电路原理

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第7章 一二阶电路时域分析

本节主要内容

- 二阶电路方程及其求解
- 二阶RLC电路的零输入响应
- 二阶RLC电路的零状态响应

Review

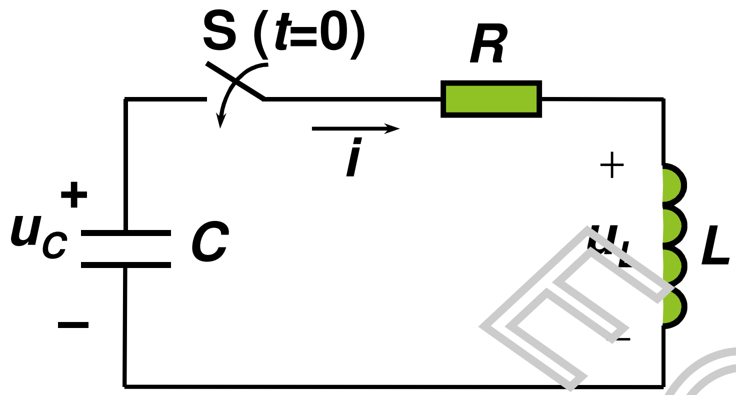
- 一阶常系数线性常微分方程
  - 齐次方程
  - 非齐次方程
  - 通解
  - 特解
- 二阶常系数线性常微分方程
  - 特解同一阶
  - 齐次通解

RLC 电路的物理特点

一阶电路只含一种储能元件，储能或是增长或是减少，在这过程中耗能元件*R*将影响过程的快慢。

含*L*和*C*的二阶电路中储能可以交换，在交换过程中，耗能元件*R*将影响过程的性质、快慢。

二阶电路的零输入响应



已知 $u_C(0^-)=U_0$ $i(0^-)=0$

求 $u_C(t)$, $i(t)$, $u_L(t)$.

解

$$u_C = Ri + u_L \qquad i = -C\frac{\mathrm{d}u_C}{\mathrm{d}t} \qquad u_L = L\frac{\mathrm{d}i}{\mathrm{d}t} = -LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}$$

$$LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2} + RC\frac{\mathrm{d}u_C}{\mathrm{d}t} + u_C = 0$$

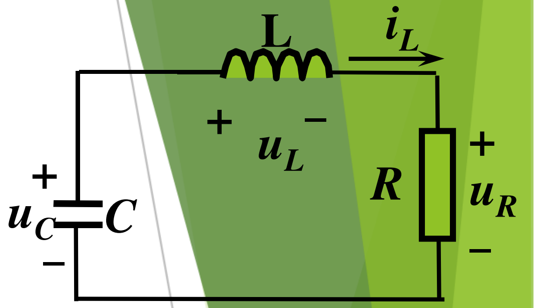
设 $u_C = A\mathrm{e}^{\lambda t}$

特征方程为 $LC\lambda^2 + RC\lambda + 1 = 0$

$$\lambda_{1,2} = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L} = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$$

关于列写方程和求初值

$$u_L + u_R = u_C$$



$$\begin{cases} u_R = Ri_L = -RC\dfrac{\mathrm{d}u_C}{\mathrm{d}t} \\ u_L = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = -LC\dfrac{\mathrm{d}^2u_C}{\mathrm{d}t^2} \end{cases}$$

$$\begin{cases} u_L = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} \\ u_R = Ri_L \\ u_C = -\dfrac{1}{C}\displaystyle\int i_L\mathrm{d}t \end{cases}$$

$$LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2} + RC\frac{\mathrm{d}u_C}{\mathrm{d}t} + u_C = 0$$

$$LC\frac{\mathrm{d}^2i_L}{\mathrm{d}t^2} + RC\frac{\mathrm{d}i_L}{\mathrm{d}t} + i_L = 0$$

$$\begin{cases} u_L = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = \dfrac{L}{R}\dfrac{\mathrm{d}u_R}{\mathrm{d}t} \\ u_C = -\dfrac{1}{C}\displaystyle\int i_L\mathrm{d}t = -\dfrac{1}{RC}\displaystyle\int u_R\mathrm{d}t \end{cases}$$

$$\begin{cases} u_R = Ri_L = \dfrac{R}{L}\displaystyle\int u_L\mathrm{d}t \\ u_C = -\dfrac{1}{C}\displaystyle\int i_L\mathrm{d}t = -\dfrac{1}{LC}\displaystyle\iint u_L\mathrm{d}t \end{cases}$$

$$LC\frac{\mathrm{d}^2u_R}{\mathrm{d}t^2} + RC\frac{\mathrm{d}u_R}{\mathrm{d}t} + u_R = 0$$

$$LC\frac{\mathrm{d}^2u_L}{\mathrm{d}t^2} + RC\frac{\mathrm{d}u_L}{\mathrm{d}t} + u_L = 0$$

$$LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+RC\frac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=0$$

$$\begin{cases} u_C(0^+)=U_0 \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=-\dfrac{1}{RC}u_R(0^+)=0 \end{cases}$$

$$LC\frac{\mathrm{d}^2i_L}{\mathrm{d}t^2}+RC\frac{\mathrm{d}i_L}{\mathrm{d}t}+i_L=0$$

$$\begin{cases} i_L(0^+)=0 \\ \left.\dfrac{\mathrm{d}i_L}{\mathrm{d}t}\right|_{t=0^+}=\dfrac{1}{L}u_L(0^+)=\dfrac{U_0}{L} \end{cases}$$

$$LC\frac{\mathrm{d}^2u_R}{\mathrm{d}t^2}+RC\frac{\mathrm{d}u_R}{\mathrm{d}t}+u_R=0$$

$$\begin{cases} u_R(0^+)=0\ \mathrm{V} \\ \left.\dfrac{\mathrm{d}u_R}{\mathrm{d}t}\right|_{t=0^+}=R\left.\dfrac{\mathrm{d}i_L}{\mathrm{d}t}\right|_{t=0^+}=\dfrac{U_0R}{L} \end{cases}$$

$$LC\frac{\mathrm{d}^2u_L}{\mathrm{d}t^2}+RC\frac{\mathrm{d}u_L}{\mathrm{d}t}+u_L=0$$

$$\begin{cases} u_L(0^+)=3\ U_0 \\ \left.\dfrac{\mathrm{d}u_L}{\mathrm{d}t}\right|_{t=0^+}=\left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}-\left.\dfrac{\mathrm{d}u_R}{\mathrm{d}t}\right|_{t=0^+}=-\dfrac{U_0R}{L} \end{cases}$$

特点：

（1）同一电路不同支路变量微分方程的特征方程完全相同

自由分量形式完全相同

（2）同一电路不同支路变量微分方程等号右端项和初值不同

强制分量和待定系数不同

（3）同一电路不同支路变量微分方程列写和初值获取难度不同

$$\lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$\boxed{\omega_0^2 = \omega_d^2 + \alpha^2}$ 衰减振荡角频率

阻尼系数 α　　ω_0^2　　自然频率

电路的自然频率不同，响应的变化规律也不同:

$R > 2\sqrt{\frac{L}{C}}$　二个不等负实根

$$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

$u_C = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

过阻尼

$R = 2\sqrt{\frac{L}{C}}$　二个相等负实根

$$\lambda = \lambda_1 = \lambda_2 = -\frac{R}{2L} = -\delta$$

$u_C = (A_1 + A_2 t) e^{\lambda t}$

临界阻尼

$R < 2\sqrt{\frac{L}{C}}$　二个共轭复根

$$\lambda_{1,2} = -\delta \pm j\omega$$

$u_C = K e^{-\delta t} \sin(\omega t + \beta)$

欠阻尼

特例　$R = 0$　二个共轭虚根

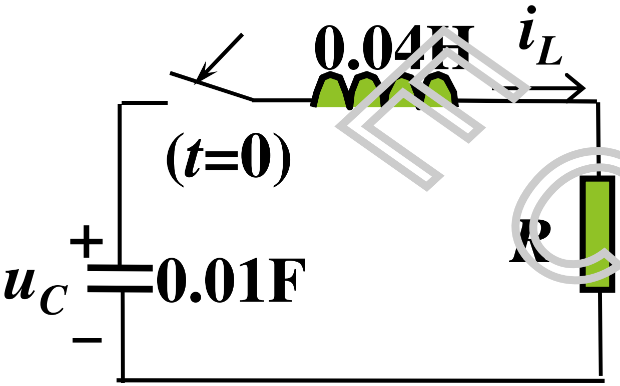
$$\lambda_{1,2} = \pm j\omega$$

$u_C = K \sin(\omega t + \beta)$

无阻尼

实例解析

R分别为5Ω、4 Ω、1 Ω、 0 Ω时求$u_C(t)$、 $i_L(t)$，$t \geq 0$ 。



$$u_C(0^-) = 3\text{V}$$

$$i_L(0^-) = 0$$

1. 列方程

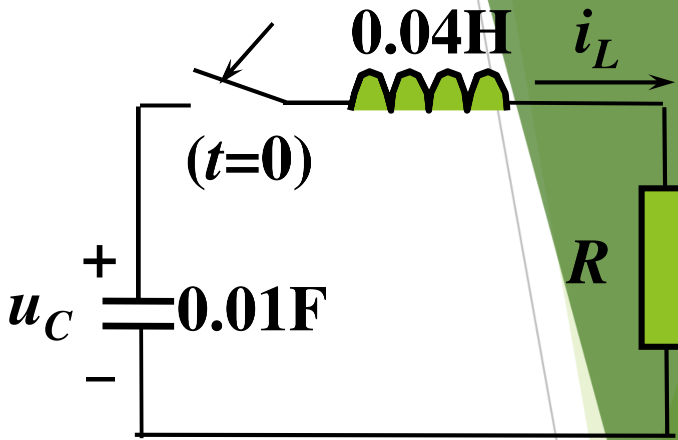
$$L\frac{\mathrm{d}i_L}{\mathrm{d}t} + Ri_L = u_C$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -i_L$$

$$\Rightarrow \quad \frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$

2. 求自由分量

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2}+\frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t}+\frac{1}{LC}u_C=0$$



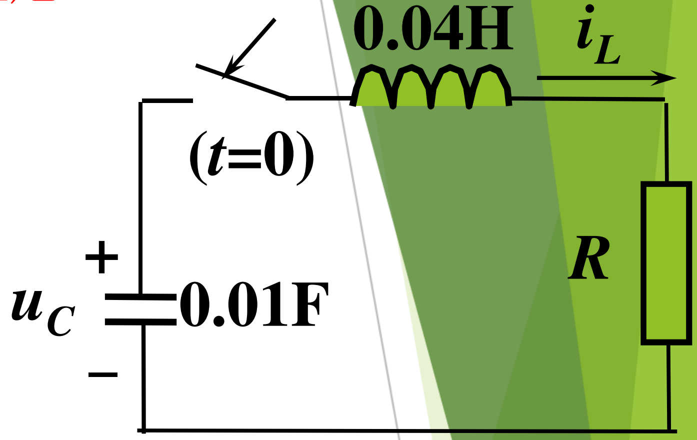
$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2}+25R\frac{\mathrm{d}u_C}{\mathrm{d}t}+2500u_C=0$$

特征方程

$$\lambda^2+25R\lambda+2500=0$$

$$b^2-4ac=625R^2-1000$$

$$b^2-4ac=625R^2-10000$$



$R=5\ \Omega$ ⇒

$$\begin{cases} b^2-4ac=5625>0 \\ \lambda_1=-25 \quad \lambda_2=-100 \\ u_C(t)=A_1\mathrm{e}^{-25t}+A_2\,\mathrm{e}^{-100t} \end{cases}$$

过阻尼

$R=4\ \Omega$ ⇒

$$\begin{cases} b^2-4ac=0 \\ \lambda_1=\lambda_2=-50 \\ u_C(t)=A_1\mathrm{e}^{-50t}+A_2t\,\mathrm{e}^{-50t} \end{cases}$$

临界阻尼

$R=1\ \Omega$ ⇒

$$\begin{cases} b^2-4ac=-9375<0 \\ \lambda_{1,2}=-12.5\pm \mathrm{j}48.4 \\ u_C(t)=K\mathrm{e}^{-12.5t}\sin(48.4t+\theta) \end{cases}$$

欠阻尼

$R=0\ \Omega$ ⇒

$$\begin{cases} \lambda_{1,2}=\pm \mathrm{j}50 \\ u_C(t)=K\sin(50t+\theta) \end{cases}$$

无阻尼

3. 将初值代入全解，确定待定系数

$u_C(0) = 3\text{V}$

$\left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+} = -\dfrac{1}{C}i_L(0^+) = 0$

$R = 5\Omega$

$$u_C(t) = A_1 e^{-25t} + A_2\, e^{-100t}$$

$$\begin{cases} A_1 + A_2 = 3 \\ -25A_1 - 100A_2 = 0 \end{cases} \Rightarrow A_1 = 4 \quad A_2 = -1$$

$$u_C(t) = 4e^{-25t} - e^{-100t}\,\text{V} \quad (t \geq 0)$$

$R = 4\Omega$

$$u_C(t) = A_1 e^{-50t} + A_2 t\, e^{-50t}$$

$$\begin{cases} A_1 = 3 \\ -50A_1 + A_2 = 0 \end{cases} \Rightarrow A_1 = 3,\ \ A_2 = 150$$

$$u_C(t) = 3e^{-50t}(1 + 50t)\,\text{V} \quad (t \geq 0)$$

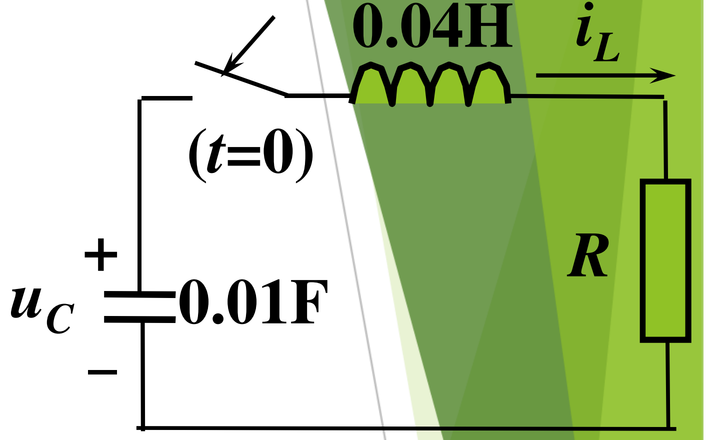
$R = 1\Omega$

$$u_C(t) = Ke^{-12.5t}\sin(48.4t + \theta)$$

$$\begin{cases} K\sin\theta = 3 \\ -12.5K\sin\theta + 48.4K\cos\theta = 0 \end{cases} \Rightarrow K = 3.1,\ \ \theta = 75.5^{\circ}$$

$$u_C(t) = 3.10e^{-12.5t}\sin(48.4t + 75.5^{\circ})\,\text{V} \quad (t \geq 0)$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -i_L$$



$R=5\Omega$

$$\begin{cases} u_C(t) = 4\mathrm{e}^{-25t} - \mathrm{e}^{-100t}\,\mathrm{V} \quad (t \geq 0) \\ i(t) = \mathrm{e}^{-25t} - \mathrm{e}^{-100t}\,\mathrm{A} \quad (t \geq 0) \end{cases}$$

$R=4\Omega$

$$\begin{cases} u_C(t) = 3\mathrm{e}^{-50t}(1-50t)\,\mathrm{V} \quad (t \geq 0) \\ i(t) = 75t\mathrm{e}^{-50t}\,\mathrm{A} \quad (t \geq 0) \end{cases}$$

$R=1\Omega$

$$\begin{cases} u_C(t) = 3.10\mathrm{e}^{-12.5t}\sin(48.4t + 75.5^\circ)\,\mathrm{V} \quad (t \geq 0) \\ i(t) = 1.55\mathrm{e}^{-12.5t}\sin 48.4t\ \mathrm{A} \quad (t \geq 0) \end{cases}$$

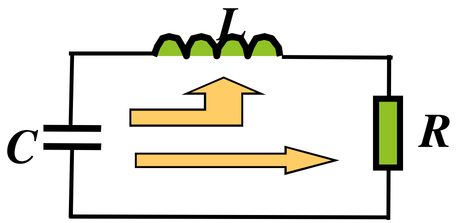
4. 波形与能量传递

$R = 5\Omega$

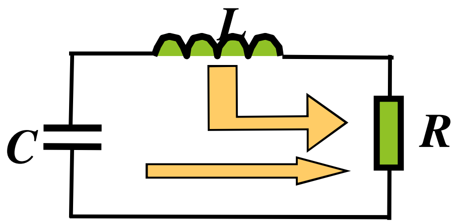
$$u_C(t) = 4e^{-25t} - e^{-100t}\,\text{V} \quad (t \geq 0)$$

$$i(t) = e^{-25t} - e^{-100t}\,\text{A} \quad (t \geq 0)$$

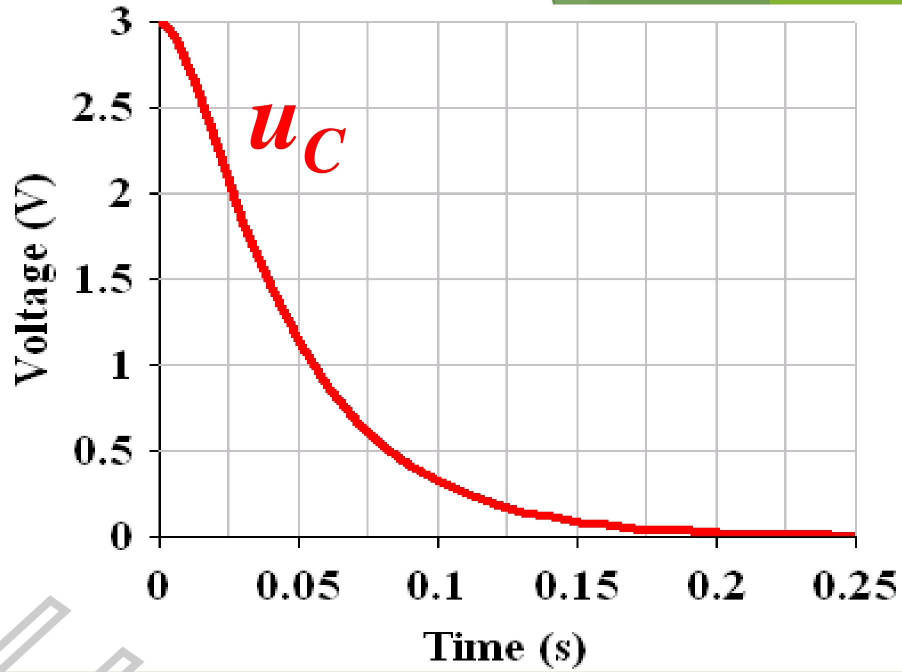
$0 < t < t_m$ u_C 减小, i 增加。

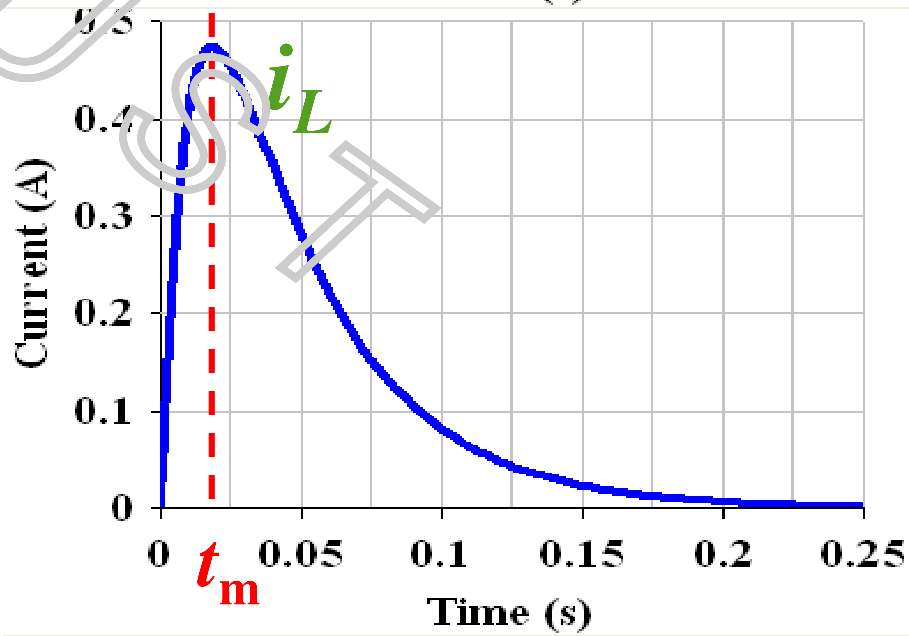


$t > t_m$ u_C 减小 , i 减小 。



过阻尼，无振荡放电



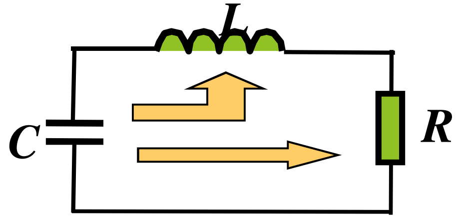


$R = 4\Omega$

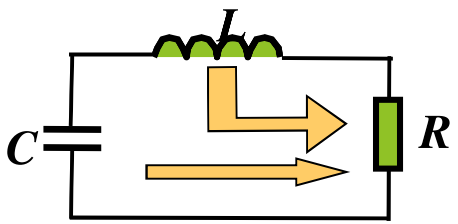
$u_C(t) = 3e^{-50t}(1+50t)\text{V} \quad (t \geq 0)$

$i(t) = 75te^{-50t}\text{A} \quad (t \geq 0)$

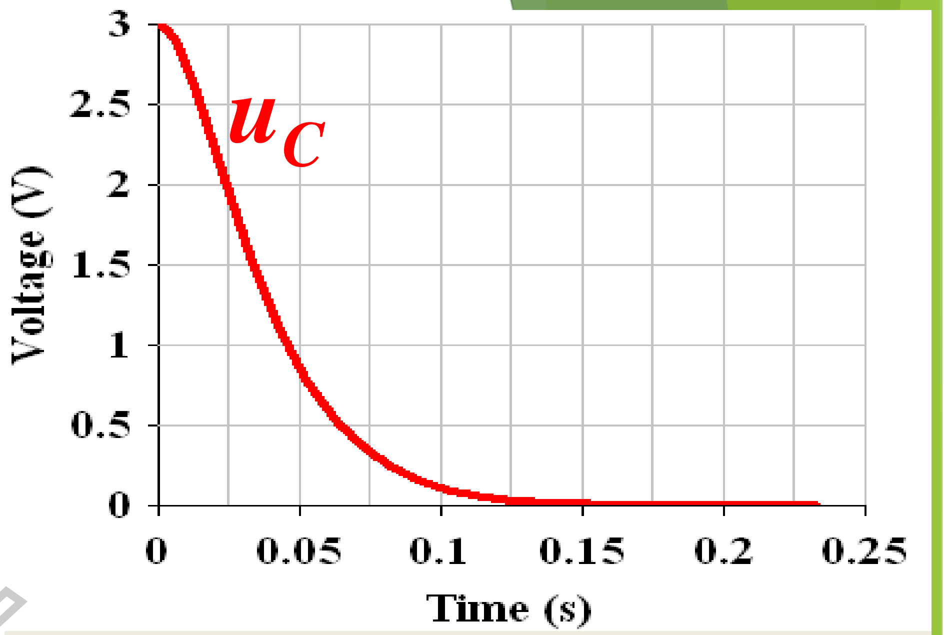
$0 < t < t_m$ u_C 减小，i 增加.

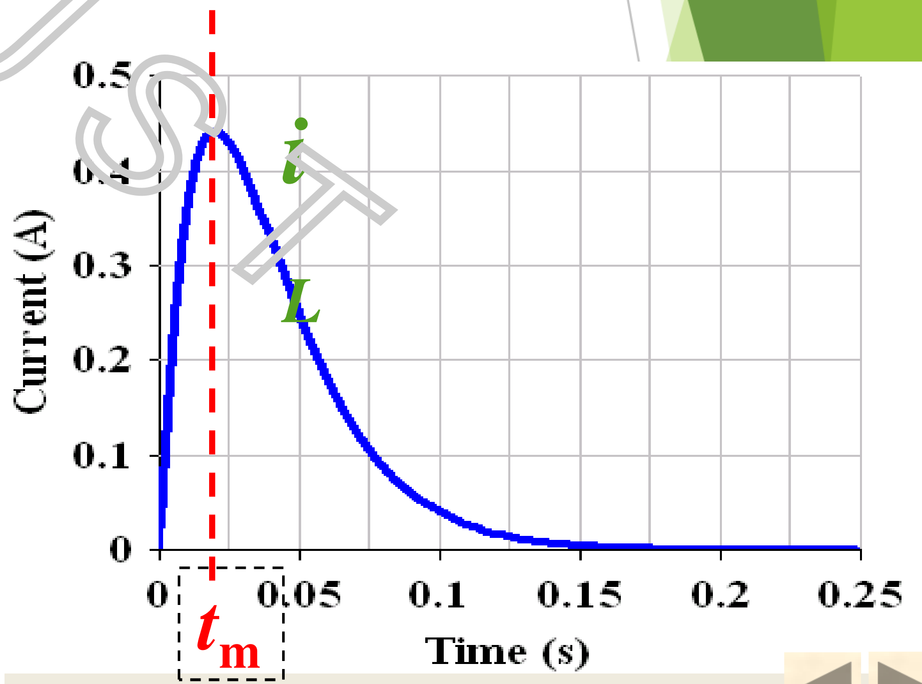


$t > t_m$ u_C 减小，i 减小.



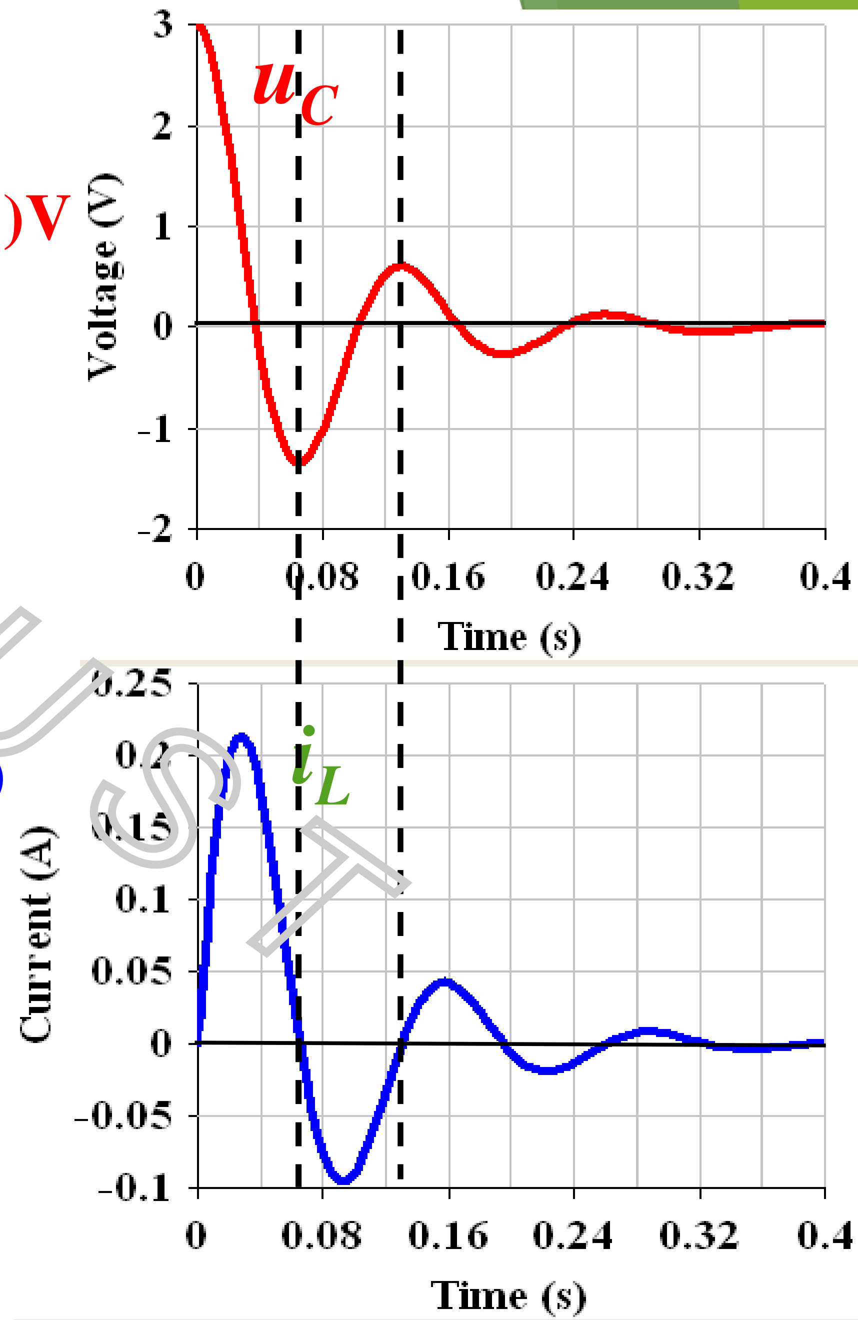
临界阻尼，无振荡放电





$R = 1\Omega$

$$u_C(t) = 3.10e^{-12.5t}\sin(48.4t + 75.5^\circ)\text{V}$$
$$(t \geq 0)$$



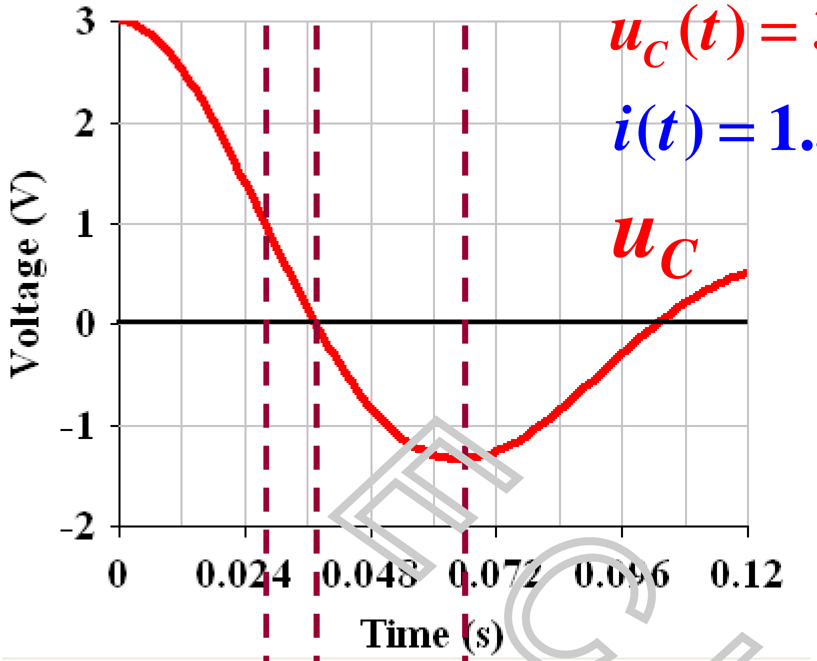
$$i(t) = 1.55e^{-12.5t}\sin 48.4t\ \text{A} \quad (t \geq 0)$$

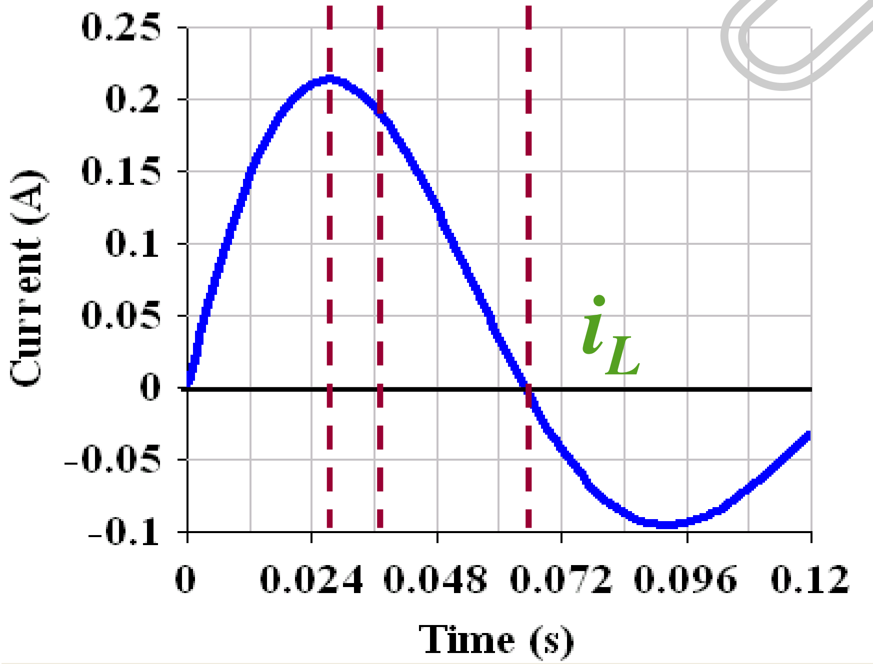
欠阻尼，振荡放电

讨论半个周期中能量的关系

$$u_C(t) = 3.10e^{-12.5t}\sin(48.4t + 75.5^\circ)\text{V}$$

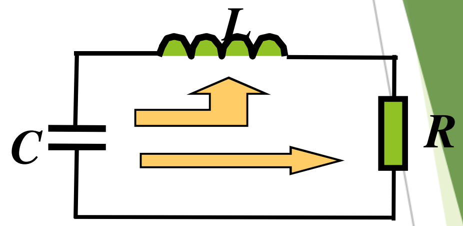
$$i(t) = 1.55e^{-12.5t}\sin 48.4t\ \text{A} \quad (t \geq 0)$$





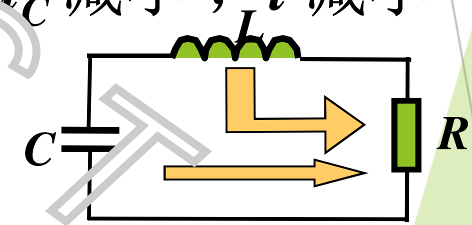
$0 \leq 48.4t \leq 90^\circ$

u_C 减小， i 增加



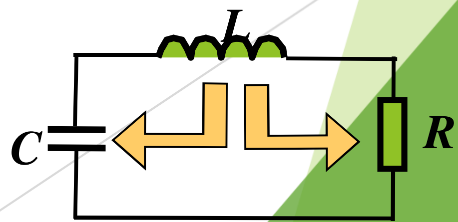
$90^\circ \leq 48.4t \leq 104.5^\circ$

u_C 减小， i 减小

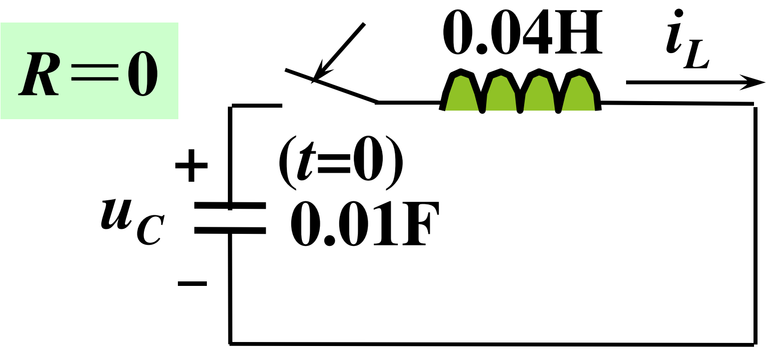


$104.5^\circ \leq 48.4t \leq 180^\circ$

$|u_C|$ 增加， i 减小



周而复始，电阻不断消耗能量，u_C i_L 衰减到零。



$$LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t}+u_C=0$$

$$p^2+2500=0 \qquad p=\pm\mathrm{j}50$$

$$u_C(t)=K\sin(50t+\theta)$$

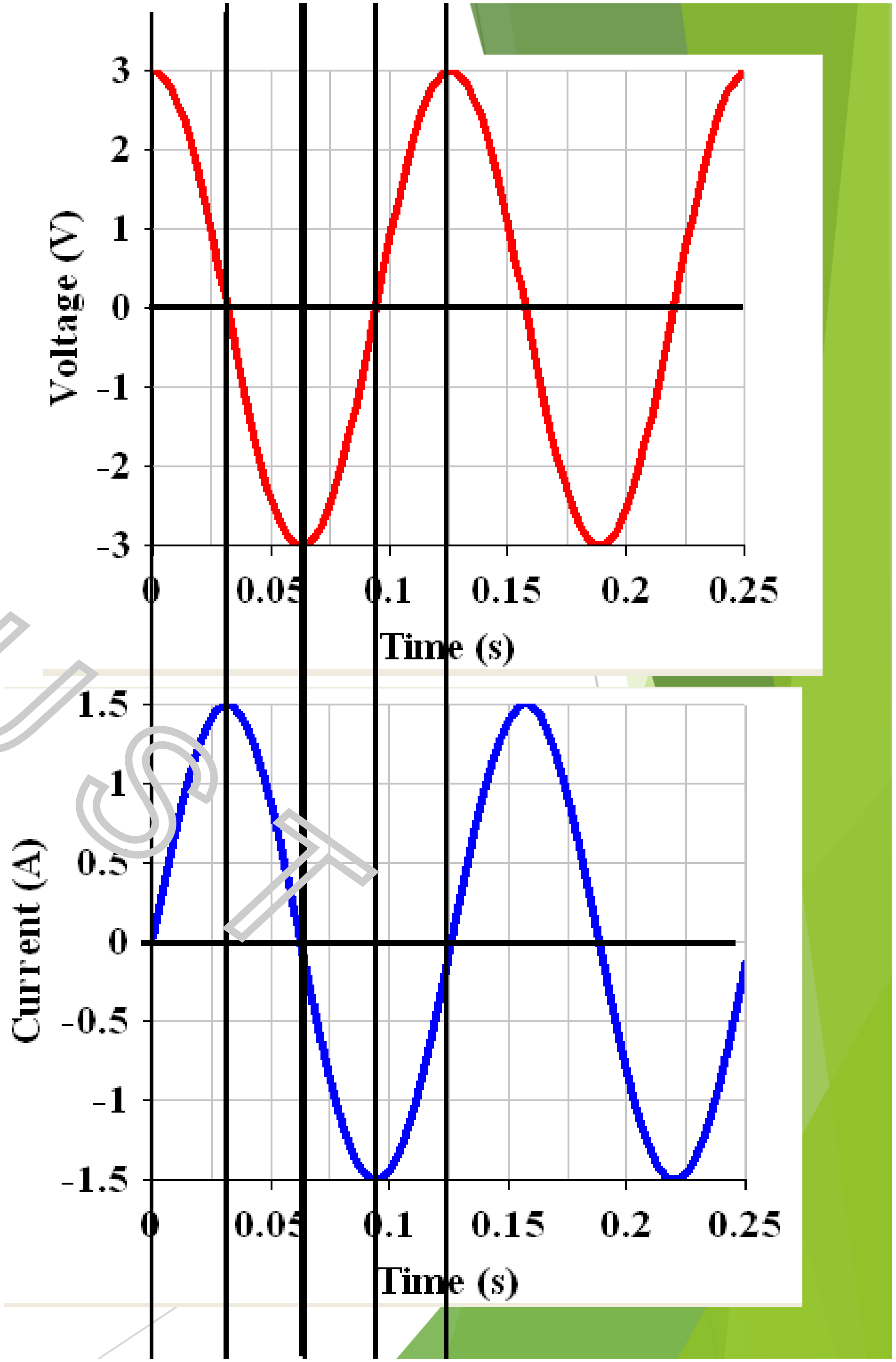
$$u_C(0)=3, \quad \left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0$$

$$K=3, \quad \theta=90^{\circ}$$

$$u_C(t)=3\cos 50t \text{ V} \qquad (t\geq 0)$$

$$i(t)=1.5\sin 50t \text{ A} \qquad (t\geq 0)$$

无阻尼振荡



有关欠阻尼二阶动态电路中3个参数的讨论：

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2}+\frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t}+\frac{1}{LC}u_C=0$$

$\frac{R}{L} \rightarrow 2\alpha$　　$\frac{1}{LC} \rightarrow \omega_0^2$

衰减系数 α

自由振荡角频率/自然角频率 ω_0

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2}+2\alpha\frac{\mathrm{d}u_C}{\mathrm{d}t}+\omega_0^2 u_C=0$$

欠阻尼

$\alpha < \omega_0$

$$s=\frac{-2\alpha\pm \mathrm{j}2\sqrt{\omega_0^2-\alpha^2}}{2}$$

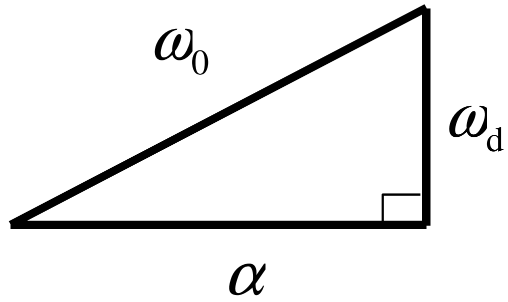
$$=-\alpha\pm\sqrt{\omega_0^2-\alpha^2}$$

$$=-\alpha\pm \mathrm{j}\omega_\mathrm{d}$$

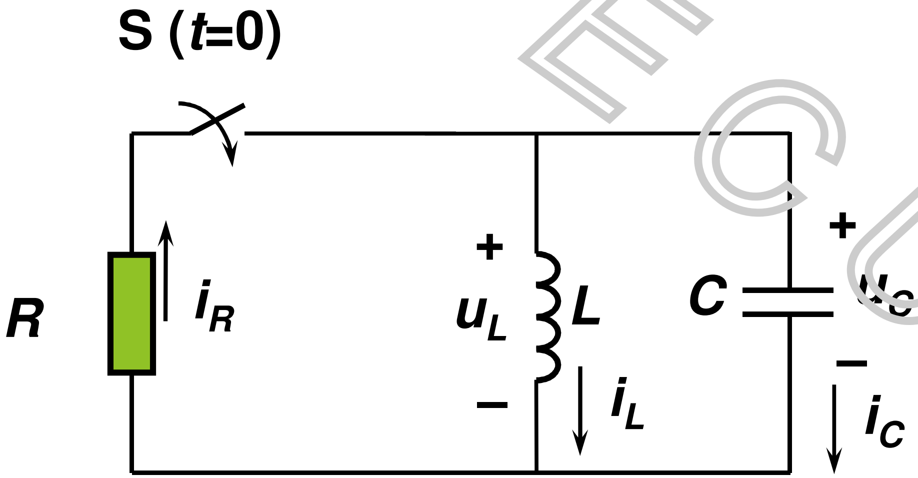
衰减振荡角频率 ω_d

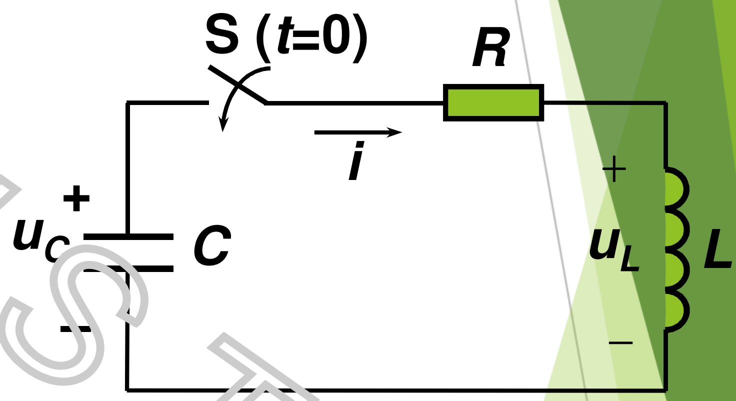
$$\boxed{\omega_0^2=\omega_\mathrm{d}^2+\alpha^2}$$

物理上稳定的系统 $\alpha>0$



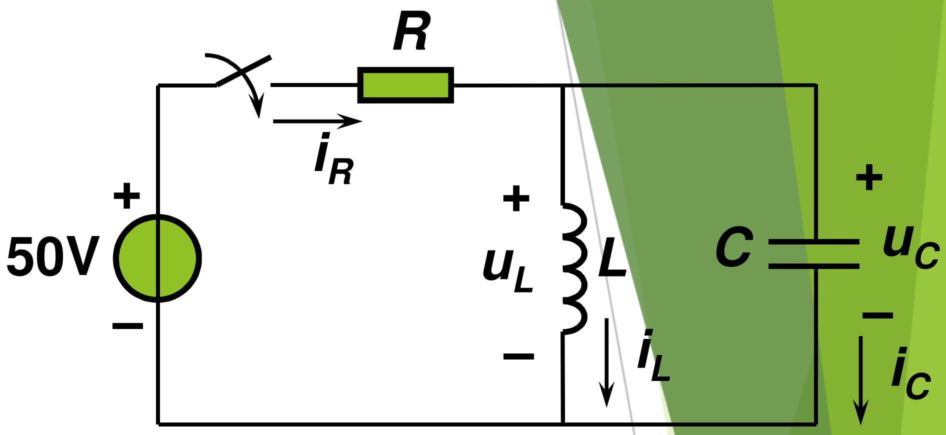
RLC并联电路----自行推导





例：已知：$i_L(0)=2\text{A}$，$u_C(0)=0$，
$R=50\Omega$， $L=0.5\text{H}$，$C=100\mu\text{F}$

求：$i_L(t)$ $i_R(t)$ 。



解：先求$i_L(t)$

(1) 列微分方程

$$\frac{50 - L\dfrac{\mathrm{d}i_L}{\mathrm{d}t}}{R} = i_L + C\frac{\mathrm{d}u_C}{\mathrm{d}t} \qquad u_C = u_L = L\frac{\mathrm{d}i_L}{\mathrm{d}t}$$

$$RLC\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + L\frac{\mathrm{d}i_L}{\mathrm{d}t} + Ri_L = 50$$

$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + 200\frac{\mathrm{d}i_L}{\mathrm{d}t} + 2\times10^4 i_L = 2\times10^4$$

$$\frac{d^2 i_L}{dt^2} + 200\frac{di_L}{dt} + 2\times10^4 i_L = 2\times10^4$$

(2) 求通解(自由分量）

特征方程 $p^2 + 200p + 20000 = 0$

特征根 $p_{1,2} = -100 \pm \mathrm{j}100$

通解 $i_L^{'}(t) = Ke^{-100t}\sin(100t+\beta)$

(3) 求特解（强制分量，稳态解）

$$i_L^{''} = 1\text{A}$$

(4) 全解

$$i_L(t) = 1 + Ke^{-100t}\sin(100t+\beta)$$

全解 $i_L(t) = 1 + Ke^{-100t}\sin(100t+\beta)$

(5) 由初值定积分常数

$i_L(0^+)$=2A， $u_C(0^+)$=0 （已知）

$$\left.\frac{di_L}{dt}\right|_{0+} = \frac{1}{L}u_L(0^+) = \frac{1}{L}u_C(0^+) = 0$$

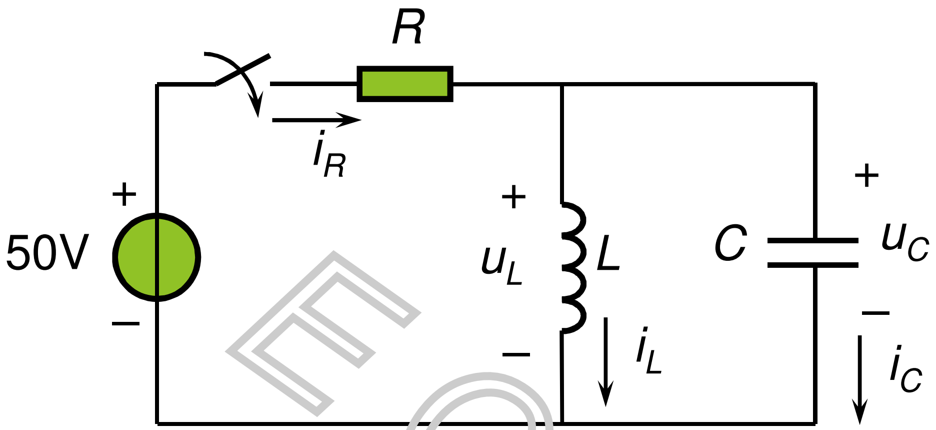
$$\frac{di_L}{dt} = -100Ke^{-100t}\sin(100t+\beta) + 100Ke^{-100t}\cos(100t+\beta)$$

$$\begin{cases} i_L(0^+) = 2 \quad \rightarrow \quad 1 + K\sin\beta = 2 \\ \left.\dfrac{di_L}{dt}\right|_{0+} = 0 \quad \rightarrow \quad -100K\sin\beta + 100K\cos\beta = 0 \end{cases}$$

解得 $K = \sqrt{2}, \beta = 45^{\circ}$

$$\therefore i_L(t) = 1 + \sqrt{2}e^{-100t}\sin(100t + 45^{\circ})\text{A} \quad (t \geq 0)$$

求 $i_R(t)$:



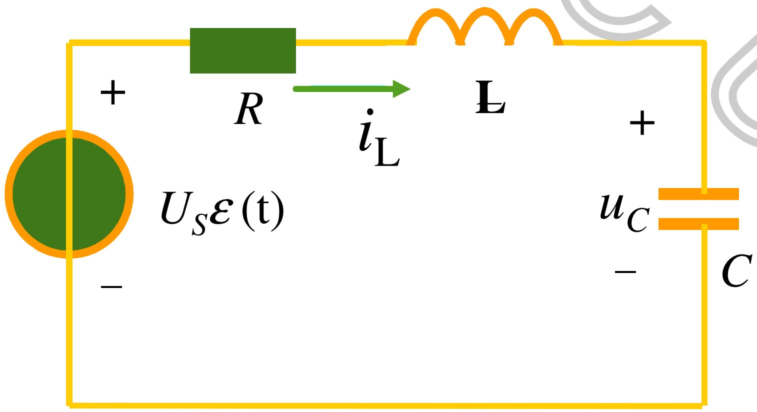
$$i_L(t) = 1 + \sqrt{2}\mathrm{e}^{-100t}\sin(100t + 45°)\mathrm{A} \quad (t \geq 0)$$

$$u_L = L\frac{\mathrm{d}i_L}{\mathrm{d}t} = -100\mathrm{e}^{-100t}\sin 100t \ \mathrm{V} \quad (t > 0)$$

$$i_R(t) = \frac{50 - u_L(t)}{50} = 1 + 2\mathrm{e}^{-100t}\sin 100t \ \mathrm{A} \quad (t > 0)$$

二阶电路的零状态响应

例　　$u_C(0_-)=0$， $i_L(0_-)=0$



$$u_C = u'_C + u''_C$$

特解　　通解

微分方程为：

$$LC\frac{d^2u_C}{dt} + RC\frac{du_C}{dt} + u_C = U_s$$

特征方程为:

$$LCP^2 + RCP + 1 = 0$$

特解: $u'_C = U_s$

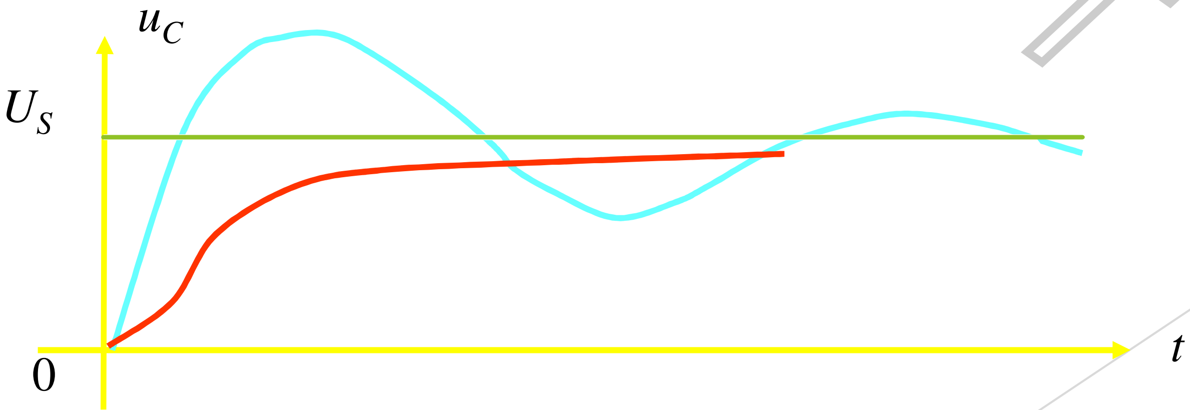
u_C解答形式为:

$$u_C = U_S + A_1 e^{p_1 t} + A_2 e^{p_2 t} \quad (p_1 \neq p_2)$$

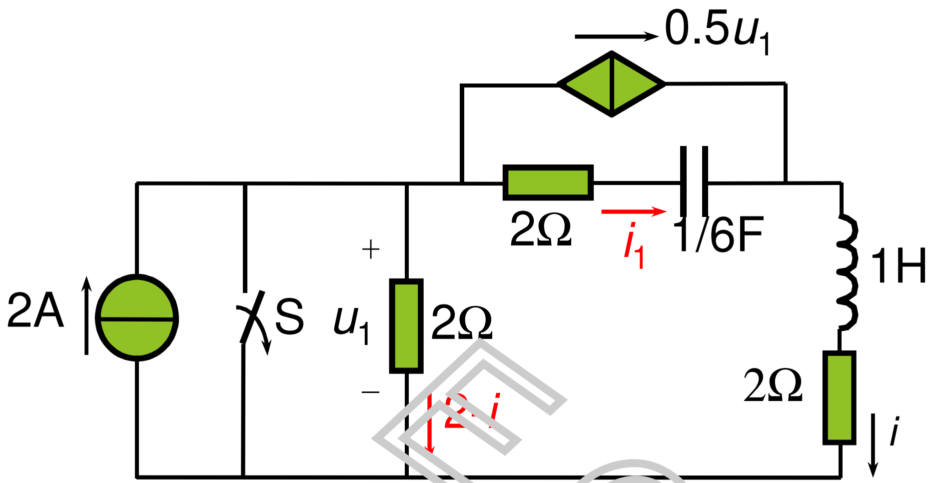
$$u_C = U_S + A_1 e^{-\delta t} + A_2 t e^{-\delta t} \quad (P_1 = P_2 = -\delta)$$

$$u_C = U_S + A e^{-\delta t} \sin(\omega t + \beta) \quad (P_{1、2} = -\delta \pm j\omega)$$

由初值 $u_C(0_+)$, $\frac{\mathrm{d}u(0_+)}{\mathrm{d}t}$ 确定二个常数



例



求左图所示电路中电流 ***i*(*t*)**的零状态响应。

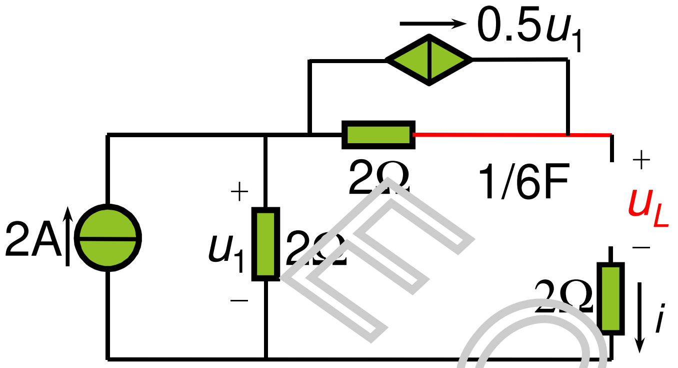
解：(1) 列写微分方程

由**KVL** $$2(2-i)=2i_1+6\int i_1\mathrm{d}t+\frac{\mathrm{d}i}{\mathrm{d}t}+2i$$

$$i_1=i-0.5u_1=i-0.5\times 2(2-i)=2i-2$$

整理得 $$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2}+8\frac{\mathrm{d}i}{\mathrm{d}t}+12i=12$$ 二阶非齐次常微分方程

(2) 求初值



0+电路模型:

$$\begin{cases} i(0^+) = i(0^-) = 0 \\ \left.\dfrac{\mathrm{d}i}{\mathrm{d}t}\right|_{0^+} = \dfrac{1}{L} u_L(0^+) \end{cases}$$

$$u_1(0^+) = 2 \times 2 = 4\mathrm{V}$$

$$u_L(0^+) = 0.5 u_1(0^+) \times 2 + u_1(0^+)$$
$$= 8\mathrm{V}$$

(3) 确定解的形式

$$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + 8\frac{\mathrm{d}i}{\mathrm{d}t} + 12i = 12$$

解答形式为:

$$i = i' + i''$$

通解i':

$p^2+8p+12=0$

$p_1=-2$ ， $p_2=-6$

$$i' = A_1 \mathrm{e}^{-2t} + A_2 \mathrm{e}^{-6t}$$

特解i'':

$i''=1\mathrm{A}$

解的形式为

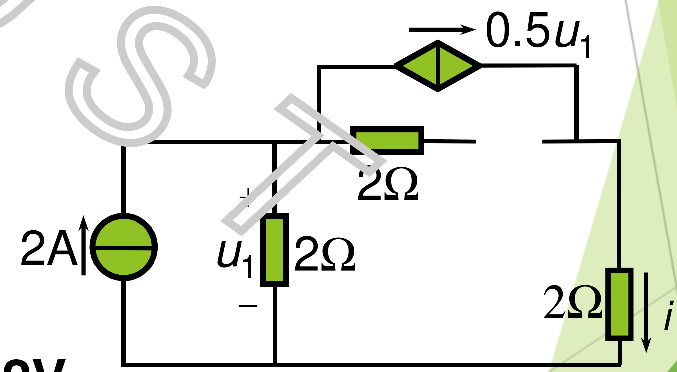
$$i(t) = 1 + A_1 e^{-2t} + A_2 e^{-6t}$$

(4) 定常数

$$\begin{cases} 0 = 1 + A_1 + A_2 \\ 8 = -2A_1 - 6A_2 \end{cases} \Rightarrow \begin{cases} A_1 = 0.5 \\ A_2 = -1.5 \end{cases}$$

$$\therefore i(t) = 1 + 0.5e^{-2t} - 1.5e^{-6t} \text{ A} \quad (t \geq 0)$$

求特解 *i*'的另一种方法:



稳态电路

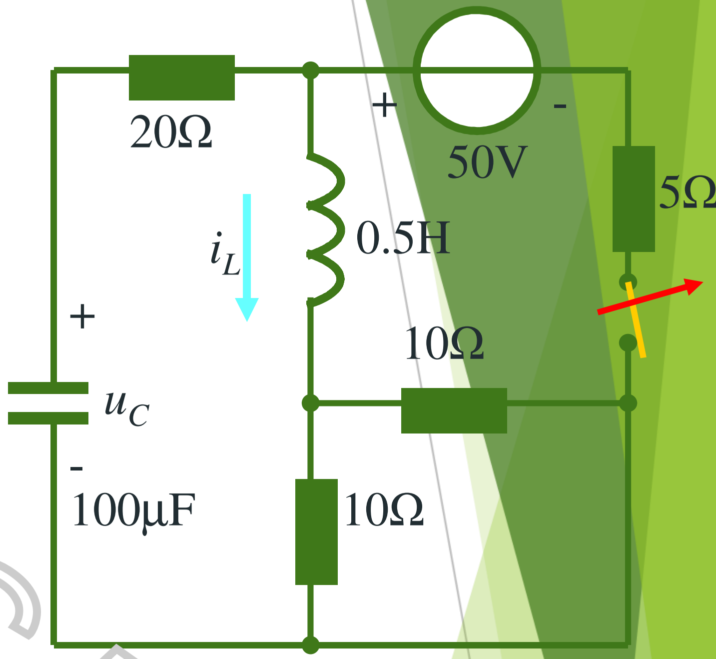
$i(\infty) = 0.5\ u_1(\infty)$

$u_1(\infty)=2(2-0.5u_1(\infty))$ $\Rightarrow$ $u_1(\infty)=2\text{V}$

$i(\infty)=1\text{A}$

例

电路如图，**t=0** 时打开开关。求u_C并画出其变化曲线。



解 （1） $u_C(0\_)=25V$

$i_L(0\_)=5A$

（2）开关打开为*RLC*串联电路，方程为：

$$LC\frac{d^2u_C}{dt}+RC\frac{du_C}{dt}+u_C=0$$

特征方程为： $50P^2+2500P+10^6=0$

$$P=-25\pm j139$$

$$u_C=Ae^{-25t}\sin(139t+\beta)$$

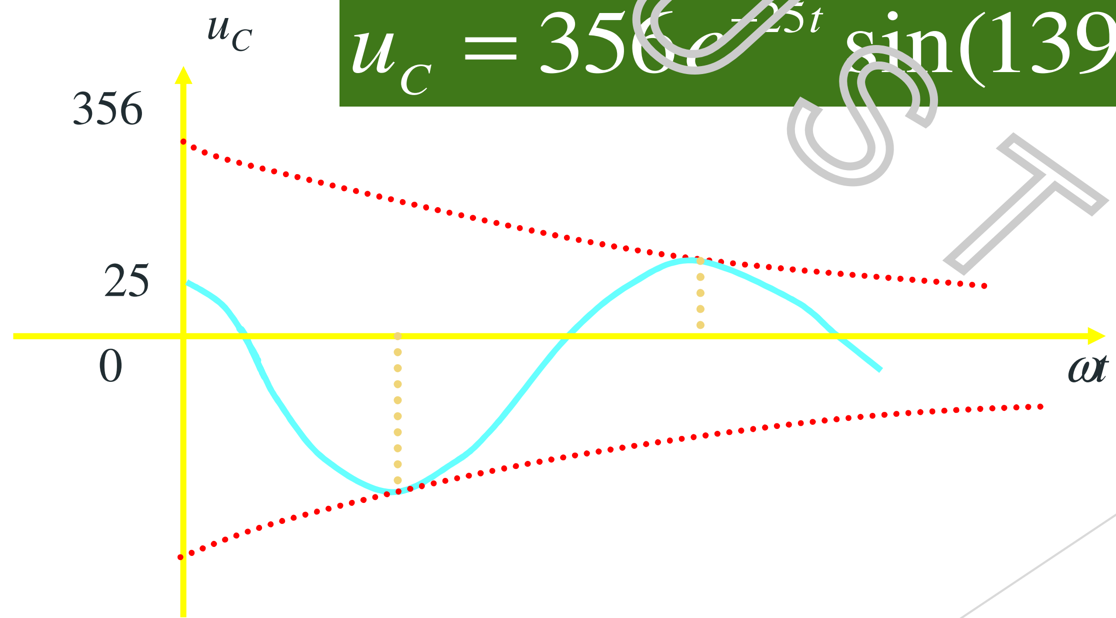
$$u_C = Ae^{-25t}\sin(139t+\beta)$$

(3) $$\begin{cases} u_C(0_+) = 25 \\ C\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\Big|_{0_+} = -5 \end{cases}$$

$$\begin{cases} A\sin\beta = 25 \\ A(139\cos\beta - 25\sin\beta) = \dfrac{-5}{10^{-4}} \end{cases}$$

$$A = 356,\quad \beta = 176^0$$

$$u_C = 356e^{-25t}\sin(139t+176^0)\mathrm{V}$$



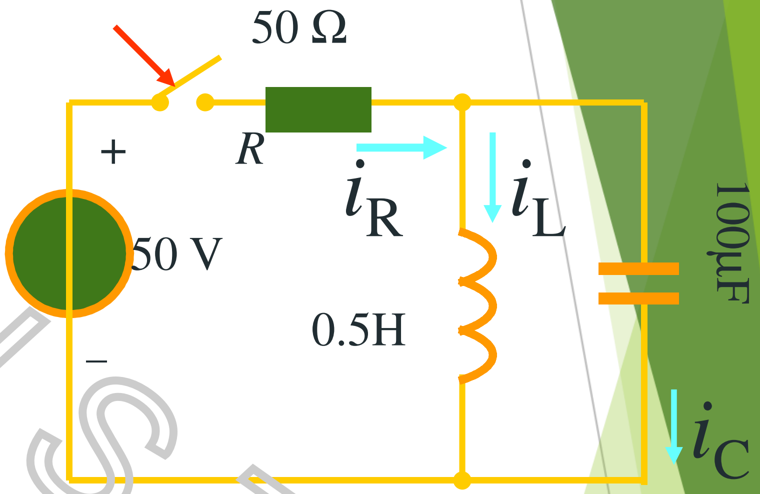
例　已知：$i_L(0_-)=2A$　$u_C(0_-)=0$　　求：i_L，i_R

解　(1) 列微分方程

应用节点法：

$$\frac{L\frac{\mathrm{d}i_L}{\mathrm{d}t}-50}{R}+i_L+LC\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2}=0$$

$$RLC\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2}+L\frac{\mathrm{d}i_L}{\mathrm{d}t}+Ri_L=50$$



(2) 求特解

$$i'_L=1\mathrm{A}$$

$$RLC\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2}+L\frac{\mathrm{d}i}{\mathrm{d}t}+Ri_L=50$$

(3) 求通解

特征方程为： $P^2+200P+20000=0$

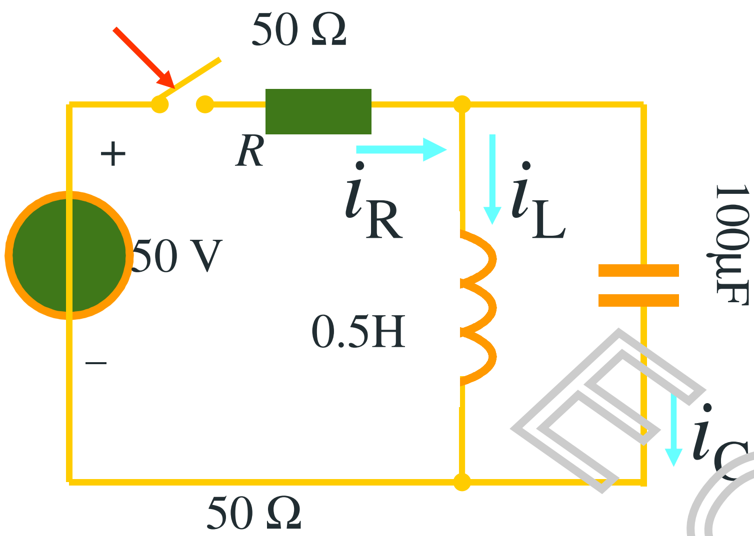
特征根为： P= -100 ±j100

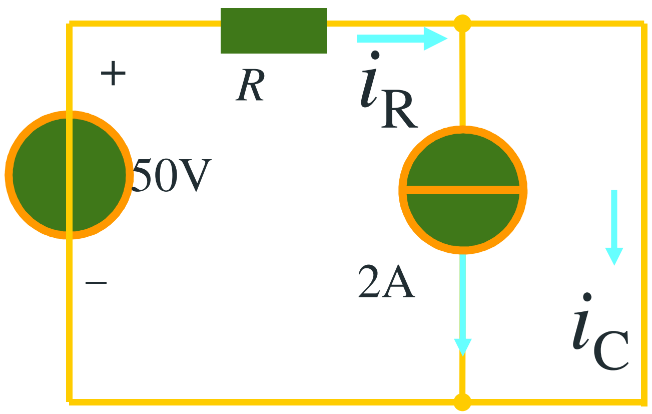
$$\rightarrow\quad i=1+Ae^{-100t}\sin(100t+\varphi)$$

(4) 定常数

$$\begin{cases}1+A\sin\varphi=2 & \leftarrow i_L(0_+)\\ 100A\cos\varphi-100A\sin\varphi=0 & \leftarrow u_L(0_+)\end{cases}$$

$$\begin{cases}\varphi=45^\circ\\ A=\sqrt{2}\end{cases}\quad\rightarrow\quad i_L=1+\sqrt{2}e^{-100t}\sin(100t+45^\circ)$$





(5) 求 i_R

$$i_R = i_L + i_C = i_L + LC\frac{d^2 i_L}{dt^2}$$

或形式写为：

$$i_R = 1 + Ae^{-100t}\sin(100t+\varphi)$$

定常数

$$\begin{cases} i_R(0_+) = 1 \quad i_C(0_+) = -1 \\ \dfrac{di_R}{dt}(0_+) = ? \quad i_R = \dfrac{50-u_C}{R} \end{cases}$$

$$\frac{di_R}{dt}(0_+) = -\frac{1}{R}\frac{du_C}{dt}(0_+) = -\frac{1}{RC}i_C(0_+) = 200$$

$$i_R = 1 + Ae^{-100t}\sin(100t + \varphi)$$

$$\begin{cases} 1 + A\sin\varphi = 1 \\ 100A\cos\varphi - 100A\sin\varphi = 200 \end{cases}$$

$$\begin{cases} \varphi = 0 \\ A = 2 \end{cases}$$

小结：

1. 二阶电路含二个独立储能元件，是用二阶常微分方程所描述的电路。
2. 二阶电路的性质取决于特征根，特征根取决于电路结构和参数，与激励和初值无关。
3. 经典法解线性二阶电路的一般步骤：

(1) 列写换路后(**t>0**)电路的微分方程并确定初始条件；

(2) 求特征根，由根的性质写出自由分量(积分常数待定)；

(3) 求强制分量(稳态分量)；

(4) 全解=自由分量+强制分量；

(5) 将初值$f(0^+)$和$f'(0^+)$代入全解，定积分常数；

(6) 讨论物理过程，画出波形等。

$$u_C = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

$$u_C = (A_1 + A_2 t)e^{\lambda t}$$

$$u_C = Ke^{-\delta t}\sin(\omega t + \beta)$$

$$u_C = K\sin(\omega t + \beta)$$