

Appendix: Simulation

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1 Extrapolation Model

- Extrapolation 1 based on GPA: want to have Y_{GPA} for all scores¹.

$$\begin{aligned} Y_{GPA,b} &= \underbrace{\alpha_1 + \beta_1 \times GPA(\%) + \gamma \times b}_{\hat{Y}_{GPA,b}} + \epsilon_1 \\ &= 10.01 + 0.09 \times GPA(\%) + 0.2 \times b + \epsilon_1 \\ &\text{where } \epsilon_1 \sim N(0, 1.002) \end{aligned} \tag{1}$$

use the estimated relationship (**estimated for $GPA > \text{cutoff}$**) to extrapolate Y_{GPA} for $GPA < \text{cutoff}$.

- Extrapolation 2 based on Quota 2 Score: want to have $Y_{s,GPA,Q2}$ for all scores. Assuming that s is continuous even below the cutoff, we can again assume a linear relationship in quota 2 score and GPA so that:

$$\begin{aligned} Y_{GPA,s,b,Q2>0} &= \underbrace{\alpha_2 + \beta_2 \times GPA(\%) + \gamma_2 \times s(\%) + \gamma_3 \times b}_{\hat{Y}_{GPA,s,b,Q2>0}} + \epsilon_2 \\ &= 9.12 - 0.04 \times GPA(\%) + 2.33 \times s(\%) + 0.3 \times b + \epsilon_2 \\ &\text{where } \epsilon_2 \sim N(0, 0.703) \end{aligned} \tag{2}$$

use the estimated relationship (**estimated for $s > \text{cutoff}$**) to extrapolate $Y_{s,GPA,Q2}$ for $s < \text{cutoff}$.

- **Test if ϵ_1 and ϵ_2 are normally distributed:** Figure 1 show distributions and Kolmogorov-Smirnov tests of residuals ϵ_1 and ϵ_2 :

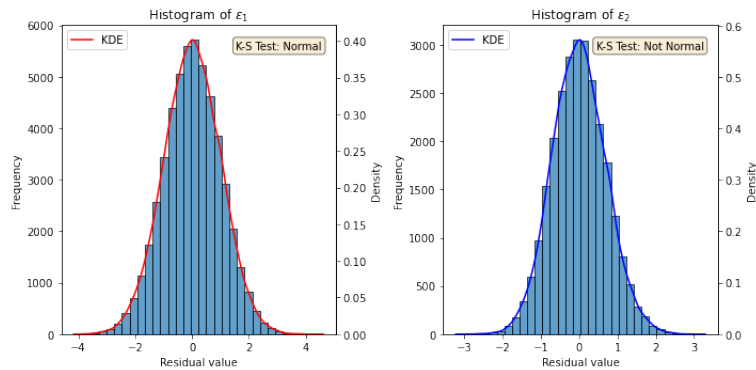
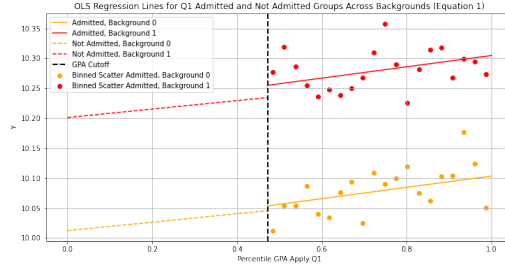


Figure 1: Distribution of ϵ_1 and ϵ_2 from Equations (1) and (2)

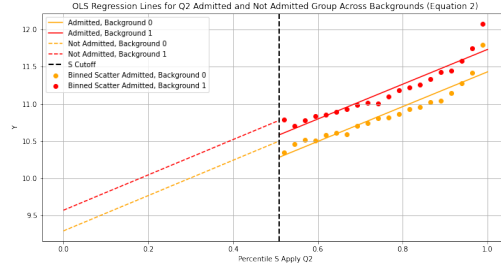
1.1 Extrapolation Plots

Note that in Figure 2a and 2b, with real data, we would not be able to fit the below-admission cutoff lines as we cannot observe the outcome of the not-admitted students.

¹Since in the empirical data we observe that the higher the GPA, the larger the earning potential.

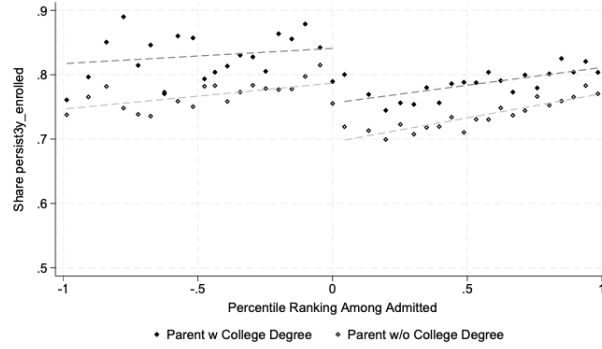


(a) OLS Regression Lines of Equation (1)



(b) OLS Regression Lines of Equation (2) using $Q2 > 0$ Sub-sample

Figure 2: Extrapolation Plots for Each Background from Equation (1) and (2)



(b) Persistence, Year 3

Figure 3: Pooled Fitted lines (Figure A9b) for Each Background in Real Data

2 Simulation Model²

Whole Sample: $N = 100000$ (Applied_Q1)

Parameters: b , GPA, and ϵ

Edu Background: $b \sim \text{Bernoulli}(p = 0.5)$

$GPA \sim \mathcal{N}(8.8, \text{sd} = 1)$ if $b = 0$, censor at 6 and 13

$GPA \sim \mathcal{N}(9.2, \text{sd} = 1)$ if $b = 1$, censor at 6 and 13

$$\epsilon_Y, \epsilon_{Q2}, \epsilon_S \sim \mathcal{N}(0, \Sigma) \text{ where } \Sigma = \begin{bmatrix} 1 & 0.5 & 0.6 \\ 0.5 & 1 & 0 \\ 0.6 & 0 & 1 \end{bmatrix}$$

$$\alpha_Y = 10, \alpha_Y^1 = 0.1, \alpha_Y^2 = 0.0, \alpha_Y^3 = 0.2$$

$$\beta_{Q2} = 0, \beta_{Q2}^1 = 0.2, \beta_{Q2}^2 = -0.05, \beta_{Q2}^3 = 0.1$$

$$\gamma_S = 0, \gamma_S^1 = 0.15, \gamma_S^2 = 0.0, \gamma_S^3 = -0.2$$

Simulate: $Y, Q2$, and S

$$Y = \alpha_Y + \alpha_Y^1 \times GPA(\%) + \alpha_Y^2 \times GPA(\%)^2 + \alpha_Y^3 \times b + \epsilon_Y$$

$$Q2 = \beta_{Q2} + \beta_{Q2}^1 \times GPA(\%) + \beta_{Q2}^2 \times GPA(\%)^2 + \beta_{Q2}^3 \times b + \epsilon_{Q2}$$

$$S = \gamma_S + \gamma_S^1 \times GPA(\%) + \gamma_S^2 \times GPA(\%)^2 + \gamma_S^3 \times b + \epsilon_S$$

GPA Cutoff: $GPA_cutoff = 8.93$

Admission:

$$\text{Admitted_Q1} = \begin{cases} 1 & \text{if } GPA \geq GPA_cutoff \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Applied_Q2} = \begin{cases} 1 & \text{if } Q2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Admitted_Q2} = \begin{cases} 1 & \text{if } S > 0 \text{ and } \text{Applied_Q2} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Objects of Interest:

For each background $b \in \{0, 1\}$:

$$P(Q2 > 0 \mid GPA(\%), Y, B = b) = \frac{P(Y \mid Q2 > 0, GPA(\%), B = b) \times P(Q2 > 0 \mid GPA(\%), B = b)}{P(Y \mid GPA(\%), B = b)}$$

$$P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) = \frac{P(Y \mid S > 0, GPA(\%), Q2 > 0, B = b) \times P(S > 0 \mid GPA(\%), Q2 > 0, B = b)}{P(Y \mid GPA(\%), Q2 > 0, B = b)}$$

$$P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b) = P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) \times P(Q2 > 0 \mid GPA(\%), Y, B = b)$$

²See Appendix for the descriptive statistics.

3 $P(Q2 > 0 \mid GPA(\%), Y, B = b)$

The first Bayes Rule:

$$P(Q2 > 0 \mid GPA(\%), Y, B = b) = \frac{P(Y \mid Q2 > 0, GPA(\%), B = b) \times P(Q2 > 0 \mid GPA(\%), B = b)}{P(Y \mid GPA(\%), B = b)} \quad (3)$$

Since in real data we only observe Y for those who were admitted, in the feasible $GPA \geq GPA_{cutoff}$ sub-sample, use local approximation instead of Bayes Rule (3) for each object of interest if $N_{GPA,Y,B} \geq \bar{N}_{GPA,Y,B}$, where $N_{GPA,Y,B}$ is the size of each $(GPA(\%), Y)$ cell for a specific background B .

1. Feasible: $\bar{N}_{GPA,Y,B} > \infty$, i.e., all from extrapolation using Bayes Rule (3)
2. Feasible: $\bar{N}_{GPA,Y,B} = 0$, i.e., local approximation without using Bayes Rule (3), but for the $GPA \geq GPA_{cutoff}$ sample, since in real data we only observe Y for those who were admitted.
3. Feasible: $\bar{N}_{GPA,Y,B} = 10$, i.e., local approximation for the $GPA \geq GPA_{cutoff}$ sample if $N_{GPA,Y,B} \geq 10$; Bayes Rule (3) from extrapolation if $N_{GPA,Y,B} < 10$ or $GPA < GPA_{cutoff}$.
4. Feasible: $\bar{N}_{GPA,Y,B} = 50$, i.e., local approximation for the $GPA \geq GPA_{cutoff}$ sample if $N_{GPA,Y,B} \geq 50$; Bayes Rule (3) from extrapolation if $N_{GPA,Y,B} < 50$ or $GPA < GPA_{cutoff}$.

3.1 $P(Y \mid Q2 > 0, GPA(\%), B = b)$

- **From Extrapolation: Equation (2) and S Integration. (Figure 5)**

Compute

$$\begin{aligned} & P[Y \in Y_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, B = b] \\ &= \sum_s P[Y \in Y_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, s(\%) \in S(\%)_{bin}, B = b] \\ & \quad * P[s(\%) \in S(\%)_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, B = b] \end{aligned} \quad (4)$$

where

- $P[Y \in Y_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, s(\%) \in S(\%)_{bin}, B = b]$: for each $GPA(\%)$ and $S(\%)$ grid, assume

$$Y_{GPA,s,b,Q2>0} \sim N(\hat{Y}_{GPA,s,b,Q2>0}, std(\epsilon_2)) \quad (5)$$

then

$$P(Y \in Y_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, s(\%) \in S(\%)_{bin}) = \Phi\left(\frac{\bar{Y}_{bin} - \mu_2}{\sigma_2}\right) - \Phi\left(\frac{Y_{bin} - \mu_2}{\sigma_2}\right) \quad (6)$$

- * μ_2 : local predicted mean ($\hat{Y}_{GPA,s,b,Q2>0}$) from Equation (2) in this $GPA(\%)$ and $S(\%)$ grid
- * σ_2 : global standard deviation of ϵ_2 estimated from Equation (2) using the observed sample above the Score cutoff.
- $P[s(\%) \in S(\%)_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, Q2 > 0, B = b]$: PMF from local approximation, shown in Figure 4.

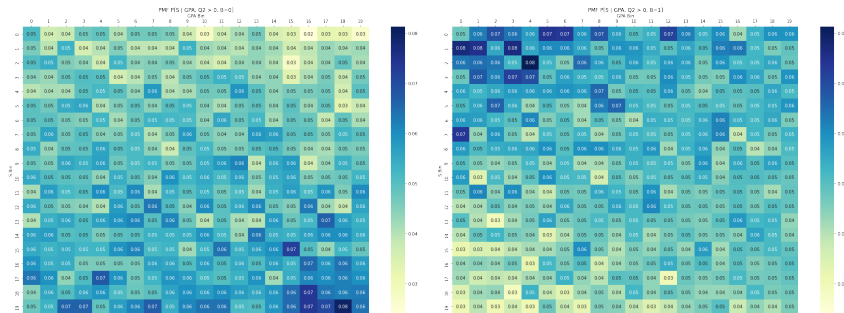


Figure 4: PMF $P[S(\%) | GPA(\%)_{bin}, Q2 > 0, B]$ from Local Approximation

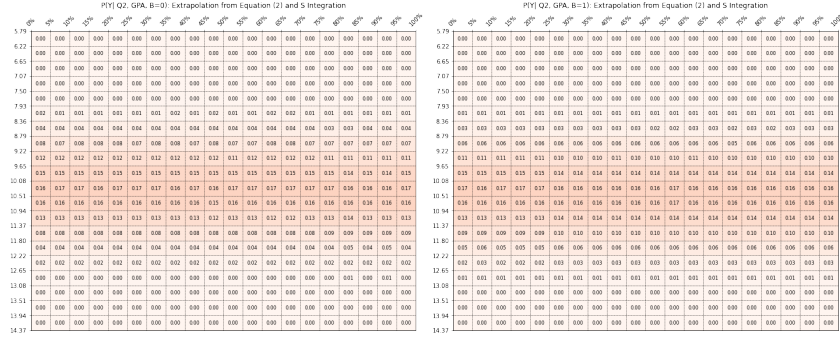


Figure 5: $P(Y|Q2 > 0, GPA, B)$ from Extrapolation

3.2 $P(Y | GPA(\%), B = b)$

- **From Extrapolation: Equation (1).** (Figure 6)

for each GPA(%) bin, assume

$$Y_{GPA,b} \sim N(\hat{Y}_{GPA,b}, std(\epsilon_2)) \quad (7)$$

then

$$P(Y \in Y_{bin} \mid GPA(\%) \in GPA(\%)_{bin}, B = b) = \Phi\left(\frac{\bar{Y}_{bin} - \mu_1}{\sigma_1}\right) - \Phi\left(\frac{Y_{bin} - \mu_1}{\sigma_1}\right) \quad (8)$$

- μ_1 : local predicted mean ($\hat{Y}_{GPA,b}$) from Equation (1) in this GPA(%) bin
- σ_1 : global standard deviation of ϵ_1 estimated from Equation (1) using the observed sample above the GPA cutoff.

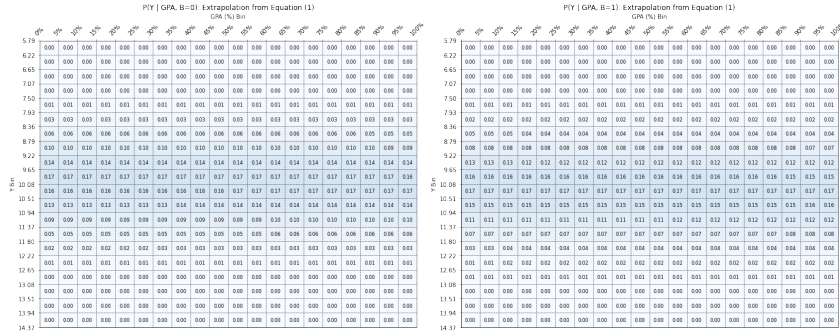


Figure 6: $P(Y|GPA, B)$ from Extrapolation

3.3 $P(Q2 > 0 \mid GPA(\%), B = b)$

The probability that a student applies to Q2 given their GPA and Background. See Figure 7.

$$P(Q2 > 0 \mid GPA(\%), B = b) = \frac{\text{Number of students who applied to Q2 in the GPA (\%) bin and Background group}}{\text{Total Number of students in the GPA (\%) bin and Background group}} \quad (9)$$

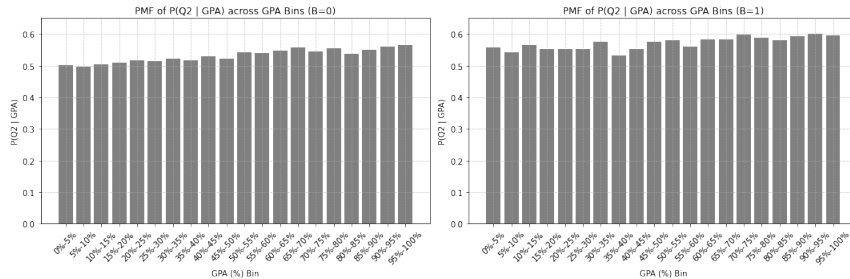
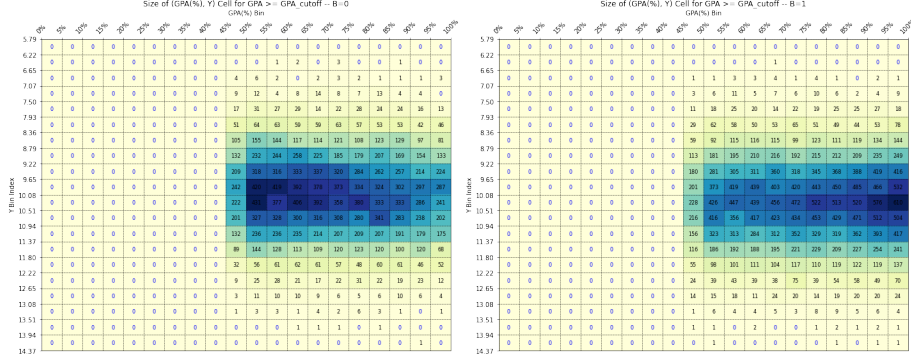
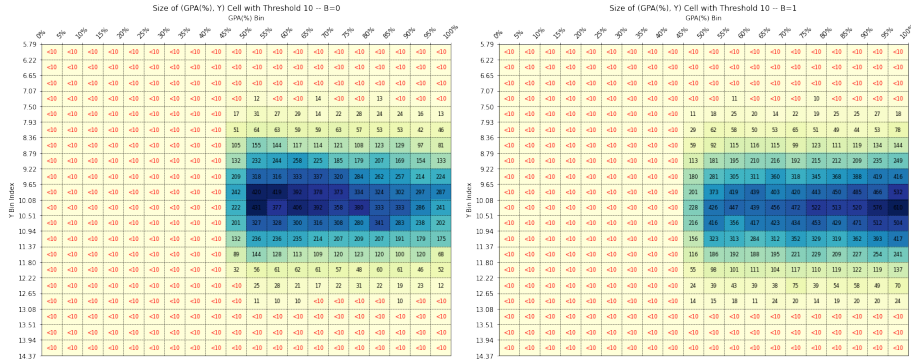


Figure 7: PMF $P(Q2 > 0 \mid GPA(\%), B)$ from Local Approximation

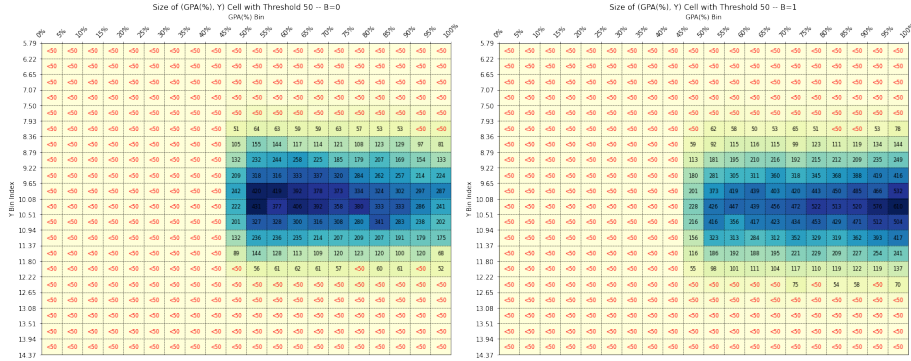
3.4 Cell Size for GPA vs. Y



(a) $GPA \geq GPA_{cut-off}$ Sub-sample: Feasible



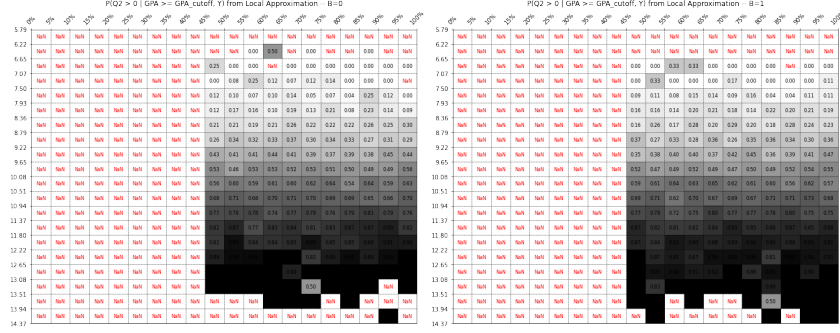
(b) $\bar{N}_{GPA,Y,B} = 10$ and $GPA \geq GPA_{cut-off}$: Feasible



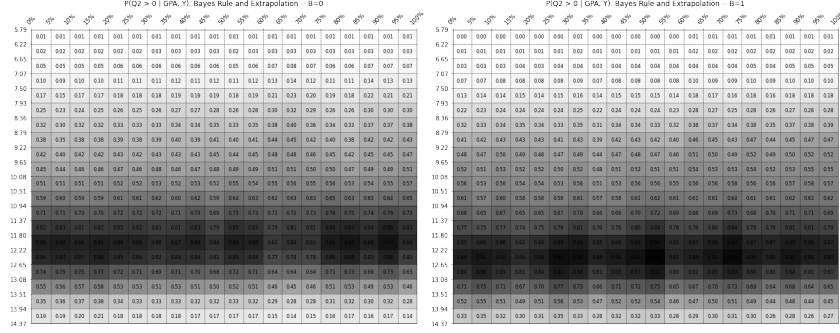
(c) $\bar{N}_{GPA,Y,B} = 50$ and $GPA \geq GPA_{cut-off}$: Feasible

Figure 8: Size of (GPA(%), Y) Cell

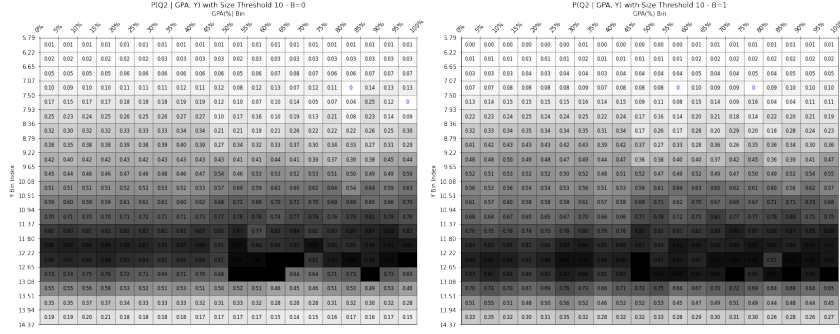
3.5 $P(Q2 > 0 \mid GPA(\%), Y, B = b)$



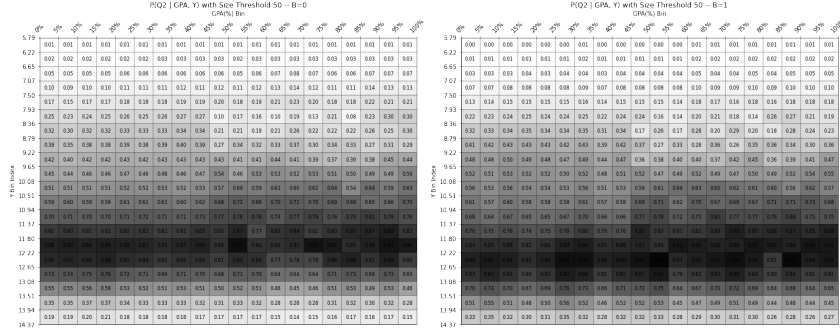
(a) Local Approximation: **Feasible**, $\bar{N}_{GPA,Y,B} = 0$ (Figure 8a)



(b) Extrapolation and Bayes Rule (3): **Feasible**, $\bar{N}_{GPA,Y,B} > \infty$



(c) **Feasible**, $\bar{N}_{GPA,Y,B} = 10$ (Figure 8b)



(d) **Feasible**, $\bar{N}_{GPA,Y,B} = 50$ (Figure 8c)

Figure 9: $P(Q2 > 0 \mid GPA(\%), Y, B)$

4 $P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b)$

The second Bayes Rule:

$$P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) = \frac{P(Y \mid S > 0, GPA(\%), Q2 > 0, B = b) \times P(S > 0 \mid GPA(\%), Q2 > 0, B = b)}{P(Y \mid GPA(\%), Q2 > 0, B = b)} \quad (10)$$

So now, in the feasible $GPA \geq GPA_{cutoff}$ & $Q2 > 0$ sub-sample, we change to use local approximation for each object of interest if $N_{GPA,Y,Q2>0} \geq \bar{N}_{GPA,Y,B,Q2>0}$, where $N_{GPA,Y,B,Q2>0}$ is the size of each $(GPA(\%), Y)$ cell for a specific background B .

1. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} > \infty$, i.e., all from extrapolation using Bayes Rule (10)
2. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} = 0$, i.e., local approximation without using Bayes Rule (10), but for the $GPA \geq GPA_{cutoff}$ sample, since in real data we only observe Y for those who were admitted
3. Feasible: $\bar{N}_{GPA,Y,B,Q2>0} = 100$, i.e., local approximation for the feasible $GPA \geq GPA_{cutoff}$ & $Q2 > 0$ sub-sample if $N_{GPA,Y,B,Q2>0} \geq 100$, Bayes Rule (10) from extrapolation if $N_{GPA,Y,B,Q2>0} < 100$ or $GPA < GPA_{cutoff}$ & $Q2 > 0$

4.1 Feasible PMF from Local Approximation (Figure 12a)

The empirical PMF approach counts the number of occurrences where $S > 0$ and divides it by the total number of occurrences in each (GPA, Y) cell for the $Q2 > 0$ and $B = b$ sub-sample. See Figure 12a.

4.2 Feasible from Extrapolation by Bayes Rule (Figure 12b)

Remember Bayes Rule (10):

$$P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) = \frac{P(Y \mid S > 0, GPA(\%), Q2 > 0, B = b) \times P(S > 0 \mid GPA(\%), Q2 > 0, B = b)}{P(Y \mid GPA(\%), Q2 > 0, B = b)}$$

1. $P(Y \mid S > 0, GPA(\%), Q2 > 0, B = b)$ from Extrapolation: Figure 10a that comes from Equation (2) using the $S > 0$ and $Q2 > 0$ sample:

$$\begin{aligned} Y_{GPA,s,b,Q2>0} &= \underbrace{\alpha_2 + \beta_2 \times GPA(\%) + \gamma_2 \times s(\%) + \gamma_3 \times b + \epsilon_2}_{\hat{Y}_{GPA,s,b,Q2>0}} \\ &= 9.12 - 0.04 \times GPA(\%) + 2.33 \times s(\%) + 0.3 \times b + \epsilon_2 \\ &\text{where } \epsilon_2 \sim N(0, 0.703) \end{aligned}$$

In the $S > 0$ and $Q2 > 0$ subsample, for each GPA (%) bin, we assume

$$Y_{GPA,S>0,b,Q2>0} \sim N\left(\hat{Y}_{GPA,S>0,b,Q2>0}, \text{std}(\epsilon_2)\right) \quad (11)$$

then

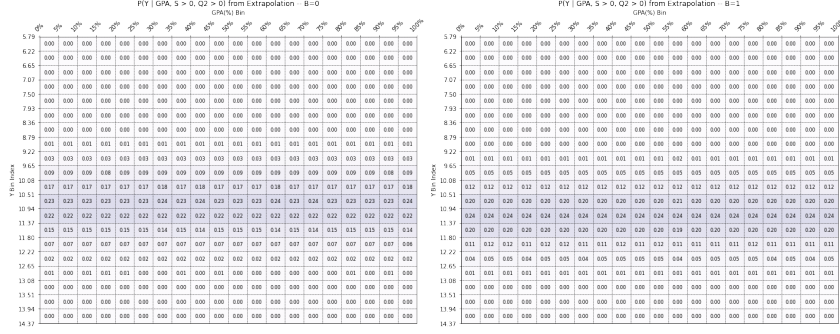
$$P(Y \in Y_{\text{bin}} \mid GPA(\%) \in GPA(\%)_{\text{bin}}, S > 0, Q2 > 0) = \Phi\left(\frac{\bar{Y}_{\text{bin}} - \mu'_2}{\sigma_2}\right) - \Phi\left(\frac{Y_{\text{bin}} - \mu'_2}{\sigma_2}\right) \quad (12)$$

where

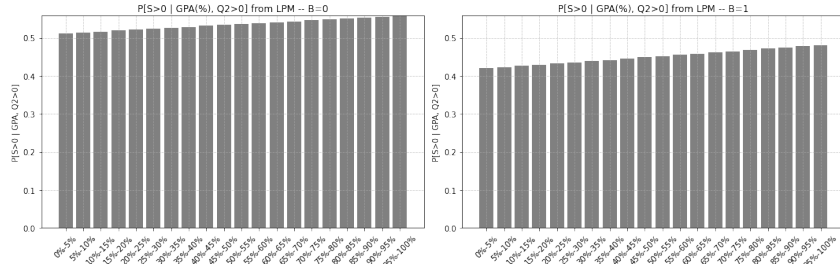
- μ'_2 : local predicted mean: $\hat{Y}_{GPA,S>0,b,Q2>0}$, which is the average of $\hat{Y}_{GPA,S,b,Q2>0}$ from Equation (2) for each $GPA(\%)$ bin.
 - σ_2 : global standard deviation of ϵ_2 estimated from Equation (2) using the observed sample above the Score cutoff.
2. $P(S > 0 \mid GPA(\%), Q2 > 0, B = b)$ from LPM: linear probability model (LPM) applied to the $Q2 > 0$ sub-sample for each background $B = b$. See Figure 10b.

$$1(S > 0, B = 0) = \underbrace{\alpha_{3,B=0} + \beta_{3,B=0} \times GPA(\%)}_{P(S>0|GPA,Q2>0,B=0) \text{ from LPM}} + \epsilon_{3,B=0} = 0.51 + 0.05 \times GPA(\%) + \epsilon_{3,B=0} \quad (13)$$

$$1(S > 0, B = 1) = \underbrace{\alpha_{3,B=1} + \beta_{3,B=1} \times GPA(\%)}_{P(S>0|GPA,Q2>0,B=1) \text{ from LPM}} + \epsilon_{3,B=1} = 0.42 + 0.06 \times GPA(\%) + \epsilon_{3,B=1} \quad (14)$$



(a) $P(Y | S > 0, GPA(\%), Q2 > 0)$ from Extrapolation



(b) $P(S > 0 | GPA(\%), Q2 > 0)$ from LPM

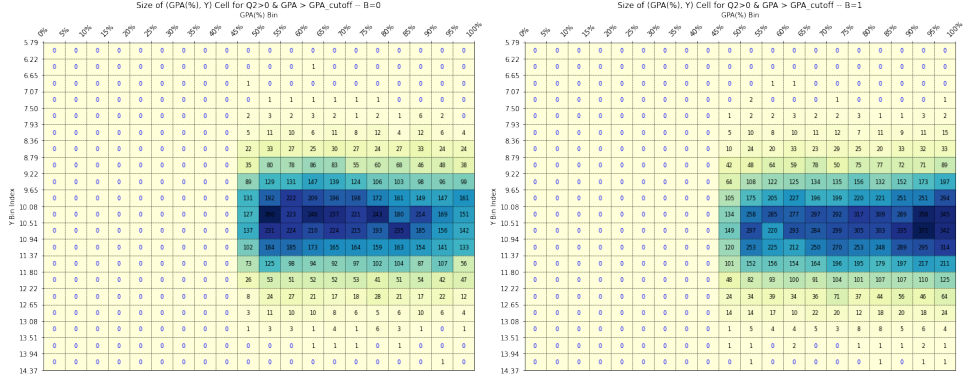
Figure 10: $P(Y | S > 0, GPA(\%), Q2 > 0, B)$ and $P(S > 0 | GPA(\%), Q2 > 0, B = b)$ from Extrapolation and LPM

3. $P(Y | GPA(\%), Q2 > 0, B = b)$ from extrapolation: same as Figure 5.
4. $P(S > 0 | GPA, Y, Q2 > 0)$ from Bayes Rule (10) where the RHS items all come from either extrapolations or LPM. See Figure 12b.

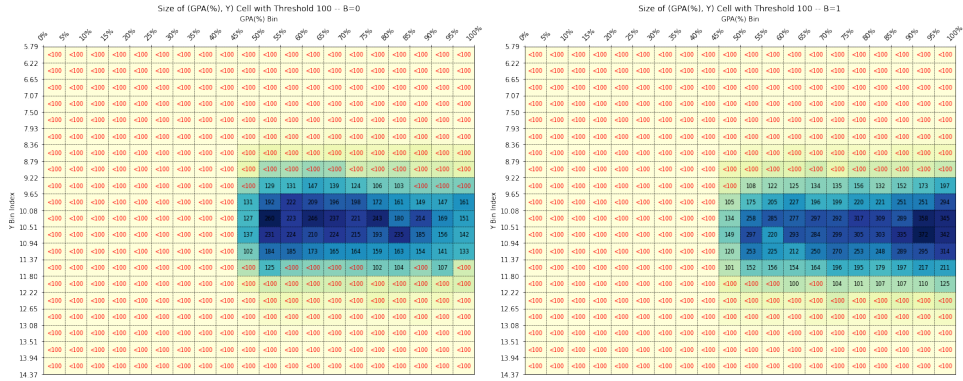
4.3 Feasible and Size Threshold 100 (Figure 12c)

Use local approximation for the feasible $GPA \geq GPA_{cutoff}$ & $Q2 > 0$ sub-sample if $N_{GPA,Y,B,Q2>0} \geq 100$; use the Bayes Rule (10) from extrapolation if $N_{GPA,Y,B,Q2>0} < 100$ or $GPA < GPA_{cutoff}$ & $Q2 > 0$. See Figure 12c.

4.4 Cell Size for GPA vs. Y when $Q2 > 0$



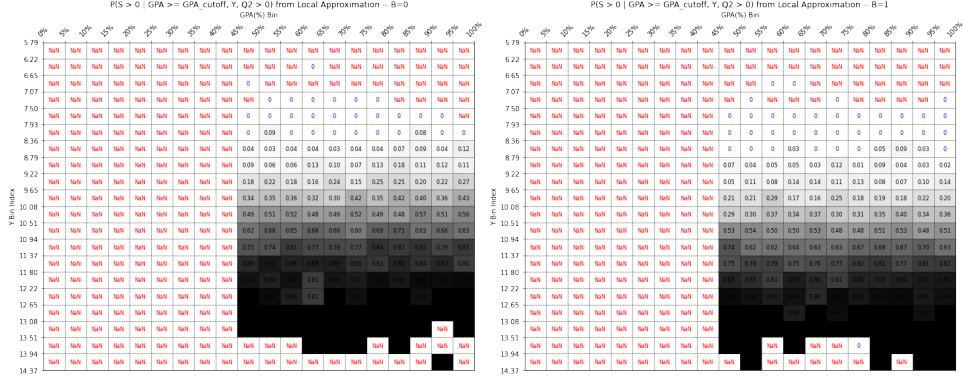
(a) $GPA \geq GPA_{cutoff}$ when $Q2 > 0$: Feasible



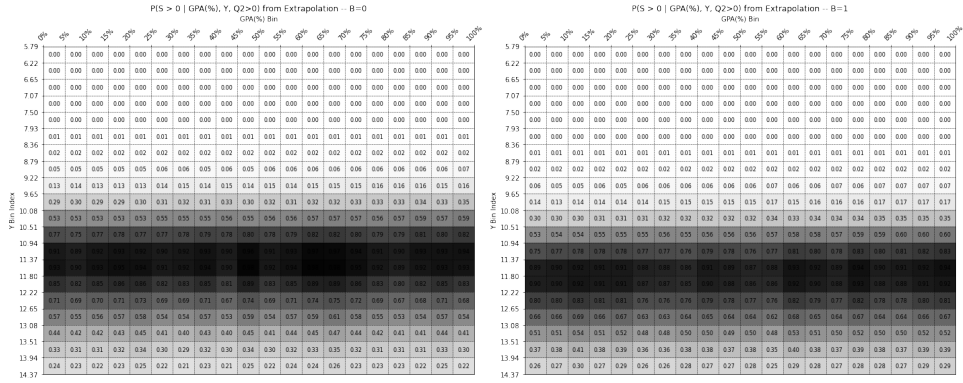
(b) $\bar{N}_{GPA,Y,B,Q2>0} = 100$ and $GPA \geq GPA_{cutoff}$: Feasible

Figure 11: Size of (GPA(%), Y) Cell for the $Q2 > 0$ Sub-sample

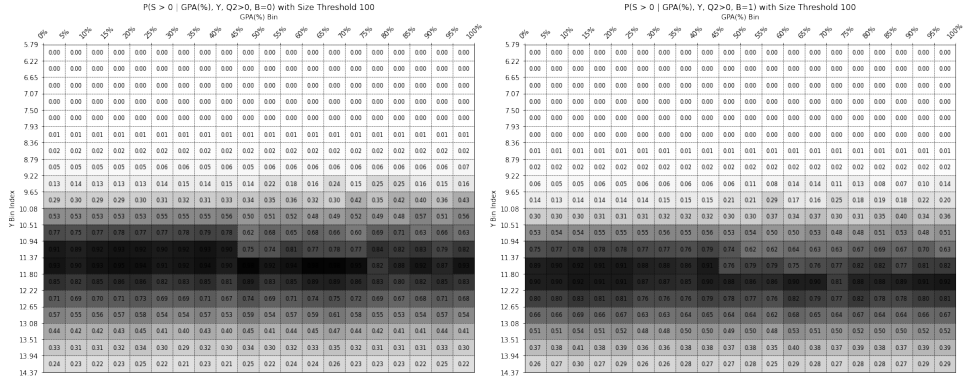
4.5 $P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b)$



(a) Local Approximation: Feasible, $\bar{N}_{GPA,Y,B,Q2>0} = 0$ (Figure 11a)



(b) Extrapolation and Bayes Rule (10): Feasible, $\bar{N}_{GPA,Y,B,Q2>0} > \infty$

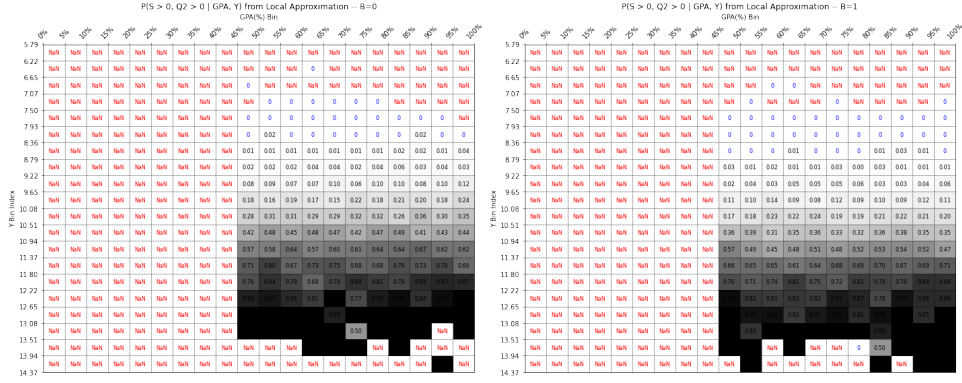


(c) Feasible, $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 11b)

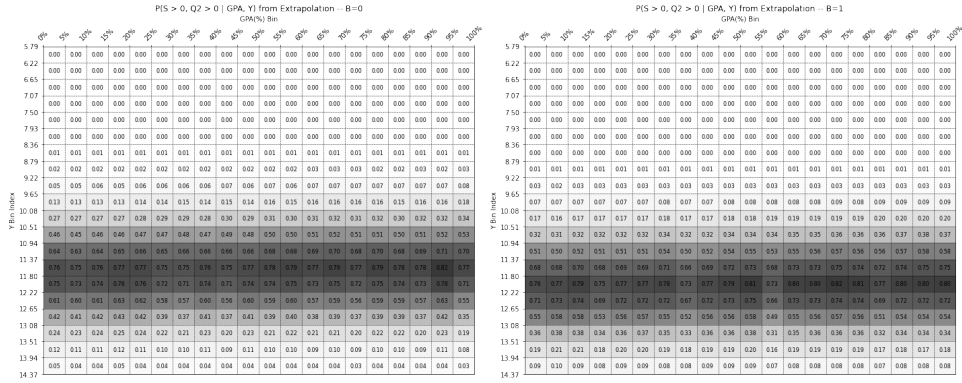
Figure 12: $P(S > 0 \mid GPA(\%), Y, Q2 > 0, B)$

5 $P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b)$

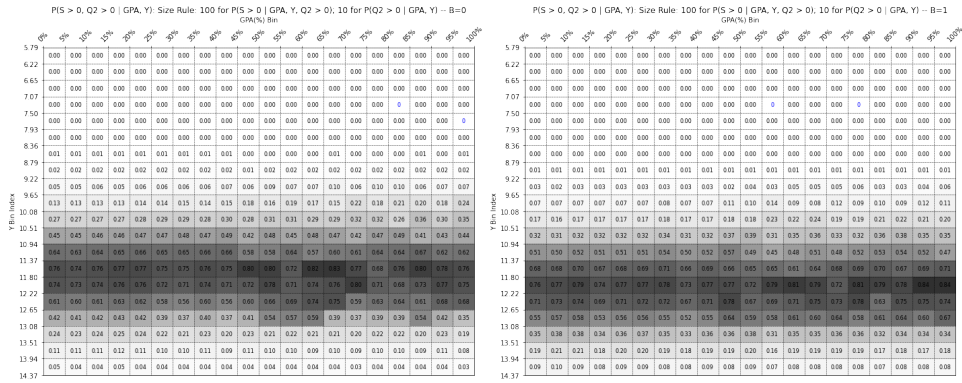
$$P(S > 0, Q2 > 0 \mid GPA(\%), Y, B = b) = P(S > 0 \mid GPA(\%), Y, Q2 > 0, B = b) \times P(Q2 > 0 \mid GPA(\%), Y, B = b) \quad (15)$$



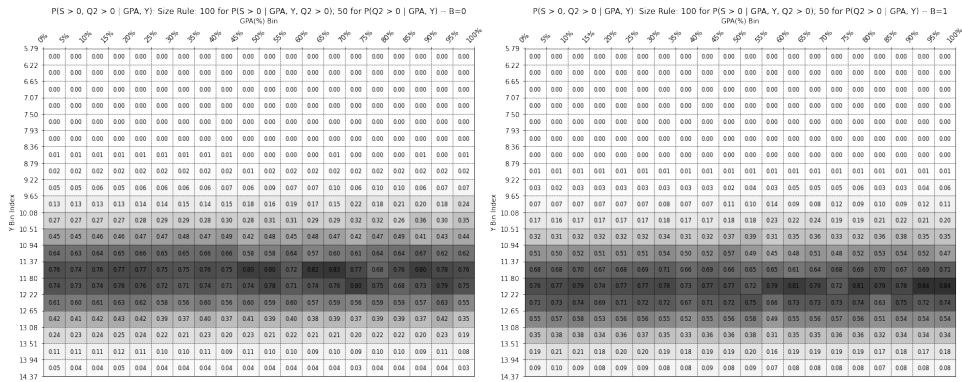
(a) Local Approximations: Feasible (Figure 9a*Figure 12a)



(b) Extrapolations and Bayes Rules: Feasible, (Figure 9b*Figure 12b)



(c) Feasible, $\bar{N}_{GPA,Y,B} = 10$ and $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 9c*Figure12c)



(d) Feasible, $\bar{N}_{GPA,Y,B} = 50$ and $\bar{N}_{GPA,Y,B,Q2>0} = 100$ (Figure 9d*Figure12c)

Figure 13: $P(S > 0, Q2 > 0 | GPA(\%), Y, B)$

6 Appendix

6.1 Descriptive Statistics of Random Seed 0 Sample

Table 1. Summary Statistics of Different Groups

| Groups | Variables | count | mean | std | min | 25% | 50% | 75% | max |
|--|----------------|--------|-------|------|-------|-------|-------|-------|-------|
| <i>Population (Applied to Q1)</i> | Background | 100000 | 0.5 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA | 100000 | 9.01 | 1.01 | 6 | 8.32 | 9.01 | 9.69 | 13 |
| | Applied_Q1 | 100000 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 100000 | 0.53 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA(%)_ApplyQ1 | 100000 | 0.5 | 0.29 | 0 | 0.25 | 0.5 | 0.75 | 1 |
| | Y | 100000 | 10.15 | 1.01 | 5.79 | 9.48 | 10.16 | 10.84 | 14.37 |
| | Q2 | 100000 | 0.13 | 1.01 | -4.71 | -0.55 | 0.13 | 0.81 | 4.12 |
| | S | 100000 | -0.02 | 1 | -4.15 | -0.7 | -0.02 | 0.65 | 4.27 |
| | Applied_Q2 | 100000 | 0.55 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | Admitted_Q2 | 100000 | 0.27 | 0.44 | 0 | 0 | 0 | 1 | 1 |
| | S(%)_ApplyQ2 | 55121 | 0.5 | 0.29 | 0 | 0.25 | 0.5 | 0.75 | 1 |
| <i>GPA >= GPA Cutoff (Admitted to Q1)</i> | Background | 52828 | 0.57 | 0.49 | 0 | 0 | 1 | 1 | 1 |
| | GPA | 52828 | 9.77 | 0.63 | 8.93 | 9.28 | 9.65 | 10.14 | 13 |
| | Applied_Q1 | 52828 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 52828 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | GPA(%)_ApplyQ1 | 52828 | 0.74 | 0.15 | 0.47 | 0.6 | 0.74 | 0.87 | 1 |
| | Y | 52828 | 10.19 | 1.01 | 6.26 | 9.52 | 10.2 | 10.87 | 14.37 |
| | Q2 | 52828 | 0.17 | 1.01 | -3.89 | -0.51 | 0.18 | 0.85 | 4.11 |
| | S | 52828 | 0 | 1 | -4.01 | -0.67 | 0 | 0.68 | 4.2 |
| | Applied_Q2 | 52828 | 0.57 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | Admitted_Q2 | 52828 | 0.28 | 0.45 | 0 | 0 | 0 | 1 | 1 |
| | S(%)_ApplyQ2 | 30125 | 0.51 | 0.29 | 0 | 0.26 | 0.51 | 0.76 | 1 |
| <i>GPA < GPA Cutoff (Not Admitted to Q1)</i> | Background | 47172 | 0.42 | 0.49 | 0 | 0 | 0 | 1 | 1 |
| | GPA | 47172 | 8.15 | 0.59 | 6 | 7.81 | 8.27 | 8.62 | 8.93 |
| | Applied_Q1 | 47172 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 47172 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | GPA(%)_ApplyQ1 | 47172 | 0.24 | 0.14 | 0 | 0.12 | 0.24 | 0.35 | 0.47 |
| | Y | 47172 | 10.11 | 1.01 | 5.79 | 9.43 | 10.11 | 10.79 | 14.33 |
| | Q2 | 47172 | 0.08 | 1 | -4.71 | -0.6 | 0.08 | 0.75 | 4.12 |
| | S | 47172 | -0.04 | 1 | -4.15 | -0.72 | -0.04 | 0.63 | 4.27 |
| | Applied_Q2 | 47172 | 0.53 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | Admitted_Q2 | 47172 | 0.26 | 0.44 | 0 | 0 | 0 | 1 | 1 |
| | S(%)_ApplyQ2 | 24996 | 0.49 | 0.29 | 0 | 0.24 | 0.49 | 0.74 | 1 |
| <i>Q2 > 0 (Applied to Q2)</i> | Background | 55121 | 0.52 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA | 55121 | 9.05 | 1.01 | 6 | 8.36 | 9.05 | 9.73 | 13 |
| | Applied_Q1 | 55121 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 55121 | 0.55 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA(%)_ApplyQ1 | 55121 | 0.51 | 0.29 | 0 | 0.26 | 0.52 | 0.76 | 1 |
| | Y | 55121 | 10.52 | 0.92 | 6.63 | 9.9 | 10.52 | 11.14 | 14.37 |
| | Q2 | 55121 | 0.85 | 0.63 | 0 | 0.35 | 0.73 | 1.22 | 4.12 |
| | S | 55121 | -0.02 | 1 | -4.15 | -0.7 | -0.02 | 0.66 | 4.27 |
| | Applied_Q2 | 55121 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q2 | 55121 | 0.49 | 0.5 | 0 | 0 | 0 | 1 | 1 |
| | S(%)_ApplyQ2 | 55121 | 0.5 | 0.29 | 0 | 0.25 | 0.5 | 0.75 | 1 |
| <i>Q2 <= 0 (Not Applied to Q2)</i> | Background | 44879 | 0.47 | 0.5 | 0 | 0 | 0 | 1 | 1 |
| | GPA | 44879 | 8.96 | 1.01 | 6 | 8.27 | 8.95 | 9.64 | 13 |
| | Applied_Q1 | 44879 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 44879 | 0.51 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA(%)_ApplyQ1 | 44879 | 0.49 | 0.29 | 0 | 0.23 | 0.48 | 0.73 | 1 |
| | Y | 44879 | 9.7 | 0.92 | 5.79 | 9.09 | 9.71 | 10.32 | 13.56 |
| | Q2 | 44879 | -0.76 | 0.59 | -4.71 | -1.09 | -0.64 | -0.3 | 0 |
| | S | 44879 | -0.02 | 1 | -3.93 | -0.7 | -0.02 | 0.65 | 4.2 |
| | Applied_Q2 | 44879 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | Admitted_Q2 | 44879 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | S(%)_ApplyQ2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>S > 0 & Q2 > 0 (Admitted to Q2)</i> | Background | 27115 | 0.48 | 0.5 | 0 | 0 | 0 | 1 | 1 |
| | GPA | 27115 | 9.06 | 1.01 | 6 | 8.38 | 9.06 | 9.75 | 13 |
| | Applied_Q1 | 27115 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 27115 | 0.55 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | GPA(%)_ApplyQ1 | 27115 | 0.52 | 0.29 | 0 | 0.27 | 0.52 | 0.77 | 1 |
| | Y | 27115 | 11 | 0.79 | 8.02 | 10.46 | 10.99 | 11.52 | 14.37 |
| | Q2 | 27115 | 0.85 | 0.63 | 0 | 0.35 | 0.73 | 1.22 | 4.12 |
| | S | 27115 | 0.79 | 0.6 | 0 | 0.32 | 0.67 | 1.14 | 4.27 |
| | Applied_Q2 | 27115 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q2 | 27115 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | S(%)_ApplyQ2 | 27115 | 0.75 | 0.14 | 0.51 | 0.63 | 0.75 | 0.88 | 1 |
| <i>GPA < GPA Cutoff & S <= 0 (Not Admitted to Q1/Q2)</i> | Background | 24375 | 0.46 | 0.5 | 0 | 0 | 0 | 1 | 1 |
| | GPA | 24375 | 8.14 | 0.59 | 6 | 7.81 | 8.26 | 8.61 | 8.93 |
| | Applied_Q1 | 24375 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | Admitted_Q1 | 24375 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | GPA(%)_ApplyQ1 | 24375 | 0.23 | 0.14 | 0 | 0.12 | 0.23 | 0.35 | 0.47 |
| | Y | 24375 | 9.65 | 0.89 | 5.79 | 9.05 | 9.66 | 10.26 | 12.74 |
| | Q2 | 24375 | 0.07 | 1.01 | -4.1 | -0.6 | 0.07 | 0.75 | 3.87 |
| | S | 24375 | -0.82 | 0.61 | -4.15 | -1.18 | -0.7 | -0.33 | 0 |
| | Applied_Q2 | 24375 | 0.53 | 0.5 | 0 | 0 | 1 | 1 | 1 |
| | Admitted_Q2 | 24375 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | S(%)_ApplyQ2 | 12899 | 0.25 | 0.15 | 0 | 0.12 | 0.25 | 0.38 | 0.51 |

6.2 Coefficients of Extrapolation Equations

| | Equtaion (1): Q1 Admitted | Equtaion (2): Q2 Admitted | Equtaion (3): Background 0 | Equation (3): Background 1 | Equation (3): Pool Background |
|-------------------------------|---------------------------|---------------------------|----------------------------|----------------------------|-------------------------------|
| background | 0.20 | 0.30 | NaN | NaN | -0.08 |
| const | 10.01 | 9.12 | 0.51 | 0.42 | 0.51 |
| percentile_GPA_applyQ1 | 0.09 | -0.04 | 0.05 | 0.06 | 0.06 |
| percentile_S_applyQ2 | NaN | 2.33 | NaN | NaN | NaN |

Figure 15: Coefficients of Equations (1)-(3)