Problem 1

There are three types of price returns: Classical Brownian Motion, Arithmetic Return System, and Log Return or Geometric Brownian Motion, and they are shown below

$$\begin{cases} r_t = P_t - P_{t-1} \\ r_t = \dfrac{P_t - P_{t-1}}{P_{t-1}} \\ r_t = \ln \dfrac{P_t}{P_{t-1}} \end{cases}$$

To simulate the results with the given normal distribution current returns:

$$\begin{cases} \sigma = 1 \\ n = 1000 \\ P_{t-1} = 100 \end{cases}$$

The result is

	$\mu(P_t)$	$\sigma(P_t)$
Classical Returns	100.01933205582233	0.9787262077473542
Arithmetic Returns	101.93320558223256	97.87262077473542
Log Returns	168.23341551794823	244.4927936930635

Based on the mathematics formular, the expected value of means and standard deviation should be

	$\widehat{\mu(P_t)}$	$\widehat{\sigma(P_t)}$
Classical Returns	$P_{t-1} = 100$	$\sigma = 1$
Arithmetic Returns	$P_{t-1} = 100$	$P_{t-1} * \sigma = 100$
Log Returns	$P_{t-1} * e^{\frac{1}{2}} = 164.87$	$P_{t-1} * \sqrt{e(e-1)} = 216.12$

We can observe that the calculated results are not too much different with the expected value.

Problem 2

The arithmetic returns of real prices data are calculated based on the formula above, and the VaRs are simulated through the following ways with the assumption of confidence level is 95%.

2.1

When using a normal distribution, the VaR is directly calculated by

$$VaR = 1.65 * \sigma_r$$

The result is 0.05455326925723376.

2.2

When using a normal distribution with an Exponentially Weighted variance, variance matrix is updated first:

$$\sigma_t^{\ 2} = \lambda \sigma_{t-1}^{\ 2} + (1-\lambda)(x_{t-1} - \bar{x})^2$$

Then, similarly, directly calculated the VaR by assuming the normal distribution, and the result is 0.03106754744474217.

2.3

If we use a MLE fitted T distribution, we simply fit the model and use PDF function with fitted parameters to calculate the result in confidence level, the result is 0.04112246904952898.

2.4

When using a fitted AR(1) model, without the assumption of normal distribution, we use Monte Carlo method and simulate 1000 points to get the VaR. The result is 0.0520304404475868.

2.5

If we choose to use a Historic Simulation, firstly we need to sort our returns and select a percentage of our data. With this selection, the VaR is 0.04121586193570658.

Problem 3

3.1

For the first method to calculate the returns, I use Arithmetic method. Then, based on the exponentially weighted covariance, the updated covariance matrix is calculated. Then, we assume that the current rate of return meets the normal distribution. The VaR of each portfolio and the total VaR can be calculated.

Portfolio A VaR	$15253.97
Portfolio B VaR	$7765.47
Portfolio C VaR	$17933.67
Total VaR	$38091.10

3.2

Then, the returns are calculated with log method, and the result changes to

Portfolio A VaR	$15289.78
Portfolio B VaR	$7799.41
Portfolio C VaR	$17891.81
Total VaR	$38158.29

For long-term data, using the log method to calculate returns is more suitable. The first reason is stabilizing the data. In addition, the log method can transform the data to normal distribution, which is good for our simulation. The accumulation of data can be easily conducted though this method. Overall, using the log method to calculate the returns has better performance.