电路原理

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11 电路的频率响应

- 网络函数
- 谐振
- 滤波器

重点：

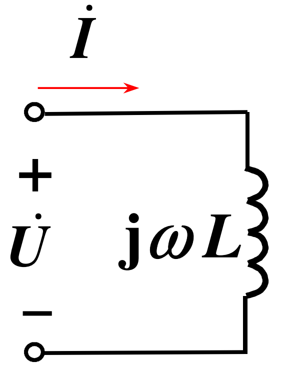
定性画频率特性

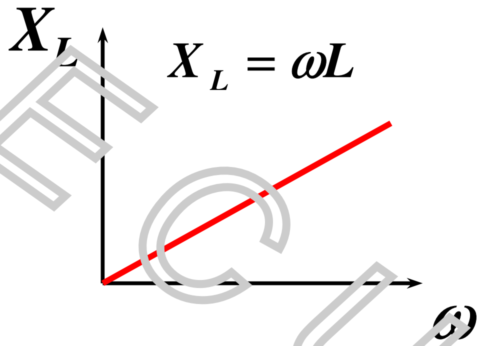
求谐振频率

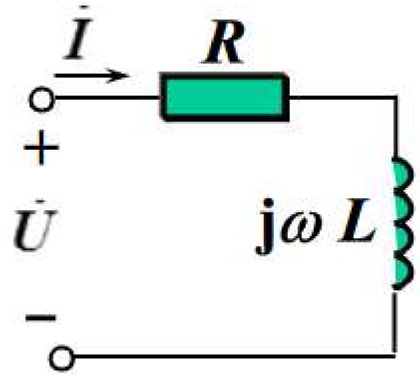
求谐振入端电阻

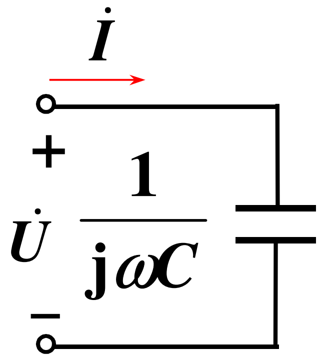
11-1 网络函数

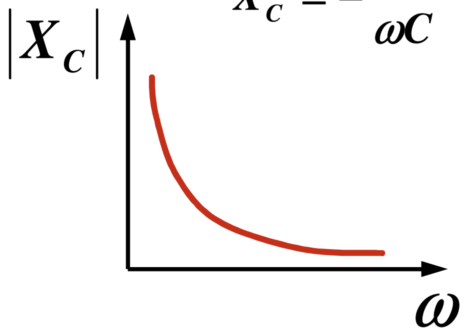
(1) 端口阻抗----响应、激励在同一端口（驱动点函数）











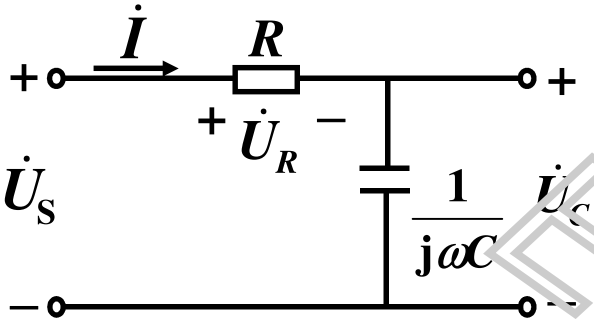
$$Z = R + j\omega L$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\varphi = \arctan \omega L / R$$

端口入端阻抗的幅值和相角都随着频率变化而变化！

(2）响应的频率特性-------响应、激励不在同一端口



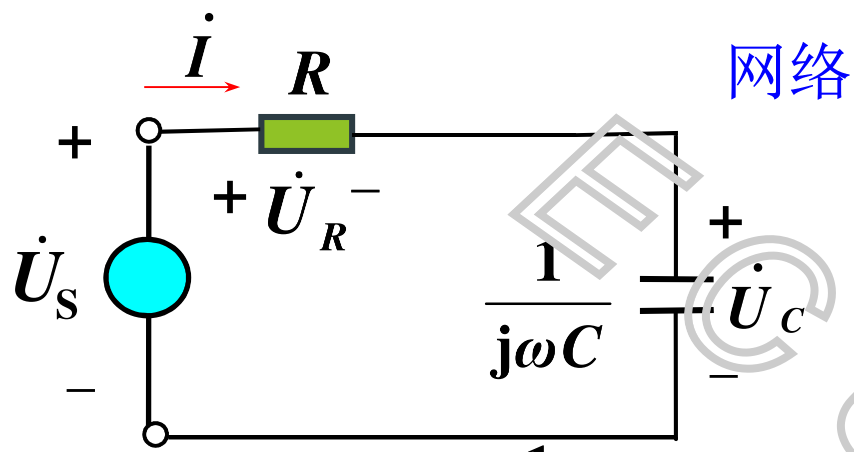
$$\dot I = \frac{U_s}{\sqrt{R^2+(1/\omega c)^2}}\angle \psi_{u_s} + \arctan\frac{1}{\omega RC}$$

$$\dot U_C = \frac{\dot U_S}{1+j\omega CR} = \frac{U_S}{\sqrt{1+(\omega RC)^2}}\angle \psi_{u_S} - \arctan(\omega RC)$$

幅频响应特性　　相频响应特性

电路稳态响应的幅值和相角都随着频率变化而变化！

(3) 传递函数



网络函数/传递函数 $H=\dfrac{\text{响应}}{\text{激励}}$

$$H(\mathrm{j}\omega)=\frac{\dot{U}_C}{\dot{U}_S}=\frac{\frac{1}{\mathrm{j}\omega C}}{R+\frac{1}{\mathrm{j}\omega C}}=\frac{1}{1+\mathrm{j}\omega CR}$$

幅频响应特性

相频响应特性

$$=\frac{1}{\sqrt{1+(\omega RC)^2}}\angle -\arctan(\omega RC)=|H(\mathrm{j}\omega)|\angle\varphi(\omega)$$

传递函数的幅值和相角都随着频率变化而变化！

正弦激励下动态电路的稳态响应随激励频率变化的特性就称为频率响应特性。

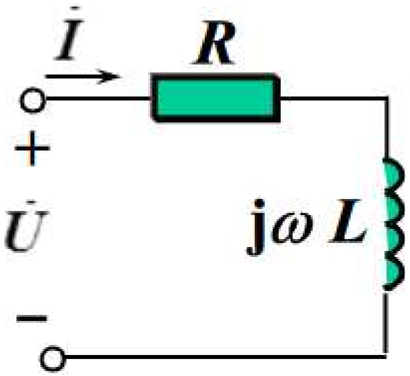
两点说明：

$$L \rightarrow \omega L \qquad C \rightarrow \frac{1}{\omega C}$$

- 电阻电路的响应不随频率而改变。

- 频响特性并非电路系统所特有。

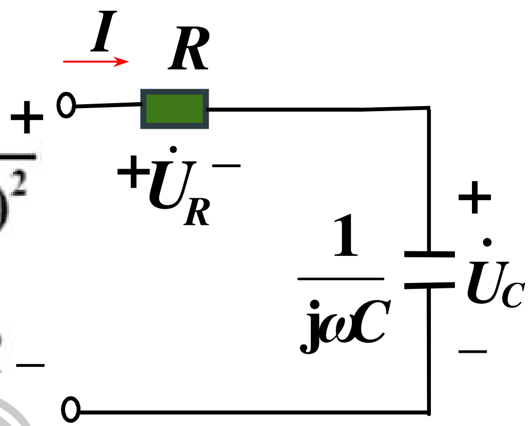
例1. 画出图示电路入端阻抗幅频特性曲线



$$Z = R + \mathrm{j}\omega L$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

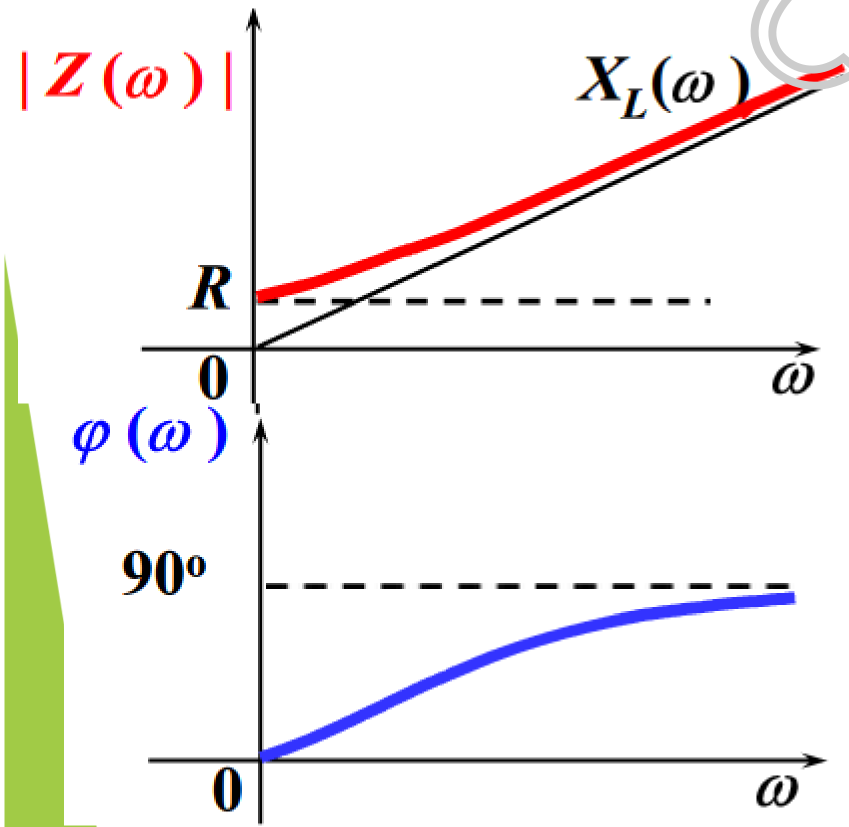
$$\varphi = \arctan \omega L / R$$

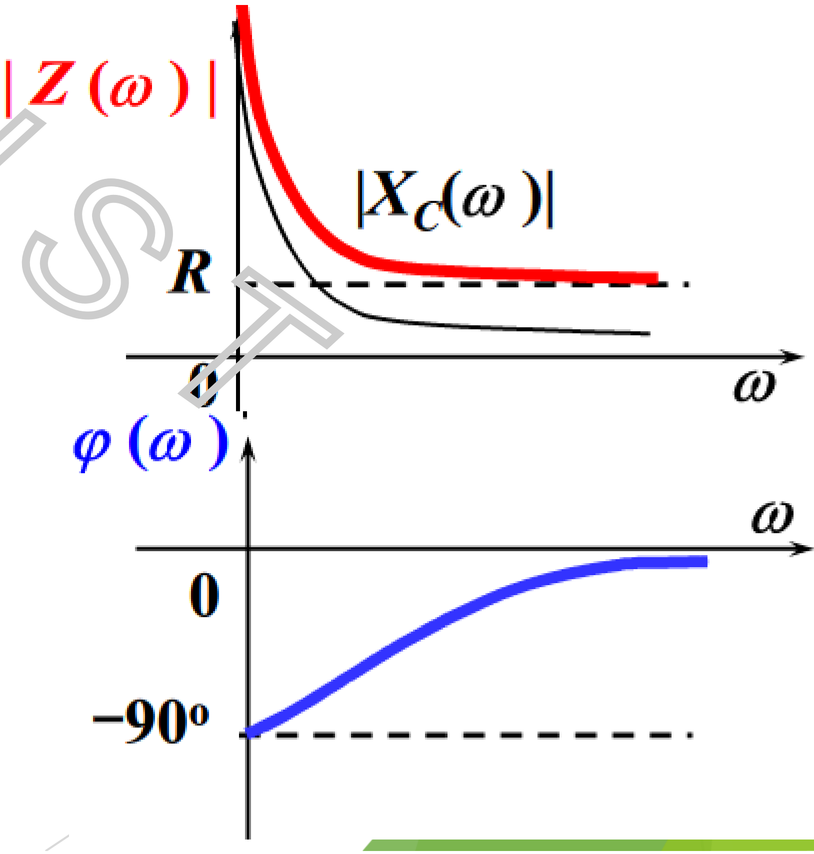


$$Z = R - \mathrm{j}\frac{1}{\omega C}$$

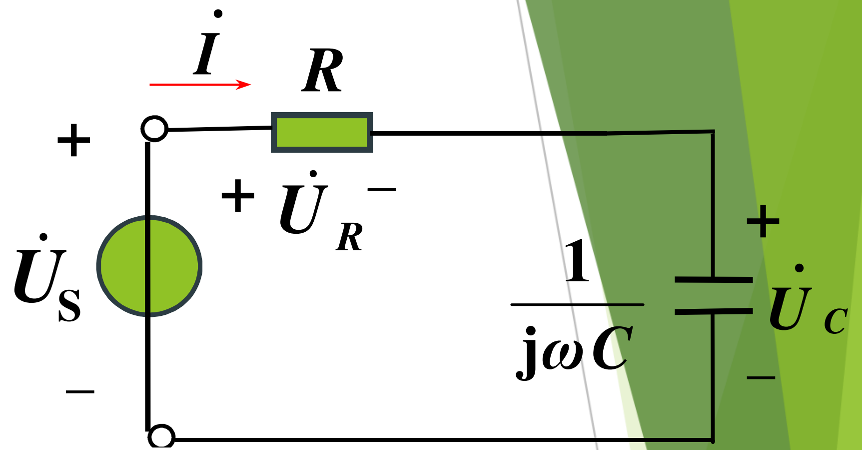
$$|Z| = \sqrt{R^2 + (\frac{1}{\omega C})^2}$$

$$\varphi = -\arctan \frac{1}{\omega CR}$$



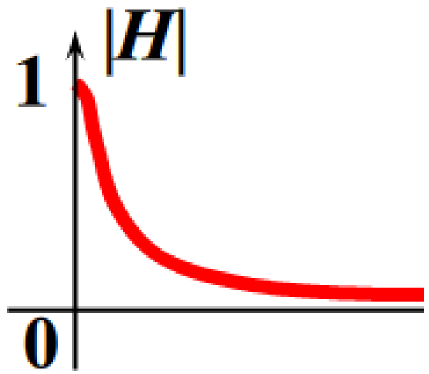


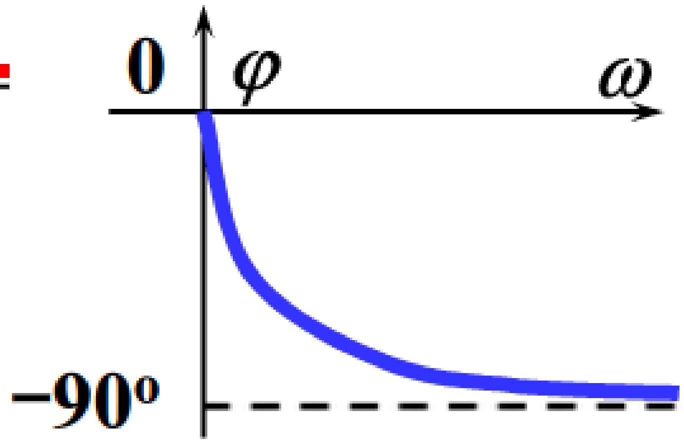
例2. 画出图示电路传递函数频率特性曲线



$$H(\omega)=|H|\angle\psi=\frac{1}{\sqrt{1+(\omega RC)^2}}\angle-\arctan(\omega RC)$$

设 RC=1

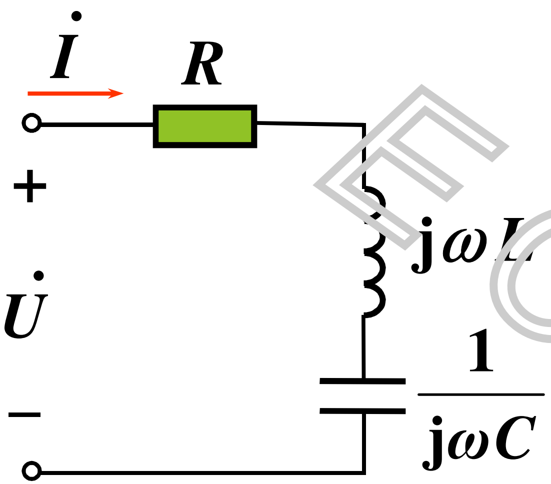




ω /(rad/s)	$\lvert H\rvert$	φ (°)
0	1	0
1	0.707	−45
5	0.196	−78.7
10	0.1	−84.3
20	0.05	−87.1
100	0.01	−89.4

11-2 *RLC* 串联谐振电路

一、什么是电路的谐振？



串联谐振

$$Z = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$

$$\omega L > \frac{1}{\omega C}$$ 感性

$$\omega L < \frac{1}{\omega C}$$ 容性

$$\omega L = \frac{1}{\omega C}$$ 阻性

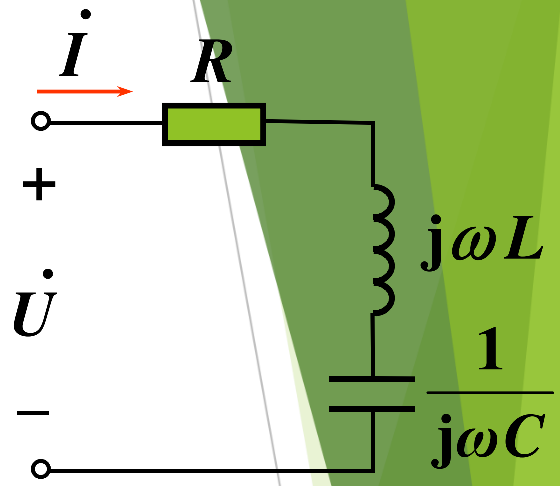
当 $X_L=|X_C|$， $\varphi=0$， 端口上电压、电流同相。电路的这种状态称为谐振。

谐振时入端阻抗为纯电阻

二、RLC串联谐振

1. 谐振条件

(1) LC 不变, 改变 ω, 使 $X_L=|X_C|$



谐振时 $\omega_0 L = \dfrac{1}{\omega_0 C}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

谐振角频率 (*resonant angular frequency*)

$$Z_0 = R$$

谐振时阻抗

重点：

定性画频率特性

求谐振频率

求谐振入端电阻

LC串联谐振相当于短路

(2) 电源频率不变，改变 L 或 C (常改变C) 。

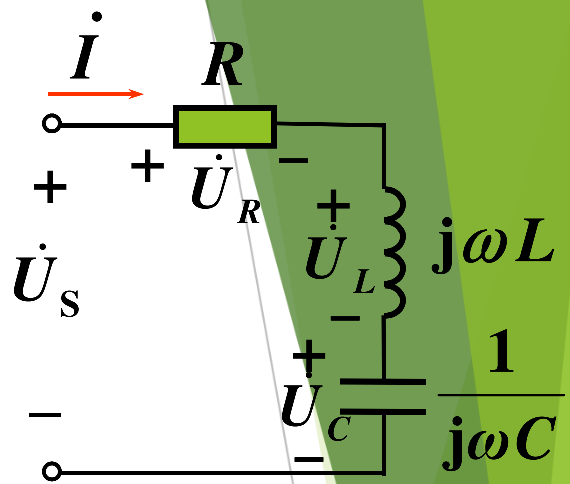
2. 串联谐振时的电压和电流

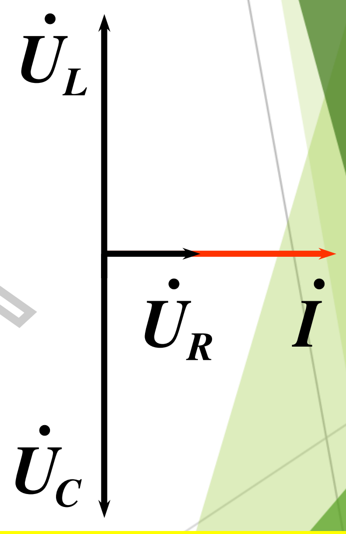
$$\dot{U}_R = R\dot{I} = \dot{U}_S$$

$$\dot{U}_L = \mathrm{j}\omega_0 L\dot{I} = \mathrm{j}\frac{\omega_0 L}{R}\dot{U}_S$$

$$\dot{U}_C = \frac{\dot{I}}{\mathrm{j}\omega_0 C} = -\mathrm{j}\frac{1}{\omega_0 C R}\dot{U}_S$$

$$\omega_0 L = \frac{1}{\sqrt{LC}}L = \sqrt{L/C} = \frac{1}{\omega_0 C}$$





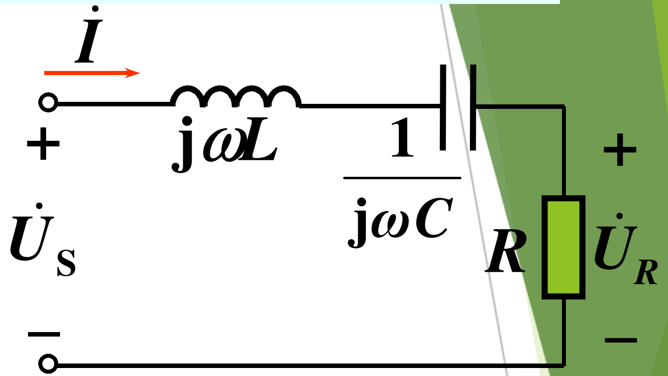
谐振时的相量图

串联谐振又称<u>电压谐振</u>

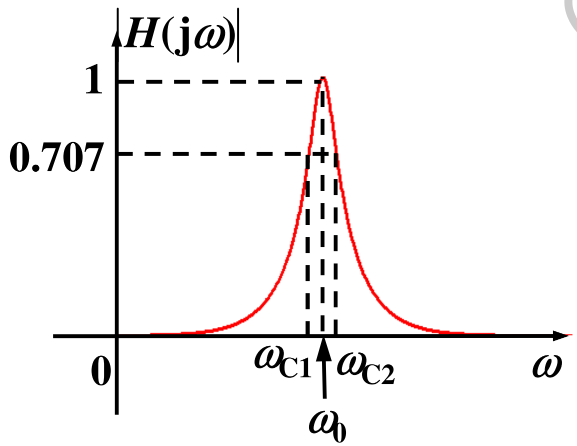
***L*、*C*上可能出现高电压**

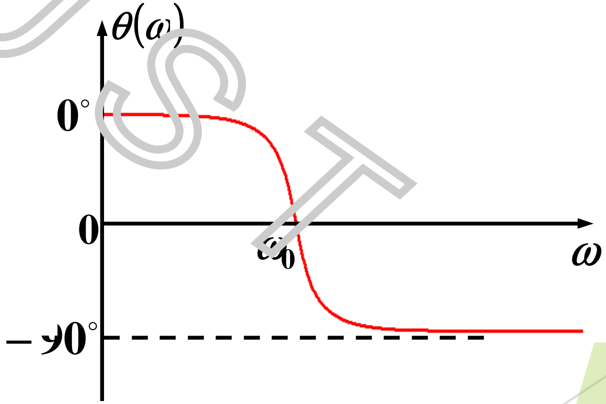
11-3 *RLC* 串联谐振电路的频率特性

(1) 网络函数的频率特性



$$H(\mathrm{j}\omega)=\frac{\dot{U}_R}{\dot{U}_S}=\frac{R}{R+\mathrm{j}\omega L+1/\mathrm{j}\omega C}=\frac{\mathrm{j}\omega CR}{1-\omega^2 LC+\mathrm{j}\omega CR}$$





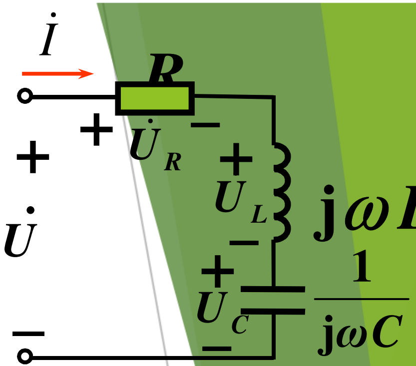
LC 带通滤波电路电路

$$\omega_0=\frac{1}{\sqrt{LC}}$$

中心频率

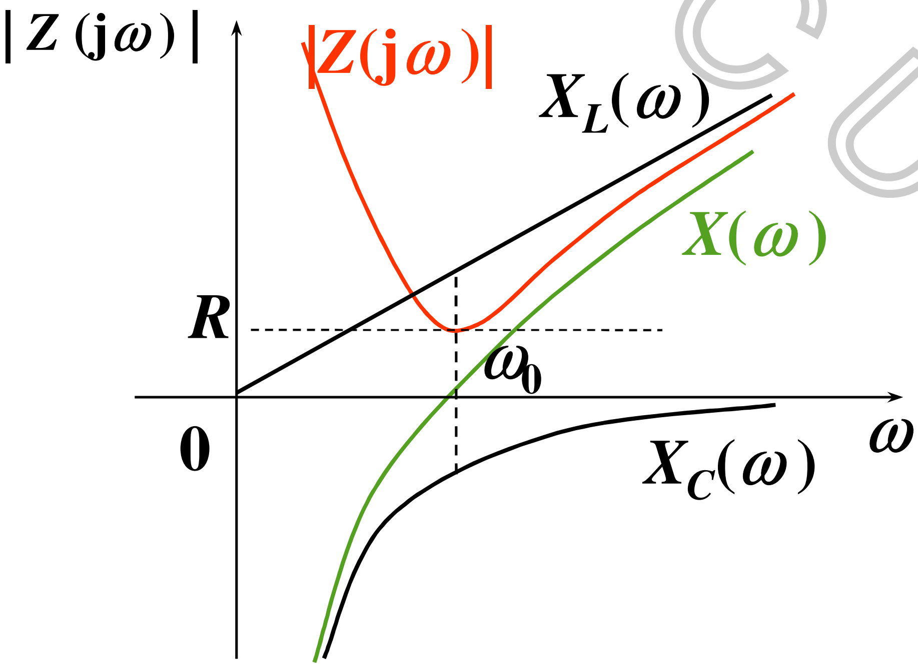
(2) 阻抗频率特性

$$Z = R + \mathrm{j}(\omega L - \frac{1}{\omega C}) = |Z(\mathrm{j}\omega)| \angle \varphi(\omega)$$

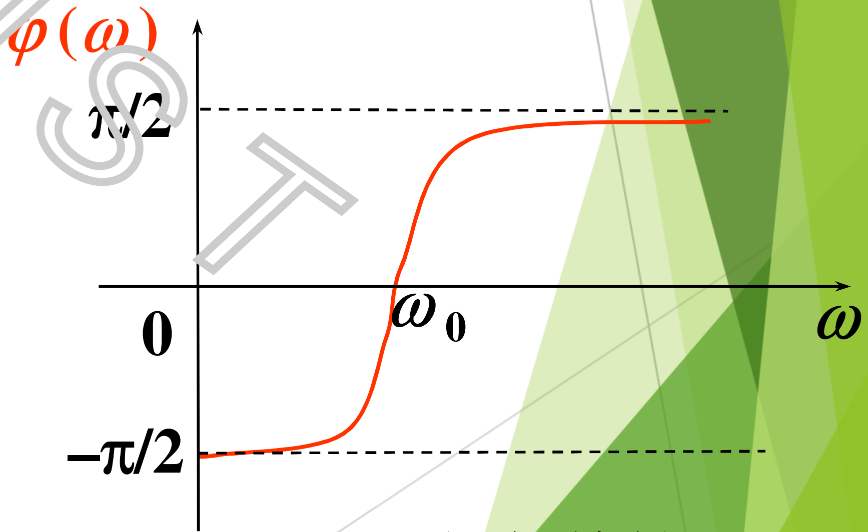


幅频特性 $|Z(\omega)| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

相频特性 $\varphi(\omega) = \arctan \dfrac{\omega L - \frac{1}{\omega C}}{R}$

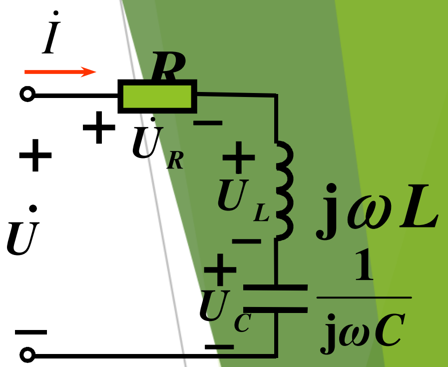


阻抗幅频特性



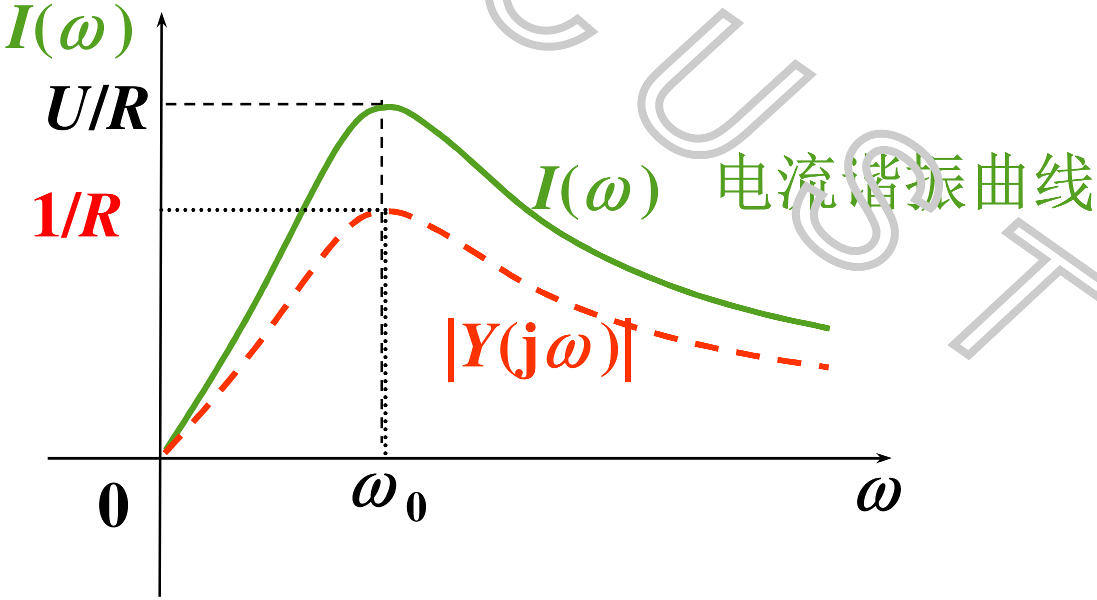
阻抗相频特性

(3) 电流频率特性



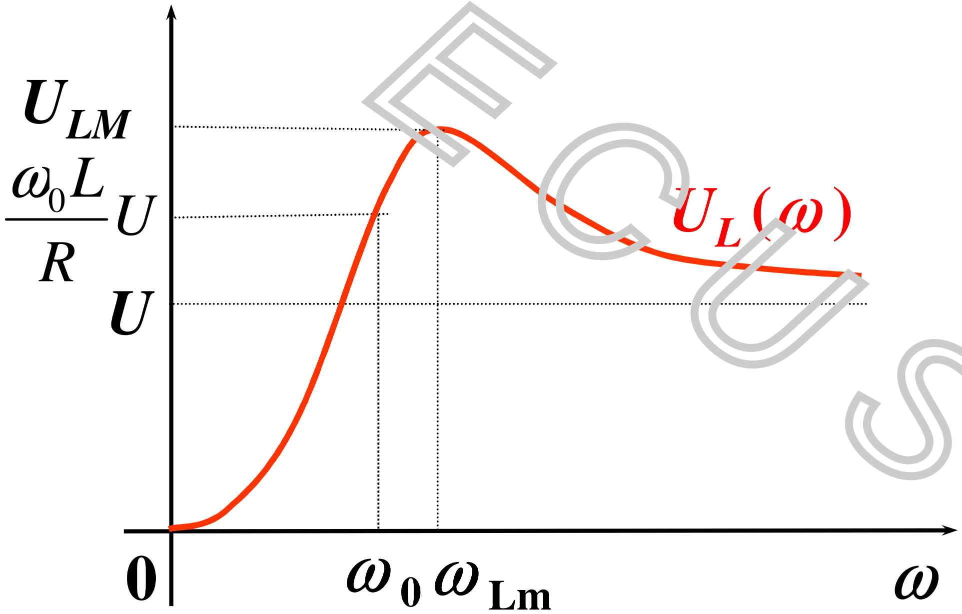
幅值关系：

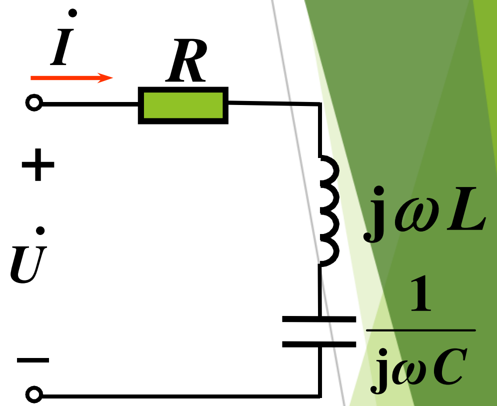
$$I(\omega)=\frac{U}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}=|Y(j\omega)|U$$

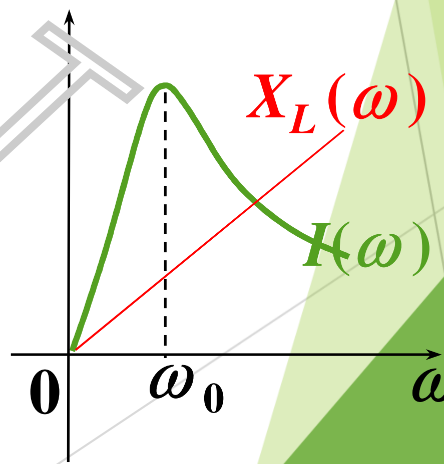


(4) $U_L(\omega)$与$U_C(\omega)$的频率特性

$$U_L(\omega)=\omega LI=\omega L\cdot\frac{U}{|Z|}=\frac{\omega LU}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$$

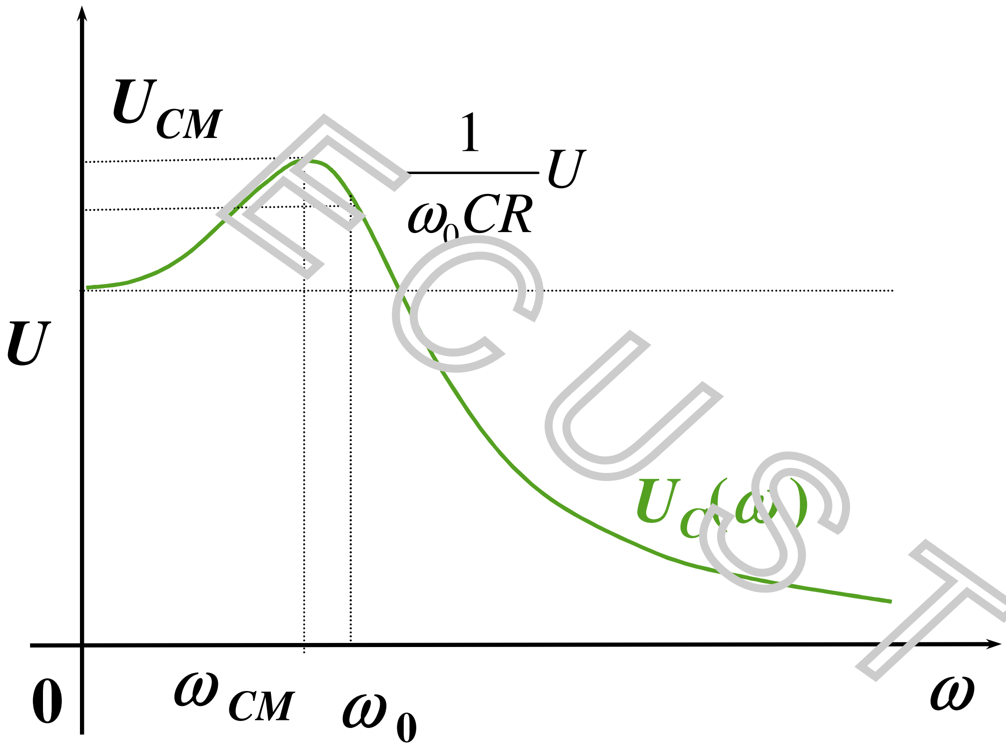


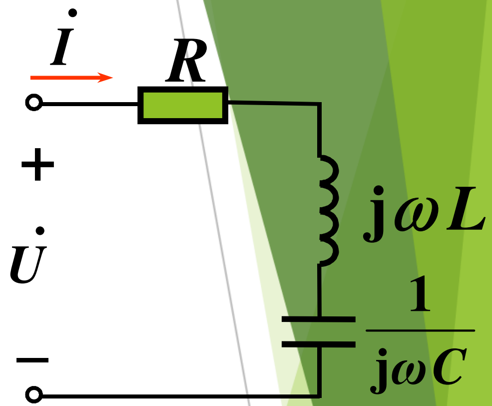


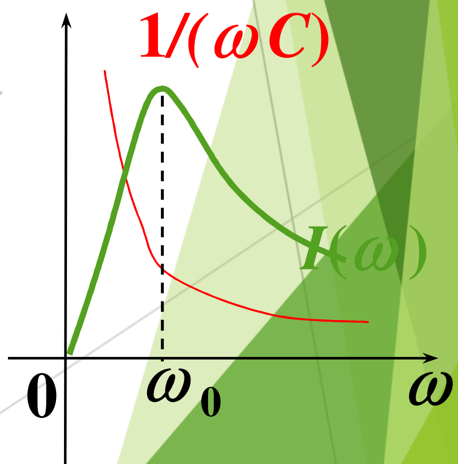


高通滤波器

$$U_C(\omega)=\frac{I}{\omega C}=\frac{U}{\omega C\sqrt{R^2+(\omega L-\dfrac{1}{\omega C})^2}}$$



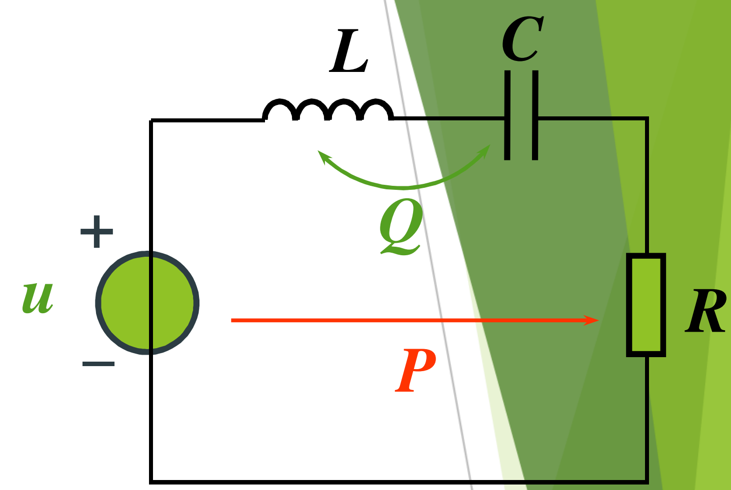




低通滤波器

$$\omega_{Lm}\bullet\omega_{Cm}=\omega_0^{2}$$

串联谐振时的能量



设 $u = U_{\mathrm{m}} \sin \omega_0 t$

则 $i = \dfrac{U_{\mathrm{m}}}{R} \sin \omega_0 t = I_{\mathrm{m}} \sin \omega_0 t$

电源发出功率

无功 $Q = UI \sin \varphi = 0$

有功 $P = UI \cos \varphi = RI^2$

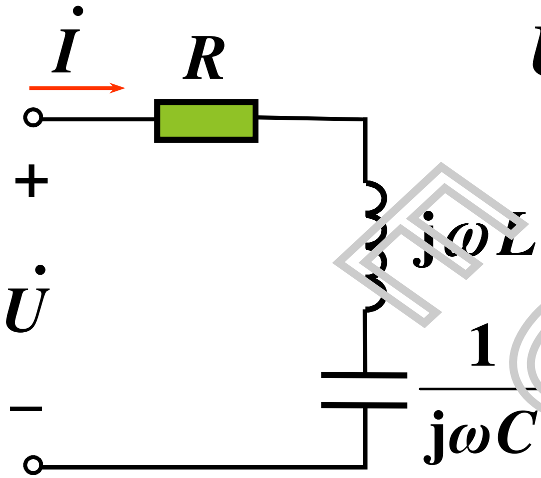
磁场能量 $w_L = \dfrac{1}{2} Li^2 = \dfrac{1}{2} LI_{\mathrm{m}}^2 \sin^2 \omega_0 t$

$$u_C = U_{C\mathrm{m}} \sin(\omega_0 t - 90^\circ) = \frac{I_{\mathrm{m}}}{\omega_0 C} \sin(\omega_0 t - 90^\circ) = -\sqrt{\frac{L}{C}} I_{\mathrm{m}} \cos \omega_0 t$$

电场能量 $w_C = \dfrac{1}{2} Cu_C^2 = \dfrac{1}{2} CU_{C\mathrm{m}}^2 \cos^2 \omega_0 t = \dfrac{1}{2} LI_{\mathrm{m}}^2 \cos^2 \omega_0 t$

衡量电路谐振程度的指标——品质因数Q

1. 从放大信号的能力来衡量谐振电路



$$\dot{U}_{L0} = j\omega_0 L\dot{I}_0 = \frac{j\omega_0 L}{R}\dot{U}$$

$$\dot{U}_{C0} = j\frac{1}{\omega_0 CR}\dot{U}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \sqrt{\frac{L}{C}}\frac{1}{R}$$

特性阻抗　　单位：Ω

(*characteristic impedance*)

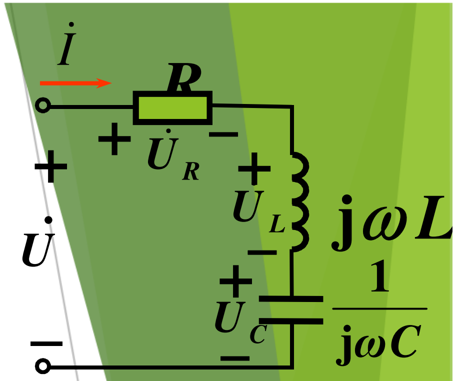
$$\rho = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

$$\dot{U}_{L0} = jQ\dot{U} \qquad \dot{U}_{C0} = -jQ\dot{U}$$

Q大 → 谐振电路对电压或电流的放大能力大

对并联谐振电路分析可以得到类似的电流放大结果。

2. 从电磁能量的转换来衡量谐振电路



$$u = U_{\max} \sin \omega t$$

$$i = \frac{U_{\max}}{R} \sin \omega t = I_{\max 0} \sin \omega t$$

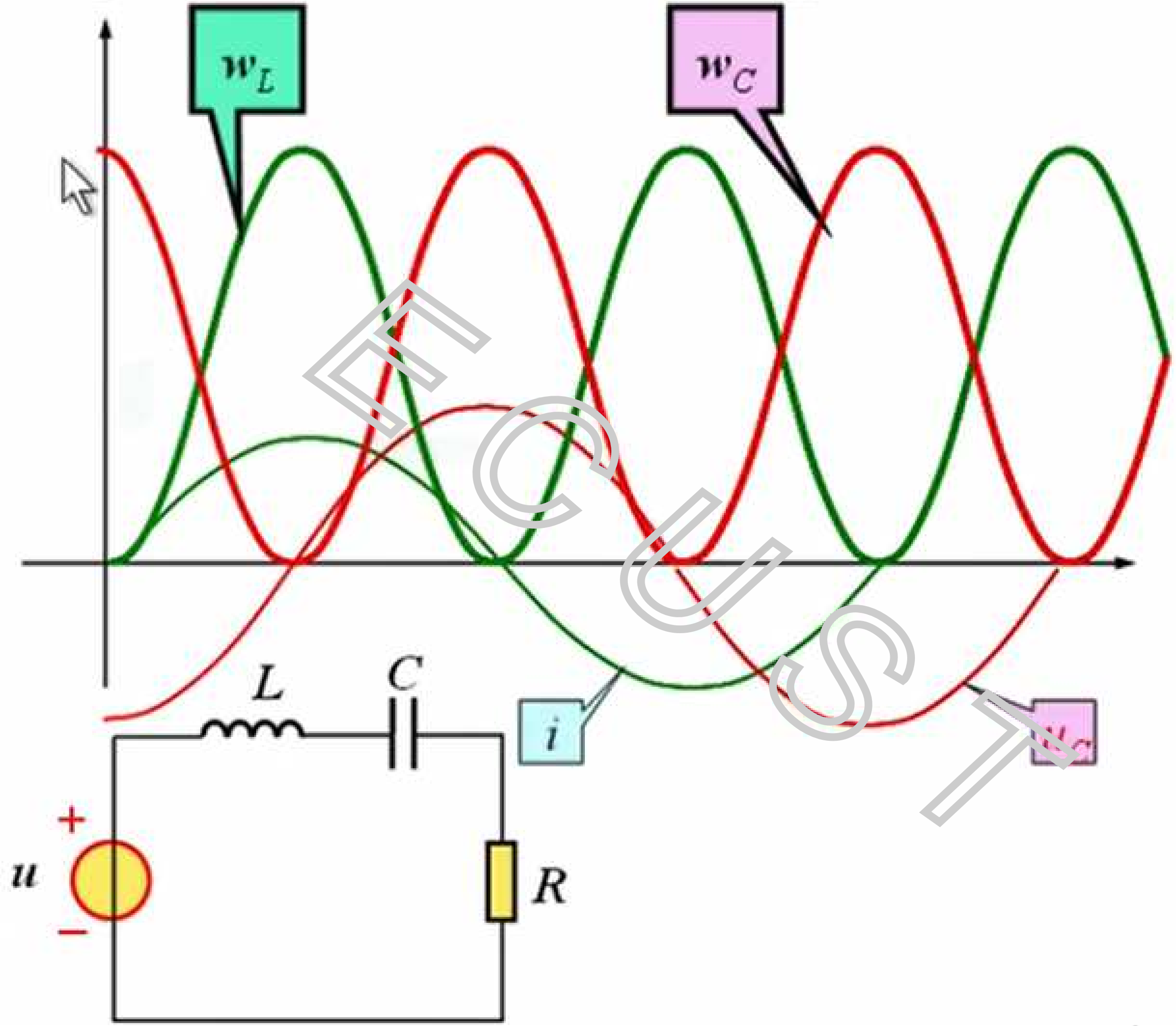
$$w_{总} = w_L + w_C = \frac{1}{2} L I^2_{\max 0} = \frac{1}{2} C U^2_{C\max 0} = L I_0^2$$

$$Q = 2\pi \frac{\text{谐振时电路中电磁场的总储能}}{\text{谐振时一周期内电路消耗的能量}}$$

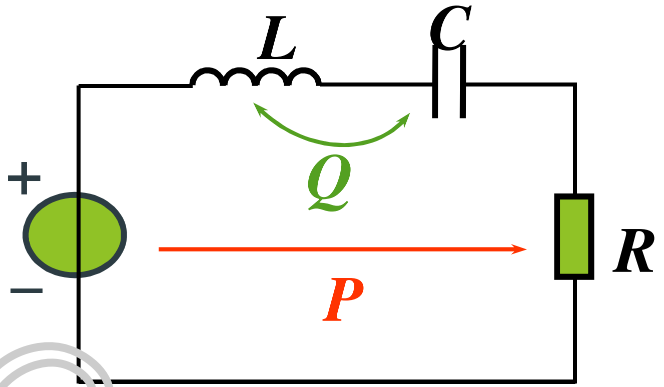
$$= 2\pi \cdot \frac{L I_0^2}{R I_0^2 T_0} = \omega_0 \cdot \frac{L I_0^2}{R I_0^2} = \frac{\omega_0 L}{R}$$

Q大 ⟶ 维持一定的储能所消耗的平均功率少

磁场能量 $w_L = \frac{1}{2} L I_m^2 \sin^2 \omega_0 t$ 电场能量 $w_C = \frac{1}{2} L I_m^2 \cos^2 \omega_0 t$



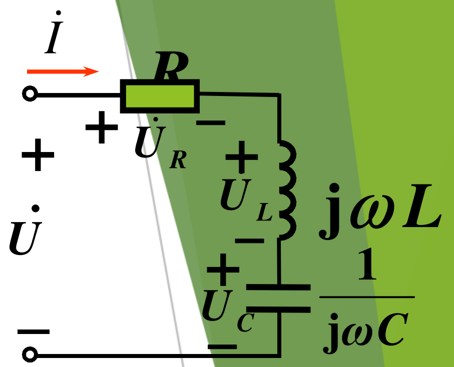
* 功率关系



$$Q=\frac{\omega_0 L}{R}=\frac{\omega_0 L I_0^2}{R I_0^2}=\frac{Q_{L0}}{P}=\frac{|Q_{C0}|}{P}$$

$$=\frac{\text{谐振时电感(或电容)中无功功率的绝对值}}{\text{谐振时电阻消耗的有功功率}}$$

3. 从频率特性来衡量谐振电路



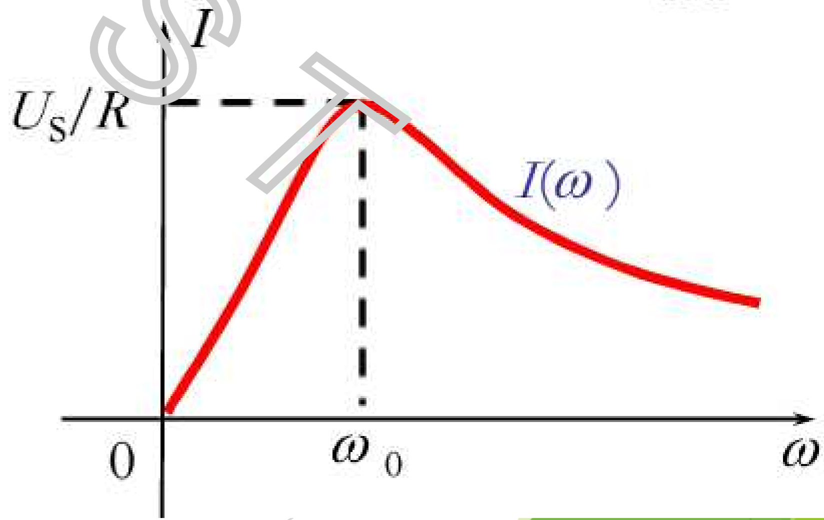
电流频率特性

$$\dot{I}=\frac{\dot{U}_S}{R+j(\omega L-\frac{1}{\omega C})}$$

幅值关系

$$I(\omega)=\frac{U_S}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}\le\frac{U_S}{R}$$

电流谐振曲线



频率特性的尖锐程度
反映了电路选择性的好坏。

如何比较不同谐振频率的
两个电路？

进行频率的归一化！！

纵轴的归一化

$$\frac{I(\omega)}{I(\omega_0)} = \frac{U/|Z|}{U/R} = \frac{R}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}-\frac{1}{\omega RC}\right)^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{\omega_0 L}{R}\cdot\frac{\omega}{\omega_0}-\frac{1}{\omega_0 RC}\cdot\frac{\omega_0}{\omega}\right)^2}} = \frac{1}{\sqrt{1+\left(Q\cdot\frac{\omega}{\omega_0}-Q\cdot\frac{\omega_0}{\omega}\right)^2}}$$

Q的定义1

横轴的归一化

$$\eta = \frac{\omega}{\omega_0}$$

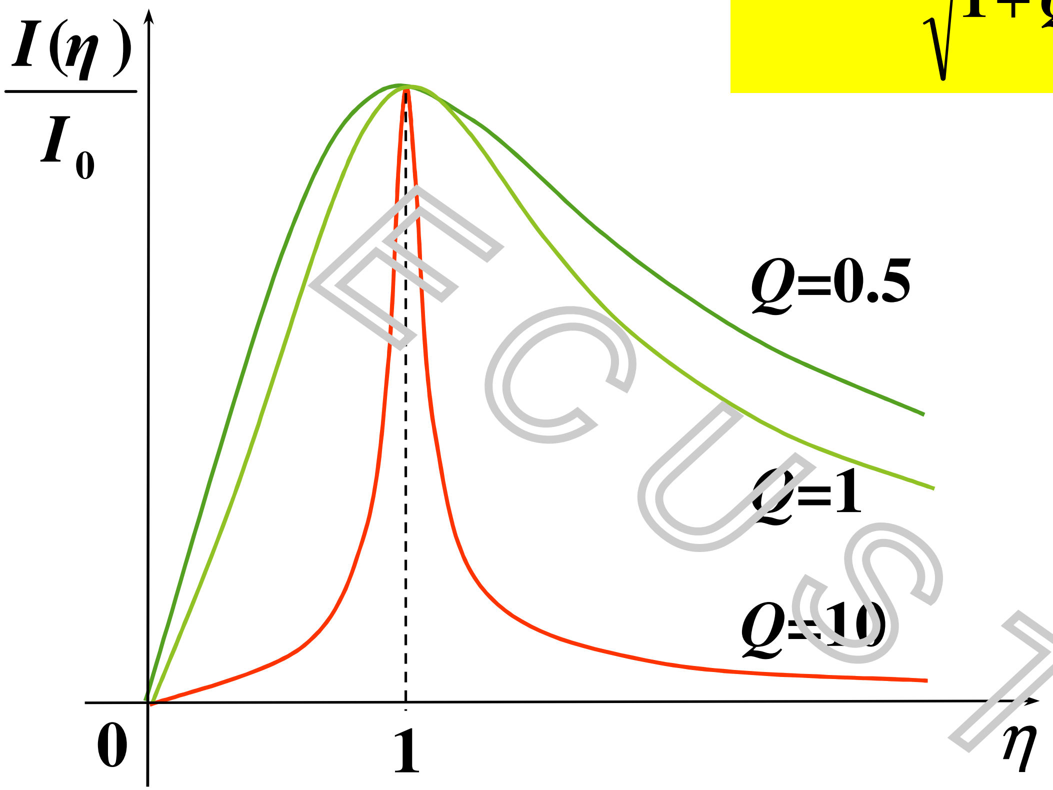
$$\frac{I(\eta)}{I_0} = \frac{1}{\sqrt{1+Q^2\left(\eta-\frac{1}{\eta}\right)^2}}$$

$\eta = 1$时，$\dfrac{I(\eta)}{I_0} = 1$

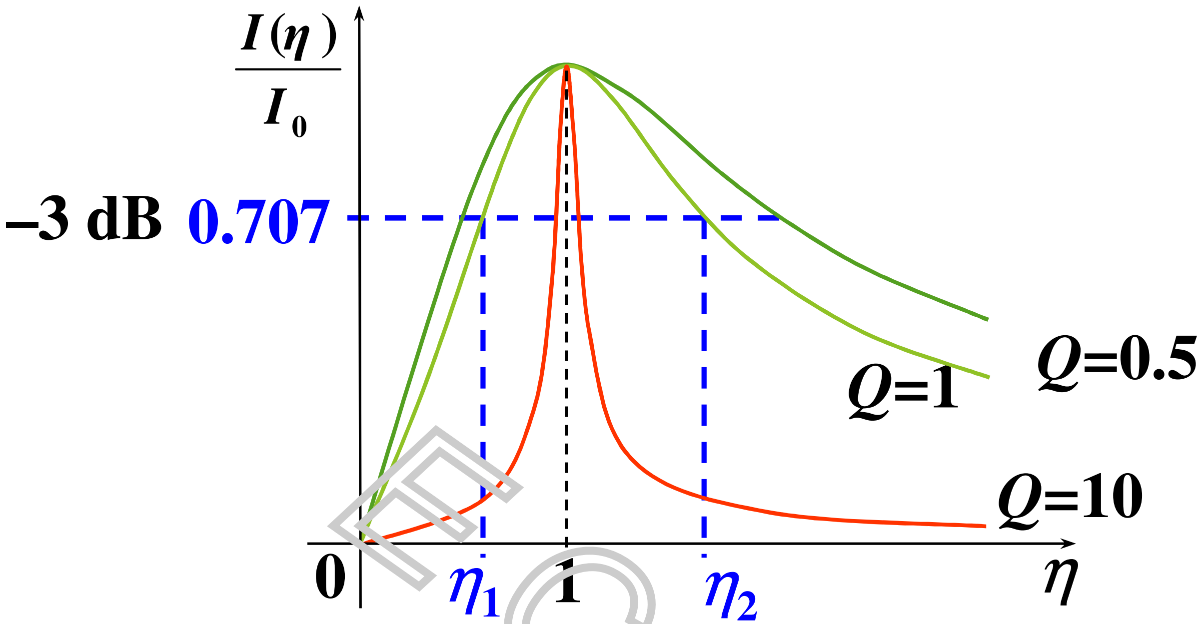
所有谐振电路都在η＝1处谐振，
谐振点的幅频特性值为1。

通用谐振频率特性

$$\frac{I(\eta)}{I_0}=\frac{1}{\sqrt{1+Q^2(\eta-\frac{1}{\eta})^2}}$$



*Q*越大，谐振曲线越尖。当稍微偏离谐振点时，曲线就急剧下降，电路对非谐振频率下的电流具有较强的抑制能力，所以选择性好。



$$\eta = \frac{\omega}{\omega_0}$$

$$\frac{I(\eta)}{I_0} = \frac{1}{\sqrt{1+Q^2(\eta-\frac{1}{\eta})^2}} = \frac{1}{\sqrt{2}}$$

$$\eta_1 = -\frac{1}{2Q} + \sqrt{\frac{1}{4Q^2}+1}$$

$$\eta_2 = \frac{1}{2Q} + \sqrt{\frac{1}{4Q^2}+1}$$

$$\eta_2 - \eta_1 = \frac{1}{Q}$$

$$Q \overset{\text{def}_3}{=} \frac{1}{\eta_2-\eta_1} = \frac{\omega_0}{\omega_2-\omega_1}$$

带宽**Band Width** (BW)

品质因数Q定义的归纳

- 从信号幅值的变化来衡量

$$Q \overset{\text{def}_1}{=} \frac{U_{L0}}{U_S} = \frac{U_{C0}}{U_S}$$

Q大 ⟶ 谐振时电容电压和电感电压大。

- 从电磁能量的转换来衡量

$$Q \overset{\text{def}_2}{=} 2\pi \frac{\text{电路中储存的电磁场总能量}}{\text{谐振时一个周期内电路消耗的能量}}$$

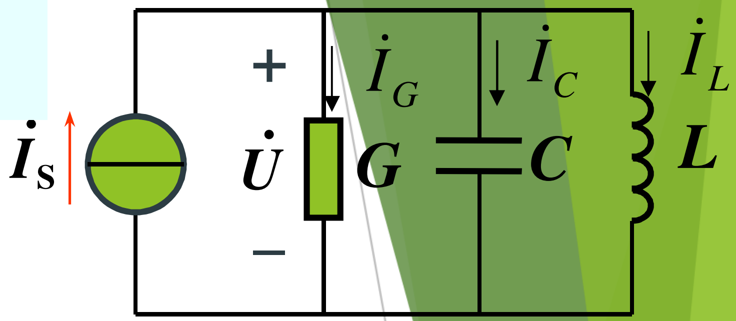
Q大 ⟶ 谐振时储能大，消耗能量少。

- 从频率特性的形状来衡量

$$Q \overset{\text{def}_3}{=} \frac{\omega_0}{\omega_2 - \omega_1}$$

Q大 ⟶ 谐振电路的选择性好

11-4 *RLC* 并联谐振电路



并联谐振时的电压和电流

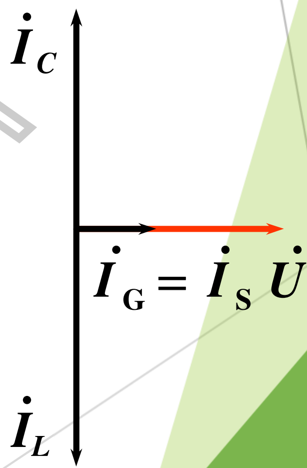
$$\dot I_G = G\dot U = \dot I_S \qquad \dot U = \frac{\dot I_S}{G}$$

$$Y = G + \mathrm{j}\left(\omega C - \frac{1}{\omega L}\right)$$

$$\dot I_L = \frac{\dot U}{\mathrm{j}\omega_0 L} = -\mathrm{j}\frac{1}{\omega_0 LG}\dot I_S$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

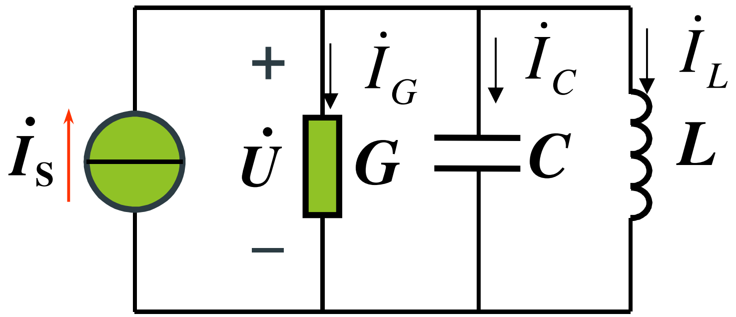
$$\dot I_C = \mathrm{j}\omega_0 C\dot U = \mathrm{j}\frac{\omega_0 C}{G}\dot I_S$$

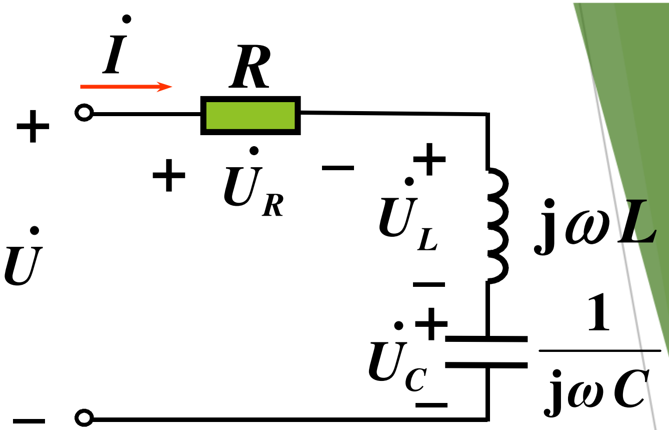


谐振时的相量图

并联谐振又称电流谐振

L、*C*上可能出现大电流





对偶

$$Y = G + j(\omega C - \frac{1}{\omega L}) \qquad Z = R + j(\omega L - \frac{1}{\omega C})$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

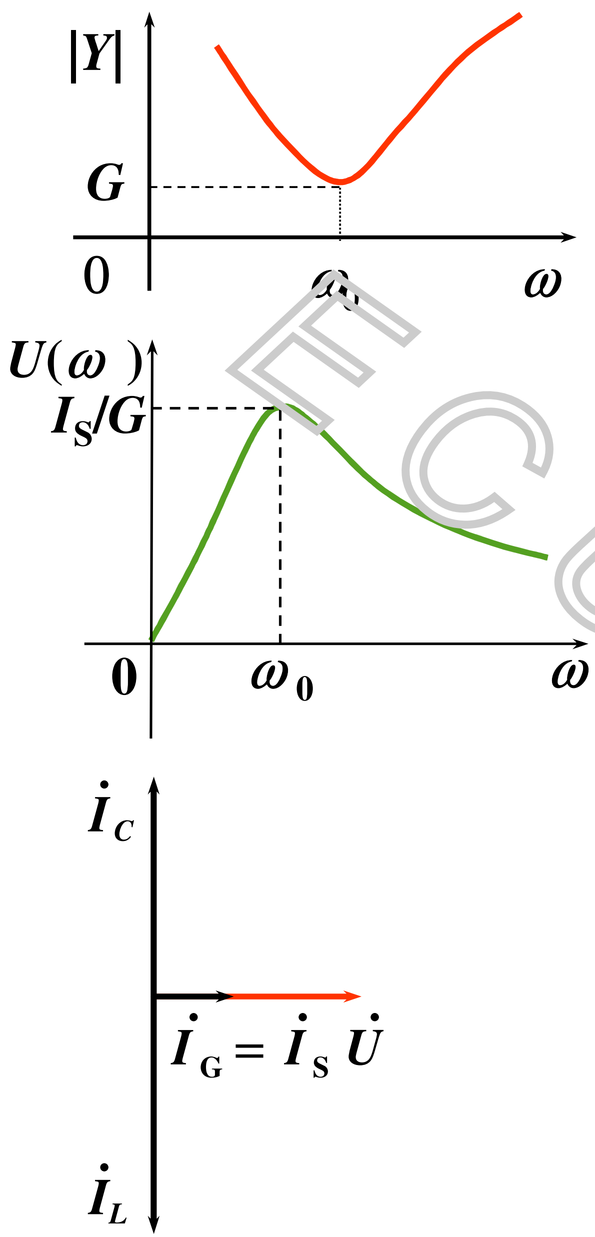
$$\boxed{Z_0 = 1/G} \qquad \boxed{Z_0 = R}$$

$$\dot{I}_L = \dot{I}_C = -j\frac{\sqrt{C/L}}{G}\dot{I}_S \qquad \dot{U}_L = \dot{U}_C = j\frac{\sqrt{L/C}}{R}\dot{U}_S$$

*LC*并联部分相当于开路 　　 *LC*串联部分相当于短路

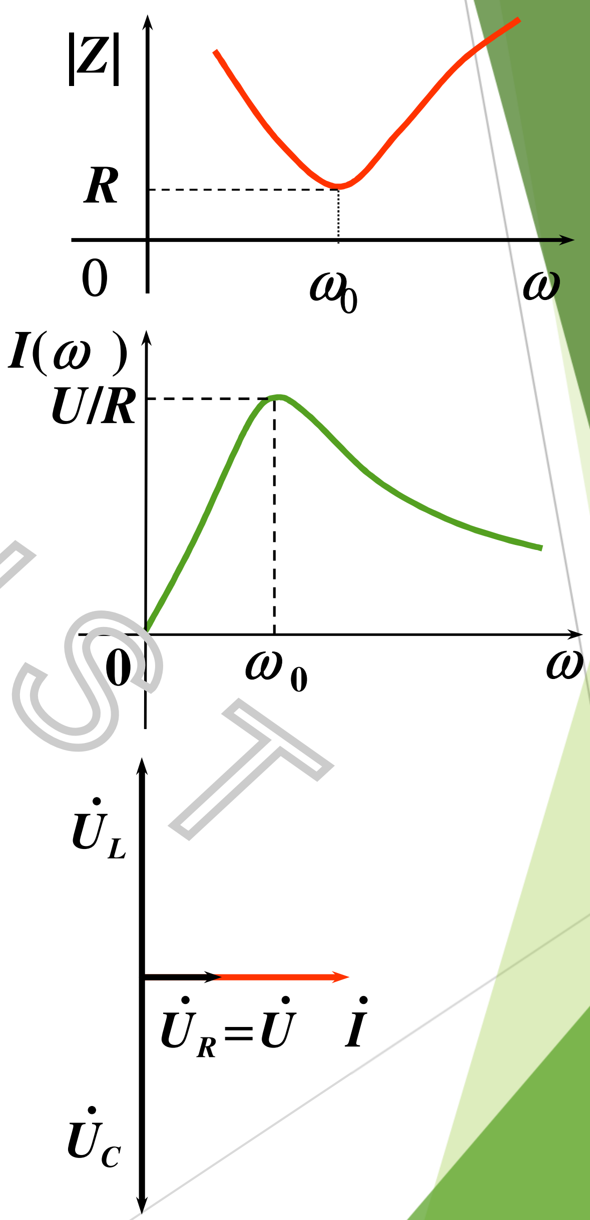
电流谐振 　　 电压谐振

$G\,C\,L$ 并联



$|Y|$
G
0
ω_0
ω
$U(\omega)$
I_S/G
0
ω_0
ω
$\dot{I}_C$
$\dot{I}_G = \dot{I}_S$ $\dot{U}$
$\dot{I}_L$

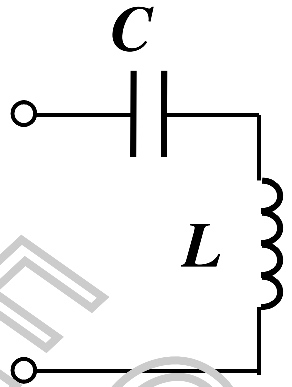
$R\,L\,C$ 串联



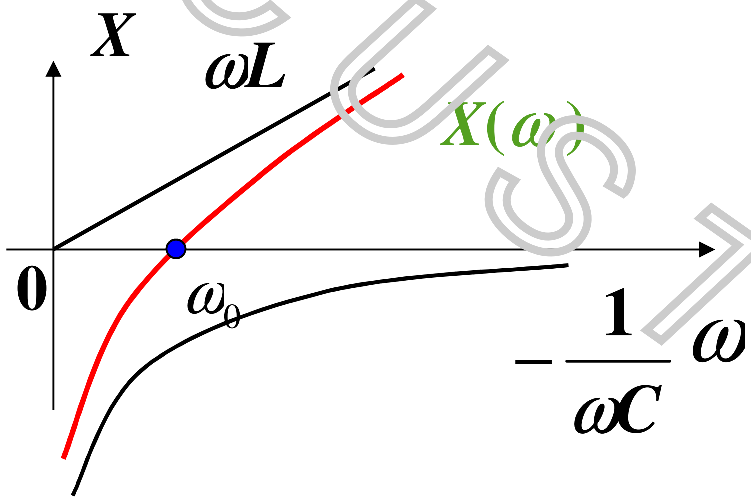
$|Z|$
R
0
ω_0
ω
$I(\omega)$
U/R
0
ω_0
ω
$\dot{U}_L$
$\dot{U}_R = \dot{U}$ $\dot{I}$
$\dot{U}_C$

由纯电感和纯电容构成的谐振电路-- LC谐振

串联谐振

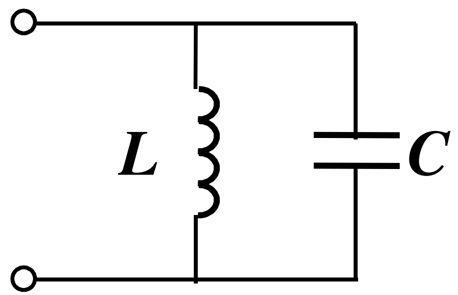


$$jX = j\left(\omega L - \frac{1}{\omega C}\right)$$



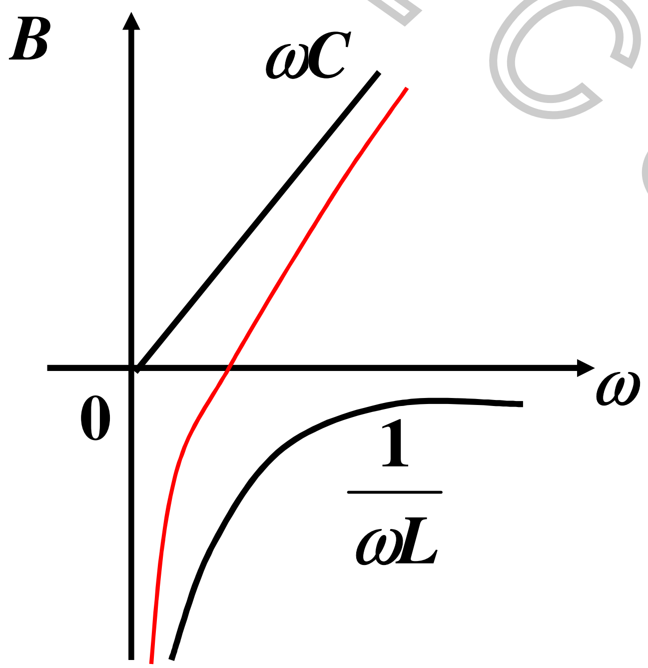
$\omega = \omega_0$时，相当于短路

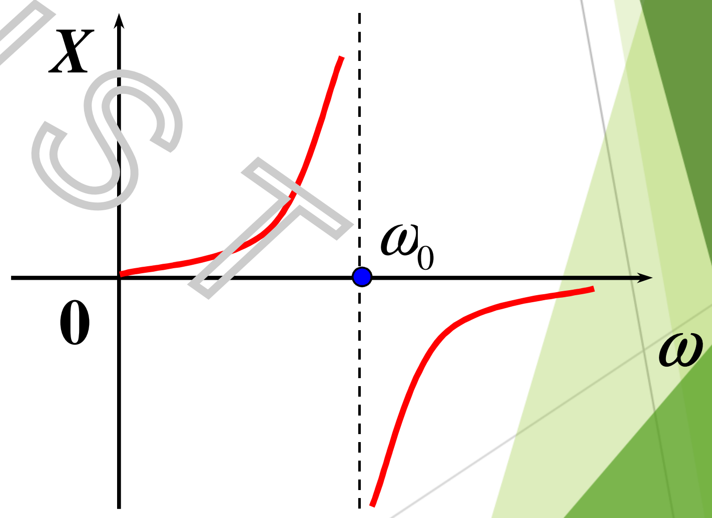
并联谐振



$$jB = \frac{1}{j\omega L} + j\omega C = j\left(\omega C - \frac{1}{\omega L}\right)$$

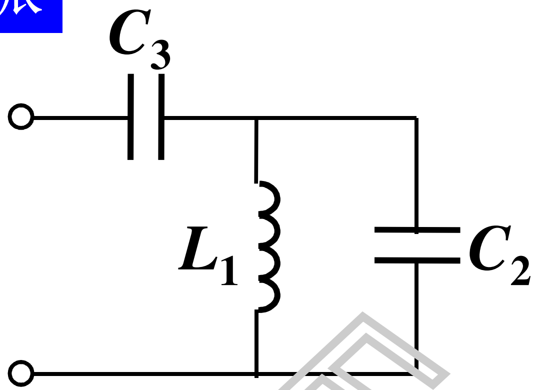
$$jX = \frac{1}{jB} = j\left(-\frac{1}{B}\right)$$

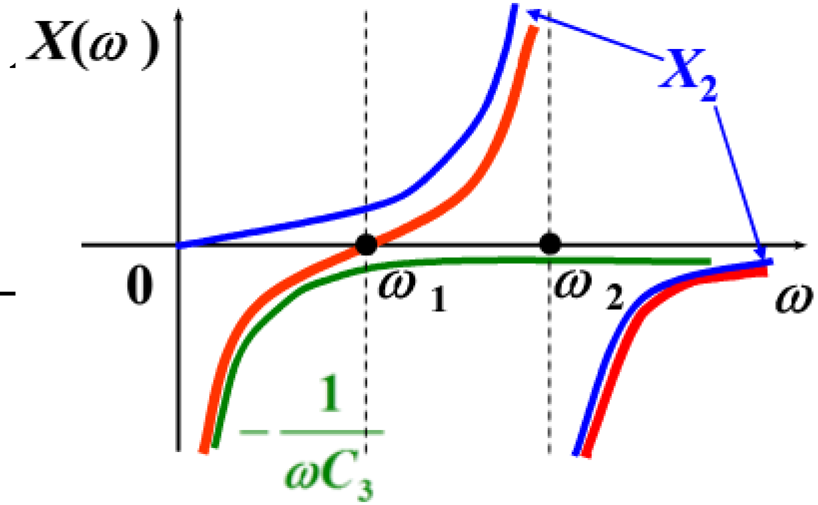




$\omega = \omega_0$ 时　　相当于开路

混联谐振





$$Z(\omega)=\frac{1}{\mathrm{j}\omega C_3}+\frac{\mathrm{j}\omega L_1\cdot\dfrac{1}{\mathrm{j}\omega C_2}}{\mathrm{j}\omega L_1+\dfrac{1}{\mathrm{j}\omega C_2}}=\frac{1}{\mathrm{j}\omega C_3}+\frac{\mathrm{j}\omega L_1}{1-\omega^2 L_1 C_2}$$

$$=-\mathrm{j}\frac{1-\omega^2 L_1(C_2+C_3)}{\omega C_3(1-\omega^2 L_1 C_2)}$$

分别令分子、分母为零，可得：

$$\omega_1=\frac{1}{\sqrt{L_1(C_2+C_3)}}$$ 串联谐振 $Z_0=0$

$$\omega_2=\frac{1}{\sqrt{L_1 C_2}}$$ 并联谐振 $Z_0=\infty$