Bayesian Social learning models

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***Abstract*—This report aims to give a brief introduction to Classical Bayesian Social Learning Filter and HMM Social Learning Filter. The respective protocols will be presented in an intuitive manner to a signal processing audience. A Bias-inducing phenomenon Data Incest, and a corresponding Incest Removal Algorithm would be introduced. Herding and Information Cascade which are two interesting consequences of introduced social learning protocol, would be compared and discussed with stimulated data. Finally, a real life social learning scenario where herding occurs is proposed.**

Keywords—Social Learning Filter, data incest, herding, information cascade

I. INTRODUCTION

A. Motivation

Human's mindset of interactive learning among society could be well captured by Social Learning. Out of which many examples can be seen as "a multiagent system trying to estimate a true underlying state(s)". Such a sensor could be seen as a sensors of linked social opinions. To see how the system as a whole trying to generate an unbiased estimate, Statistical Signal Processing could be a good start.

B. Main Resultes and Organization

In the first part, a well-known protocol[2] of Classical Social Learning would be intuitively introduced. Under the context of which, Data Incest, as a sign of misinformation propagation, together with its influences on the Bayesian Estimations and its Removal Algorithm would be illustrated using real-world examples. The first example was a Human Subjects Experiment conducted by the Department of Phycology of University of British Columbia[3] and detailly demonstrated by [4]. The second example considers a recommendation system on a social network, generated by using pseudo parameters.

In the second part, the previous protocol of Social Learning Filter is generalized and a popular protocol of HMM Social Learning Filter[1] is derived. Two interesting behaviors of the current and previous protocol, namely Herding and Information Cascade would then be defined. With various self-formulated stimulations performed, these two consequences would be meticulously examined and their differences would be intuitively explained.

In the third part, the interaction between Herding and Exit Selection during Evacuation would be explored and some interesting conclusions drown by some former studies would be presented.

II. CLASSICAL SOCIAL LEANRNING FILTER

A. Classical Social Learning Protocol

In Classical Social Learning[1], a multiagent system aims to given an unbiased estimate of one underlying, finite state random variable($x \in X = \{1,2,3 \ldots, k\}$), with a known prior distribution π_0. We assume that each agent will make an action in a predetermined order ($k = 1,2, \ldots$), and that each agent would have access to public belief π_{k-1}at iteration k. The classical learning protocol can be defined as follows[2].

Step1) ***Private observation***: At time k, agent k records Private observation y_k from observation distribution shown below. We assume that $Y = \{1,2, \ldots, k\}$ is finite. In this step, the information observed has nothing to do with an agent's objective judgement.

$$B_{iy} = P(y|x=i), i \in \mathbb{X}.$$

Step2) ***Private belief***: Using the public belief π_{k-1}, agent k updates its private posterior belief $\eta_k(i) = P(x_k = i \mid a_1, \dots, a_{k-1}, y_k)$ using Bayes Formula. Note that there is a one iteration delay in the update of π_k, as the update of public belief happens at last (Step 4). Where,

$$\eta_k = \frac{B_{y_k}\pi}{\mathbf{1}'_X B_y \pi}, \quad B_{y_k} = \mathrm{diag}(P(y_k|x=i), i \in \mathbb{X})$$

Step3) ***Short-sighted action***: Agent k takes an action $A = \{1,2,\dots,k\}$ to minimize the expected cost, such that. "Short-sighted" can be explained such that agents only take into account of what has already happened. Such that

$$\begin{aligned} a_k &= \arg\min_{a \in \mathcal{A}} \mathbf{E}\{c(x,a)|a_1,\dots,a_{k-1},y_k\} \\ &= \arg\min_{a \in \mathcal{A}} \{c'_a \eta_k\}. \end{aligned}$$

Step4) ***Public belief update***: Given the last action $a_k$and the latest public belief π_{k-1}, every subsequent agent perform social learning according to the social learning filter. This can be written as

$$\pi_k = T(\pi_{k-1}, a_k), \qquad T(\pi, a) = \frac{R'i_a \pi}{\sigma(\pi, a)} \quad R'i_a = \mathrm{diag}(P(a|x=i,\pi), i \in \mathbb{X})$$

In this filter, $\sigma(\pi, a)$ is the normalization term. $P(a|x = i, \pi)$ can be written as $\sum_{y \in Y} P(a|y,\pi)\, P(y|x_k = i)$ by the Law of Total probability. Moreover, intuitively $P(a|y,\pi) = 1$only when a in this case is the optimal action.

B. Ituitive Example of Classical Social Learning

This protocol can be generalized into numerous real-life examples. Consider an online-polling system such as Yelp. The system would be aiming to estimate the true performance of different restaurants. Each agent would eat at a restaurant and acquire a private observation, taking into how others think about this restaurant (public belief), a particular agent would be able to form a private, posterior belief of the restaurant. Bearing in mind of the reputation system(cost function, how their rating would be trusted), the agent would take an action minimizing the expected cost.

Then, once the actions are shown on the system, and the information contained by these discontinuous actions (only hard decisions were shown publicly) would propagate, the public belief of one particular restaurant would be updated.

Obviously, in this one simple example, numerous assumptions were made, besides original assumptions raised when the protocol was defined.

It was assumed that time is not a factor, which is to say the restaurants' performances do not change over time. It was assumed that each agent has the same cost function, it was assumed that each agent has the same observation distribution, it was assumed that each agent was equally affected by the public belief, etc. I want to point readers to another paper in more of a psychology field, however greatly demonstrate the natures of social learning[5].

C. Definition of Data Incest

Data incest can be seen as a form, or a source of misinformation propagation. It occurs dur to unintentionally reuse of the same, previous action of agents[4].

It is to say that the identical previous action, has been falsely assumed as independent. This easily leads overconfidence in estimations, therefore bias.

D. Ituitive Example of Data Incest

Building up on the previous Yelp example from I(B), assuming we have only memoryless agents, let us consider this situation [Fig. 1]. Agent 1 ate at one particular restaurant and left a positive comment. agent 2 saw the previous comment (took the updated public belief) and became more prone to leave a positive comment. Suppose agent 2 did leave a positive comment for the sake of argument. When agent 1 wants to leave a comment again, his action would double confirmed by other two positive comments that are already in the system. He/ She is even more likely to leave a positive comment again, disregarding the real observation.

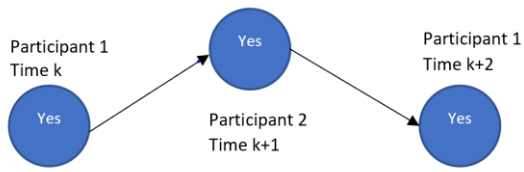

Fig. 1 Potential Data Incest in Yelp

In this sense, the first comment of agent 1 is reused during the second and the third ratings, and it was falsely assumed by agents that all comments were independent.

E. Real Life Example of Data Incest

Take another real-life example elaborated in [3] and [4]. A interactive perceptual trial was performed on student from University of British Columbia 1658 times. In each of the trial, one of the two students would be asked to answer a perceptual question, given access to their partners' previous answer. This mimics the Classical Social Learning protocol, and let the Action, Observation state, True state Space only consist of two discrete states. Also let each single trail terminates when the response of both participants do not change for three consecutive runs.

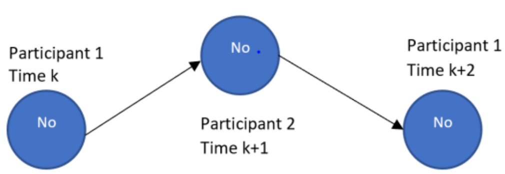

Fig. 2 Potential Data Incest Pattern

Consider two types of information flow shown by Fig. 2 and Fig. 1. Please note here "Yes" or "No" were chosen as an option just for the sake of argument and do not carry any real-life meanings.

Assuming the pattern starts at time k, for both of the patterns, the action committed by participant one influenced the action made by Participant 2 at time $k+1$. Therefore the action committed at $k+2$ by participant 1 could have been influenced by the very first action at time k. At least one of the two information propagation patterns were found in 79% of the experiments. Furthermore, 29% out of all the experiments; not only one of the two patterns occurred, but "participant 1" have shown a change of decision at time $k+1$. Assuming the second change of decision has nothing to do with his/ her observation, Data Incest in this experiment modifies individual's decision in almost one third of all cases.

F. Data Incest Removal Algorism

Let us firstly define our system by a sequence of time dependent DAG, such that each Vertex is an agent at time k, an edge $(\mathrm{n}', \mathrm{n}'')$ exist between two Vertex if and only if the information of n' reaches n''.

We can then define $A_n(i,j) = 1$ if there is a single hop path from i to j, define $T_n(i,j) = 1$ if there exists a path from i to j. We also define two matrices $\mathrm{H_n}$ and $\mathrm{F_n}$ as $H_n = \{m : A_n(m,n) = 1\}$, $F_n = \{m : T_n(m,n) = 1\}$.

At last, we are safe to say that H_n denotes the set of previous nodes m that commute with n with a single hop, whereas F_n denotes the set of nodes that eventually arrive at n. It is obvious that T_n is upper triangular with ones on the diagonal and therefore invertible.

Our objective would be provide each node n with the true Posterior Distribution which could be denoted as $\pi^0_{\mathrm{n-1}}(\mathrm{i}) = \mathrm{P}(\mathrm{x} = \mathrm{i}|\{\mathrm{a_m},\ \mathrm{m} \in \mathrm{F_n}\})$. It could be interpreted as "a prior distribution that takes into account of actions from all the connected agents". The Incest Free π_{n-1} can be written as follows

$$l_{n-1}(i) = \sum_{m \in H_n} w_n(m) l_m(i)$$
$$l_m(i) = log\pi_m(i)$$
$$l_{n-1}(i) = log\pi_{n-1}(i)$$
$$w_n(m) = T_{n-1}^{-1} t_n$$

Please note that t_n is the first $n-1$ elements of the n^{th} column.

The algorism has a necessary and sufficient condition, which can be shorted in words as “The adjacency matrix of the directed graph was given for exact incest removal at each stage”. Which is pretty intuitive, how can the information given to a node be tuned, if we do not even know where did it get its information from?

G. *Incest Removal Performance*

Let us access the Incest Removal Algorism by comparing a system’s estimation before and after applying the algorism. We consider a simpler, Belief Propagation[4] system shown by Fig. 3.

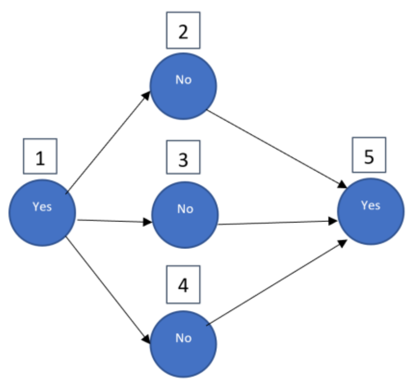

Fig. 3 Simple Belief Propagation System

The system consisting of five agents and each of them takes a naïve prior $\pi_{n-} = \frac{\prod_{m \in H_n} \pi_m}{\mathbf{1}' \prod_{m \in H_n} \pi_m}$, which is just a normalized aggregation of their adjacent agents’ posterior distribution. The system then follows Step1, Step2 and Step3 as stated in Classical Bayesian Social Learning protocol. Set the system parameter as $B = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$, $\pi_0 = (0.5,\ 0.5)'$, real states $X = \{\text{Yes}, "No"\}$, Observed states $Y = \{\text{Yes}, "No"\}$ and we also assume the real state $\mathrm{X} = "No"$.

It could be easily verified that the posterior distribution $\pi_5(\text{"Yes"}) = 0.8, \pi_5(\text{"No"}) = 0.2$. This is to say that even though most of the agents correctly agree with that the True State should be “No”, the agent 5 would still be significantly biased against state “Yes”.

Given that all information of five agents could be sampled (necessary and sufficient condition), after applying the introduced algorism, we get that the Incest Free estimate as $P(X = \text{"No"} | y_1, \ldots, y_5) = 0.96$, which confidently givens the correct estimation of the real underlying state.

III. HMM SOCIAL LEARNING FILTER

Recall in the Yelp case in Classical Social Learning Filter, that we had to assume that the each restaurant’s performance stay the same over time. Now we do not have to as we generalize the original protocol to HMM Social Learning Filter, so that the underlying state is viewed as a Markov Chain changing according to its Transition Matrix. Now we redefine the original Protocol.

A. *HMM Social Learning Protocol*

In HMM Social Learning[1], a multiagent system aims to given an unbiased estimate of the underlying state of a Hidden Markov Chain with a finite state space($x \in X = \{1,2,3 \ldots, k\}$), with a known prior distribution π_0, based on their noisy observations $\mathrm{y_k}$. We assume that each agent will make an action in a predetermined order ($k = 1,2, \ldots$), and that each agent would have access to public belief π_{k-1} at iteration k. The HMM learning protocol can be defined as follows[2]. Some detailed illustration mentioned before during Classical Learning Filter will be omitted here.

Step1) ***Private observation***: At time k, agent k records Private observation y_k from observation distribution shown below. We assume that $Y = \{1,2, \ldots, k\}$ is finite. The observation distribution can be written the same as before as follows

$$B_{iy} = P(y|x = i), i \in \mathbb{X}.$$

Step2) ***Private belief***: Using the public belief π_{k-1}, agent k updates its private posterior belief $\eta_k(\mathrm{i}) = P(x_k = i \mid a_1, \ldots, a_{k-1}, y_k)$ using Bayes Formula. Note that there is a one iteration delay in the update of π_k, as the update of public belief happens at last (Step 4). Where,

$$\eta_k = \frac{B_{y_k} P' \pi}{1'_X B_y P' \pi}, \quad \text{where } B_{y_k} = \text{diag}(P(y_k | x = i), i \in \mathbb{X}).$$

Step3) ***Short-sighted action***: Agent k takes an action $A = \{1,2,\dots,k\}$ to minimize the expected cost, such that. "Short-sighted" can be explained such that agents only take into account of what has already happened. Such that

$$a_k = \arg\min_{a\in\mathcal{A}} \mathbf{E}\{c(x,a)|a_1,\dots,a_{k-1},y_k\} \\ = \arg\min_{a\in\mathcal{A}}\{c_a'\eta_k\}.$$

Step4) ***Public belief update***: Given the last action a_k and the latest public belief π_{k-1}, every subsequent agent perform social learning according to the social learning filter. This can be written as

$$\pi_k = T(\pi_{k-1}, a_k), \quad \text{where } T(\pi,a) = \frac{R_a^\pi P'\pi}{\sigma(\pi,a)} \quad R'i_a = \operatorname{diag}(P(a|x=i,\pi), i\in\mathbb{X})$$

This protocol is almost exactly the same as the Classical Social Learning protocol, except in Step 2 and Step 4, where the Transition Matrix P was added to capture the dynamic nature of HMM.

B. *Herding & Information Cascade Definition*

Serving as two interesting and important features of both learning protocol, Herding and Information Cascade would be discussed and compared in details. They are respectively defined as follows.

- A herd of agents takes place at time k', if the actions of all agents after time k' is identical, i.e. $a_k = a_{k'}$ for all time $k > k'$, for a sufficiently long period.

- An information cascade occurs at time k', if the public belief of all agents after k' are identical, i.e. $\pi_k = \pi_{k'}$ for all time $k > k'$.

By definition, we can tell that an Information Cascade would inevitably lead to an Herd of agents, however the reverse is obviously not true.

As the public freezes in a way the agents probability $P(a_k|y_k,\pi_{k-1})$ becomes independent of the private observations. Then we can know from the protocol that $P(a_k = a|x_k = i,\pi_{k-1})$ becomes $P(a_k = a|\pi_{k-1})$. Given also that the public belief π freezes, the action of agents freezes. In both of the described protocols, the Information Cascade occurs within finite time with probability 1. The proof follows elementary application of Martingale Convergence Theorem[1].

C. *Numerical Example of Herding & Information Cascade*

The behavior of Information Cascade would be discussed and compared separately in both protocols using stimulated data points.

a) Under the Context of Classical Social Learning

Some typical plots of stimulated data were shown above in Fig. 4, Fig. 5 and Fig. 6. The system parameters applied were identical except for the cost functions. The underlying state was fixed at $X = \{1\}$, observed states $Y \in \{1, 2\}$ and the observation distribution was $B = \begin{pmatrix}0.9 & 0.1\\0.1 & 0.9\end{pmatrix}$. The cost functions associated with each plot (from top to bottom) are $\begin{pmatrix}0.01 & 1\\1 & 0\end{pmatrix}$, $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ and $\begin{pmatrix}0.05 & 1\\1 & 0\end{pmatrix}$. Please note that π_1 in this case denote the public belief of true state is 1.

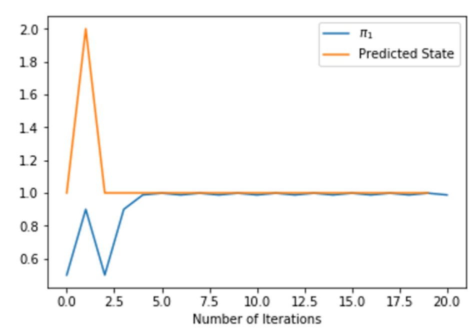

Fig. 4 *Cost Function =* $\begin{pmatrix}0.01 & 1\\1 & 0\end{pmatrix}$

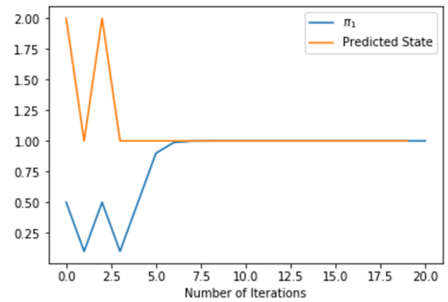

Fig. 5 Cost Function = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

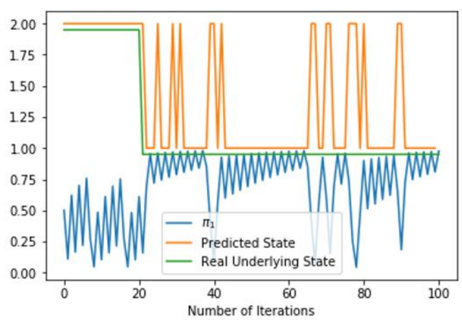

Figure 6 HMM social learning filter

By definition, we can easily tell that Herding occurred in the first plot, both Herding of Agents and Information Cascade occurred in the second plot, whereas neither occurred in the third plot. This could be intuitively explained by their cost functions.

Consider a simple real-life example of gambling. Let consider "1" as "gambling", "2" as "not gambling". Although gambling can lead to utility surplus in some cases (B is not diagonal with 1 s), however every sensible person understands gambling is not an optimal solution (True state is "1"). However, life is not perfect, this fact could be captured by the "0.01" from the first cost function. Our thoughts of gambling and winning, are revealed by the little gaps between π_1 and 1. When life is as perfect as winning gambling all the time, our belief of "not gambling" would be perfectly solid (second plot). One the other hand, if life is really not going well, as getting the true state right still cost too much (third cost function), gambling (state 2) may often be an optimal choice.

b) Under the Context of HMM Social Learning

One typical stimulation of HMM social learning filter is shown above. Now, individual agents observe the two state Markov Chain $X \in \{1,2\}$ in noises, yields observations $Y \in \{1,2\}$ The system parameters applied were specifically tuned as $B = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 0 \\ 0.1 & 0.9 \end{pmatrix}$, cost $c = c(i, a) = \begin{pmatrix} 1 & 1.67 \\ 6.1 & 0.6 \end{pmatrix}$ for illustration purpose.

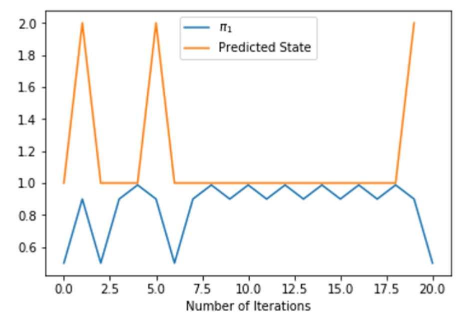

Fig. 6 Cost Function = $\begin{pmatrix} 0.05 & 1 \\ 1 & 0 \end{pmatrix}$

An Information Cascade did not occur in this particular case within a 100 iterations. On the other hand, the system was always able to quickly detect the change in underlying state quickly. The Herding of agents typically occur when $\pi_1 \in (0, \pi_1^*)$ with optimal action "2" or when $\pi_1 \in (\pi_1^{*\prime}, 1)$ with optimal action "1". Finding out empirical, explicit π_1^* and $\pi_1^{*\prime}$would be possible merely by replicating the same system stimulation, however they would be meaningless given the pseudo parameters.

IV. Real Life Herding: Exit Selection during Evacuation

A. *Introduction*

Herding was examined as one of the behaviors or characteristics of interest in [6] and [7]. We focus more on explaining herding behavior, and therefore [7]. In the context of Evacuation, the agent may simply choose an Exit because the most people have chosen that particular exist. The public belief overrides the individual observation and thinking, which may let the agents skip systematic evaluation of situation and miss the best exit.

B. Modelling and Conclusions

Random Utility Models was built on data collected by Lovreglio et al. (2014a) according to methodology that is based on "a priori" face-to-face interviews. More specifically, a Mixed Binary Logit Model whose parameters are assumed to be randomly distributed was applied. This method serves better for the a probabilistic approach to model Herding, and being able to capture the variations in agents' actions.

Omitting unnecessary modelling details, the study double confirms the herding behavior in exit choice. It concludes that based on available subjects, the greater is the difference of the number of people close to the most crowded exit and the least crowded exit, the lower is the probability of exhibiting herding behaviors. Their model also indicates that when the an Exit has no people, an agent is very likely to herd, due to the high perceived-uncertainty (The exist could be either the best or the worst).

V. Conclusions and Acknowledgement

This reports give a brief introduction to Social Learning in the context of Classical Learning Filter and HMM Learning Filter. Data incest and corresponding Removal Algorism were introduced as a common problem and solution in both of the cases. Information Cascade and Herding, are defined, examined and compared using stimulated data.

Most of the material introduced and discussed, are based on or derivative from two great papers [4][8], where many extensions of introduced ideas and other ideas were provided in tutorial fashion.

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