Appendix: Simulation

Hongyu Mou

January 7, 2024

1 Extrapolation Model

- Extrapolation 1 based on GPA: want to have Y_{GPA} for all scores[1].

$$\begin{aligned} \boldsymbol{Y_{GPA,b}} &= \underbrace{\alpha_1 + \beta_1 \times GPA(\%) + \gamma \times b}_{\boldsymbol{\hat{Y}_{GPA,b}}} + \epsilon_1 \\ &= 10.01 + 0.09 \times GPA(\%) + 0.2 \times b + \epsilon_1 \\ &\text{where } \boldsymbol{\epsilon_1 \sim N(0, 1.002)} \end{aligned} \tag{1}$$

use the estimated relationship (estimated for $GPA >$ cutoff) to extrapolate Y_{GPA} for $GPA <$ cutoff.

- Extrapolation 2 based on Quota 2 Score: want to have $Y_{s,GPA,Q2}$ for all scores. Assuming that s is continuous even below the cutoff, we can again assume a linear relationship in quota 2 score and GPA so that:

$$\begin{aligned} \boldsymbol{Y_{GPA,s,b,Q2>0}} &= \underbrace{\alpha_2 + \beta_2 \times GPA(\%) + \gamma_2 \times s(\%) + \gamma_3 \times b}_{\boldsymbol{\hat{Y}_{GPA,s,b,Q2>0}}} + \epsilon_2 \\ &= 9.12 - 0.04 \times GPA(\%) + 2.33 \times s(\%) + 0.3 \times b + \epsilon_2 \\ &\text{where } \boldsymbol{\epsilon_2 \sim N(0, 0.703)} \end{aligned} \tag{2}$$

use the estimated relationship (estimated for $s >$ cutoff) to extrapolate $Y_{s,GPA,Q2}$ for $s <$ cutoff.

- **Test if ϵ_1 and ϵ_2 are normally distributed:** Figure 1 show distributions and Kolmogorov-Smirnov tests of residuals ϵ_1 and ϵ_2:

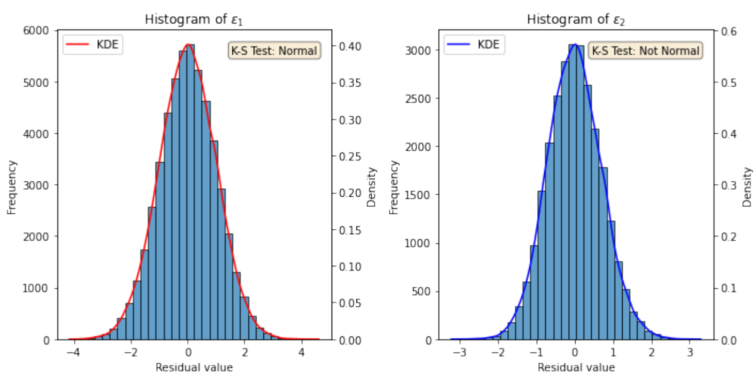
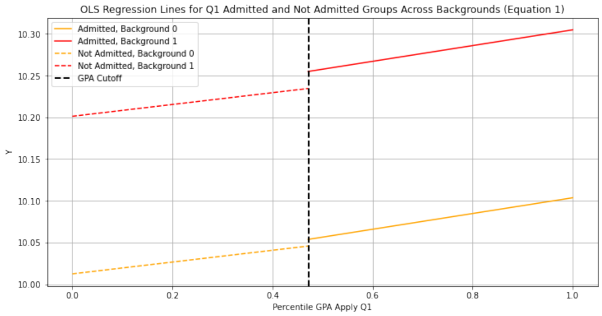


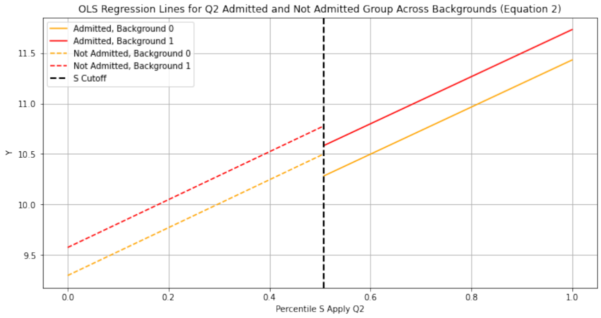
Figure 1: Distribution of ϵ_1 and ϵ_2 from Equations (1) and (2)

[1] Since in the empirical data we observe that the higher the GPA, the larger the earning potential.

1.1 Extrapolation Plots

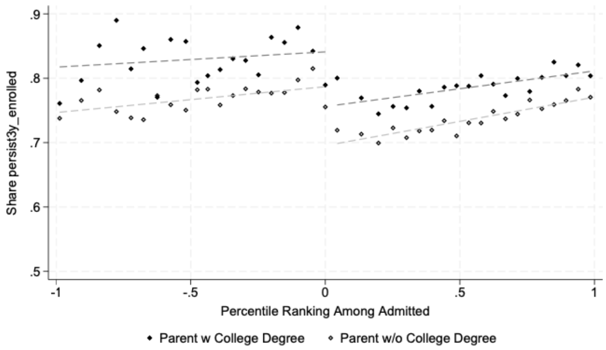


(a) OLS Regression Lines of Equation (1)



(b) OLS Regression Lines of Equation (2) using $Q2 > 0$ Sub-sample

Figure 2: Extrapolation Plots for Each Background from Equation (1) and (2)



(b) Persistence, Year 3

Figure 3: Example Fitted lines (Figure A9b) for Each Background in Real Data

2 Simulation Model[2]

Whole Sample: $N = 100000$ (Applied_Q1)

Parameters: b, GPA, and ϵ

Edu Background: $b \sim$ Bernoulli(p = 0.5)

GPA $\sim \mathcal{N}(8.8, \text{sd} = 1)$ if $b = 0$, censor at 6 and 13

GPA $\sim \mathcal{N}(9.2, \text{sd} = 1)$ if $b = 1$, censor at 6 and 13

$$\epsilon_Y, \epsilon_{Q2}, \epsilon_S \sim \mathcal{N}(0, \Sigma) \text{ where } \Sigma = \begin{bmatrix} 1 & 0.5 & 0.6 \\ 0.5 & 1 & 0 \\ 0.6 & 0 & 1 \end{bmatrix}$$

$\alpha_Y = 10, \alpha_Y^1 = 0.1, \alpha_Y^2 = 0.0, \alpha_Y^3 = 0.2$

$\beta_{Q2} = 0, \beta_{Q2}^1 = 0.2, \beta_{Q2}^2 = -0.05, \beta_{Q2}^3 = 0.1$

$\gamma_S = 0, \gamma_S^1 = 0.15, \gamma_S^2 = 0.0, \gamma_S^3 = -0.2$

Simulate: $Y, Q2,$ and S

$$Y = \alpha_Y + \alpha_Y^1 \times GPA(\%) + \alpha_Y^2 \times GPA(\%)^2 + \alpha_Y^3 \times b + \epsilon_Y$$
$$Q2 = \beta_{Q2} + \beta_{Q2}^1 \times GPA(\%) + \beta_{Q2}^2 \times GPA(\%)^2 + \beta_{Q2}^3 \times b + \epsilon_{Q2}$$
$$S = \gamma_S + \gamma_S^1 \times GPA(\%) + \gamma_S^2 \times GPA(\%)^2 + \gamma_S^3 \times b + \epsilon_S$$

GPA Cutoff: GPA_cutoff = 8.93

Admission:

$$\text{Admitted\_Q1} = \begin{cases} 1 & \text{if GPA >= GPA\_cutoff} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Applied\_Q2} = \begin{cases} 1 & \text{if } Q2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Admitted\_Q2} = \begin{cases} 1 & \text{if } S > 0 \text{ and Applied\_Q2} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Objects of Interest:

For each background $b \in \{0, 1\}$:

$$P(Q2 > 0 \mid GPA(\%), Y, B = b) = \frac{P(Y \mid Q2 > 0, GPA(\%), B = b) \times P(Q2 > 0 \mid GPA(\%), B = b)}{P(Y \mid GPA(\%), B = b)}$$

$$P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) = \frac{P(Y \mid S > 0, GPA(\%), Q2 > 0, B = b) \times P(S > 0 \mid GPA(\%), Q2 > 0, B = b)}{P(Y \mid GPA(\%), Q2 > 0, B = b)}$$

$$P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b) = P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) \times P(Q2 > 0 \mid GPA(\%), Y, B = b)$$

[2]See Appendix for the descriptive statistics.

3 $P(Q2 > 0 \mid GPA(\%), Y, B = b)$

The first Bayes Rule:

$$P(Q2 > 0 \mid GPA(\%), Y, B = b) = \frac{P(Y \mid Q2 > 0, GPA(\%), B = b) \times P(Q2 > 0 \mid GPA(\%), B = b)}{P(Y \mid GPA(\%), B = b)} \tag{3}$$

Since in real data we only observe Y for those who were admitted, in the feasible $GPA >= GPA_{cutoff}$ sub-sample, use local approximation instead of Bayes Rule (3) for each object of interest if $N_{GPA,Y,B} \geq \bar{N}_{GPA,Y,B}$, where $N_{GPA,Y,B}$ is the size of each $(GPA(\%), Y)$ cell for a specific background B.

1. Feasible: $\bar{N}_{GPA,Y,B} > \infty$, i.e., all from extrapolation using Bayes Rule (3)
2. Feasible: $\bar{N}_{GPA,Y,B} = 0$, i.e., local approximation without using Bayes Rule (3), but for the $GPA >= GPA_{cutoff}$ sample, since in real data we only observe Y for those who were admitted.
3. Feasible: $\bar{N}_{GPA,Y,B} = 10$, i.e., local approximation for the $GPA >= GPA_{cutoff}$ sample if $N_{GPA,Y,B} \geq 10$; Bayes Rule (3) from extrapolation if $N_{GPA,Y,B} < 10$ or $GPA < GPA_{cutoff}$.
4. Feasible: $\bar{N}_{GPA,Y,B} = 50$, i.e., local approximation for the $GPA >= GPA_{cutoff}$ sample if $N_{GPA,Y,B} \geq 50$; Bayes Rule (3) from extrapolation if $N_{GPA,Y,B} < 50$ or $GPA < GPA_{cutoff}$.

3.1 $P(Y \mid Q2 > 0, GPA(\%), B = b)$

- **From Extrapolation**: Equation (2) and S Integration. **(Figure 5)**

  Compute

$$\begin{aligned} &P[Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, B = b] \\ =& \sum_s P\left[Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, s(\%) \in S(\%)_{\text{bin}}, B = b\right] \\ &* P\left[s(\%) \in S(\%)_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, B = b\right] \end{aligned} \tag{4}$$

  where

  - $P\left[Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, s(\%) \in S(\%)_{\text{bin}}, B = b\right]$: for each GPA(%) and S(%) grid, assume

$$Y_{GPA,s,b,Q2>0} \sim N(\hat{Y}_{GPA,s,b,Q2>0}, std(\epsilon_2)) \tag{5}$$

    then

$$P(Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, s(\%) \in S(\%)_{\text{bin}}) = \Phi\left(\frac{\overline{Y}_{\text{bin}} - \mu_2}{\sigma_2}\right) - \Phi\left(\frac{\underline{Y}_{\text{bin}} - \mu_2}{\sigma_2}\right) \tag{6}$$

    * $\boldsymbol{\mu_2}$: local predicted mean $(\hat{Y}_{GPA,s,b,Q2>0})$ from Equation (2) in this GPA(%) and S(%) grid
    * $\boldsymbol{\sigma_2}$: global standard deviation of ϵ_2 estimated from Equation (2) using the observed sample above the Score cutoff.
  - $P\left[s(\%) \in S(\%)_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, Q2 > 0, B = b\right]$: PMF from local approximation, shown in Figure 4.

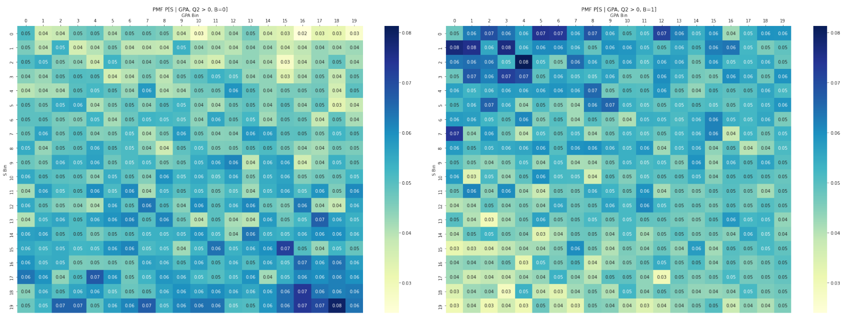


Figure 4: PMF $P[S(\%)|GPA(\%), Q2 > 0, B]$ from Local Approximation

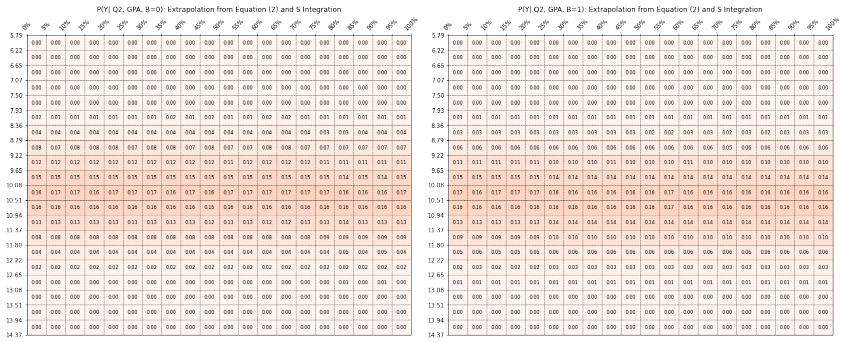


Figure 5: $P(Y|Q2 > 0, GPA, B)$ from Extrapolation

3.2 $P(Y \mid GPA(\%), B = b)$

- **From Extrapolation**: Equation (1). **(Figure 6)**

  for each GPA(%) bin, assume

$$Y_{GPA,b} \sim N(\hat{Y}_{GPA,b}, std(\epsilon_2)) \tag{7}$$

  then

$$P(Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, B = b) = \Phi\left(\frac{\overline{Y}_{\text{bin}} - \mu_1}{\sigma_1}\right) - \Phi\left(\frac{\underline{Y}_{\text{bin}} - \mu_1}{\sigma_1}\right) \tag{8}$$

  - $\boldsymbol{\mu_1}$: local predicted mean ($\hat{Y}_{GPA,b}$) from Equation (1) in this GPA(%) bin
  - $\boldsymbol{\sigma_1}$: global standard deviation of ϵ_1 estimated from Equation (1) using the observed sample above the GPA cutoff.

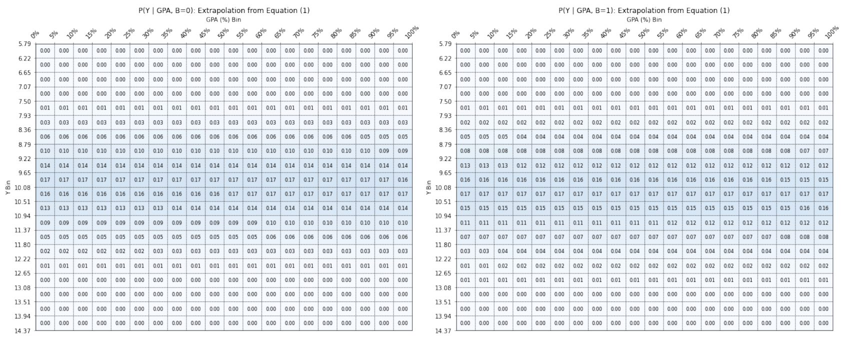


Figure 6: $P(Y|GPA, B)$ from Extrapolation

3.3 $P(Q2 > 0 \mid GPA(\%), B = b)$

The probability that a student applies to Q2 given their GPA and Background. See Figure 7.

$$P(Q2 > 0 \mid GPA(\%), B = b) = \frac{\text{Number of students who applied to Q2 in the GPA (\%) bin and Background group}}{\text{Total Number of students in the GPA (\%) bin and Background group}} \tag{9}$$

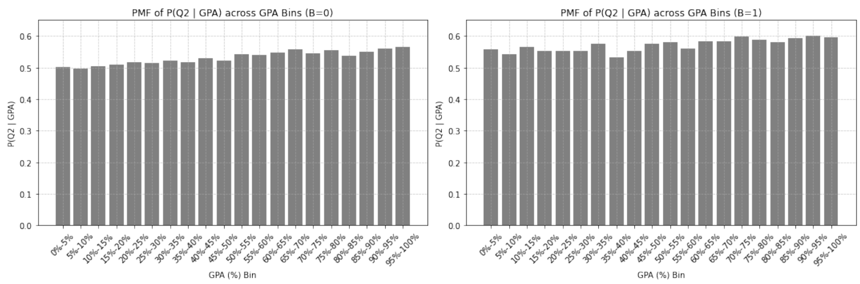
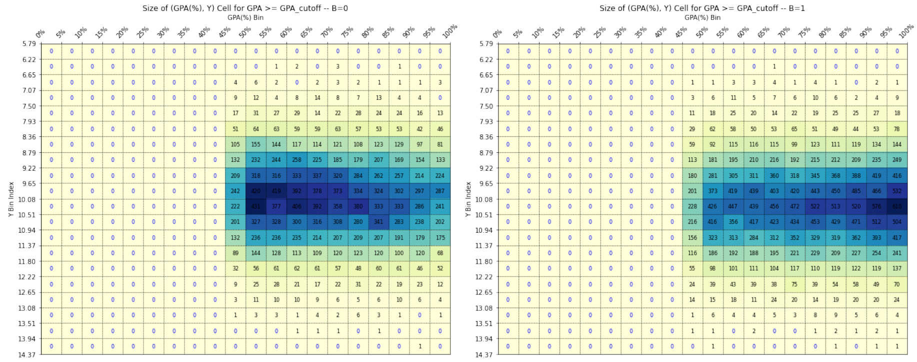
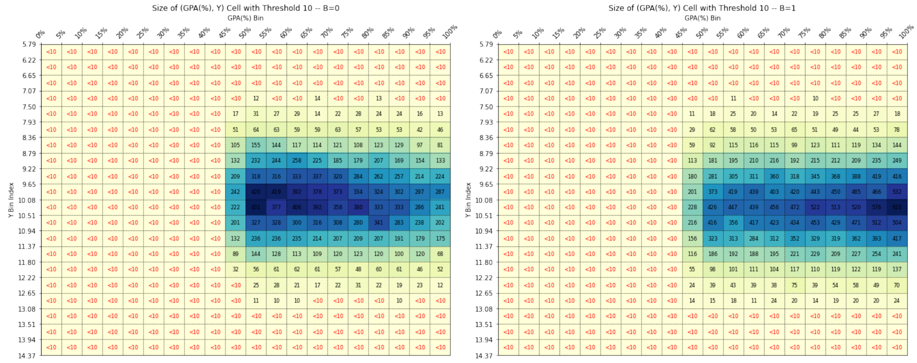


Figure 7: PMF $P(Q2 > 0 \mid GPA(\%), B)$ from Local Approximation

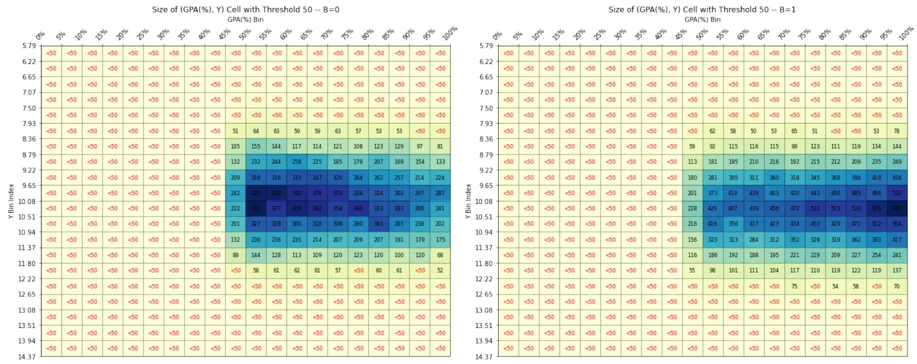
3.4 Cell Size for GPA vs. Y



(a) $GPA >= GPA_{cutoff}$ Sub-sample: **Feasible**



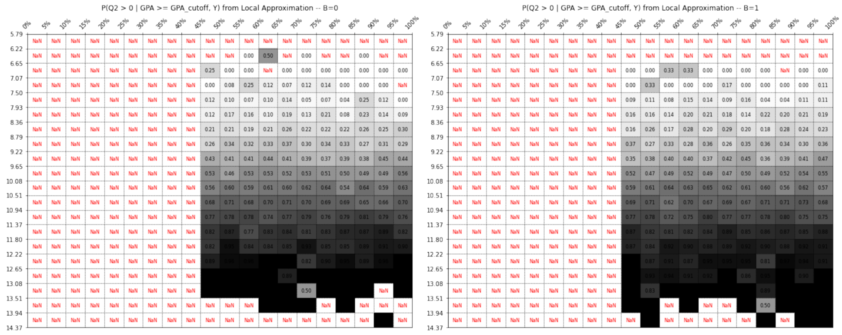
(b) $\bar{N}_{GPA,Y,B} = 10$ and $GPA >= GPA_{cutoff}$: **Feasible**



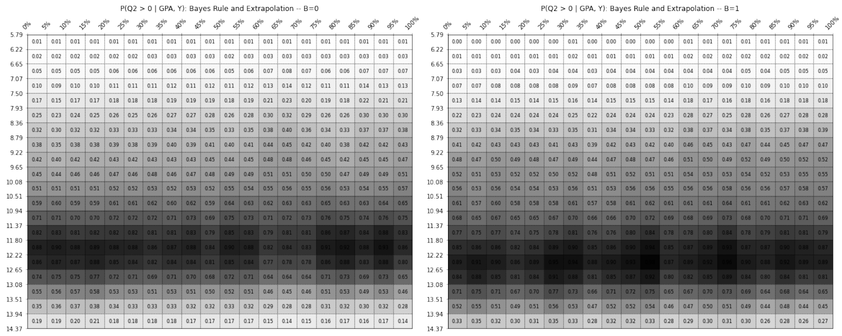
(c) $\bar{N}_{GPA,Y,B} = 50$ and $GPA >= GPA_{cutoff}$: **Feasible**

Figure 8: Size of $(GPA(\%), Y)$ Cell

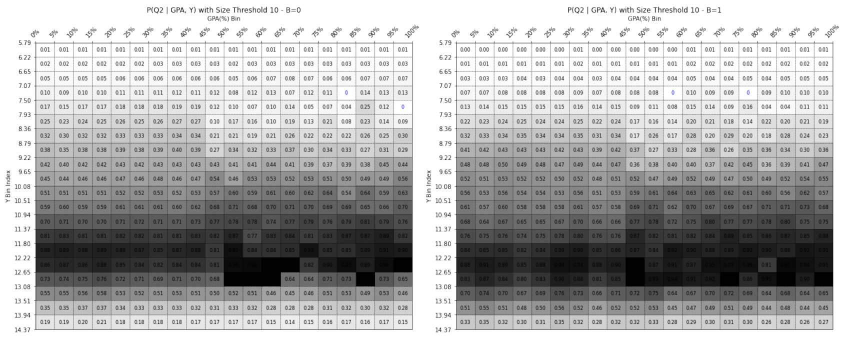
3.5 $P(Q2 > 0 \mid GPA(\%), Y, B = b)$



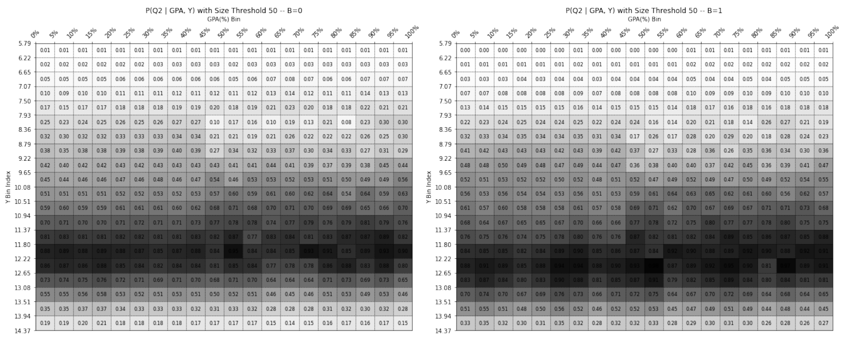
(a) Local Approximation: **Feasible**, $\bar{N}_{GPA,Y,B} = 0$ (Figure 8a)



(b) Extrapolation and Bayes Rule (3): **Feasible**, $\bar{N}_{GPA,Y,B} > \infty$



(c) **Feasible**, $\bar{N}_{GPA,Y,B} = 10$ (Figure 8b)



(d) **Feasible**, $\bar{N}_{GPA,Y,B} = 50$ (Figure 8c)

Figure 9: $P(Q2 > 0 \mid GPA(\%), Y, B)$

4 $P(S>0 \mid GPA(\%), Y, Q2>0, B=b)$

The second Bayes Rule:

$$P(S>0 \mid GPA(\%), Y, Q2>0, B=b) = \frac{P(Y \mid S>0, GPA(\%), Q2>0, B=b) \times P(S>0 \mid GPA(\%), Q2>0, B=b)}{P(Y \mid GPA(\%), Q2>0, B=b)} \tag{10}$$

So now, in the feasible $GPA >= GPA_{cutoff}$ & $Q2>0$ sub-sample, we change to use local approximation for each object of interest if $N_{GPA,Y,Q2>0} \geq \bar{N}_{GPA,Y,B,Q2>0}$, where $N_{GPA,Y,B,Q2>0}$ is the size of each $(GPA(\%), Y)$ cell for a specific background B.

1. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} > \infty$, i.e., all from extrapolation using Bayes Rule (10)
2. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} = 0$, i.e., local approximation without using Bayes Rule (10), but for the $GPA >= GPA_{cutoff}$ sample, since in real data we only observe Y for those who were admitted
3. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} = 100$, i.e., local approximation for the feasible $GPA >= GPA_{cutoff}$ & $Q2>0$ sub-sample if $N_{GPA,Y,B,Q2>0} \geq 100$, Bayes Rule (10) from extrapolation if $N_{GPA,Y,B,Q2>0} < 100$ or $GPA < GPA_{cutoff}$ & $Q2>0$

4.1 Feasible PMF from Local Approximation (Figure 12a)

The empirical PMF approach counts the number of occurrences where $S>0$ and divides it by the total number of occurrences in each (GPA, Y) cell for the $Q2>0$ and $B=b$ sub-sample. See Figure 12a.

4.2 Feasible from Extrapolation by Bayes Rule (Figure 12b)

Remember Bayes Rule (10):

$$P(S>0 \mid GPA(\%), Y, Q2>0, B=b) = \frac{P(Y \mid S>0, GPA(\%), Q2>0, B=b) \times P(S>0 \mid GPA(\%), Q2>0, B=b)}{P(Y \mid GPA(\%), Q2>0, B=b)}$$

1. $P(Y \mid S>0, GPA(\%), Q2>0, B=b)$ from Extrapolation: Figure 10a that comes from Equation (2) using the $S>0$ and $Q2>0$ sample:

$$\begin{aligned}
\mathbf{Y_{GPA,s,b,Q2>0}} &= \underbrace{\alpha_2 + \beta_2 \times GPA(\%) + \gamma_2 \times s(\%) + \gamma_3 \times b}_{\hat{\mathbf{Y}}_{\mathbf{GPA,s,b,Q2>0}}} + \epsilon_2 \\
&= 9.12 - 0.04 \times GPA(\%) + 2.33 \times s(\%) + 0.3 \times b + \epsilon_2 \\
&\text{where } \boldsymbol{\epsilon_2 \sim N(0, 0.703)}
\end{aligned}$$

In the $S>0$ and $Q2>0$ subsample, for each GPA (%) bin, we assume

$$Y_{GPA,S>0,b,Q2>0} \sim N\left(\hat{Y}_{GPA,S>0,b,Q2>0}, \text{std}\left(\epsilon_2\right)\right) \tag{11}$$

then

$$P\left(Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, S>0, Q2>0\right) = \Phi\left(\frac{\bar{Y}_{\text{bin}} - \mu_2'}{\sigma_2}\right) - \Phi\left(\frac{\underline{Y}_{\text{bin}} - \mu_2'}{\sigma_2}\right) \tag{12}$$

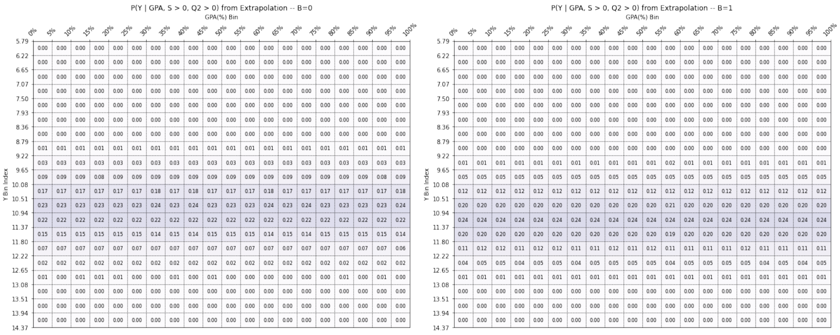
where

- $\boldsymbol{\mu_2'}$: local predicted mean: $\hat{Y}_{GPA,S>0,b,Q2>0}$, which is the average of $\hat{Y}_{GPA,S,b,Q2>0}$ from Equation (2) for each $GPA(\%)$ bin.
- $\boldsymbol{\sigma_2}$: global standard deviation of ϵ_2 estimated from Equation (2) using the observed sample above the Score cutoff.

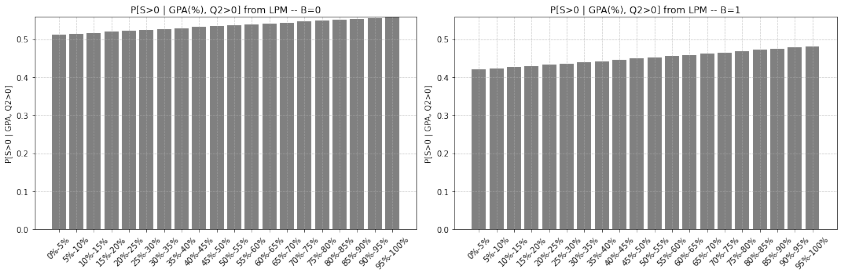
2. $P(S>0 \mid GPA(\%), Q2>0, B=b)$ from LPM: linear probability model (LPM) applied to the $Q2>0$ sub-sample for each background $B=b$. See Figure 10b.

$$1(S>0, B=0) = \underbrace{\alpha_{3,B=0} + \beta_{3,B=0} \times GPA(\%)}_{P(S>0|GPA,Q2>0,B=0)\ \textbf{from LPM}} + \epsilon_{3,B=0} = 0.51 + 0.05 \times GPA(\%) + \epsilon_{3,B=0} \tag{13}$$

$$\mathbb{1}(S>0, B=1) = \underbrace{\alpha_{3,B=1} + \beta_{3,B=1} \times GPA(\%)}_{P(S>0|GPA,Q2>0,B=1) \text{ from LPM}} + \epsilon_{3,B=1} = 0.42 + 0.06 \times GPA(\%) + \epsilon_{3,B=1} \quad (14)$$



(a) $P(Y|S>0, GPA(\%), Q2>0)$ from Extrapolation



(b) $P(S>0 \mid GPA(\%), Q2>0)$ from LPM

Figure 10: $P(Y \mid S>0, GPA(\%), Q2>0, B)$ and $P(S>0 \mid GPA(\%), Q2>0, B=b)$ from Extrapolation and LPM

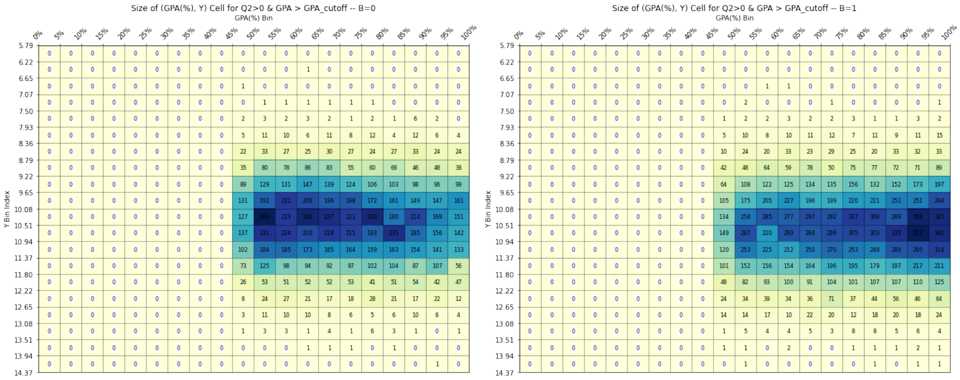
3. $P(Y \mid GPA(\%), Q2>0, B=b)$ from extrapolation: same as Figure 5.

4. $P(S>0 \mid GPA, Y, Q2>0)$ from Bayes Rule (10) where the RHS items all come from either extrapolations or LPM. See Figure 12b.

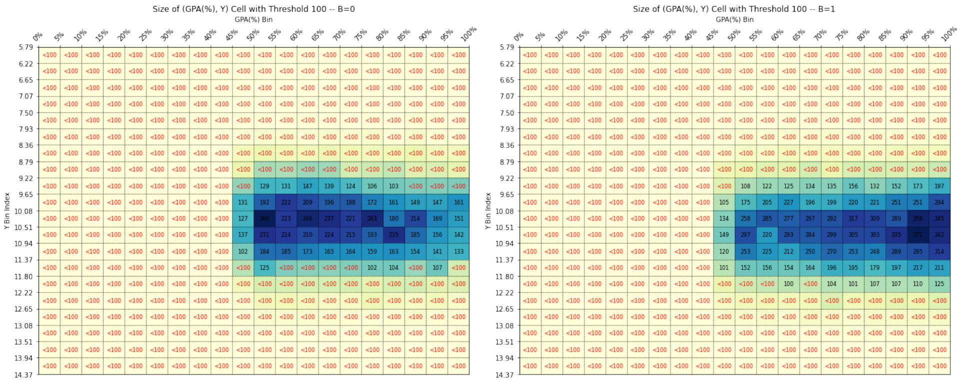
4.3 Feasible and Size Threshold 100 (Figure 12c)

Use local approximation for the feasible $GPA >= GPA_{cutoff}$ & $Q2>0$ sub-sample if $N_{GPA,Y,B,Q2>0} \geq 100$; use the Bayes Rule (10) from extrapolation if $N_{GPA,Y,B,Q2>0} < 100$ or $GPA < GPA_{cutoff}$ & $Q2>0$. See Figure 12c.

4.4 Cell Size for GPA vs. Y when $Q2 > 0$



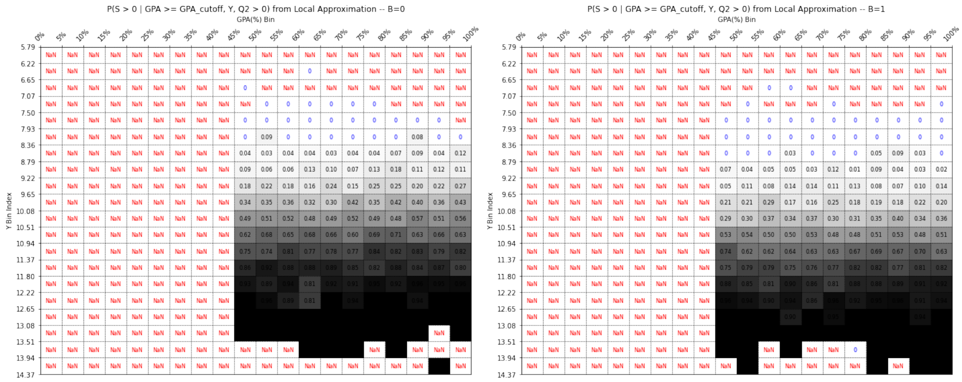
(a) $GPA >= GPA_{cutoff}$ when $Q2 > 0$: **Feasible**



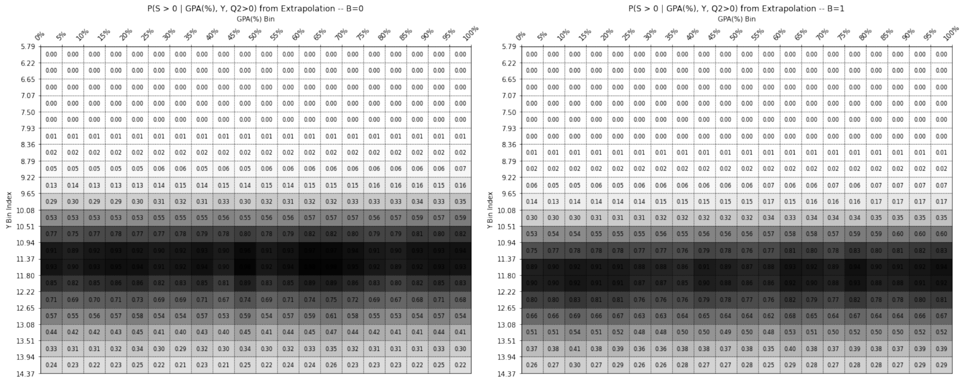
(b) $\bar{N}_{GPA,Y,B,Q2>0} = 100$ and $GPA >= GPA_{cutoff}$: **Feasible**

Figure 11: Size of $(GPA(\%), Y)$ Cell for the $Q2 > 0$ Sub-sample

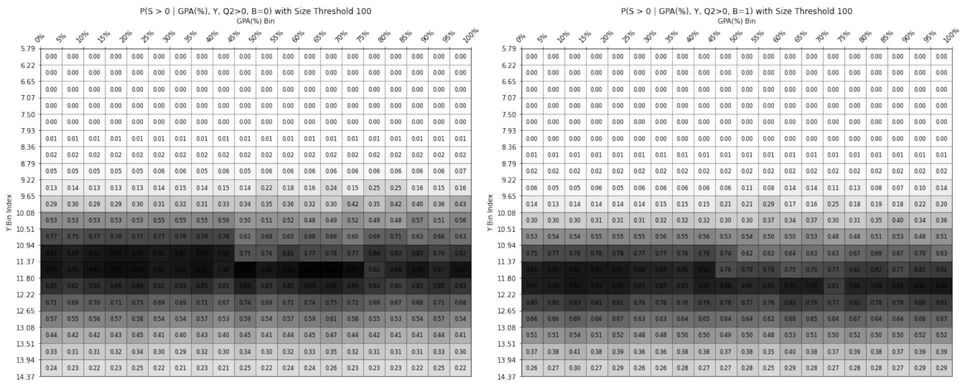
4.5 $P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b)$



(a) Local Approximation: **Feasible**, $\bar{N}_{GPA,Y,B,Q2>0} = 0$ (Figure 11a)



(b) Extrapolation and Bayes Rule (10): **Feasible**, $\bar{N}_{GPA,Y,B,Q2>0} > \infty$

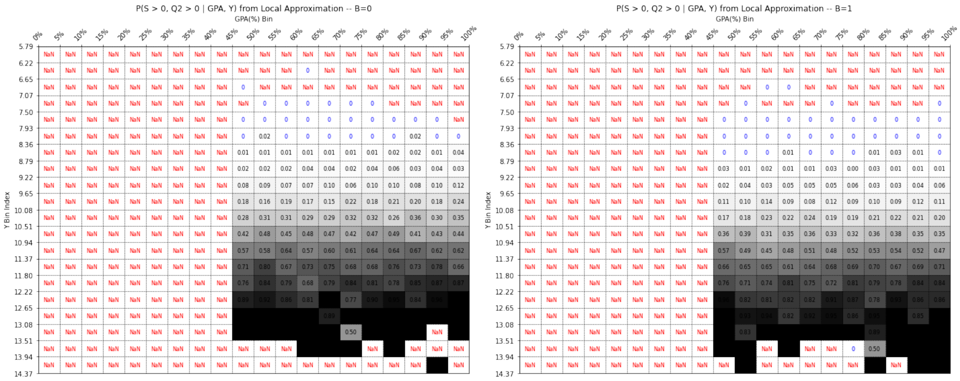


(c) **Feasible**, $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 11b)

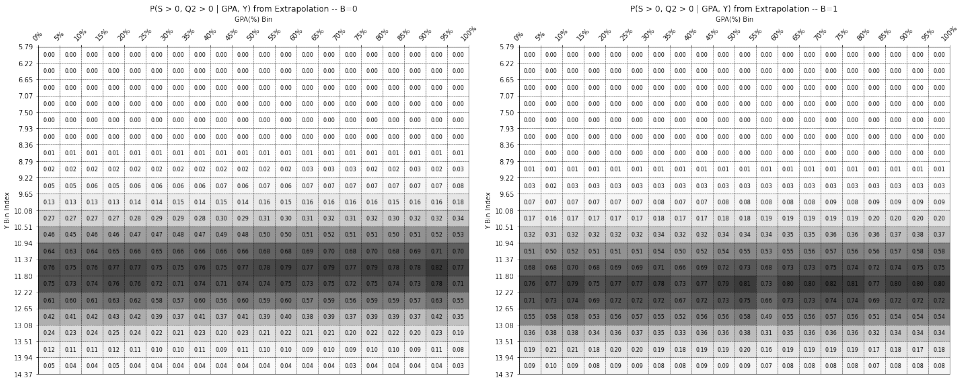
Figure 12: $P(S > 0 | GPA(\%), Y, Q2 > 0, B)$

5 $P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b)$

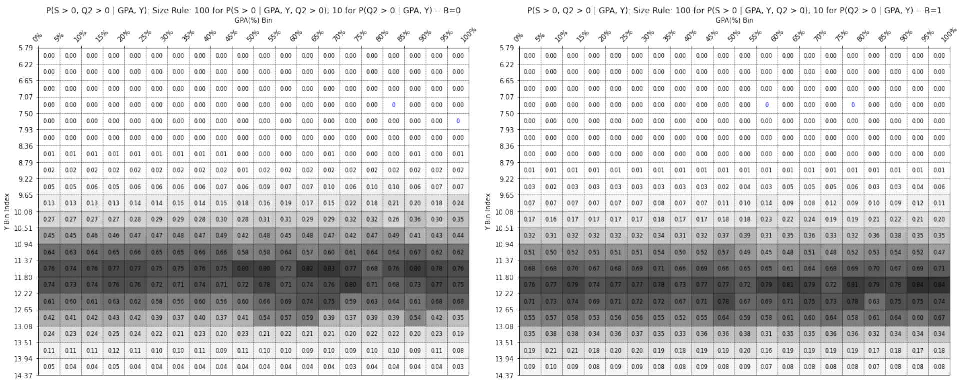
$$P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b) = P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) \times P(Q2 > 0 \mid GPA(\%), Y, B = b) \tag{15}$$



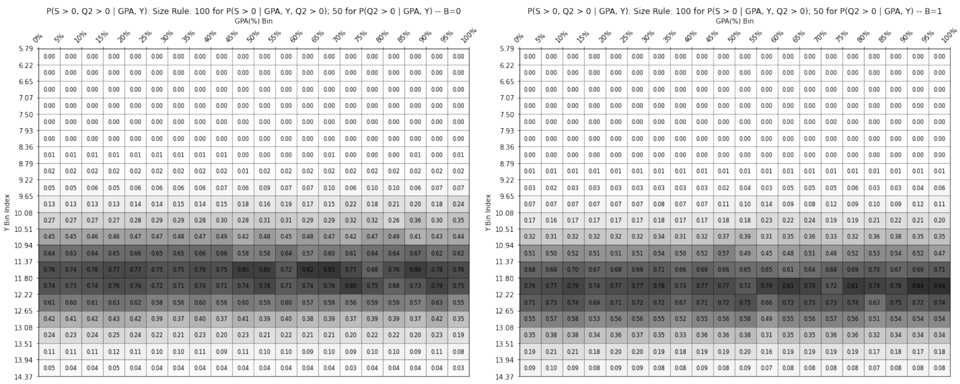
(a) Local Approximations: **Feasible** (Figure 9a*Figure 12a)



(b) Extrapolations and Bayes Rules: **Feasible**, (Figure 9b*Figure 12b)



(c) **Feasible**, $\bar{N}_{GPA,Y,B} = 10$ and $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 9c*Figure12c)



(d) **Feasible**, $\bar{N}_{GPA,Y,B} = 50$ and $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 9d*Figure12c)

Figure 13: $P(S > 0, Q2 > 0 \mid GPA(\%), Y, B)$

6 Appendix

6.1 Descriptive Statistics of Random Seed 0 Sample

Table 1. Summary Statistics of Different Groups

Groups	Variables	count	mean	std	min	25%	50%	75%	max
Population (Applied to Q1)	Background	100000	0.5	0.5	0	0	1	1	1
	GPA	100000	9.01	1.01	6	8.32	9.01	9.69	13
	Applied_Q1	100000	1	0	1	1	1	1	1
	Admitted_Q1	100000	0.53	0.5	0	0	1	1	1
	GPA(%)_ApplyQ1	100000	0.5	0.29	0	0.25	0.5	0.75	1
	Y	100000	10.15	1.01	5.79	9.48	10.16	10.84	14.37
	Q2	100000	0.13	1.01	-4.71	-0.55	0.13	0.81	4.12
	S	100000	-0.02	1	-4.15	-0.7	-0.02	0.65	4.27
	Applied_Q2	100000	0.55	0.5	0	0	1	1	1
	Admitted_Q2	100000	0.27	0.44	0	0	0	1	1
	S(%)_ApplyQ2	55121	0.5	0.29	0	0.25	0.5	0.75	1
GPA>=GPA Cutoff (Addmitted to Q1)	Background	52828	0.57	0.49	0	0	1	1	1
	GPA	52828	9.77	0.63	8.93	9.28	9.65	10.14	13
	Applied_Q1	52828	1	0	1	1	1	1	1
	Admitted_Q1	52828	1	0	1	1	1	1	1
	GPA(%)_ApplyQ1	52828	0.74	0.15	0.47	0.6	0.74	0.87	1
	Y	52828	10.19	1.01	6.26	9.52	10.2	10.87	14.37
	Q2	52828	0.17	1.01	-3.89	-0.51	0.18	0.85	4.11
	S	52828	0	1	-4.01	-0.67	0	0.68	4.2
	Applied_Q2	52828	0.57	0.5	0	0	1	1	1
	Admitted_Q2	52828	0.28	0.45	0	0	0	1	1
	S(%)_ApplyQ2	30125	0.51	0.29	0	0.26	0.51	0.76	1
GPA<GPA Cutoff (Not Addmitted to Q1)	Background	47172	0.42	0.49	0	0	0	1	1
	GPA	47172	8.15	0.59	6	7.81	8.27	8.62	8.93
	Applied_Q1	47172	1	0	1	1	1	1	1
	Admitted_Q1	47172	0	0	0	0	0	0	0
	GPA(%)_ApplyQ1	47172	0.24	0.14	0	0.12	0.24	0.35	0.47
	Y	47172	10.11	1.01	5.79	9.43	10.11	10.79	14.33
	Q2	47172	0.08	1	-4.71	-0.6	0.08	0.75	4.12
	S	47172	-0.04	1	-4.15	-0.72	-0.04	0.63	4.27
	Applied_Q2	47172	0.53	0.5	0	0	1	1	1
	Admitted_Q2	47172	0.26	0.44	0	0	0	1	1
	S(%)_ApplyQ2	24996	0.49	0.29	0	0.24	0.49	0.74	1
Q2>0 (Applied to Q2)	Background	55121	0.52	0.5	0	0	1	1	1
	GPA	55121	9.05	1.01	6	8.36	9.05	9.73	13
	Applied_Q1	55121	1	0	1	1	1	1	1
	Admitted_Q1	55121	0.55	0.5	0	0	1	1	1
	GPA(%)_ApplyQ1	55121	0.51	0.29	0	0.26	0.52	0.76	1
	Y	55121	10.52	0.92	6.63	9.9	10.52	11.14	14.37
	Q2	55121	0.85	0.63	0	0.35	0.73	1.22	4.12
	S	55121	-0.02	1	-4.15	-0.7	-0.02	0.66	4.27
	Applied_Q2	55121	1	0	1	1	1	1	1
	Admitted_Q2	55121	0.49	0.5	0	0	0	1	1
	S(%)_ApplyQ2	55121	0.5	0.29	0	0.25	0.5	0.75	1
Q2<=0 (Not Applied to Q2)	Background	44879	0.47	0.5	0	0	0	1	1
	GPA	44879	8.96	1.01	6	8.27	8.95	9.64	13
	Applied_Q1	44879	1	0	1	1	1	1	1
	Admitted_Q1	44879	0.51	0.5	0	0	1	1	1
	GPA(%)_ApplyQ1	44879	0.49	0.29	0	0.23	0.48	0.73	1
	Y	44879	9.7	0.92	5.79	9.09	9.71	10.32	13.56
	Q2	44879	-0.76	0.59	-4.71	-1.09	-0.64	-0.3	0
	S	44879	-0.02	1	-3.93	-0.7	-0.02	0.65	4.2
	Applied_Q2	44879	0	0	0	0	0	0	0
	Admitted_Q2	44879	0	0	0	0	0	0	0
	S(%)_ApplyQ2	0	0	0	0	0	0	0	0
S>0 & Q2>0 (Addmitted to Q2)	Background	27115	0.48	0.5	0	0	0	1	1
	GPA	27115	9.06	1.01	6	8.38	9.06	9.75	13
	Applied_Q1	27115	1	0	1	1	1	1	1
	Admitted_Q1	27115	0.55	0.5	0	0	1	1	1
	GPA(%)_ApplyQ1	27115	0.52	0.29	0	0.27	0.52	0.77	1
	Y	27115	11	0.79	8.02	10.46	10.99	11.52	14.37
	Q2	27115	0.85	0.63	0	0.35	0.73	1.22	4.12
	S	27115	0.79	0.6	0	0.32	0.67	1.14	4.27
	Applied_Q2	27115	1	0	1	1	1	1	1
	Admitted_Q2	27115	1	0	1	1	1	1	1
	S(%)_ApplyQ2	27115	0.75	0.14	0.51	0.63	0.75	0.88	1
GPA<GPA Cutoff & S<=0 (Not Addmitted to Q1/Q2)	Background	24375	0.46	0.5	0	0	0	1	1
	GPA	24375	8.14	0.59	6	7.81	8.26	8.61	8.93
	Applied_Q1	24375	1	0	1	1	1	1	1
	Admitted_Q1	24375	0	0	0	0	0	0	0
	GPA(%)_ApplyQ1	24375	0.23	0.14	0	0.12	0.23	0.35	0.47
	Y	24375	9.65	0.89	5.79	9.05	9.66	10.26	12.74
	Q2	24375	0.07	1.01	-4.1	-0.6	0.07	0.75	3.87
	S	24375	-0.82	0.61	-4.15	-1.18	-0.7	-0.33	0
	Applied_Q2	24375	0.53	0.5	0	0	1	1	1
	Admitted_Q2	24375	0	0	0	0	0	0	0
	S(%)_ApplyQ2	12899	0.25	0.15	0	0.12	0.25	0.38	0.51

Figure 14: Descriptive Statistics

6.2 Coefficients of Extrapolation Equations

	Equtaion (1): Q1 Admitted	Equtaion (2): Q2 Admitted	Equtaion (3): Background 0	Equation (3): Background 1	Equation (3): Pool Background
background	0.20	0.30	NaN	NaN	-0.08
const	10.01	9.12	0.51	0.42	0.51
percentile_GPA_applyQ1	0.09	-0.04	0.05	0.06	0.06
percentile_S_applyQ2	NaN	2.33	NaN	NaN	NaN

Figure 15: Coefficients of Equations (1)-(3)