$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

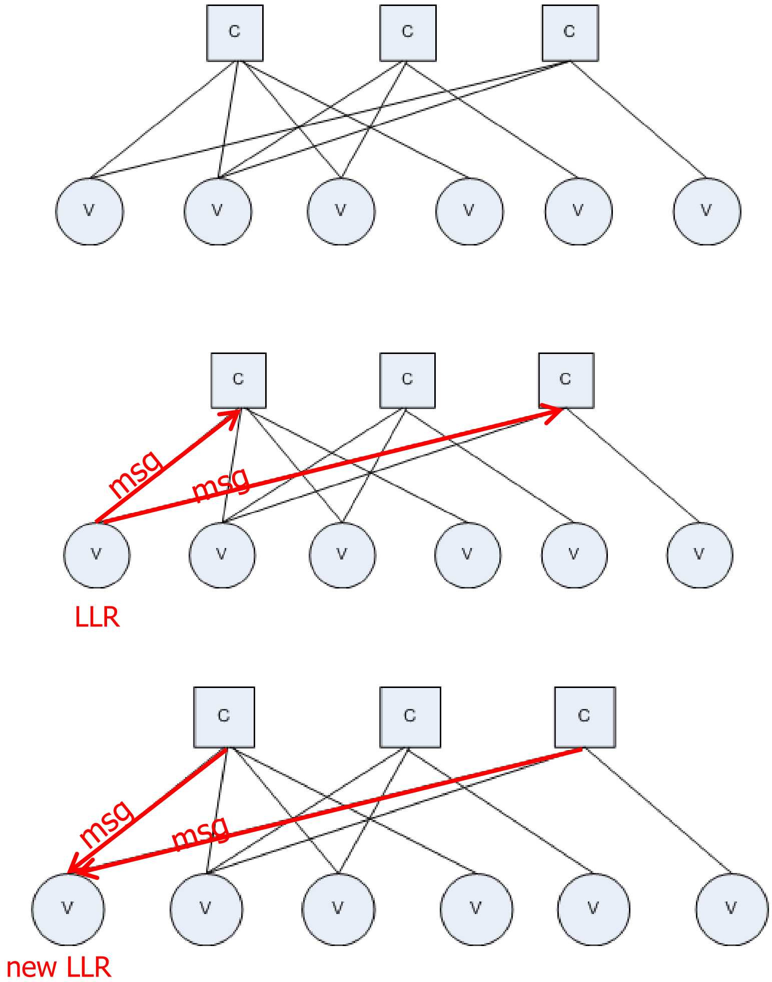
$$\mathbf{d} = \mathbf{c} \cdot \mathbf{G} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [\underbrace{1 \quad 0 \quad 1}_{\mathbf{c}} \quad \underbrace{0 \quad 1 \quad 1}_{\mathbf{w}}]$$

$$\mathbf{G} \cdot \boldsymbol{H}^{\boldsymbol{T}} = \mathbf{0}$$

$$\boldsymbol{H}^{\boldsymbol{T}} \cdot \boldsymbol{c} = \mathbf{0}$$

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_0 \oplus c_1 \oplus c_2 \oplus c_3 = 0$$
$$c_1 \oplus c_2 \oplus c_4 = 0$$
$$c_0 \oplus c_1 \oplus c_5 = 0$$


C
V
msg
msg
LLR
msg
msg
new LLR

(1) Initialization: For $i \in \{1, \dots, M\}, j \in \{1, \dots, N\}$

$$\alpha_j^0 = \ln \frac{P(VN_j = 0|y_j)}{P(VN_j = 1|y_j)} = \frac{2y_j}{\sigma^2}$$

$$\alpha_{i,j}^0 = \alpha_j^0$$

(2) Update CN: $\forall\, CN_i, i \in \{1, \dots, M\}$ do

$$\beta_{i,j}^n = \mathrm{sgn}(\beta_{i,j}^n) \cdot |\beta_{i,j}^n|$$

where

$$\mathrm{sgn}(\beta_{i,j}^n) = \prod_{j' \in \boldsymbol{\mathcal{N}}(i) \setminus j} \mathrm{sgn}(\alpha_{i,j'}^{n-1})$$

$$|\beta_{i,j}^n| = \bigwedge_{j' \neq j} (\alpha_{i,j'}^{n-1})$$

(3) Update VN: $\forall\, VN_j, j \in \{1, \dots, N\}$

$$\alpha_{i,j}^n = \alpha_{i,j}^0 + \sum_{i' \in \boldsymbol{\mathcal{M}}(j) \setminus i} \beta_{i',j}^n$$

(4) Update a posteriori LLR:

$$\alpha_j^n = \alpha_{i,j}^0 + \sum_{i' \in \boldsymbol{\mathcal{M}}(j)} \beta_{i',j}^n$$

(5) Repeat from (2) to (4) until the maximum number of iteration is reached, the estimated codeword is then given by:

$$\hat{c}_j = \begin{cases} 0, & if\ \alpha_j^n > 0 \\ 1, & else \end{cases}$$

Algorithm	$\bigwedge_{j' \neq j} \left(\alpha_{i,j'}^{n-1} \right)$
SP	$\Phi\left(\sum_{j' \in \mathcal{N}(i) \backslash j} \Phi(\left\lvert \alpha_{i,j'}^{n-1} \right\rvert) \right)$ $\Phi(x) = -\log(\tanh\left(\frac{x}{2}\right))$
MS	$\min_{j' \in \mathcal{N}(i) \backslash j} \{ \left\lvert \alpha_{i,j'}^{n-1} \right\rvert \}$
OMS	$\max\{ \min_{j' \in \mathcal{N}(i) \backslash j} \{ \left\lvert \alpha_{i,j'}^{n-1} \right\rvert \} - \beta, 0 \}$ $\beta \geq 0$
NMS	$\lambda \cdot \min_{j' \in \mathcal{N}(i) \backslash j} \{ \left\lvert \alpha_{i,j'}^{n-1} \right\rvert \}$ $\lambda < 1$