Heat Exchange Analysis and Improvements of Building

Group 8

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Abstract

Thermal effects and heat exchange in buildings constructed in the last century are essential for those built in regions where the temperature vary rapidly and have a significant air draft. This report models heat exchange inside and outside buildings and proposes several ways to improve thermal efficiency. This report then uses equations combined with diagrams to produce the best methods for amelioration.

1 Introduction

Buildings constructed in the 1960s are often very energy inefficient due to materials, structure [1], and lack of thermal insulation. This effectively means that for most buildings, a lot of energy spent heating the building is wasted through the ground [2], walls [3], and windows [4].

Stopping this energy loss and improving the heating system can also be expensive. First, changing the material and structure of a building built in the twentieth century could be difficult, as some of them are fragile partially due to being affected by climate change and pollution, as well as other effects of time [5]. Secondly, due to the high thermal transmittance of the whole building, the inside radiator would consume more electricity and other types of power than modern houses [6]. In addition, some buildings are protected by the government so that only slight modifications can be made to them, which also causes barriers [7]. So finding the cheapest way to do so effectively is beneficial.

In this model report, the first goal is to model the heat flows between the building and the outside with differential equations. In order to simplify the research process, we assume that the shape of the building is cube, that all inside furniture has the same thermal mass, the external temperature T_{e} remains unchanged, the internal temperature $T_{\mathrm{i}}(t)$ changed uniformly and that the air density ρ_a will not change with temperature.

Secondly, we will study different variables of the building (such as materials, structure, and area of windows / doors) and variables of the heating system (such as working hour, energy source and displacement, etc.). Then a new model should be created to judge the best reasonable combination of variables above, thus obtaining the best way to improve the building.

2 Heat exchange model

For a basic model, we assume that the building only has one room. To be more realistic, the further model in the appendix will consider the condition of two rooms. **Figure** 1 shows the basic exchange of indoor and outdoor heat. According to the law of energy conservation

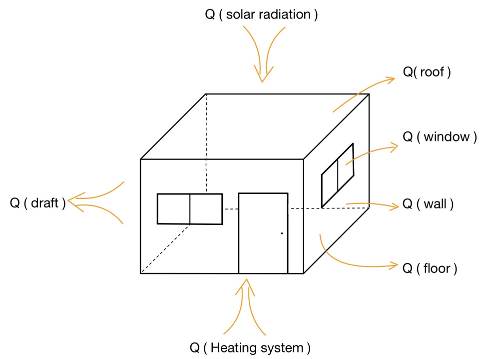

Figure 1: Heat exchange

$$\Delta Q = C\frac{\mathrm{d}T_{\mathrm{i}}(t)}{\mathrm{d}t} = Q_I(t) - Q_L(t) + Q_S(t_{\mathrm{day}}), \tag{1}$$

where C is the **thermal mass** of the entire room, $Q_I(t)$ is the heat generated by the indoor heating system, $Q_S(t_{day})$ is the heat generated by solar radiation, and $Q_L(t)$ is the loss of indoor heat.

2.1 Solar radiation $Q_S(t_{\mathrm{day}})$

The solar radiation is the constant part of heating the building. The effects of the Sun on the house will be modeled with the following assumptions:

- The light source (Sun) can be modeled as directional
- The light source moves only on a 3-dimensional vertical plane through the house in parallel to at least one of the walls.
- The light source moves uniformly though the sky in a 12-hour period, and disappears for then next 12-hour period
- The light source provides a constant power per unit area when directly above

The heat generated by a directional radiation at angle θ radians to horizontal on vertical and horizontal surfaces can be expressed as

$$L_{horizontal} = P \cdot l^2 \cdot (1 - \Gamma) \cdot \sin\theta, \tag{2}$$

$$L_{vertical} = P \cdot l^2 \cdot (1 - \Gamma) \cdot |\cos\theta|, \tag{3}$$

where Γ is the reflection coefficient of the walls and P is the power provided by the light source per unit area.

Therefore given the total heat generated on the house by the light source can be modeled as

$$L_{total} = P \cdot l^2 \cdot \left[(1 - \Gamma) \cdot \sin\theta + (1 - \Gamma) \cdot |\cos\theta|\right]. \tag{4}$$

Angle θ can be expressed as

$$\theta = \frac{\pi}{43200} t_{day},$$

where T is the time of day considered. Therefore, after considering that $Q_S(t_{day})$ must be 0 at night, we reach the expression

$$Q_s(t_{day}) = Pl^2 \left[(1-\Gamma) \cdot \sin \frac{\pi}{43200} t_{day} + (1-\Gamma) \cdot |\cos \frac{\pi}{43200} t_{day}|\right] \cdot H(43200 - t_{day} \bmod 86400) \tag{5}$$

where $H(x)$ is the Heaviside step function. If we investigated heat exchange on a single day, this term could be considered as a constant coefficient.

2.2 Indoor heat loss $Q_L(t)$

$$Q_L(t) = \left[\sum U_i A_i + \frac{ACH \cdot L^3 \cdot \rho \cdot c_p}{3600}\right] \cdot [T_\mathrm{i}(t) - T_\mathrm{e}], \tag{6}$$

where U_i is the **U-value** of specific material expressed as $U = \frac{1}{R}$, A_i is the area of the specific material involved in the exchange of heating and ACH is the Air Changes per hour (an industry standard), which could be expressed as

$$\mathbf{ACH} = Q_C/V_r,$$

where Q_c is the airflow rate (m^3/h) and V_r (m^3) is the volume of the room. If there is no heating system inside the building and other outside radiator resources, we could use equation (7) to express heat exchange and variation in indoor temperature.

$$\Delta Q = -C \cdot \frac{\mathrm{d}T_\mathrm{i}}{\mathrm{d}t} = \left[\sum U_i A_i + \frac{ACH \cdot V_r \cdot \rho \cdot c_p}{3600}\right] \cdot [T_\mathrm{i}(t) - T_\mathrm{e}] \tag{7}$$

Use $T_\mathrm{i}(0) = T_{\mathbf{init}}$ to solve the ODE equation (8) and get the following:

$$T_\mathrm{i}(t) = T_\mathrm{e} + (T_\mathrm{init} - T_\mathrm{e})e^{-t\tau}, \tag{8}$$

where

$$\tau = \frac{\sum U_i A_i + \frac{1}{3600} \cdot ACH \cdot V_r \cdot \rho \cdot c_p}{C}$$

is the **time coefficient** which could describe the speed of variation in temperature over time. It is easy to find that the indoor temperature will become close to the outdoor temperature with time. However, for more general cases, we consider the indoor heating system and other sources of heat.

$$C \cdot \frac{\mathrm{d}T_\mathrm{i}(t)}{\mathrm{d}t} = Q_I(t) + Q_S(t_{day}) - \left[\sum U_i A_i + \frac{ACH \cdot V_r \cdot \rho \cdot c_p}{3600}\right] \cdot [T_\mathrm{i}(t) - T_\mathrm{e}] \tag{9}$$

Solve the ODE equation (10),

$$T_\mathrm{i}(t) = e^{-t\tau} \left[\int_0^t e^{-t\tau}[\tau\ T_\mathrm{e} + \frac{Q_I(t) + Q_S(t_{day})}{C}]\ \mathrm{d}t\right] \tag{10}$$

Note that if we do not consider indoor heating, the equation above will become equation 8.

3 Indoor Heating system Model

In order to improve the living conditions inside the building, we considered an indoor heating system. We assume that the temperature does not change with height and vary uniformly, there is no convection of indoor heat, the radiation is the only heating system inside the building. So we do not need to consider the placement of the radiator and take the underfloor heating system

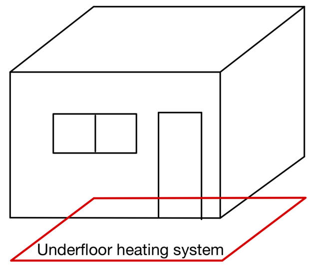

Figure 2: Underfloor heating system

to simplify the modeling process (**Figure** 2). The heating of the underfloor radiator is continuous and uniform. We could apply the first law of thermodynamics to the entire value of the room and use the relationship: **Rate of heat input = Rate of heat storage by air** to obtain the equation:

$$Q_S(t_{day}) + Q_u(t) \cdot U_f - Q_L(t) = m_a c_p \frac{\mathrm{d}T_\mathrm{i}(t)}{\mathrm{d}t} \tag{11}$$

where $Q_u(t)$ is the heat provided by the underfloor heating system, U_f is the **U-value** of the floor, and m_a is the mass of indoor air. Applying the equation: $m_a = \rho V = \rho A_f H$, where A_f is the area of the floor, and solving the equation with initial condition at $t = 0$, $T(0) = T_{\mathbf{init}}$,

$$T_\mathrm{i}(t) = T_{\mathbf{init}} + \frac{Q_s(t_{day}) + Q_u(t) \cdot U_f - Q_L(t)}{m_a c_p} \cdot t \tag{12}$$

We could set a moderate temperature T_m that is higher than the initial temperature to obtain the time t_u of the underfloor radiator used to heat the building.

$$t_u = \frac{T_m - T_\mathrm{init}}{Q_s(t_{day}) + Q_u(t) \cdot U_f - Q_L(t)} \cdot (m_a\ c_p) \tag{13}$$

4 Methods to Improve Thermal Insulation

According to the basic equation (6), increasing the thermal resistance of the building would lower the overall thermal transmittance, thus decreasing the heat loss $Q_\mathrm{L}(t)$ and improving the thermal efficiency. So we will consider improving the R-value of different parts of the building, which contain the wall, windows, and roof. We could then improve indoor heating by upgrading the radiator system while considering floor material and heating methods. Noting that all choices of material made in the following would consider costs as well.

4.1 Improve the wall

As the building built in the last century could be fragile and difficult to rebuild, we decided to attach a high heat resistance layer to the bricks instead of directly changing the wall material (**Figure** 3). In addition, to avoid decreasing living space, the heat-resistance layer should be thick. We could use the **Material Index** M_Li to select the best material. The heat resistance R_Li of a specific layer material is

$$R_\mathrm{Li} = \frac{L_\mathrm{l}}{k_\mathrm{Li}} \tag{14}$$

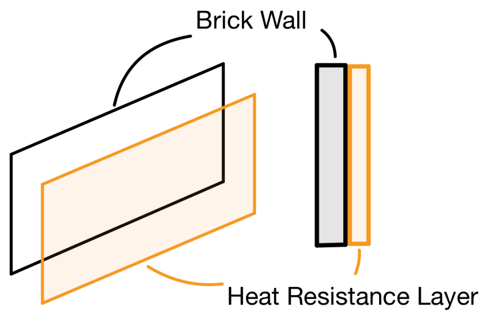

Figure 3: Heat Resistance Layer

Material	C_{Lm} (£/kg)	$\rho_{Li}(kg/m^3)$	$k_{Li}(W/m \cdot K)$	M_{Li}
Aero-gel	50	70	0.015	0.019
VIP	20	180	0.004	0.069
PUR	2	30	0.020	0.833
XPS	1	25	0.025	1.600
GPS*	1	20	0.020	2.500
Rock Wool	1	100	0.030	0.333
Glass Wool	0.5	50	0.030	1.333
Foam Glass	2	120	0.038	0.110
Cellulose	0.5	40	0.035	1.428
Foamed Concrete	0.3	300	0.050	0.222

Table 1: Common Layer Material

where L_l is the thickness of the layer and k_i is the heat conductivity $(W/m \cdot K)$. Calculating the cost of the specific material C_{Li}:

$$C_{\mathrm{Li}} = C_{\mathrm{Lm}} \cdot \rho_{\mathrm{Li}} \ L \ A \tag{15}$$

where C_{Lm} is the cost per kilogram $(/kg)$. According to Equation (12), to prevent heat from leaking from inside to outside, the U value of the layer material should be lower, so that the heat resistance R_{Li} should be higher and C_{Li} should be lower. Assume $M' = R_{\mathrm{Li}} \ / \ C_{\mathrm{Li}}$ should also be higher :

$$M_{\mathrm{Li}}{}' = \frac{R_{\mathrm{Li}}}{C_{\mathrm{Li}}} = \frac{L/k_{\mathrm{Li}}}{C_{\mathrm{Lm}} \cdot \rho AL} = \frac{1}{C_{\mathrm{Lm}} \cdot \rho_{\mathrm{Li}} \ k_{\mathrm{Li}} \ A}$$

Then we could obtain the final material index of the specific material:

$$M_{\mathrm{Li}} = (C_{\mathrm{Lm}} \cdot \rho_{\mathrm{Li}} \ k_{\mathrm{Li}})^{-1} \tag{16}$$

According to **Table** 1 of common building materials and their coefficient, **graphite polystyrene** (GPS) is the most suitable material for the heat resistance layer which is enhanced with graphite particles and thus improves its thermal performance by reflecting and absorbing radiant heat[8].

4.2 Glaze the Windows

Because, this building was built in the 1960s, we assume that the windows are only single glazed; therefore, a possible modification would be to double glaze them, something that is common practice nowadays to reduce thermal transmittance.

Layers of Glazing	U_{Wi} (W/m^2)	C_{mW} $(£/m^2)$	M_{Wi}
Single Glazed	4.8	150	720
Double Glazed	2.25	300	675
Triple Glazed*	0.5	500	250

Table 2: Glazed Windows

Types of Roof Insulation	U_{roof} (W/m^2)	C_{roof} $(/m^2)$	M_{ri}
No Insulation	2.5	0	n/a
Mineral Wool*	0.035	4.37	0.153
Reflective Foil Insulation	0.038	10.27	0.390
Phenolic Foam	0.02	11.14	0.223

Table 3: Table showcasing different types of roof insulation and their properties

Some typical U values for single, double and triple glazed windows and their price points are outlined in **Table** 2. For better thermal resistance and lower price, assume the material index of the glazed windows M_{Wi}, which should be lower:

$$M_{\text{Wi}} = U_{\text{Wi}} \cdot C_{\text{mW}} \tag{17}$$

According to Table 2, the **triple-glazed window** is the most suitable choice for windows. This type of glazing incorporates three different layers of glass with spaces for gas between all of them, which results in extremely low thermal transmittance.

4.3 Improve the Insulation in the Roof

Insulation in the roof would be another way to reduce the thermal transmittance of the building, as a large proportion of the heat lost in any building is through the roof. **Table** 3 shows different types of insulation and their respective U-values, price points, and their resulting material index. Like with windows, the roof material index

$$M_{\text{ri}} = U_{\text{roof}} \cdot C_{\text{roof}} \tag{18}$$

with the lowest value found being **mineral wool** (0.153).

4.4 Improve the Underfloor Radiator

For underfloor radiator, we will consider two parts: Floor and Radiator. As the heating system is under the floor (**Figure** 4), its behavior would be affected by the heat resistance (R value) of the floor material. Similarly to the first method, we use the material index M_R to select the best material while considering costs and behavior.

4.4.1 Selecting floor material

In order to transmit more heat from the heating system, according to equation (12), the U-value of the floor should be higher so that the heat resistance R_{Fi} should be low.

$$R_{\text{Fi}} = \frac{L_F}{k_{\text{Fi}}} \tag{19}$$

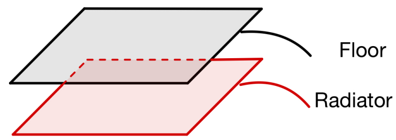

Figure 4: Underfloor Heating system

where L_F is the thickness of the floor and k_{Fi} is the heat conductivity. In addition, **Formaldehyde Emission**, which typically increases with temperature [9], should also be considered for better living conditions. The emission $E(T)$ could be expressed as:

$$E(T) = E_{\text{F0}} \cdot \exp\left(-\frac{E_a}{R\ T_s}\right) \tag{20}$$

where E_{F_0} is the reference emission at standard temperature, E_a is the activation energy for formaldehyde emission (J/mol), R is the constant gas, and T_s is the absolute temperature. Then calculate the cost of floor material C_{Fi}:

$$C_{\text{Fi}} = C_{Fm} \cdot \rho_{\text{Li}} Fi\ V_F \tag{21}$$

where C_{Fm} is the cost per kilogram and V_F is the volume of the floor. In addition, the floor should not expand too much while heating, which could obtain:

$$\Delta V = \beta_{\text{Fi}} \cdot V_F \cdot \Delta T \tag{22}$$

where β_{Fi} is the volume expansion coefficient (K^{-1}). According to our goal, both R_{Fi}, $E(T)$, and C_{Fi} should be lower. Assume $M_{\text{Fi}}{}' = 1/(E(T) \cdot C_{\text{Fi}} \cdot R_{\text{Fi}})$ to be higher.

$$M_{\text{Fi}}{}' = \frac{1}{E(T) \cdot C_{\text{Fi}} \cdot R_{\text{Fi}}} = \frac{k_{Fi} \cdot \beta \cdot \Delta T \cdot exp\ [\ E_a/RT\ ]}{L \cdot C_{\text{Fm}} \cdot \rho_{\text{Fi}} \cdot \Delta V \cdot E_{\text{F0}}}$$

Obtain the final material index of the specific floor material M_{Fi}:

$$M_{\text{Fi}} = \frac{k_f \cdot \beta}{C_{\text{Fm}} \cdot \rho_{\text{Fi}} \cdot E_{\text{F0}}} \tag{23}$$

According to **Table** 4, the **Ceramic Tile** is the best material for the floor.

4.4.2 Improve radiator

To improve the radiator while considering the costs of electricity and the system itself, we will give three types of radiator with different heating methods, energy costs, and fabrication costs, and use indexes to express them (**Table** 5). We define $P_u(t)$ as the power density (W/m^2) of the heating system per area, A_f is the entire area of the floor, C_e is the cost of electricity per kWh and C_p would be the fabrication costs. Then obtain the entire cost C_w.

$$C_w(t) = P_u(t) \cdot A_f \cdot C_e \cdot t + C_p \cdot A_f \tag{24}$$

Then obtaining the expression for costs of the entire unit area $C_w(t)\ '$ shows working less than 6.256 hours, the heating cable costs less, while working more than 6.256 hours, the Carbon Crystal Heating costs less.

Material	k_{Fi} $(W/m \cdot K)$	β_{Fi} (K^{-1})	C_{Fm} $(£/kg)$	ρ_{Fi} (kg/m^3)	E_{F0}	M_{Fi}
Ceramic Tile *	1.0	5	0.8	1800	0.0001*	34.722
Marble	2.0	3	1.5	2500	0.0001*	16.000
Granite	1.7	4	1.0	2600	0.0001*	26.154
Solid Wood	0.12	30	2.5	600	0.0200	0.12
Engineered Wood	0.12	40	1.5	700	0.0300	0.15
Laminate Floor	0.15	50	1.2	800	0.0800	0.0117
PVC	0.15	80	1.0	1400	0.0200	0.0429
Rubber Floor	0.14	100	2.0	900	0.0200	0.0222
Cork Floor	0.05	150	3.0	250	0.0100	0.0800
Bamboo Floor	0.15	150	3.0	250	0.0200	0.0600

* Several materials don't have formaldehyde emission, use 0.0001 instead of 0 to simplify.

Table 4: Common Floor Material

Radiator	$P_u(t)(W/m^2)$	C_p $(£/m^2)$	C_e $(£/kWh)$	$C_w(t)\ '$
Heating Cable	100	76.8		$C_w(t)_1\ ' = 25.0 \cdot t + 76.8$
Carbon Fiber Heating	80	119	0.25	$C_w(t)_2\ ' = 20.0 \cdot t + 119$
Carbon Crystal Heating	50	155		$C_w(t)_3\ ' = 12.5 \cdot t + 155$

Table 5: Radiators

$$\begin{cases} C_w(t)_3 > C_w(t)_2 > C_w(t)_1\ ,\ t \in (0, 4.8) \\ C_w(t)_2 > C_w(t)_3 > C_w(t)_1\ ,\ t \in (4.8, 6.256) \\ C_w(t)_2 > C_w(t)_1 > C_w(t)_3\ ,\ t \in (6.256, \infty) \end{cases}$$

5 Results

We are going to be using the basic model with only one thermal mass to get results, we will take some average values for the thermal transmittance of the different components, compare the change in temperature of the building between those and the improved values we found in the previous section, and finally use equation (10) to plot several comparison graphs of temperature variation with time to get the best thermal improvement method to the building.

Figures 5, 6, and 8 show that the building with lower **ACH**, lower U-value and higher radiator power would get a higher maximum temperature after the heating system was running for a while. Figure 7 shows that different external temperatures would affect the maximum internal temperature and the rate of increase while the building reaches the maximum.

6 Conclusion

This report shows the heating exchange between the interior and exterior of the building and the variables that would affect the variation of the indoor temperature while the radiator is working. We use equation (10) as the basic model of thermal exchange which considers the indoor heating system, solar radiation, and air draft. Focus on the key point: U-value of the construction materials and the heating system, we apply the equation of the material index and the heating

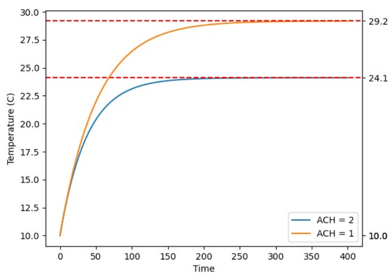

Figure 5: ACH Comparison

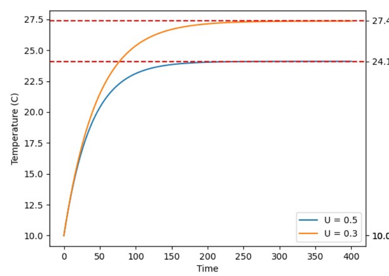

Figure 6: U-value Comparison

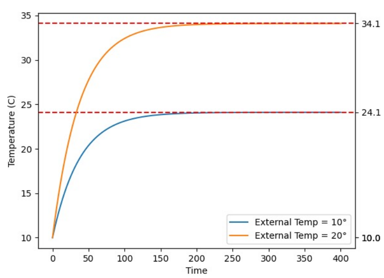

Figure 7: T_e Comparison

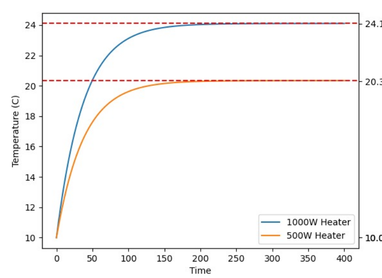

Figure 8: Heating system Comparison

costs (24) separately to conclude the best improvement of the building while taking costs into account. The results of how these variables affect the temperature rise process have been taken into the basic model and shown in the diagrams above in the **Results** part. In summary, the building should add a heat resistance layer made of graphite polystyrene, replace windows with triple-glazed ones, place wool insulation inside the roof, use ceramic tile floor to substitute for the wood one, and apply the carbon-fiber radiator to heat the room.

However, the limitation of the model is obvious, the report does not account for different rooms having different specific heat capacities, and it assumes that given the same amount of heat energy, both rooms will heat and cool at the same rate, which is a major limiting factor. Also in the model, the heat gained from the Sun has to be a constant for a general solution to be possible, which means that it does not account for the Sun at different times of day.

So in the further study, we could use the partial differential equation to demonstrate the variation and distribution of temperature with time and height under the influence of different types of radiators [10] (partly shown in the **Appendix**), consider a building with two rooms or attic with different thermal masses and model the heat exchange between them[11], conclude how the heat of solar radiation changes with the movement of the sun, and think about different types of energy supplement, thus making the building more environmentally friendly and sustainable. In addition, we should also consider the transformation of the building for other uses to reduce the negative impact of thermal inefficiency while increasing economic benefits[12].

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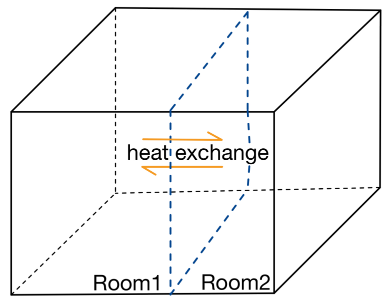

Figure 9: Indoor heat exchange

Appendix

1 Further study 1: Heat convection in the underfloor heating model

In a more realistic condition, **one - dimensional heat convection** would occur while the underfloor heating system works. All the assumptions are the same as the 'Indoor Heating system Model', and we could use a one-dimensional heat convection equation to model changes in heat with time and height.

$$\frac{\partial T_{\mathrm{i}}(z,t)}{\partial t} = \alpha \frac{\partial^2 T_{\mathrm{i}}(z,t)}{\partial z} \tag{25}$$

where z is the height to the floor and α is the thermal diffusivity, defined as:

$$\alpha = \frac{k}{\rho_a \ c_p}$$

where k is the thermal conductivity of the room. The unit of thermal diffusivity is m^2/s, representing the rate at which temperature disturbances propagate through a material. We then need to establish the initial condition. The temperature of the floor is T_f.

$$T_f = T_{init} - Q_u R_f.$$

It is easy to get the temperature at time 0 and height 0 is T_f, which could be expressed as $T(0,0) = T_f$. Then we could use an experiment to obtain the boundary condition to solve the partial differential equation.

2 Further study 2: Building with two rooms inside

In Model 2, we will consider the building with two rooms of the same size that have different thermal masses C_1 and C_2, which are closely related to the material inside the room **(Figure 9)**. So in addition to heat exchange between the outside and inside of the building, we should also take heat exchange between rooms into account. In order to simplify the model process, we assume that the initial temperature of two single rooms is the same and is all equal to $T_{\mathbf{init}}$, and have the same material of the exterior wall, floor, ceiling, and window area. In general cases, the temperature of two rooms would be different and the loss of indoor heat Q_i would occur, which could be expressed by the equation:

$$\text{For Room 1: } Q_{i1} = U_{\mathbf{iw}} A_{\mathbf{iw}} \cdot [T_{\mathrm{i}}(t)_1 - T_{\mathrm{i}}(t)_2], \tag{26}$$

Nomenclature

Abbreviations

ACH Air changes per hour (times/h)
GPS Graphite polystyrene material

Variables

β_{Fi} Volume expansion coefficient
ΔQ Internal and external heat exchange
Γ Reflection coefficient of the wall
τ Time coefficient which describes the speed of variation in temperature
A_f Area of the floor
A_i Area of the specific material involved in heat exchange
C Thermal mass of the entire room
$C_{\mathbf{roof}}$ Cost of the roof per kilogram
C_{Fi} The cost of floor material
C_e Cost of electricity per kWh
C_p Underfloor radiator fabrication costs
C_{Fm} Cost per kilogram of the floor
C_{Li} Cost of the specific material
C_{Lm} Cost per kilogram of the layer material
C_{mW} Cost per kilogram of the layer material
E_a Activation energy for formaldehyde emission
E_{F_0} Reference emission at standard temperature
k_{Fi} Heat conductivity of the floor
k_i Heat conductivity
$L_{\mathbf{horizontal}}$ Heat generated by solar radiation on horizontal surface
$L_{\mathbf{vertical}}$ Heat generated by solar radiation on vertical surface
L_F Thickness of the floor
Ll Thickness of the layer
M_{ri} Material index of the roof
m_a Mass of the indoor air
M_{Li} Material index of heat resistance layer
M_{Wi} Material index of the glazed window
P Power provided by the light source per unit area
$P_u(t)$ Power density of the heating system per area
Q_c Airflow rate
$Q_I(t)$ Heat generate from indoor heating system
$Q_L(t)$ Heat loss from the building
$Q_s(t_{day})$ Heat from solar radiation
$Q_u(t)$ Heat provided by the underfloor radiator
R Constant gas
R_{Fi} Heat resistance of the floor
R_{Li} Heat resistance of specific layer material
T_{init} Initial temperature inside the building
$U_{\mathbf{roof}}$ U value of the roof
U_f U-value of the floor
$U_i(t)$ U-value of specific material
U_{Wi} U value of the window
V_F Volume of the floor
V_r Volume of the air in the room
T_m Moderate temperature
t_u Time of the radiator used to heat the building

Function

$E(T)$ Formaldehyde emission
$H(x)$ Heaviside step function

$$\text{For Room 2: } Q_{i2} = U_{\mathbf{iw}} A_{\mathbf{iw}} \cdot [T_{\mathrm{i}}(t)_2 - T_{\mathrm{i}}(t)_1], \tag{27}$$

where $U_{\mathbf{iw}}$ and $A_{\mathbf{iw}}$ is the U-value and the area of the indoor wall. Then we need to rewrite the conclusion heat exchange equation for rooms 1 and 2,

- Room 1:

  $$\Delta Q_1 = C_1 \cdot \frac{dT_{\mathrm{i}}(t)_1}{dt} = Q_I(t) + Q_S(t_{day}) - Q_L(t)_1 - U_{\mathbf{iw}} A_{\mathbf{iw}}\ [T_{\mathrm{i}}(t)_1 - T_{\mathrm{i}}(t)_2]$$

  which could be simplify into:

  $$\frac{dT_{\mathrm{i}}(t)_1}{dt} = -(\tau_1 + \mu_1)\ T_{\mathrm{i}}(t)_1 + \mu_1\ T_{\mathrm{i}}(t)_2 + (u_1 + \tau_1 \cdot T_{\mathrm{e}}), \tag{28}$$

- Room 2: The equation for room 2 is similar to the first one,

  $$\frac{dT_{\mathrm{i}}(t)_2}{dt} = -(\tau_2 + \mu_2)\ T_{\mathrm{i}}(t)_2 + \mu_2\ T_{\mathrm{i}}(t)_1 + (u_2 + \tau_2 \cdot T_{\mathrm{e}}) \tag{29}$$

With equations (16) and (17), we could obtain the general solution of the ODE equation:

$$T_{\mathrm{i}}(t)_1 = A\ exp(\lambda_1) + B\ exp(\lambda_2) + \frac{\mu_2\ w_1 - v_1\ w_2 + w_2}{v_1\ v_2 - \mu_1\ \mu_2} \tag{30}$$

$$T_{\mathrm{i}}(t)_2 = C\ exp(\lambda_3) + D\ exp(\lambda_4) + \frac{\mu_1\ w_2 - v_2\ w_1 + w_1}{v_1\ v_2 - \mu_1\ \mu_2} \tag{31}$$

where $A\ B\ C\ D$ are the coefficients,

$$\lambda_1 = \lambda_3 = \frac{1}{2}[(v_1 + v_2) + \sqrt{(v_1 - v_2)^2 + 4\ \mu_1\mu_2}]$$

$$\lambda_2 = \lambda_4 = \frac{1}{2}[(v_1 + v_2) - \sqrt{(v_1 - v_2)^2 + 4\ \mu_1\mu_2}]$$

$$\tau_{1,2} = \frac{\sum U_i A_i + \frac{1}{3600} \cdot ACH \cdot V \cdot \rho \cdot c_p}{C_{1,2}}$$

$$u_{1,2} = \frac{Q_I(t) + Q_S(t_{day})}{C_{1,2}}$$

$$v_{1,2} = -(\tau_{1,2}\ +\ \mu_{1,2})$$

$$w_{1,2} = u_{1,2}\ +\ \tau_{1,2} \cdot T_{\mathrm{e}}$$

$$\mu_{1,2} = \frac{U_{iw}\ A_{iw}}{C_{1,2}}$$