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第9章 正弦稳态分析

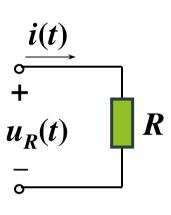
----正弦激励下动态电路的稳态响应

- 9.1 阻抗和导纳
- 9.2 相量图
- 9.3 正弦稳态电路分析
- 9.4 正弦电路功率

9.1 阻抗和导纳

一、元件特性的相量形式

1. 电阻



已知
$$i(t) = \sqrt{2}I\sin(\omega t + \psi)$$

$$u_R(t) = Ri(t) = \sqrt{2}RI\sin(\omega t + \psi)$$

相量形式:

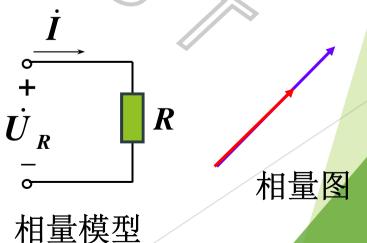
$$\dot{I} = I \angle \psi$$

有效值关系: $U_R = RI$

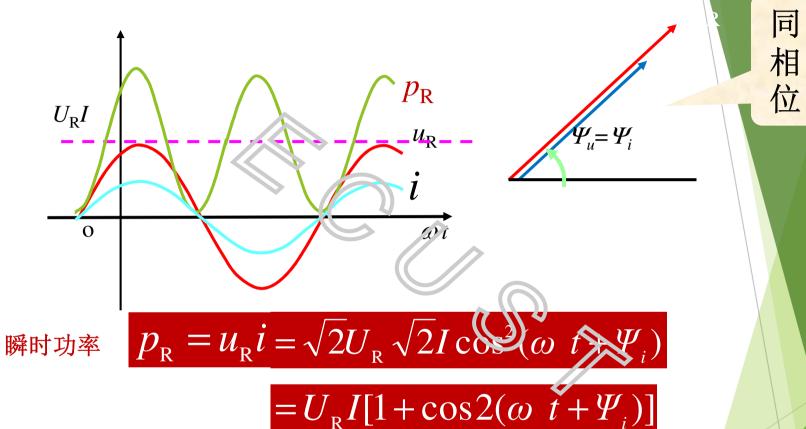
$$\dot{U}_R = RI \angle \psi$$

相位关系: u,i同相

相量关系



波形图及相量图



瞬时功率以2ω交变,始终大于零,表明电阻始终吸收功率

2. 电感

时域

频域

$$i(t) = \sqrt{2}I\sin\omega t$$

$$u(t) = L \frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$=\sqrt{2}\omega LI\cos\omega t$$

$$= \sqrt{2}\omega L I \sin(\omega + 9\%^{\circ})$$

$$\dot{I} = I \angle 0^{\circ}$$

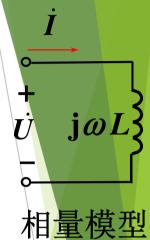
$$\dot{U} = j\omega L \dot{I}$$

有效值关系

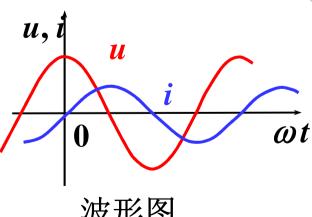
$$U = \omega L I$$

相位关系

u 超前:90°



相量图



波形图

$$U=\omega LI$$

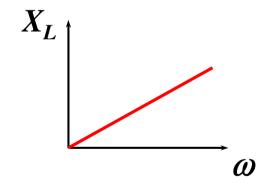
$$X_L = U/I = \omega L = 2\pi f L$$
, 单位: Ω

 $\omega L \times \frac{u}{i}$ $\omega L \times \frac{\dot{U}}{i}$

感抗(inductive reactance)

感抗的物理意义:

- (1) 表示限制电流的能力;
- (2) 感抗和频率成正比。

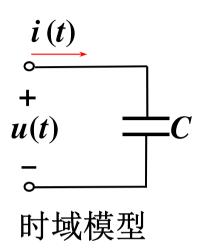


$$\omega = 0($$
直流), $X_L = 0$, 短路;

$$\omega \rightarrow \infty$$
, $X_L \rightarrow \infty$, 开路;

(3) 由于感抗的存在使电流落后电压。

3. 电容



时域

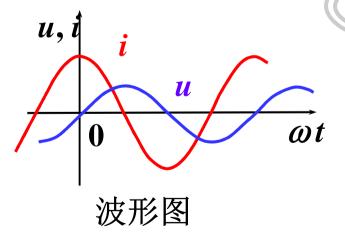
$$u(t) = \sqrt{2}U \sin \omega t$$

$$du(t)$$

$$\mathbf{d}t$$

$$= \sqrt{2}\omega CU \cos \omega i$$

$$= \sqrt{2}\omega CU \sin(\omega t + 95^{\circ})$$



频域

$$\dot{U} = U \angle 0^{\circ}$$

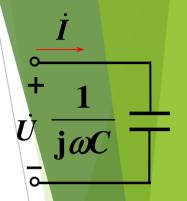
$$\dot{I} = j\omega C \dot{U}$$

有效值关系

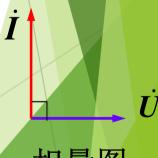
$$I=\omega C U$$

相位关系

i 超剪u 90°



相量模型



相量图

$$I=\omega CU$$

$$\frac{U}{I} = \frac{1}{\omega C}$$

$$X_C = -\frac{1}{\omega C}$$

容抗 (capacitive reactance)

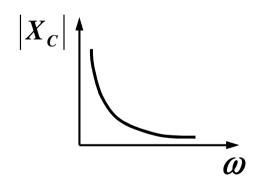
错误的写法

$$\frac{1}{\omega C} \times \frac{u}{i}$$

$$\frac{1}{\omega C} \times \frac{\dot{U}}{\dot{I}}$$

容抗的物理意义:

- (1) 表示限制电流的能力;
- (2) 容抗的绝对值和频率成反比。



$$\omega = 0$$
(直流), $|X_{\rm C}| \to \infty$, 隔直作用;

$$\omega \rightarrow \infty$$
, $X_C \rightarrow 0$, 旁路作用;

(3) 由于容抗的存在使电流领先电压。

二、电路定律的相量形式和电路的相量模型

1. 基尔霍夫定律的相量形式

$$\sum i(t) = 0 \qquad \Rightarrow \qquad \sum \dot{I} = 0$$

$$\sum u(t) = 0 \qquad \Rightarrow \qquad \sum \dot{U} = 0$$

2. 电路元件的相量关系

$$u = Ri$$

$$\dot{U} = Ri$$

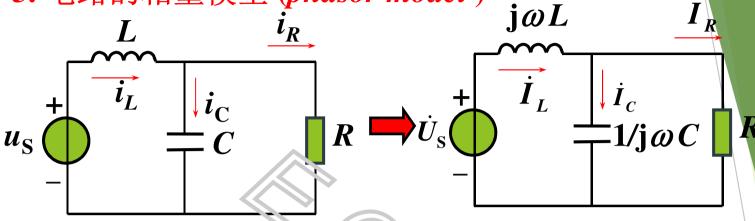
$$u = L \frac{\mathrm{d}i}{\mathrm{d}t}$$

$$\dot{U} = j\omega Li$$

$$u = \frac{1}{C} \int i \, \mathrm{d}t$$

$$\dot{U} = \frac{1}{j\omega C} i$$

3. 电路的相量模型 (phasor model)



时域电路

$$\begin{cases} i_{L} = i_{C} + i_{R} \\ L \frac{\mathrm{d}i_{L}}{\mathrm{d}t} + \frac{1}{C} \int i_{C} \mathrm{d}t = u_{S} \\ R i_{R} = \frac{1}{C} \int i_{C} \mathrm{d}t \end{cases}$$

时域列写微分方程

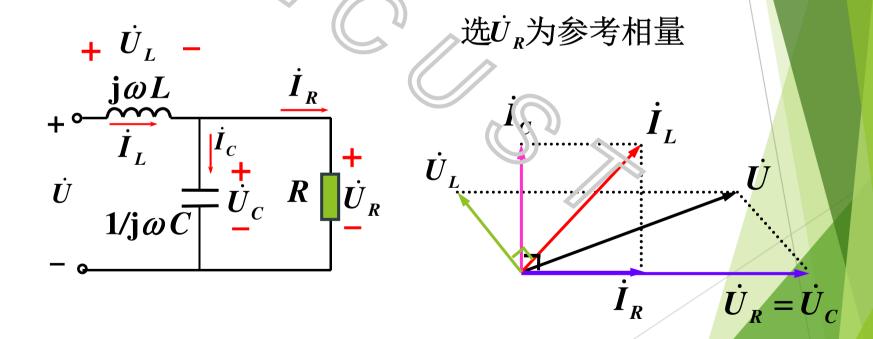
相量模型

$$\begin{vmatrix}
\dot{I}_{L} = \dot{I}_{C} + \dot{I}_{R} \\
\dot{\mathbf{j}} \omega \dot{I}_{L} + \dot{\mathbf{j}} \omega \dot{C} \dot{I}_{C} = \dot{U}_{S} \\
R\dot{I}_{R} = \frac{1}{\dot{\mathbf{j}} \omega \dot{C}} \dot{I}_{C}
\end{vmatrix}$$

相量形式代数方程

4. 相量图(phasor diagram)

- (1) 同频率的正弦量才能表示在同一个相量图中;
- (2) 相量以 ω 角速度逆时针方向旋转;
- (3) 选定一个参考相量(设初相位为零)。



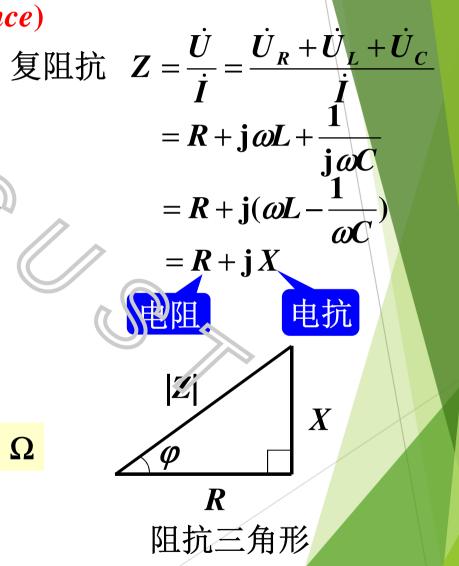
三、复阻抗和复导纳

1. 复阻抗(complex impedance)

$$Z = R + jX = |Z| \angle \varphi$$

$$|Z| = \frac{U}{I}$$
 阻抗模 单位: Ω

$$\varphi = \psi_u - \psi_i$$
 阻抗角



具体分析一下 RLC 串联电路:

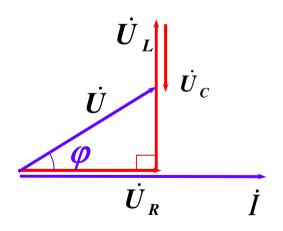
 $Z=R+j(\omega L-1/\omega C)=|Z|\angle\varphi$

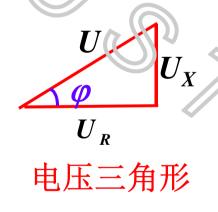
 $\omega L > 1/\omega C$, X>0, $\varphi>0$, 电压领先电流, 电路显感性;

 $\omega L < 1/\omega C$, X < 0 . $\varphi < 0$,电压落后电流,电路呈容性;

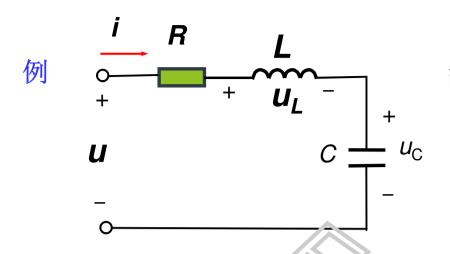
 $\omega L=1/\omega C$, X=0 , $\omega=0$, 电压与电流同相,电路呈电阻性。

画相量图: 选电流为参考向量($\omega L > 1/\omega C$)





$$U = \sqrt{U_R + U_X}$$
 $|Z|$
 R
阻抗三角形



已知: R=15 Ω , L=0.3mH, C=0.2 μ F, $u=5\sqrt{2}\sin(\omega t+60^\circ)\mathrm{V}, f=3 imes10^4\mathrm{Hz}$ 求 i , u_R , u_L , u_C \circ

解 其相量模型为

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{5\angle 60^{\circ}}{33.54\angle 63.4^{\circ}} = 0.149\angle - 3.4^{\circ} \text{ A}$$

$$\dot{U}_{R} = R\dot{I} = 15\times 0.149\angle - 3.4^{\circ} = 2.235\angle - 3.4^{\circ} \text{ V}$$

$$\dot{U}_{L} = \mathbf{j}\omega L\dot{I} = 56.5\angle 90^{\circ} \times 0.149\angle - 3.4^{\circ} = 8.42\angle 86.4^{\circ} \text{ V}$$

$$\dot{U}_{C} = -\mathbf{j}\frac{1}{\omega C}\dot{I} = 26.5\angle - 90^{\circ} \times 0.149\angle - 3.4^{\circ} = 3.95\angle - 93.4^{\circ} \text{ V}$$

则

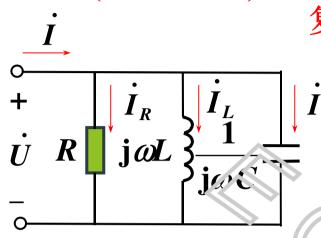
$$i = 0.149\sqrt{2}\sin(\omega t - 3.4^{\circ}) \text{ A}$$
 $u_R = 2.235\sqrt{2}\sin(\omega t - 3.4^{\circ}) \text{ V}$
 $u_L = 8.42\sqrt{2}\sin(\omega t + 86.6^{\circ}) \text{ V}$
 $u_C = 3.95\sqrt{2}\sin(\omega t - 93.4^{\circ}) \text{ V}$

 \dot{U}_{c} \dot{U}_{L} \dot{U}_{c} \dot{U}_{L} \dot{U}_{L} \dot{U}_{R} \dot{I} 相量图

 U_L =8.42V>U=5V,分电压大于总电压,

为什么?

2. 复导纳(admittance)



复导纳
$$Y = \frac{\dot{I}}{\dot{U}} = \frac{\dot{I}_R + \dot{I}_L + \dot{I}_C}{\dot{U}}$$
 \dot{I}_C

$$= \frac{1}{R} + \frac{1}{\dot{j}\omega L} + \frac{1}{\dot{j}\omega C}$$

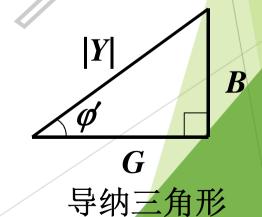
$$= G - \dot{j}\frac{1}{\omega L} + \dot{j}\omega C$$

$$= G + \dot{j}(B_L + B_C)$$

$$= G + \dot{j}B$$

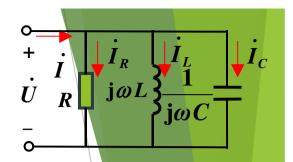
$$Y = \frac{\dot{I}}{\dot{U}} = G + jB = |Y| \angle \varphi'$$

$$\begin{cases} |Y| = \frac{I}{U} & \text{导纳的模} \quad \stackrel{\text{单位: S}}{} \\ \varphi' = \psi_i - \psi_u & \text{导纳角} \end{cases}$$



具体分析一下 RLC 并联电路

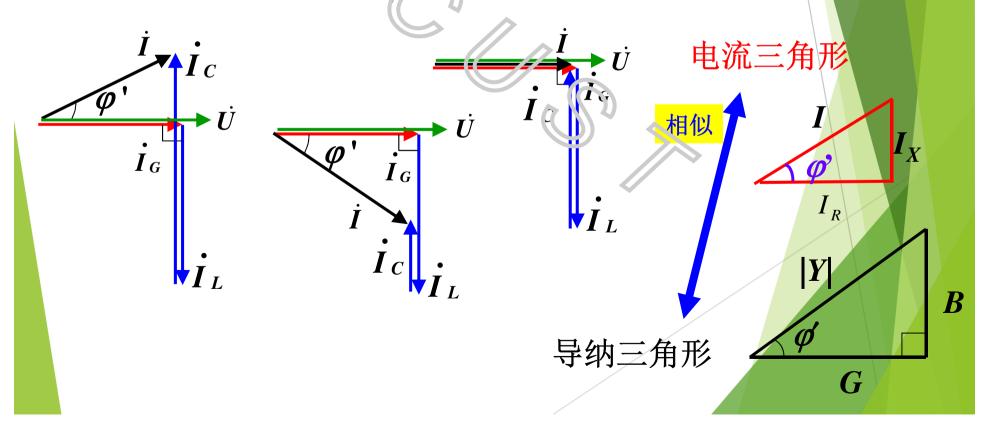
$$Y=G+j(\omega C-1/\omega L)=|Y|\angle\varphi'$$



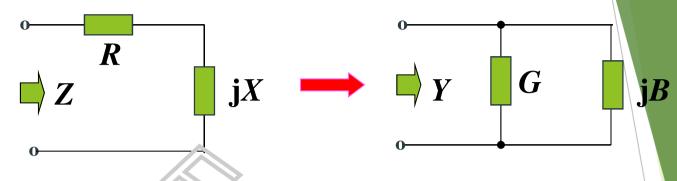
 $\omega C > 1/\omega L$, B>0, $\varphi'>0$, 电压落后电流, 电路呈容性;

 $\omega C < 1/\omega L$, $B < \omega$, $\varphi' < 0$, 电压领先电流,电路呈感性;

 $\omega C=1/\omega L$,B=0 , $\omega'=0$,电压与电流同相,电路呈阻性。



3. 复阻抗和复导纳的等效变换



$$Z = R + jX = |Z| \angle \varphi \implies Y = G + jB = |Y| \angle \varphi'$$

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = C + jB$$
 $Y = \frac{1}{Z}$

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = G + jB$$

$$\therefore G = \frac{R}{R^2 + X^2}, \quad B = \frac{-X}{R^2 + X^2}$$

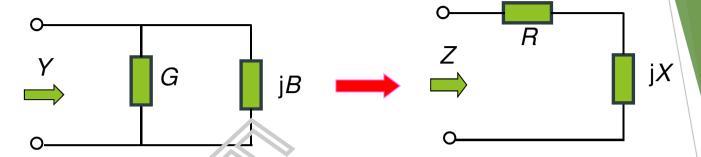
$$Y = \frac{1}{Z}$$

$$|Y| = \frac{1}{|Z|}$$

$$|Y| = \frac{1}{|Z|}$$

一般情况 $G \neq 1/R$ $B \neq 1/X$

同样, 若由Y变为Z, 则有:



$$Y = G + jB = |Y| \angle \varphi$$

$$Z = R + jX = |Z| \angle \varphi$$

$$Z = \frac{1}{Y} = \frac{1}{G + jB} = \frac{G - jB}{G^2 + B^2} = R + jX$$

$$\therefore R = \frac{G}{G^2 + B^2}, \quad X = \frac{-B}{G^2 + B^2}$$

$$|Y| = \frac{1}{|Z|}, \quad \varphi = -\varphi'$$

4. 阻抗串、并联

$$\dot{U} = \dot{U}_1 + \dot{U}_2 = \dot{I}Z_1 + IZ_2 = \dot{I}(Z_1 + Z_2)$$

两个阻抗串联

两个阻抗并联 等效导纲

$$Y$$
 \dot{U} Z_1 I_1 I_2 I_2 I_2 I_3 I_4 I_5 I_5

等效阻抗
$$Z = Z_1 + Z_2$$
 分压公式 $\dot{U}_1 = \frac{Z_1}{Z_1 + Z_2} \dot{U}$, $\dot{U}_2 = \frac{Z_2}{Z_1 + Z_2} \dot{U}$

$$Y = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{Z_1 + Z_2}{Z_1 Z_2}$$

等效阻抗
$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

分流公式
$$\dot{I}_1 = \frac{Z_2}{Z_1 + Z_2} \dot{I}, \dot{I}_2 = \frac{Z_1}{Z_1 + Z_2} \dot{I}$$

17个阻抗串联

等效阻抗
$$Z = \sum_{k=1}^{n} Z_k$$

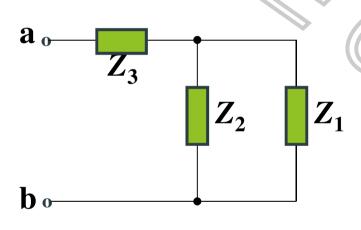
分压公式
$$\dot{U}_k = \frac{Z_k}{\sum_{k=1}^n Z_k} \dot{U}$$
 $(k = 1, 2, \dots, n)$

n个导纳并联

$$\dot{I}_{k} = \frac{Y_{k}}{\sum_{n=1}^{k} I} \quad (\lambda = 1, 2, \dots n)$$

例1 已知
$$Z_1$$
=10+j6.28Ω Z_2 =20-j31.9 Ω Z_3 =15+j15.7 Ω 求 Z_{ab} $^{\circ}$

$$Z_{ab} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = Z_3 + Z_3$$



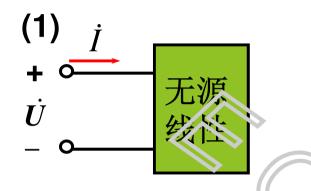
$$Z = \frac{(10 + \text{j6.28})(20 - \text{j31.9})}{10 + \text{j6.28} + 20 - \text{j31.9}}$$

$$= \frac{11.81 \angle 32.13^{\circ} \times 37.65 \angle - 57.61^{\circ}}{39.45 \angle - 40.5^{\circ}}$$

$$= 13.89 + \text{j2.86}\Omega$$

$$\therefore Z_{ab} = Z_3 + Z = 15 + j15.7 + 10.89 + j2.86$$
$$= 25.89 + j18.56 = 31.9 \angle 35.6^{\circ} \Omega$$

小结:



$$Z = \frac{\dot{U}}{\dot{I}}$$
 $\dot{U} = \dot{I}Z$ 相量形式 $Y = \frac{\dot{I}}{\dot{U}}$ $\dot{I} = Y\dot{U}$ 欧姆定理

- (2) Z是与u,i无关的复数。
- (3) 根据Z、Y可确定无源二端网络的注能
- (4) 一般情况Z、Y均是 ω 的函数

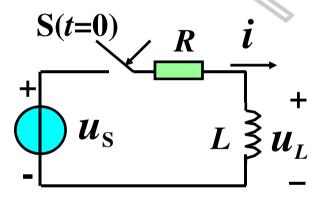
9-3 用相量法分析电路的正弦稳态响应

步骤: ① 画出电路的相量模型

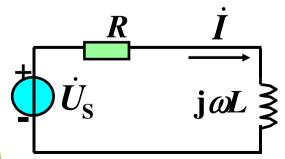
$$R, L, C o$$
复阻抗 $i, u o \dot{U}, \dot{I}$

② 列相量代数方程

例2 正弦激励下的过渡过程。



用相量法求i(∞)



$$u_{\rm S} = U_{\rm m} \sin(\omega t + \psi_{\rm u}) \quad i(0) = 0 \quad \Re : \quad \dot{i}(t)$$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)|_{0^+}]e^{-\frac{t}{\tau}}$$

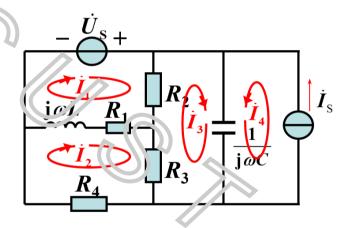
$$\dot{I} = \frac{\sqrt{2} - \psi_u}{\sqrt{R^2 + (\omega L)^2} \angle \arctan\left(\frac{\omega L}{R}\right)}$$

$$I = \frac{\sqrt[K]{\sqrt{2}}}{\sqrt{R^2 + (\omega L)^2}} \quad \varphi = \psi_u - \arctan\left(\frac{\omega L}{R}\right)$$

$$i(\infty) = \sqrt{2}I\sin(\omega t + \varphi)$$
 $i(\infty)|_{0^{+}} = \sqrt{2}I\sin\varphi$

$$i = \left(\sqrt{2}I\sin(\omega t + \varphi) - \sqrt{2}I\sin\varphi e^{-\frac{Rt}{L}}\right)\varepsilon(t)$$

例3 列写电路的回路电流方程。



解:

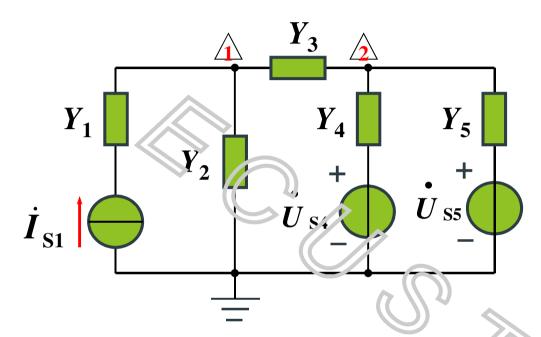
$$(R_{1} + R_{2} + j\omega L)\dot{I}_{1} - (R_{1} + j\omega L)\dot{I}_{2} - R_{2}\dot{I}_{3} = \dot{U}_{S}$$

$$(R_{1} + R_{3} + R_{4} + j\omega L)\dot{I}_{2} - (R_{1} + j\omega L)\dot{I}_{1} - R_{3}\dot{I}_{3} = 0$$

$$(R_{2} + R_{3} + \frac{1}{j\omega C})\dot{I}_{3} - R_{2}\dot{I}_{1} - R_{3}\dot{I}_{2} + \frac{1}{j\omega C}\dot{I}_{4} = 0$$

$$\dot{I}_{4} = \dot{I}_{S}$$

例4 列写电路的节点电压方程

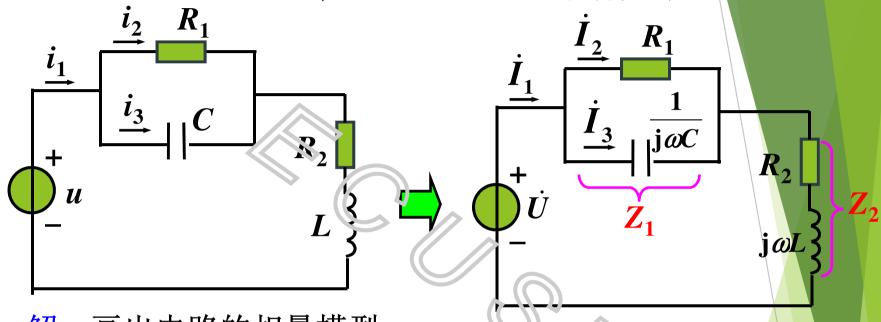


解:

$$(Y_2 + Y_3)\dot{U}_1 - Y_3\dot{U}_2 = \dot{I}_{S1}$$

$$-Y_3\dot{U}_1 + (Y_3 + Y_4 + Y_5)\dot{U}_2 = Y_4\dot{U}_{S4} + Y_5\dot{U}_{S5}$$

例 5 已知: $R_1 = 1000\Omega$, $R_2 = 10\Omega$, L = 500mH, C = 10μF, U = 100V, ω = 314 rad/s。求各支路电流。



解: 画出电路的相量模型

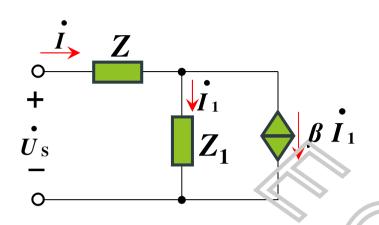
$$Z_{1} = \frac{R_{1}(\frac{1}{j\omega C})}{R_{1} + \frac{1}{j\omega C}} = \frac{1000 \times (-j318.5)}{1000 - j318.5} = \frac{318.5 \times 10^{3} \angle -90^{\circ}}{1049 \angle -17.67^{\circ}}$$
$$= 303.6 \angle -72.32^{\circ} = 92.20 - j289.3 \Omega$$
$$Z_{2} = R_{2} + j\omega L = 10 + j157 \Omega$$

$$\dot{I}_1 = \frac{\dot{U}}{Z} = \frac{100\angle 0^{\circ}}{167.2\angle -52.31^{\circ}} = 0.598\angle 52.3^{\circ} \text{ A}$$

$$\dot{I}_{2} = \frac{\sqrt{j\omega C}}{R_{1} + \frac{1}{j\omega C}} \dot{I}_{1} = \frac{-j318.5}{1049 \angle -17.67^{\circ}} \times 0.598 \angle 52.3^{\circ} = 0.182 \angle -20.0^{\circ} \text{ A}$$

$$\dot{I}_{3} = \frac{R_{1}}{R_{1} + \frac{1}{j\omega C}} \dot{I}_{1} = \frac{1000}{1049 \angle -17.67^{\circ}} \times 0.598 \angle 52.3^{\circ} = 0.570 \angle 70.0^{\circ} \text{ A}$$

已知: $Z=10+j50\Omega$, $Z_1=400+j1000\Omega$ 。



问: β 等于多少时, \dot{I}_1 和 \dot{U}_S 相位差90°?

 βI_1 分析:找出 I_1 和 U_S 关系: $\dot{U}_S = Z_{in}I_1$

 Z_{in} 实部为零, \dot{U}_{s} 与 \dot{I}_{1} 相位差为90°.

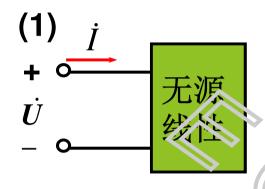
M:
$$\dot{U}_S = Z\dot{I} + Z_1\dot{I}_1 = Z(1+\beta)\dot{I}_1 + Z_1\dot{I}_1$$

$$\frac{\dot{U}_{S}}{\dot{I}_{1}} = (1+\beta)Z + Z_{1} = 410 + 10\beta + \mathbf{j}(50 + 50\beta + 1000)$$

$$\Leftrightarrow 410 + 10\beta = 0 \quad , \quad \beta = -41$$

$$\frac{U_{\rm S}}{\dot{I}_{\rm i}}$$
 = -j1000 故电流领先电压 90°.

小结:



$$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egin{aligned} egin{aligned} egin{aligned} eg$$

- (2) Z是与u,i无关的复数。
- (3) 根据Z、Y可确定无源二端网络的注能
- (4) 一般情况Z、Y均是 ω 的函数

用相量图分析电路

1. 定性画相量图方法

关键: 选择合适参考相量

串联电路以电流为参考相量,并联电路以电压为参考相量。

明白元件和支路的电压、电流相量关系:

R: 电压与电流同相

: ↓ L: 电压超前电流90° 支路:

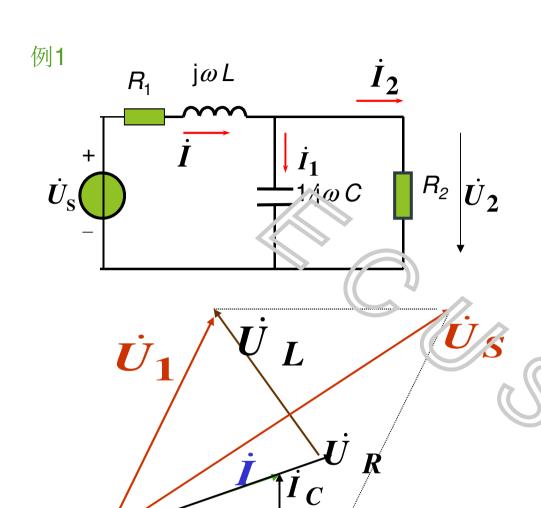
C: 电流超前电压90°

RL支路: 电压超前电流 ϕ

角

RC支路: 电流超前电压 φ 角

 $90^{\circ} > \phi > 0^{\circ}$



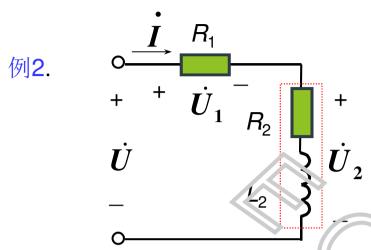
以Ú2为参考相量

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U}_1 = \dot{U}_R + \dot{U}_L$$

$$\dot{U}_S = \dot{U}_1 + \dot{U}_2$$

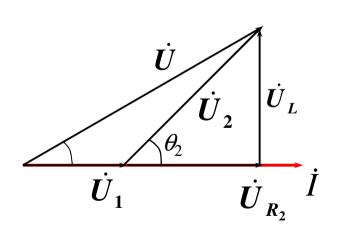
$$\dot{U}_{\rm S} = \dot{U}_1 + \dot{U}_2$$



已知: U=115V , U_1 =55.4V , U_2 =80V , R_1 =32 Ω , f=50Hz

 \dot{U}_2 求:线圈的电阻 R_2 和电感 L_2 。

解一用相量图分析。



$$U_R = U_2 \cos \theta = 33.92V$$
 $R = \frac{U_R}{L} = 19.6\Omega$

$$I = U_{1} / R_{1} = 55.4 / 32 = 1.73A$$

$$U^{2} = U_{1}^{2} + \tilde{U}_{2}^{2} - 2U_{1}U_{2}\cos(180^{\circ} - \theta_{2})$$

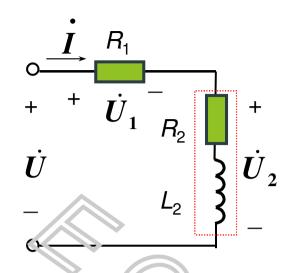
$$= U_{1}^{2} + U_{2}^{2} + 2U_{1}U_{2}\cos\theta_{2}$$

$$\therefore \theta_{2} = 64.9^{\circ}$$

$$U_{L} = U_{2}\sin\theta = 72.46V$$

$$X_{L} = \frac{U_{L}}{I} = 41.88\Omega$$

$$L = X_{2} / (2\pi f) = 0.133H$$



解二
$$I = U_1 / R_1 = 55.4 / 32 = 1.73 A$$

$$I = \frac{115}{\sqrt{(32 + R_2)^2 + (\omega L_2)^2}} = 1.73$$

$$I = \frac{80}{\sqrt{R_2^2 + (\omega L_2)^2}} = 1.73$$

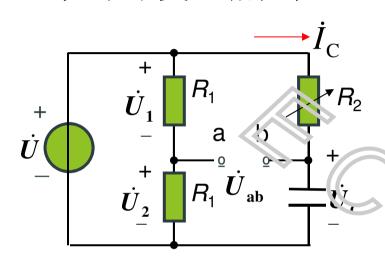
$$I = \frac{80}{\sqrt{R_2^2 + (\omega L_2)^2}} = 1.73$$

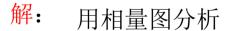
解得:

$$R_2 = 19.58\Omega, L_2 = \frac{41.86}{2\pi f} = 0.133 \text{H}.$$

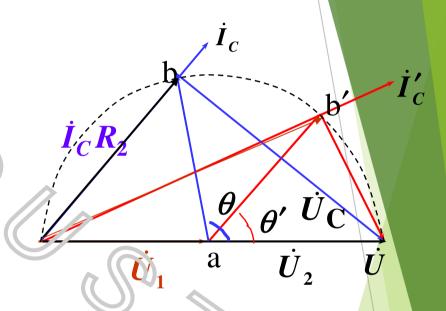
例3. 移相桥电路。当 R_2 由 $0\rightarrow\infty$ 时,

 \dot{U} ab大小不变,相位从 $180^{\circ} \rightarrow 0$ ^{说明其工作原理。}



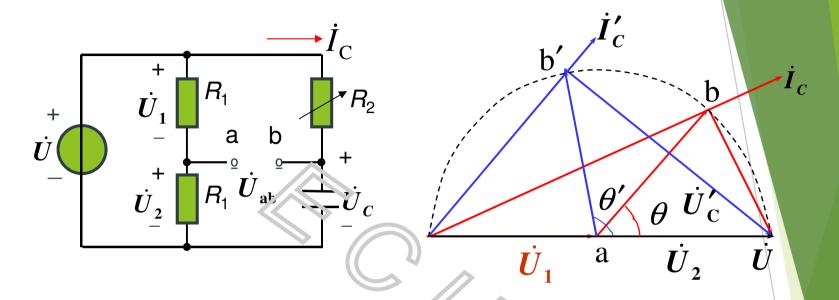


- 1)选*Ü*为参考相量
- 2) \dot{U}_1 , \dot{U}_2 与 \dot{U} 同相
- $3)\dot{I}_C$ 超前 $\dot{U}\varphi$ 角



$$4)\dot{U}_{ab} = -\dot{U}_1 + \dot{I}_C R_2$$

5)改变R2(红线示)



由相量图可知,当 R_2 改变, \dot{U}_{ab} 大少不变,相位改变;

 $\exists R_2 \uparrow$, $|\theta| \downarrow$.

 $\stackrel{\text{\tiny \perp}}{=} R_2 \rightarrow \infty$. $\theta = 0^\circ$.

 $\stackrel{\text{def}}{=} R_2 = 0, \quad \theta = 180^\circ;$

θ为移相角,移相范围:180°~0°