Suppose the distribution of demand D satisfies Assumption 1. Then, there exists a unique $Q^O \geq q$ that maximizes $\Pi_{\mathscr{B}}(Q)$ for a given interest rate r. Denote $S^* = F^{-1}(1 - \frac{Rc}{p})$; then, we have*

$$
Q^O(q) = \left\{ \begin{array}{ll} Q^*, & \textit{if } q \leq S^* \\ 0, & \textit{if } q > S^* \end{array} \right\},
$$

where Q^* is the unique solution to the equation

$$
\overline{F}(Q) - \overline{F}(\alpha_T Q - \beta_T)\alpha_T = 0,
$$

and

$$
\alpha_T = \frac{c}{p} + \frac{rv(\theta^T - 1)}{p\theta^T(\theta - 1)}, \qquad \beta_T = \frac{rvq}{p}\left(\frac{T}{\theta - 1} - \frac{\theta^T - 1}{\theta^T(\theta - 1)^2}\right) + \frac{cq}{p}
$$

and T is defined in (10) for any Q.

Theorem 1 indicates that Q^* is critically determined by T^* because a longer optimal planning horizon can lead to inflated financing cost and reduce the order quantity. Nevertheless, T^* is affected by the optimal quantity Q^* at the same time according to Equation (10). To obtain a pair of Q^* and T^* that are consistent with each other, we utilize the uniqueness of the optimal quantity and provide an iterative approach in Algorithm 1 to jointly solve for Q^* and T^*.

For the lender, a total $M(Q) = rvA(Q) + c(Q - q)$ amount of loan is due in the final period, which consists of the cumulative interest $rvA(Q)$ and the original capital $c(Q - q)$. When the repayment received

---

Algorithm 1: Derivation of the Optimal Inventory Level

---

input : q, p, c, v, r

intialize $T = 0$, $\tilde{T}^* = 1$

while $T \neq \tilde{T}^*$ **do**

  $T = T + 1$

  Solve FOC

$$\overline{F}(Q) - \overline{F}(\alpha_T Q - \beta_T)\alpha_T = 0$$

  to obtain the best response price Q_T^*

  Solve

$$\tilde{T}^* = \left\lfloor \log\left(\frac{Q_T^*}{q}(\theta - 1) + 1\right) / \log\theta \right\rfloor$$

end while

output: $T^* = \tilde{T}^*$, $Q^* = Q_T^*$

---

by the lender satisfies $pD < M(Q)$, the lender returns $pDx_{T^*}/M(Q)$ products back to the borrower and salvages the rest of the inventory, obtaining $s(x_{T^*} - pDx_{T^*}/M(Q))$ in revenue. As a result, the lender's objective function can be written as

$$\Pi_{\mathscr{L}} = \max_R \mathbb{E}\left[\min(pD, M(Q)) + sx_{T^*}\left(1 - \frac{pD}{M(Q)}\right)^+\right] - vr_f A(Q) - c(Q - q). \tag{11}$$

Similar to the single-period scenario, the lender sets the interest rate R to maximize profit. As the optimal interest rate and the optimal pledging quantity do not have explicit expressions, we resort to Figure 2 to demonstrate the lender's and borrower's equilibrium behaviors. Specifically, Panel (a) of Figure 2 shows the optimal pledging behavior according to Proposition 5. Depending on the initial inventory q, Panel (a) shows that the borrower may choose to start pledging at four and six periods ahead of the demand realization stage. Additionally, after the initial adjustment period, the borrower pledges an increasing quantity in the following periods at a rate of $v/c = 120\%$. Panel (b) demonstrates the relationship between the optimal planning horizon and the objective inventory Q^O. The borrower can postpone the pledging decision as q increases, which translates into reduced financing costs and further leads to a higher objective inventory Q^O. Panel (c) presents the optimal interest rate and the optimal cumulative quantity $A(Q^O)$ under different q values. Due to the discrete nature of the planning period, α_T and β_T change when the optimal planning horizon changes from T to $T + 1$. As a result, both the optimal interest rate and the corresponding order quantity exhibit several kinks as q increases. Notably, different from the single-period case, in which the borrower's cumulative pledge quantity is bounded by the initial inventory q, in Panel (c), when q is small, the borrower is able to repeatedly pledge their on-hand inventory over multiple periods to eventually stockpile considerably more product than she initially possesses due to the flexibility offered by the IBF scheme. Additionally, the interest rate jumps every time the planning horizon is reduced by one period, and the most significant increase occurs when the borrower has sufficient inventory to switch from repeated pledging to

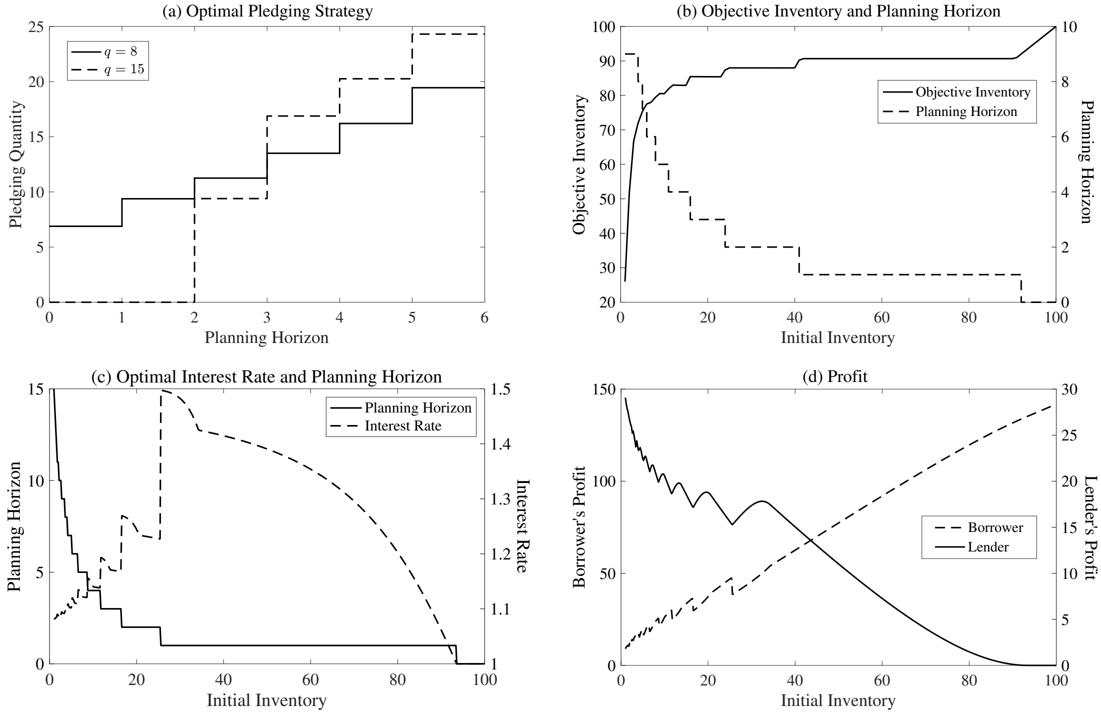

Figure 2: **Equilibrium outcomes in the multiperiod scenario.** Panel (a) demonstrates the optimal borrowing quantity z_t in each time period of the borrower. The objective inventory is set to be 100, and the initial inventory is chosen as 8 and 15, respectively. Panel (b) illustrates the optimal objective inventory Q^O and the borrowing period T for a borrower facing a fixed interest rate $R = 1.1$. Panel (c) shows the optimal interest rate set by the lender and the corresponding planning horizon. Panel (d) illustrates the borrower's and lender's profits. For all panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1$ and $R_f = 1$.

one-shot pledging. That is, when the initial inventory level is sufficiently high, the optimal interest rate becomes identical to that in the case of the single-period model. Finally, Panel (d) illustrates the profit of the borrower and the lender under different q values. Other than the dips around the kinks, which are caused by the jump in the planning horizon, the borrower enjoys a higher profit overall when q increases as the total financing costs decrease, which in turn dampens the lender's profit.

3.3 Supply Chain Efficiency under IBF

In this section, we study the welfare implications of IBF on each stakeholder and the whole supply chain. We are particularly interested in the efficiency gain from IBF and whether the large retailer has financial incentives to allow the borrower to stockpile. To this end, we compare the surplus of each party under the following scenarios: 1) the small retailer has no access to external financing at all; 2) the small retailer has access to IBF but is prohibited from pledging inventory purchased on loan; 3) the small retailer has access to IBF and is allowed to stockpile; and 4) the small retailer has free and unlimited access to loans. Denoting these four cases as *Base Case*, *Single IBF*, *Multi IBF* and *First Best*, respectively, we present the surplus comparisons under these scenarios in Figure 3.

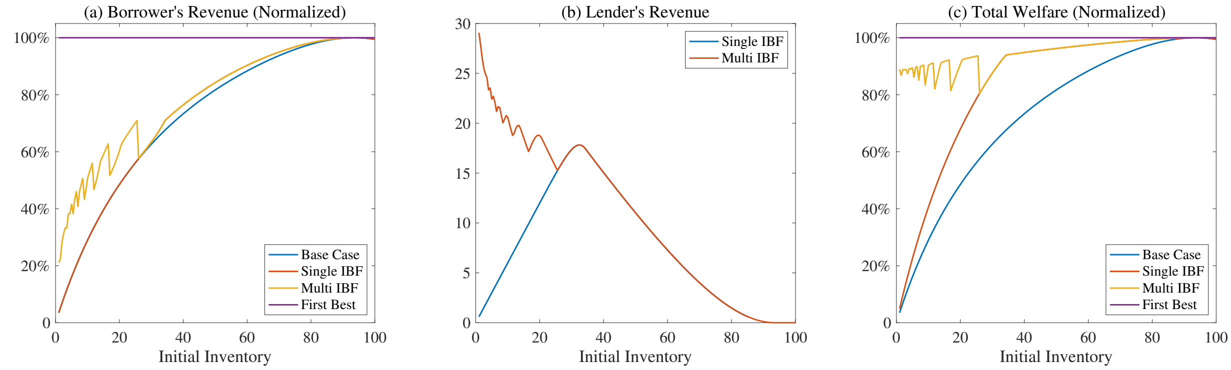

Figure 3: **Surplus comparisons for financing through IBF.** Panel (a) shows the normalized borrower surplus, panel (b) shows the lender profits, and panel (c) illustrates the normalized supply chain welfare. For all the panels, the parameter values are $c = 1, v = 1.2, p = 1.5, s = 1.1$, $R_f = 1$ and $D \sim \mathcal{N}(100, 15^2)$.

In Panel (a) of Figure 3, we observe that the borrower enjoys the highest profit when stockpiling through IBF. Nevertheless, when the borrower's initial inventory level is low, the profit difference between the *Single IBF* and *Multi IBF* scenarios is not large. This can be explained as follows. According to Panel (b) in Figure 2, a low initial inventory implies a long planning horizon. In such case, the incurred interest expense over the planning horizon becomes substantial, making the borrower set a lower objective inventory level. Meanwhile, in the absence of stockpiling, the value of using IBF at low inventory level is minimal. This is because, knowing the small retailer would likely pledge all her inventory, the lender sets the interest rate sufficiently high to make the borrower indifferent between *Single IBF* and the *Base Case*. This is why, according to Panel (b), the lender is able to extract the most surplus when the borrower has low initial inventory. Notably, Panel (b) indicates that the lender generates significantly higher profit when the borrower stockpiles, suggesting that despite charging a lower interest rate, the lender enjoys a profit gain from a significantly increased order quantity compared to the *Single IBF* case. Importantly, this setting provides a justification for the current contract terms that allow the borrower to pledge products that are purchased on loan. Finally, Panel (c) illustrates that the current IBF scheme can substantially improve the supply chain efficiency, achieving more than 90% of the total welfare compared to the first best scenario.

3.4 Discussion of Model Assumptions

In this section, we discuss the key assumptions in modeling the inventory-based financing scheme.

Demand realized at the last period. In our model, we assume stochastic demand occurs at the last period T, but not in $t = 1, 2, ..., T-1$. This reflects the fact that stockpiling behavior tends to spike ahead of major shopping holidays, which we empirically illustrate in Section 4.2 using data from JD Finance. Given the huge spike in sales during shopping holidays (Hsu 2017) and that customers may strategically delay their purchase to wait for holiday deals (Baucells et al. 2017), the demand incurred during the planning horizon

Table 1: Descriptive Statistics

	Mean	Std.Dev	Min	Max
Loan Amount (in Million Yuan)	0.55	0.89	1.50×10^{-3}	9.06
Credit Limit of Retailer (in Million Yuan)	16.17	21.12	1.00	67.00
Interest Rate ($\times10^{-2}$)	14.34	1.82	7.00	17.00
Product Valuation (in Yuan)	466.80	704.77	8.58	4620.39

can be negligible, especially when the lead time and the resulting planning horizon are short. Alternatively, our assumption also naturally holds when the product has a low contribution margin or when the firm does not reinvest the revenue generated during the planning horizon to stockpile.

Single SKU. In our analysis, we assume the retailer pledges and sells only a single type of product. However, retailers in reality may hold and pledge multiple products through IBF. In such cases, the large retailer can view these products as a 'representative product' or a bundle, whose total sales is the convolution of the sales from each individual product in the bundle. Thus, our model also applies to such scenarios.

4 Empirical Analysis

In this section, we search for empirical evidence to verify the theoretical results derived from Section 3. The datasets we use to test our theory are obtained from JD Finance, which has been implementing the IBF scheme with retailers since June 2016. In what follows, we first provide an overview of our datasets. Then, we leverage the loan data to investigate the existence and magnitude of stockpiling behavior. Finally, we study how the interest rate set by JD Finance affects small retailers' borrowing patterns.

4.1 Data Description

We use two datasets for our empirical analysis. The first dataset contains detailed information on all 934 loans JD Finance issued from June 20th, 2016 to April 10th, 2018. Specifically, we observe for each loan the credit limit of the borrower, the loan principle, the interest rate, the date of issue, and the corresponding due date. In addition, the dataset documents the actual payment date and the corresponding payment amount. Furthermore, we see whether a loan defaults, and in case of default, the default amount and the liquidation value. The second dataset contains product-level information. We observe the product type[5] and the specific SKU pledged in each loan. For each SKU, we also observe the quantity pledged and the product valuation. Summary statistics are presented in Table 1.

Our datasets are drawn from the loan contracts, which include detailed information on loan-related terms such as dates of loan origination and repayment, loan amount, interest rate, and the quantity and valuation of

[5]Each product belongs to one of the seven product categories: Consumer Electronics, Sports and Outdoors, Clothing, Computer Accessories, Sneakers, Cell Phone, Kitchen Accessories.

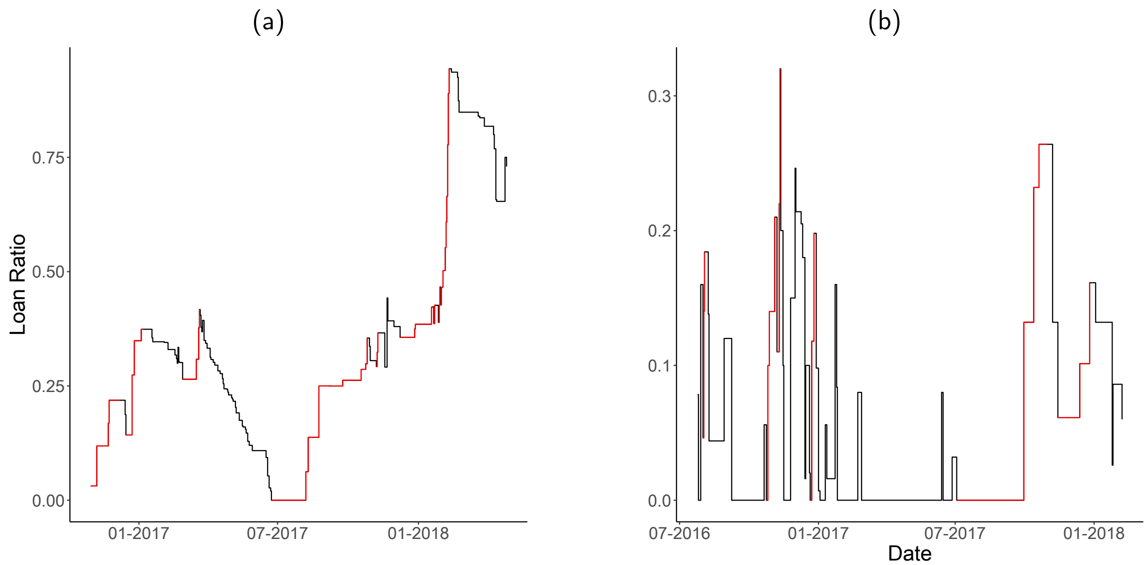

Figure 4: **Trajectory of outstanding loans.** These plots trace the normalized loan ratio (outstanding loan amount divided by the credit limit) of two independent retailers. The identified stockpiling behaviors are colored in red.

the pledged products. Nevertheless, variables such as the production cost, the price and the demand forecast of the product are exclusively known to the borrower and are not observed in the data. According to our interview with JD Finance's manager, such information is either imprecise, as the product is sold through multiple channels, or is shielded from the lender, despite the dominant position and the strong bargaining power the lender has. As a result, our empirical study focuses primarily on loan borrowing patterns and the impact of interest rate on borrowing behaviors, rather than verifying the model predictions on the optimal interest rates and the exact borrowing quantity, which hinges upon such unobserved variables.

4.2 Model-Free Evidence for Stockpiling Behavior

In this section, we present evidence that borrowers indeed stockpile under the IBF scheme. By definition, to stockpile is to order consecutively in preparation for upcoming high demand. We thus search for stockpiling behavior by identifying borrowing patterns that satisfy the following definition.

Definition 1 (Stockpiling) *A user of the IBF scheme is considered to be stockpiling if she borrows multiple loans consecutively without repayment to the lender in the meantime.*

Proposition 5 demonstrates that when faced with high future demand, the borrower has to repeat the pledge-purchase cycle multiple times to reach the objective inventory level. Thus, Definition 1 aims to distinguish the consecutive borrowing behavior of stockpiling from the one-shot borrowing pattern, i.e., borrowing followed by immediate repayment. As we observe the time stamps for the borrowing and repayment of each loan in our datasets under JD Finance's IBF scheme, we can map the evolution of the normalized outstanding loan for each borrower. We present two outstanding loan trajectories from two IBF users in Figure 4, in which loans related to stockpiling are colored in red. We see that stockpiling behavior is prevalent, and borrowers may stockpile starting with the first time they apply for a loan through IBF. Specifically, the small retailer displayed in panel (a) of Figure 4 exclusively borrows to stockpile, while the one in panel (b) uses a mixture of single-period and multi-period loans. Combining Definition 1 with the loan trajectories, we have identified 87 stockpiling incidences, which correspond to 25.4% of the total number of loans and 26.5% of the total loan amount. These small retailers borrow 2.7 times on average when they stockpile, and some stockpiling incidences can involve as many as nine consecutive loans.

Additionally, we study when borrowers resort to IBF and engage in stockpiling behavior. To this end, we map the loan origination dates on the timeline in Figure 5.[6] Panel (a) presents a heat map of all the loan origination dates under the IBF scheme (which are identical to the dates when borrowers actually receive the loan). We observe that the usage of IBF peaks in October and January, which are the months proceeding Singles Day, the largest online shopping event in China, which occurs on November 11th, and Chinese New Year, which is typically in February. Loans applied during these two months account for 51.4% of the total loan incidences and 47.7% of the total loan amount. The fact that retailers expect a flood of orders during these two shopping seasons explains the aggressive usage of IBF in these two months: to have the inventory properly built up by the holidays given the lead time, small retailers have to pledge their inventory to place new orders one month ahead so that the products will arrive on time. Meanwhile, we plot exclusively the initiations of stockpiling in Panel (b). Interestingly, there is a noticeable shift in the loan borrowing date when we compare Panel (b) to Panel (a). When small retailers engage in stockpiling instead of one-shot borrowing, 57.5% start pledging two to three months ahead of these holidays, from as early as August and December. This result is consistent with Proposition 5; that is, when the upcoming demand and the resulting objective inventory level are much larger than the current on-hand inventory, the borrowers would start pledging T^* periods before the demand is realized. The spike in loan applications before major holidays calls for the lender to guarantee liquidity to avoid a cash crunch and also to set an appropriate interest rate to reduce risk and maximize profit.

Meanwhile, given the prevalence of stockpiling behavior, it is crucial for the lender to recognize the borrower's inventory level and discern if a loan application is part of a stockpiling plan in order to set a proper interest rate. According to Panel (c) of Figure 2, the optimal interest rate is nonlinear in the borrower's initial inventory level. Assuming that all loans are one-time would make the lender set an interest rate that results in the small retailer setting a suboptimal objective inventory level, which jeopardizes the lender's profit, as well as supply chain efficiency. In reality, it can be difficult for the lender to determine if a loan is associated with stockpiling. Our results suggest that when accurate information about borrower's

[6]Note that as our data cover the usage of IBF from June 2016 to April 2018, we lack April and June data to make these two months comparable to the rest of the months. Nevertheless, the relative magnitude of the borrowing frequency remains valid for the rest of the ten months.

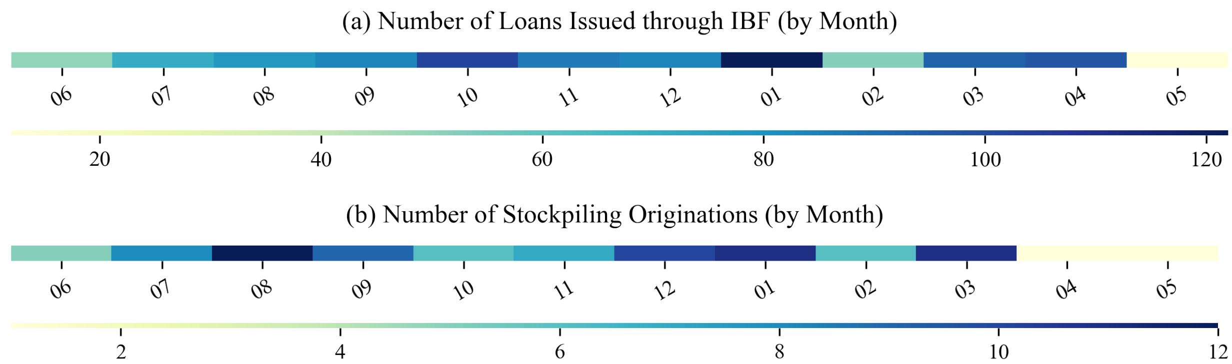

Figure 5: **Temporal distribution of borrowing and stockpiling behavior.** Panel (a) illustrates the number of loans issued across 12 months. Panel (b) summarizes the number of stockpiling originations within 12 months.

demand is not available, the month of the year can serve as a good predictor of stockpiling behavior.

4.3 Implications of Interest Rate on Borrowing Behaviors

In Section 4.2, we demonstrate that small retailers exhibit different borrowing patterns (i.e., one-shot loan or stockpiling) under IBF. In this section, we further investigate how such borrowing patterns are affected by the interest rate set by the lender. To facilitate hypothesis development, we resort to our theoretical results from Propositions 1 and 5 to numerically demonstrate the optimal borrowing behavior under various interest rates. On the basis of Figure 6, we summarize the impact of interest rates on borrowing patterns in the following two hypotheses:

Hypothesis 1 *For loans associated with stockpiling activity, an increased interest rate leads to a shortened planning horizon.*

Hypothesis 2 *For one-time loans, a decreased interest rate leads to a lower borrowing amount.*

Hypothesis 1 states that the planning horizon shortens as the optimal interest rate increases, as manifested in Figure 6. The most common strategy to test Hypothesis 1 is to use the following regression:

$$T_{it} = \beta_0 + \beta_1 r_{it} + \gamma q_{it} + \epsilon_{it}\,, \tag{12}$$

where T_{it} is the planning horizon for stockpile loans that start during month t and r_{it} is the interest rate of loan i borrowed in month t. In addition, according to Proposition 5 and Equation (11), the initial inventory affects both r_{it} and T_{it} (we present causal directed acyclic graph in Figure C.1). Thus, we also include q_{it}, borrower i's inventory level when she applies for the loan in month t, into the regression. Nevertheless, there are two issues in regression (12) that can lead to biased β_1, the effect from interest rate that we intend to

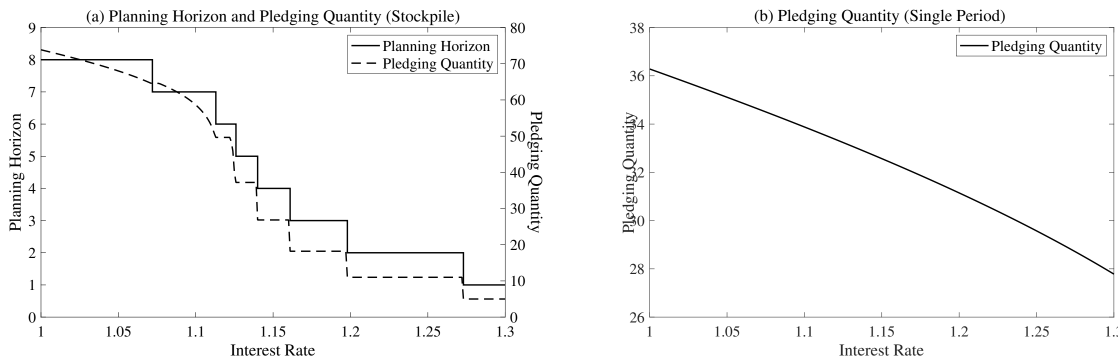

Figure 6: **The effect of interest rate on borrowing patterns.** Panel (a) illustrates the optimal planning horizon and pledging quantity in response to changing interest rates for multiperiod loans. Panel (b) demonstrates the optimal loan amount under different interest rates for single-period loans.

uncover. The first issue is unobserved confounding; that is, we do not directly observe initial inventory q_{it} in our dataset. In fact, acquiring precise information about the borrower's inventory level and dynamically updating it overtime can be a common challenge faced by the lender in a retailer-initiated IBF scheme, as the borrowers may not be willing to share their inventory data as they do in buyer-supplier relationships. To address this issue, we approximate q_{it} by $g_1(CONTROL_{it})$, where $g_1(\cdot)$ is a nonparametric function to be estimated and $CONTROL_{it}$ is a vector of control variables, including the industry fixed effects, borrower location fixed effects, monthly fixed effects, product valuation and the logarithm of the borrower's credit limit. According to Rumyantsev and Netessine (2007), a firm's inventory level depends critically on the location, specific industry, month of the year and product valuation. In addition, the inventory decision is affected by the company size, which is reflected by the credit limit (Fewings 1992, Rumyantsev and Netessine 2007). This way, $g_1(CONTROL_{it})$ captures the product-specific features, the firm-specific features and the temporal variations in q_{it}. While controlling for $g_1(CONTROL_{it})$ may not be sufficient to completely eliminate the bias caused by unobserved q_{it}, it helps to attenuate the magnitude of the bias.

The second issue is that, according to Proposition 5, q_{it} affects T_{ij} and r_{ij} in a highly nonlinear fashion. Take q_{it} and T_{ij} as an example. If we denote the relationship between q_{it} and T_{ij} as $T_{ij} = g_2(q_{it})$, then compounded with the unknown approximation function $g_1(\cdot)$, we have $T_{ij} = g_2(g_1(CONTROL_{it}))$, which results in an even more complex dependency between $CONTROL_{it}$ and T_{ij}. Assuming a linear relationship will thus accentuate the bias in β_1. To address the unknown dependency between $CONTROL_{it}$ and T_{it} (and also r_{it}) and obtain unbiased estimates for the effect of r_{it}, we take advantage of a recent development in the econometric literature and apply the double machine learning (DML) technique proposed by Chernozhukov et al. (2018). DML allows for a nonparametric characterization of the impact of $CONTROL_{it}$ while reducing overfitting of the nonparametric model and minimizing the regularization bias of the coefficient of r_{it}.

As an overview, the estimation process of DML involves the following steps. We first rewrite Equation (12) as a partially linear model with the impact of $CONTROL_{it}$ modeled through a nonparametric function.

Then, we construct a 5-fold random partition of the entire sample. The sample splitting reduces the over-fitting bias from the fitting stage. Given a partition k, we predict $T_{it,-k}$ from $CONTROL_{it,-k}$ and $r_{it,-k}$ from $CONTROL_{it,-k}$, where the subscript $_{-k}$ indicates that the fitting is performed over the four partitions other than partition k. Notably, both predictive tasks can be accomplished by arbitrary machine learning algorithms such as boosting (Freund et al. 1996) and random forest (Ho 1995). Based upon machine learning estimators, we derive the residual terms $\tilde{T}_{it,k}$ and $\tilde{r}_{it,k}$ in partition k and conduct linear regression in partition k to estimate the average treatment effect $\hat{\theta}_{0k}$. Lastly, to minimize the regularization bias, the final estimator is constructed as the average estimated effect across all partitions $\hat{\theta}_0 = \sum_{k=1}^{5} \hat{\theta}_{0k}$

Specifically, to capture the unknown functional relationships between $CONTROL_{it}$ and $\tilde{T}_{it}$ and between $CONTROL_{it}$ and $\tilde{r}_{it}$, we rewrite Equation (12) as a partially linear regression model as:

$$T_{it} = \theta_0 r_{it} + g_0(CONTROL_{it}) + U_{it}, \tag{13}$$

$$r_{it} = m_0(CONTROL_{it}) + V_{it}. \tag{14}$$

Here, $g_0(\cdot)$ and $m_0(\cdot)$ are nonparametric functions that capture the impact of $CONTROL_{it}$ on T_{it} and r_{it}, respectively.[7] The error terms U_{it} and V_{it} satisfy the conditional independence assumption $E[U_{it}|r_{it}, CONTROL_{it}] = 0$ and $E[V_{it}|CONTROL_{it}] = 0$. Equation (13) is our main estimation equation, in which θ_0 is the impact of the interest rate on the borrowing period that we would like to infer when r_{it} is exogenous conditional on the control variables. Equation (14) keeps track of confoundedness, namely, the dependence of interest rate r_{it} on the control variables. Based on the conditional independence assumption of the error terms, we rearrange Equation (13) as

$$T_{it} - E[T_{it}|CONTROL_{it}] = \theta_0(r_{it} - E[r_{it}|CONTROL_{it}]) + U_{it}. \tag{15}$$

To fit the two conditional expectations in Equation 16, we use sample-splitting and cross-fitting techniques to mitigate the overfitting bias and the regularization bias, as suggested in Chernozhukov et al. (2018). To this end, we randomly partition our data into five equally sized subsamples. For each partition k, we apply nonparametric machine learning methods to the remaining partitions to estimate the conditional expectation $q_0(CONTROL_{it,-k}) = E[T_{it,-k}|CONTROL_{it,-k}]$ and $m_0(CONTROL_{it,-k}) = E[r_{it,-k}|CONTROL_{it,-k}]$ as $\hat{q}_0(CONTROL_{it,-k})$ and $\hat{m}_0(CONTROL_{it,-k})$. In our analysis, we conduct the first-stage estimation via two tree-based machine learning algorithms, namely, random forest and XGBoost (Chen and Guestrin 2016), to assess the robustness of the estimation results.

For cross-fitting, we apply the trained machine learning estimators from the previous step, $\hat{q}_0(\cdot)$ and $\hat{m}_0(\cdot)$, to partition k. Denoting $\tilde{T}_{it,k} = T_{it,k} - \hat{q}_0(CONTROL_{it,k})$ and $\tilde{r}_{it} = r_{it} - \hat{m}_0(CONTROL_{it,k})$,

[7] Here, $g_0(\cdot) = g_2(g_1(\cdot))$

Table 2: Impact of Interest Rate on Stockpiling

	(1)	(2)	(3)	(4)
	Period (Random Forest)	Period (XGBoost)	Log Loan (Random Forest)	Log Loan (XGBoost)
Coeff. of the Interest Rate	-19.34^{*}	-18.41^{**}	-14.43^{***}	-16.69^{***}
	(9.72)	(6.78)	(3.75)	(3.76)
Number of Observations	$68^{\dagger}$	68	339	339

† There are 87 stockpiling incidences in our data, yet 19 of the 87 incidences have missing values in credit limit or/and product valuation and are excluded from the estimation.

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Equation (16) becomes

$$\tilde{T}_{it,k} = \theta_{0k}\tilde{r}_{it,k} + U_{it,k} \tag{16}$$

in which the desired coefficient θ_{0k} can be obtained via standard linear regression. By repeating the fitting of the conditional expectations and the regression estimation across all partitions, we obtain the average effect as $\hat{\theta}_0 = \sum_{k=1}^{5} \hat{\theta}_{0k}$. We report the estimated $\hat{\theta}_0$ in Table 2, with the corresponding standard errors in parentheses. Specifically, Columns (1)-(2) report the estimation results when we use different machine learning methods to predict T_{it} and r_{it} from the control variables. A higher interest rate significantly reduces the borrowing period across both specifications, with XGBoost yielding a more significant results.[8] Specifically, a 1% increase in the interest rate would shorten the planning horizon by approximately 18% of the lead time. Thus, Hypothesis 1 is supported.

Hypothesis 2 suggests that small retailers borrow less when facing a lower interest rate. This is manifested in Panels (b) in Figure 6. To formally test (H2), we again resort to the same DML strategy, but we use the logarithm of the loan amount as the dependent variable in the partially linear regression, which is expressed as

$$\begin{aligned} LOG\_LOAN_{it} &= \theta_0 + \theta_1 r_{it} + g_0(CONTROL_{it}) + U_{it}, \\ r_{it} &= m_0(CONTROL_{it}) + V_{it}. \end{aligned} \tag{17}$$

In Columns (3)-(4) of Table 2, we observe uniformly large and negative point estimates, which suggest that lower interest rates lead to higher borrowing amounts. Specifically, a 1% increase in the interest rate leads to approximately 15% decrease in the loan amount. Thus, Hypothesis 2 is also supported.

5 Discussion, Limitations and Concluding Remarks

In this paper, we study a novel, retailer-initiated inventory-based financing (IBF) scheme. We first construct a game-theoretical model, in which a small retailer (i.e., the borrower) has a single chance to pledge her

[8] This finding is consistent with the previous literature that reports XGBoost rendering the best outcomes in fitting and prediction tasks (Fu et al. 2021).

inventory in exchange for capital. The small retailer then uses the loan to purchase additional products to meet future demand. We characterize the small retailer's optimal pledging quantity and the interval for the optimal interest rate. We then extend our single-period model to a multiperiod framework, in which the small retailer stockpiles by repeatedly pledging her on-hand inventory ordered in previous periods using loans from the IBF scheme. In this case, we first characterize the borrower's profit-maximizing decisions, including the objective inventory, the optimal planning horizon and the amount of inventory to be pledged in each period. We then explore the optimal interest rate the large retailer should set through numerical analysis. We demonstrate that the innovative IBF scheme to ease retailers' budget constraints can improve supply chain efficiency and the profit of each party within the supply chain.

Complementing our theoretical analysis, we study small retailers' borrowing behavior leveraging a loan dataset from JD Finance. We first verify that the stockpiling behavior under the IBF scheme predicted by our model and Fu et al. (2021) not only exists but is actually prevalent, accounting for 26.5% of the total loan amounts borrowed through IBF during the years 2016 to 2018. We also demonstrate that stockpiling activities spike one to two months ahead of major shopping seasons and holidays. Last, we use double machine learning to address the nonlinear effect from the small retailer's initial inventory, an unobserved confounder in our analysis. Consistent with our theoretical predictions, when facing a higher interest rate, the small retailer borrows less for one-shot loans and has a shorter planning horizon for multiperiod loans. By understanding how and when small retailers use IBF, our empirical results provide guidance for the lender's interest rate decision-making process and liquidity management plan.

Interestingly, under the IBF scheme, the loan amount a small retailer is initially able to obtain is mostly determined by her on-hand inventory but not the credit limit set by the large retailer, which is usually rarely reached. As a result, the small retailer may need to go through several pledge-purchase cycles to achieve the desired inventory level. This inconvenience arises because the two parties in the IBF scheme are two retailers that are not in a vertical collaboration within a supply chain, making well-studied financing schemes, such as reverse factoring, not applicable. Therefore, the small retailer must rely on collateral to secure the loan, as he does not have full information about the trustworthiness of the small retailer.

Retailer-initiated IBF as a supply chain financing solution has emerged only in recent years. As a result, few studies have investigated the contract design of the retailer-initiated IBF scheme in the field of operations management. As one of the first works on this topic, our study is subject to several limitations. Practically, it is typically challenging for large retailers to acquire precise information about the borrower's inventory position and sales prospects. The lack of end-to-end data could lead to suboptimal interest rate decisions that deviate from the model prediction. With better information and data sharing between the borrower and lender, future research can check whether the lender's interest rate and small retailers' pledging decisions are consistent with the theoretical predictions and, if not, identify the potential causes of the deviations. Our work can also be extended by studying how stockpiling activities, in conjuncture with market volatility, inject uncertainty into the repayment date, which imposes additional challenges for the lender to salvage the product and may alter the salvage value of the product. Finally, when other financing alternatives, for example, commercial loans and trade credit, are made available to the small retailer, it would be interesting to compare which financing scheme is preferred under various market conditions.

References

Alan, Y. and V. Gaur (2018). Operational investment and capital structure under asset-based lending. *Manufacturing & Service Operations Management*.

Baucells, M., N. Osadchiy, and A. Ovchinnikov (2017). Behavioral anomalies in consumer wait-or-buy decisions and their implications for markdown management. *Operations Research 65*(2), 357–378.

Buzacott, J. A. and R. Q. Zhang (2004). Inventory management with asset-based financing. *Management Science 50*(9), 1274–1292.

Caldentey, R. and X. Chen (2009). The role of financial services in procurement contracts. *The Handbook of Integrated Risk Management in Global Supply Chains*, 289–326.

Chen, T. and C. Guestrin (2016). Xgboost: A scalable tree boosting system. In *Proceedings of the 22nd acm sigkdd international conference on knowledge discovery and data mining*, pp. 785–794.

Chernozhukov, V., D. Chetverikov, M. Demirer, E. Duflo, C. Hansen, W. Newey, and J. Robins (2018). Double/debiased machine learning for treatment and structural parameters.

Devalkar, S. K. and H. Krishnan (2019). The impact of working capital financing costs on the efficiency of trade credit. *Production and Operations Management 28*(4), 878–889.

Fewings, D. R. (1992). A credit limit decision model for inventory floor planning and other extended trade credit arrangements. *Decision Sciences 23*(1), 200–220.

Foley, C. F., A. Raman, and N. C. Craig (2012). Inventory-based lending industry note. *Harvard Business School Case* (612-057).

Freund, Y., R. E. Schapire, et al. (1996). Experiments with a new boosting algorithm. In *icml*, Volume 96, pp. 148–156. Citeseer.

Fu, K., X. Gong, V. N. Hsu, and J. Xue (2021). Dynamic inventory management with inventory-based financing. *Production and Operations Management*.

Fu, R., M. Aseri, P. Singh, and K. Srinivasan (2021). "un" fair machine learning algorithms. *Management Science*.

Ho, T. K. (1995). Random decision forests. In *Proceedings of 3rd international conference on document analysis and recognition*, Volume 1, pp. 278–282. IEEE.

Hsu, T. (2017). Alibaba's singles day sales hit new record of $25.3 billion. *New York Times 10*.

Hsu, V. and J. Wu (2019). The financing role of inventory: Evidence from china's metal industries. *Working Paper, The Chinese University of Hong Kong*.

Huang, R., G. Lai, X. Wang, and W. Xiao (2022). Lending to third-party sellers with platform loan. *Available at SSRN*.

Iancu, D. A., N. Trichakis, and G. Tsoukalas (2017). Is operating flexibility harmful under debt? *Management Science 63*(6), 1730–1761.

Jing, B., X. Chen, and G. G. Cai (2012). Equilibrium financing in a distribution channel with capital constraint. *Production and Operations Management 21*(6), 1090–1101.

Kouvelis, P. and F. Xu (2021). A supply chain theory of factoring and reverse factoring. *Management Science*.

Kouvelis, P. and W. Zhao (2011). The newsvendor problem and price-only contract when bankruptcy costs exist. *Production and Operations Management 20*(6), 921–936.

Kouvelis, P. and W. Zhao (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research 60*(3), 566–580.

Martínez-de Albéniz, V. and D. Simchi-Levi (2013). Supplier–buyer negotiation games: Equilibrium conditions and supply chain efficiency. *Production and Operations Management 22*(2), 397–409.

Peura, H., S. A. Yang, and G. Lai (2017). Trade credit in competition: A horizontal benefit. *Manufacturing & Service Operations Management 19*(2), 263–289.

Rui, H. and G. Lai (2015). Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management 24*(6), 934–946.

Rumyantsev, S. and S. Netessine (2007). What can be learned from classical inventory models? a cross-industry exploratory investigation. *Manufacturing & Service Operations Management 9*(4), 409–429.

Tanrisever, F., H. Cetinay, M. Reindorp, and J. C. Fransoo (2012). Value of reverse factoring in multi-stage supply chains. *Available at SSRN 2183991*.

Tunca, T. I. and W. Zhu (2018). Buyer intermediation in supplier finance. *Management Science 64*(12), 5631–5650.

Van der Vliet, K., M. J. Reindorp, and J. C. Fransoo (2015). The price of reverse factoring: Financing rates vs. payment delays. *European Journal of Operational Research 242*(3), 842–853.

Wu, A., B. Huang, and D. M.-H. Chiang (2014). Support SME suppliers through buyer-backed purchase order financing.

Yang, S. A. and J. R. Birge (2018). Trade credit, risk sharing, and inventory financing portfolios. *Management Science 64*(8), 3667–3689.

Appendix for
Retailer-Initiated Inventory-Based Financing

A Proofs for Propositions and Theorems

A.1 Proof for Proposition 1

Proof. The borrower's objective function is

$$\begin{aligned}
\Pi_{\mathscr{B}}(z) &= \mathbb{E}\left[p\min\left(D, q+\frac{v}{c}z\right) - vRz\right]^+ \\
&= \int_0^{q+\frac{v}{c}z} (px - vRz)^+ f(x)dx + \int_{q+\frac{v}{c}z}^{\infty} (pq + p\frac{v}{c}z - vRz)^+ f(x)dx \\
&= \int_{\frac{vR}{p}z}^{q+\frac{v}{c}z} (px - vRz) f(x)dx + (pq + \frac{v}{c}pz - vRz)\overline{F}(q+\frac{v}{c}z) \\
&= \int_{\frac{vR}{p}z}^{q+\frac{v}{c}z} p\overline{F}(x)dx.
\end{aligned}$$

By taking its derivative, we obtain

$$\frac{\partial \Pi_{\mathscr{B}}(z)}{\partial z} = \overline{F}(q+\frac{v}{c}z)\frac{pv}{c} - \overline{F}(\frac{vR}{p}z)Rv. \tag{A.1}$$

Case 1: $q \leq c/(c+v)S^*$. Then

$$\frac{\partial \Pi_{\mathscr{B}}(z)}{\partial z} \geq \overline{F}(q+\frac{v}{c}z)\frac{pv}{c} - Rv \geq \overline{F}(q+\frac{v}{c}q)\frac{pv}{c} - Rv \geq \overline{F}(S^*)\frac{pv}{c} - Rv = 0,$$

where we use the fact that $\overline{F}$ is monotonically decreasing with $\overline{F}(0) = 1$. Thus, $\Pi_{\mathscr{B}}(z)$ is monotonically increasing in $[0, q]$; hence, in this case, $z^O(q) = q$.

Case 2: $c/(c+v)S^* < q < S^*$. Then, we have

$$\left.\frac{\partial \Pi_{\mathscr{B}}}{\partial z}\right|_{z=0} = \overline{F}(q)\frac{pv}{c} - Rv > 0.$$

On the other hand, from the third requirement in Assumption 1, there exists a threshold M such that $\frac{\partial \Pi_{\mathscr{B}}}{\partial z} < 0$ when $z > M$. This, along with the first assumption in Assumption 1, proves that there exists a unique solution $z^*(q) > 0$ to equation (A.1). Additionally, $\Pi_{\mathscr{B}}(z)$ increases in $[0, z^*(q)]$ but decreases in $[z^*(q), \infty]$. Hence. the optimal solution would be $z^O(q) = \min\{q, z^*(q)\}$.

Case 3: $q \geq S^*$. Then, we have $\left.\frac{\partial \Pi_{\mathscr{B}}}{\partial z}\right|_{z=0} \leq 0$. Note that under Assumption 1, the derivative $\frac{\partial \Pi_{\mathscr{B}}}{\partial z}$ would approach zero from the negative side as $z \to \infty$. Hence, $\frac{\partial \Pi_{\mathscr{B}}}{\partial z} < 0$, and we have $z^O(q) = 0$.

A.2 Proof for Proposition 2

Proof. The borrower's objective function in this case can be written as

$$\begin{aligned}\Pi_{\mathscr{B}}^{H}(z) &= \mathbb{E}\left[p\min\left(D, q+\frac{v}{c}z\right) - vRz\right] \\ &= \int_0^{q+\frac{v}{c}z}(px - vRz)\,f(x)dx + (pq + p\frac{v}{c}z - vRz)\overline{F}(q+\frac{v}{c}z) \\ &= -vRz + \int_0^{q+\frac{v}{c}z} p\overline{F}(x)dx.\end{aligned}$$

Hence, its derivative with respect to z is

$$\frac{\partial \Pi_{\mathscr{B}}^{H}(z)}{\partial z} = -vR + \frac{vp}{c}\overline{F}(q+\frac{v}{c}z),$$

which is monotonically decreasing with respect to z.

Case 1: $q \leq \frac{c}{c+v}Q^*$. Under this case, we have $\overline{F}(q+\frac{v}{c}z) \geq \overline{F}(\frac{c+v}{c}q) \geq \overline{F}(Q^*) = \frac{cR}{p}$, which indicates that $\frac{\partial \Pi_{\mathscr{B}}^{H}(z)}{\partial z} \geq 0$ when $z \in [0, q]$. Hence, $\Pi_{\mathscr{B}}^{H}(z)$ achieves its maximum at $z = q$.

Case 2: $\frac{c}{c+v}Q^* < q < Q^*$. Then, $\frac{\partial \Pi_{\mathscr{B}}^{H}(z)}{\partial z}$ has a single root $z^* = \frac{c}{v}(Q^* - q) \in [0, q]$. Therefore, $\Pi_{\mathscr{B}}^{H}(z)$ is increasing in $[0, z^*]$ and decreasing in $[z^*, q]$. Hence, $\Pi_{\mathscr{B}}^{H}(z)$ achieves its maximum at $z = z^*$.

Case 3: $q \geq Q^*$. Then, we can have $\overline{F}(q+\frac{v}{c}z) \leq \overline{F}(Q^*) = \frac{cR}{p}$, which means $\frac{\partial \Pi_{\mathscr{B}}^{H}(z)}{\partial z} \leq 0$ in $[0, q]$. Hence, $\Pi_{\mathscr{B}}^{H}(z)$ achieves its maximum at $z = 0$.

A.3 Proof for Proposition 3

Proof. Clearly, we only need to prove the case when $\frac{c}{c+v}Q^* < q \leq Q^*$. Suppose the solution z^* satisfies

$$F(q+\frac{v}{c}z^*) = \frac{p-Rc}{p} + \frac{Rc}{p}F\left(\frac{vRz^*}{p}\right).$$

Because we assume $p \geq vR$, we further have $F\left(\frac{vRz^*}{p}\right) \leq F(z^*) \leq F(q) \leq F(Q^*) = 1 - Rc/p$. Hence,

$$F(q+\frac{v}{c}z^*) \leq \frac{p-Rc}{p} + \frac{Rc}{p}(1-\frac{Rc}{p}) = \frac{p^2-R^2c^2}{p}.$$

Note that the heuristic policy z^H satisfies $F(q+\frac{v}{c}z^H) = \frac{p-Rc}{p}$. Thus, we can conclude that

$$\begin{aligned}z^* - z^H &= \frac{c}{v}\left[(q+\frac{v}{c}z^*) - (q+\frac{v}{c}z^H)\right] \\ &\leq \frac{c}{v}\left[F^{-1}\left(\frac{p^2-R^2c^2}{p}\right) - F^{-1}\left(\frac{p-Rc}{p}\right)\right].\end{aligned}$$

Our proposition follows from the fact that $z^O - z^H = \min\{z^*, q\} - z^H \leq z^* - z^H$.

A.4 Proof for Proposition 4

Proof. The lender's objective function can be written as

$$\begin{aligned}
\Pi_{\mathscr{L}}(R,v) &= \mathbb{E}\left[\min(pD, vRz) + s\left(z - \frac{p}{vR}D\right)^+\right] - vR_f z, \\
&= \int_0^{\frac{vRz}{p}} \left(px + s(z - \frac{px}{vR})\right) f(x)dx + \int_{\frac{vRz}{p}}^{\infty} vRzf(x)dx - vR_f z \\
&= \int_0^{\frac{vRz}{p}} \left(p - \frac{sp}{vR}\right) xf(x)dx + (s - vR)zF(\frac{vRz}{p}) + v(R - R_f)z \\
&= -\left(1 - \frac{s}{vR}\right) p \int_0^{\frac{vRz}{p}} F(x)dx + v(R - R_f)z
\end{aligned}$$

When $R > \frac{p}{c}\overline{F}(q)$, we have $z^* = 0$ by Proposition 1, which means $\Pi_{\mathscr{L}}(R,v) = 0$.

When $R \le \frac{p}{c}\overline{F}(\frac{c+v}{c}q)$, we have $z^* = q$ by Proposition 1. Thus, we have $\frac{\partial z^*}{\partial R} = 0$, and we can calculate $\frac{\partial \Pi_{\mathscr{L}}}{\partial R}$ as

$$\begin{aligned}
\frac{\partial \Pi_{\mathscr{L}}(R,v)}{\partial R} &= -\frac{sp}{vR^2}\int_0^{\frac{vRz}{p}} F(x)dx - (vz - \frac{sz}{R})F(\frac{vRz}{p}) + vz \\
&\ge -\frac{sp}{vR^2}\int_0^{\frac{vRz}{p}} F(x)dx + \frac{sz}{R}F(\frac{vRz}{p}) \\
&\ge -\frac{sp}{vR^2}F(\frac{vRz}{p})\frac{vRz}{p} + \frac{sz}{R}F(\frac{vRz}{p}) = 0.
\end{aligned}$$

Hence, $\Pi_{\mathscr{L}}$ is monotonically increasing in R, and this case cannot be optimal.

Combining the above two statements, we find the optimal R^* has the bound

$$R^* \in \left[\frac{p}{c}\bar{F}\left(\frac{c+v}{c}q\right), \frac{p}{c}\bar{F}(q)\right]. \tag{A.2}$$

When $q \to 0$, we have $\bar{F}\left(\frac{c+v}{c}q\right) \to 1$ and $\bar{F}(q) \to 1$, which yields $R^* \to \frac{p}{c}$ by A.2.

A.5 Proof for Proposition 5

Proof. We first prove that if $w_0 = 0$, then $w_t^* = 0$ for every $t = 0, 1, \ldots, T$. This is because if the retailer's initial cash position is $w_0 = 0$, then holding positive cash means the retailer must be pledging some extra quantity rather than buying the exact amount of u_t at some time point. However, since the retailer can pledge any quantity at any time point, it is better for her to pledge the amount needed and save the rest as the product; in this way, she would not be charged interest for the extra quantity pledged. In fact, for every pledging strategy $\{u_t, z_t\}$ with w_t as its cash position, we construct a corresponding policy $\{u_t', z_t'\}$ that satisfies $z_t' = cu_t/v$ and $u_t' = u_t$. By induction, it is easy to see that $vq_t' = vq_t + w_t$ for every $t = 0, 1, \ldots, T$; hence, this policy is feasible. The total inventory in these two cases will be the same because $x_T + q_T = q + \sum_{t=0}^{T-1} u_t = x_T' + q_T'$. However, we obtain $w_t' = 0$ and $x_t' \le x_t$ for $t = 0, 1, 2, \ldots, T$, which means the retailer will pay less interest to the platform, indicating the profit will be higher under policy $\{u'_t, z'_t\}$.

Based on the statement above, we can further write the state dynamics as

$$q_{t+1} = q_t + (\theta - 1)z_t \tag{A.3}$$

$$x_{t+1} = x_t + z_t, \tag{A.4}$$

where $\theta = \frac{v}{c}$. For a given T and Q, the objective function would be

$$B_0^T(q, 0, 0) = \mathbb{E}\left[p \min(D, Q) - vx_T - rv\Sigma_{t=1}^T x_t\right]^+ = \mathbb{E}\left[p \min(D, Q) - c(Q - q) - rvA(Q)\right]^+.$$

Hence, maximizing $B_0^T(q, 0, 0)$ is equivalent to minimizing $A(Q) = \sum_{t=1}^T x_t = \sum_{t=0}^{T-1}(T - t)z_t$. Therefore, the dynamic programming problem can be written as a linear programming problem

$$\begin{aligned}
\min_{z_i} \quad & \sum_{t=0}^{T-1}(T - t)z_t \\
s.t. \quad & x_T + q_T = Q \\
& 0 \le z_t \le q_t \qquad t = 0, 1, 2, \dots, T - 1.
\end{aligned}$$

Note that from the state dynamics A.3 and A.4, we have $x_T + q_T = q + \sum_{t=0}^{T-1} z_t$ and $q_t = q + (\theta - 1)\sum_{i=0}^{t-1} z_i$. Thus, we can further write the problem as

$$\begin{aligned}
\min_{z_i} \quad & \sum_{t=0}^{T-1}(T - t)z_t \\
s.t. \quad & q + \sum_{t=0}^{T-1} z_t = Q \\
& z_0 \le q \\
& z_t \le q + (\theta - 1)\sum_{i=0}^{t-1} z_i \qquad t = 1, 2, \dots, T - 1. \\
& z_t \ge 0 \qquad t = 0, 1, 2, \dots, T - 1
\end{aligned} \tag{A.5}$$

The Lagrangian of this problem is

$$L(\boldsymbol{z}, \mu, \boldsymbol{\lambda}, \boldsymbol{\kappa}) = \sum_{t=0}^{T-1}(T - t)z_t + \mu\left(q + \sum_{t=0}^{T-1} z_t - Q\right) + \lambda_0(z_0 - q) + \sum_{t=1}^{T-1}\lambda_t\left(z_t - q - (\theta - 1)\sum_{i=0}^{t-1} z_i\right) - \sum_{t=0}^{T-1}\kappa_t z_t.$$

The first-order condition is

$$\frac{\partial L}{\partial z_t} = T - t + \mu + \lambda_t - (\theta - 1)\sum_{i=t+1}^{T-1}\lambda_t - \kappa_t = 0.$$

Subtraction of the t-th and $(t+1)$-th equations yields

$$1 + \lambda_t - \theta\lambda_{t+1} = \kappa_t - \kappa_{t+1}. \tag{A.6}$$

If $\lambda_t > 0$, then by complementary slackness, we have $z_t = q + (\theta - 1)\sum_{i=0}^{t-1} z_i > 0$. Again by complementary slackness of κ_t, we have $\kappa_t = 0$. Thus, to satisfy Equation (A.6), we must have $\lambda_{t+1} > 0$. By induction, we can have $\lambda_i > 0$ for every $i \geq t$. Hence, there exists $0 \leq T_0 \leq T-1$ such that $\lambda_t = 0$ when $t \leq T_0$ and $\lambda_t > 0$ when $t > T_0$.

For every $t \leq T_0 - 1$, we have $\lambda_t = \lambda_{t+1} = 0$, so the left side of equation A.6 is 1, which indicates that $\kappa_t > 0$ and $z_t = 0$. For $t > T_0$, since $\lambda_t > 0$, we have $z_t = q + (\theta - 1)\sum_{i=0}^{t-1} z_i$ by complementary slackness. As a result, the optimal solution of linear programming A.5 has the following structure:

$$\begin{aligned} z_t &= 0, \qquad t = 0, 1, \ldots, T_0 - 1 \\ z_{T_0+1} &= q + (\theta - 1) z_{T_0} \\ z_t &= \theta z_{t-1}, \qquad t = T_0 + 2, T_0 + 2, \ldots, T - 1. \end{aligned}$$

Thus, from the constraint

$$\sum_{t=0}^{T-1} z_t = \sum_{t=T_0+1}^{T} \theta^{t-T_0-1} z_{T_0+1} + z_{T_0} = \frac{\theta^{T-T_0} - 1}{\theta - 1}(q + (\theta-1) z_{T_0}) + z_{T_0} = Q - q,$$

we have

$$z_{T_0} = \frac{1}{\theta^{T-T_0}}\left[Q - \frac{\theta^{T-T_0} - 1}{\theta - 1} q\right].$$

From the constraint $0 \leq z_{T_0} \leq q$, we have

$$T - T_0 = \left\lfloor \frac{\log\left(\frac{Q}{q}(\theta - 1) + 1\right)}{\log \theta} \right\rfloor := T^*,$$

which proves the statement in the theorem. Furthermore, we can calculate $A(Q)$ as

$$A(Q) = \sum_{t=0}^{T-1} (T - t) z_t = \frac{q(\theta^{T^*} - 1)}{(\theta - 1)^2} - \frac{qT^*}{\theta - 1} + z_0 \frac{\theta^{T^*} - 1}{\theta - 1}.$$

A.6 Proof for Theorem 1

Denote $M(Q) = c(Q-q) + rvA(Q)$; then, the borrower's objective function can be written as:

$$
\begin{aligned}
\Pi_{\mathscr{B}}(Q) &= \mathbb{E}\left[p\min(D,Q) - c(Q-q) - rvA(Q)\right]^{+} \\
&= \mathbb{E}\left[p\min(D,Q) - M(Q)\right]^{+} \\
&= \int_0^Q (px - M(Q))^{+} f(x)dx + (pQ - M(Q))^{+}\overline{F}(Q) \\
&= \begin{cases} 0 & pQ < M(Q) \\ p\int_{M(Q)/p}^{Q} \overline{F}(x)dx & pQ \geq M(Q) \end{cases}
\end{aligned}
$$

Without loss of generality, we consider only the region where $Q \geq M(Q)/p$ in the following text. By direct calculation, we can see that $M(Q)$ is linear in Q. In fact, $M(Q)$ can be written as:

$$
\begin{aligned}
M(Q) &= \left[c + rv\frac{\theta^T - 1}{\theta^T(\theta-1)}\right]Q - \left[rv\left(\frac{T}{\theta-1} - \frac{\theta^T-1}{\theta^T(\theta-1)^2}\right) + c\right]q \\
&:= \alpha_T pQ - \beta_T p.
\end{aligned}
$$

As a result, the derivative of $\Pi_{\mathscr{B}}(Q)$ is

$$
\frac{\partial \Pi_{\mathscr{B}}(Q)}{\partial Q} = p\left(\overline{F}(Q) - \overline{F}\left(\alpha_T Q - \beta_T\right)\alpha_T\right). \tag{A.7}
$$

By means of basic calculus, we can see that $\alpha_T > 0, \beta_T > 0$ and α_T, β_T are monotonically increasing with respect to T. (Here, the assumption $\theta = v/c > 1$ is used.) This further implies that α_T and β_T are monotonically increasing with respect to Q. We next prove that function (A.7) has at most a single root. In fact, by Assumption 1, we have that function (A.7) has at most a single root for any fixed T. We now prove by contradiction. Suppose it has two roots $Q_1 < Q_2$. Then, because the planning horizon T is increasing in Q, we must have $T_1 < T_2$ for these two pledging periods, respectively. This further indicates that $\alpha_{T_1} < \alpha_{T_2}$ and $\beta_{T_1} < \beta_{T_2}$. We next show that from $\alpha_{T_1} < \alpha_{T_2}$ and $\beta_{T_1} < \beta_{T_2}$, we would obtain $Q_1 > Q_2$, which is a contradiction.

Denote

$$
\begin{aligned}
G_1(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_1}Q - \beta_{T_1}\right)\alpha_{T_1}, \\
G_2(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_2}Q - \beta_{T_2}\right)\alpha_{T_2}, \\
G_3(Q) &= \overline{F}(Q) - \overline{F}\left(\alpha_{T_2}Q - \beta_{T_1}\right)\alpha_{T_2}.
\end{aligned}
$$

We already have Q_1, Q_2 is the solution to $G_1(Q), G_2(Q)$. It is easy to see that $G_3(Q_2) > G_2(Q_2) = 0$. Note that G_3 approaches zero from the negative side as $Q \to \infty$; hence, $G_3(Q)$ would also have a root, which we denote as Q_3. We next show that $Q_2 < Q_3 < Q_1$.

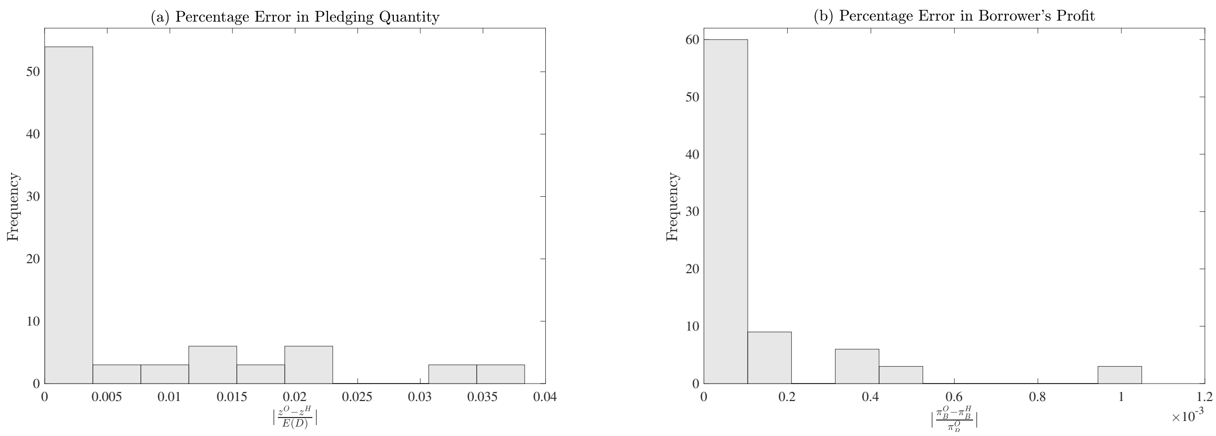

Figure B.1: **Performance gap between the optimal and heuristic solutions.** Panel (a) illustrates the difference in quantity that the borrower pledges. Panel (b) shows the difference in the borrower's profit.

In fact, from Assumption 1, the equation

$$\overline{F}(\alpha_{T_1}Q - \beta_{T_1})\alpha_{T_1} - \overline{F}(\alpha_{T_2}Q - \beta_{T_1})\alpha_{T_2} = 0 \tag{A.8}$$

also has at most a single solution. If there is a solution Q_0 to (A.8), then because we have that $x\overline{F}(x-m)$ is unimodal under Assumption 1, we would obtain

$$\overline{F}(\alpha_{T_1}Q_0 - \beta_{T_1})\alpha_{T_1} = \overline{F}(\alpha_{T_2}Q_0 - \beta_{T_1})\alpha_{T_2} \geq \overline{F}(Q_0 - \beta_{T_1}) \geq \overline{F}(Q_0).$$

Hence, $G_1(Q_0) < 0$. Additionally, we would have $G_1(Q) > G_3(Q)$ when $Q < Q_0$ and $G_1(Q) \geq G_3(Q)$ when $Q \geq Q_0$. If there is no solution to (A.8), we would naturally obtain $G_1(Q) > G_3(Q)$. As a result, we have $Q_3 < Q_1$ either way. On the other hand, by the monotonicity of $\overline{F}$, we have $0 = G_2(Q_2) < G_3(Q_2)$. This implies $Q_2 < Q_3$. Thus, we achieve $Q_2 < Q_3 < Q_1$, which is a contradiction to the fact that $Q_1 < Q_2$.

We now have proven that (A.7) has at most a single root. Similar to the case in the single period, $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}$ would approach zero from the negative side as $Q \to \infty$ according to the third assumption in Assumption 1. Hence, $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}$ having a solution in $[q, \infty)$ is equivalent to $\frac{\partial \Pi_{\mathscr{B}}}{\partial Q}\big|_{Q=q} \geq 0$, which is further equivalent to $q \leq S^*$, where $S^* = F^{-1}(1 - \frac{Rc}{p})$. As a result, when $q \leq S^*$, $\Pi_{\mathscr{B}}(Q)$ is increasing in $[q, Q^*]$ and decreasing in (Q^*, ∞), where Q^* is the root of function (A.7). Moreover, when $q > S^*$, we have $\Pi_{\mathscr{B}}(Q)$ is decreasing in $[q, \infty)$. This gives us the statement in the theorem.

B Numerical Analysis

To simulate the percentage difference in the equilibrium outcomes between the optimal and heuristic scenarios, we experiment using the following parameter combinations: $\mu = 100$, $\sigma = (10, 30, 50)$, $v = (0.5, 1, 1.5)$, $R = (1.05, 1.1, 1.15)$, $c = 1$, $p = (1.1, 1.5, 1.9)$. We observe that the resulting errors in both quantity and profit are small.

C The Directed Acyclic Graph Representation of Equation (12)

As the large retailer moves first in the Stackleberg game by setting the interest rate after observing the initial inventory, and the small retailer decides the planning horizon in response to the interest rate, the causal relationship can be summarized as follows.

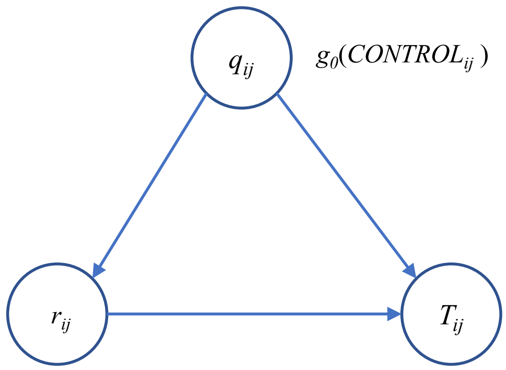

Figure C.1: **Causal relationship between r_{ij} and T_{ij}.**