HIGH RESOLUTION IONOSPHERIC TOMOGRAPHY THROUGH ORTHOGONAL DECOMPOSITION

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ABSTRACT

Using data from several ground stations tracking a single satellite pass, the methods of computerized tomography can be used to construct images that show a cross section of ionospheric electron density. Due to physical limitations of the imaging system, the images obtained when traditional computerized tomography algorithms are applied to the ionospheric tomography problem exhibit very poor vertical resolution. However, the vertical resolution can be improved through the use of *a priori* information. This paper presents a new method of introducing *a priori* information into the reconstruction algorithm using orthogonal decomposition. The central idea presented here is to constrain the image space in such a way as to limit the vertical distribution of the reconstructed image to a space consisting of distributions based on *a priori* information, while allowing the horizontal distribution to vary with as much freedom as possible.

1. INTRODUCTION

The distribution of electron density in the ionosphere exhibits a complicated structure and dynamic behavior that has been studied for many years. There are several ways to measure ionospheric total electron content (TEC), one of which is the differential Doppler technique. This technique yields a measurement from which the value of the line integral of electron density along the path of propagation can be obtained. In computerized tomography, each point of data on a projection is the line integral of the parameter of interest through the region where the distribution of the parameter is to be imaged. Thus, if there are several ground stations and a satellite orbit that lie in a plane, then the value of the line integral of electron density along the propagation paths between the ground stations and the satellite orbit can be obtained. This is exactly the form of data used in computerized tomography; therefore, computerized tomography methods can be used to reconstruct an image of a cross section of ionospheric electron density [1]–[3].

The practical feasibility of ionospheric tomography has been demonstrated in several recent studies [4],[5]. However, there are several difficulties associated with this application of tomography [3], [6]–[9]:

- The range of view angles is limited, i.e. the angle that each data ray makes with the vertical is limited, and none of the rays pass horizontally through the ionosphere.
- The number of data points in each projection can be no greater than the number of ground stations, since any two data points from one ground station must be at different angles.
- The data points within each projection are unevenly distributed due to the way in which artificial projections must be formed from individually acquired data.

Therefore, the standard algorithms of computerized tomography have only limited success when applied to ionospheric tomography.

Practical ionospheric tomography algorithms must use *a priori* information to minimize the effects of incomplete data. However, in many of the algorithms currently used for ionospheric tomography, the *a priori* vertical distribution is very strongly dependent on the *a priori* information. In addition, most ionospheric tomography algorithms resolve only large scale structures and underestimate the magnitude of the peak electron density. Therefore, continued development of reconstruction algorithms is still the key to improving the resolution of ionospheric tomography systems.

2. DISCUSSION

2.1. Orthogonal Decomposition

The orthogonal decomposition approach to tomography is a theoretical framework that unifies several other standard algorithms, such as algebraic reconstruction technique (ART), filtered back projection (FBJ), and direct Fourier method (DFM). In orthogonal decomposition, the image is expressed as a weighted sum of a set of orthonormal basis functions.

$$f(\theta, r) = \sum_i a_i \phi_i(\theta, r) \tag{1}$$

where θ is latitude, r is the distance from the center on the earth, $f(\theta, r)$ is the source image, $\{\phi_i(\theta, r)\}$ are a set of orthonormal functions, and $\{a_i\}$ are the unknown weights. The solution of the tomographic problem consists of calculating the weight associated with each image domain basis function [9],[10].

It has been shown that if complete projection data are available, and if the orthonormal basis spans the entire image domain, then the projections of the orthonormal basis functions in the image domain form an orthonormal set in the projection domain [9],[10]. Ionospheric tomography corresponds to the case where complete projections are not available, so the projections of the image domain basis functions are not orthogonal.

2.2. Resolution and *A Priori* Information

For ionospheric tomography, if the reconstruction algorithm does not utilize *a priori* information, then resolution in the vertical direction will be very poor, regardless of what algorithm is used. On the other hand, if the reconstruction depends too strongly on *a priori* information, then some of the information contained in the data may be ignored by the reconstruction algorithm. In addition, if a stochastic model is used, then the solution usually represents a weighted average between the correct solution and an *a priori* model.

A priori information is entered into the orthogonal decomposition algorithm by choosing the basis functions so that the solution is constrained to lie in the space spanned by a set of model ionospheres. The model ionospheres should be chosen to span the space of all reasonable solutions to the reconstruction problem. Since most of the *a priori* information is in the vertical direction, it is natural to use separable basis functions for the image domain. Each basis function can be expressed as

$$\phi_{ij}(\theta, r) = \xi_i(\theta)\eta_j(r) \tag{2}$$

where the set $\{\phi_{ij}(x, y)\}$ is orthonormal if each of the sets $\{\xi_i(\theta)\}$ and $\{\eta_j(r)\}$ are orthonormal.

The set $\{\xi_i(\theta)\}$ must satisfy

$$\int_{\theta_1}^{\theta_2} \xi_m(\theta)\xi_n(\theta)\, d\theta = \begin{cases} 1, & \text{if } m = n; \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

and the set $\{\eta_j(r)\}$ must satisfy

doesn't make sense—since vertical uses a priori, wont' it be correlated?

$$\boxed{\int_{r_1}^{r_2} \eta_m(r)\eta_n(r) r\, dr = \begin{cases} 1, & \text{if } m = n; \\ 0, & \text{otherwise.} \end{cases} \quad (4)}$$

Note that there is a radial weighting factor that must be included in the construction of $\{\eta_j(r)\}$ to take into account the curvature of the imaging region.

For ionospheric tomography, since horizontal resolution is generally good, *a priori* information is needed primarily for the vertical direction. Therefore, the horizontal basis functions should span the entire space in the horizontal direction. For example, pixel basis functions, trigonometric basis functions, or Legendre polynomials may be used. The horizontal resolution of the reconstruction depends, in part, on the number of basis functions used.

If the vertical basis functions span the entire space in the vertical direction, then the vertical resolution will be very poor, since no *a priori* information has been used. Instead, the vertical basis functions should be based on a set of model ionospheres that spans the space of all reasonable vertical profiles. In this way, the space in which the reconstruction will lie is limited based upon *a priori* information.

Let the samples of a set of model ionosphere profiles be entered as the columns of the matrix B. Let R be a diagonal matrix with the same number of rows as B where each diagonal entry is proportional to the radial coordinate of the corresponding sample in B. Then the product $R^{\frac{1}{2}}B$ may be decomposed using the singular value decomposition as follows:

the radial correction was our problem

$$R^{\frac{1}{2}}B = U\Sigma V^T \tag{5}$$

where the columns of U are an orthonormal basis for the column space of $R^{\frac{1}{2}}B$, the columns of V are an orthonormal basis for the row space of $R^{\frac{1}{2}}B$, and Σ is a diagonal matrix of singular values. Then samples of $\eta_j(r)$ are obtained from the j^{th} column of U_R, defined as

$$U_R = R^{-\frac{1}{2}}U. \tag{6}$$

The set $\{\eta_j(r)\}$ will be orthogonal with a radial weighting factor, since

$$U_R^T R U_R = U^T U = I. \tag{7}$$

The vertical resolution of the reconstruction is enhanced, because the solution is restricted in the vertical direction to a space of reasonable solutions. On the other hand, the solution is free to lie anywhere in the space of reasonable solutions without being weighted toward one particular most probable solution.

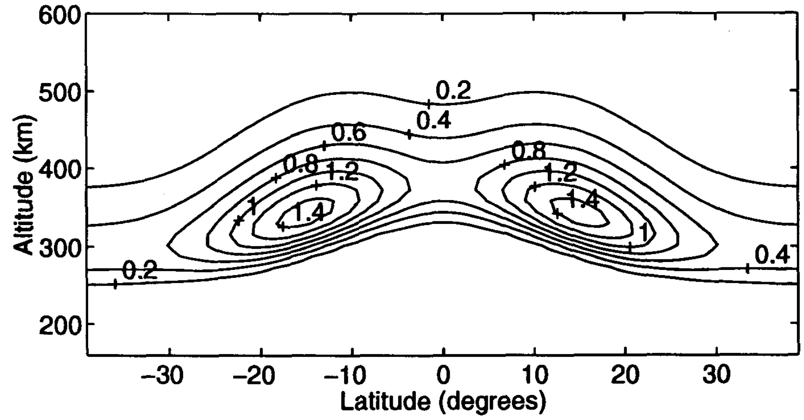

Figure 1: Simulated data.

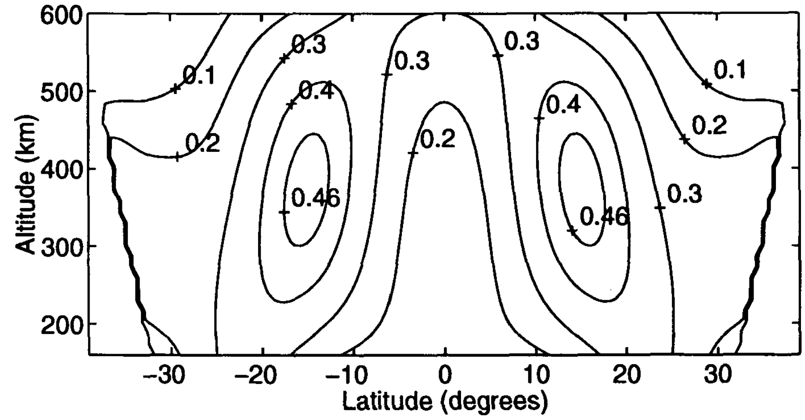

Figure 2: Reconstruction using Legendre polynomials.

3. SIMULATIONS

Figure 1 shows the simulated data set used in the reconstructions of Figure 2, Figure 3, and Figure 4. The data image shown in Figure 1 was numerically integrated for receiver positions every 4 degrees from -28 to 28 degrees latitude, and satellite positions every 2 degrees from -39 to 39 degrees latitude.

In all of the reconstructions that follow, 3 basis functions in the vertical direction and 21 basis functions in the horizontal direction are used. The horizontal basis functions are Legendre polynomials, so that there is no *a priori* information in the horizontal direction. The difference between each of the reconstructions that follow is in the choice of vertical basis functions and in the amount of *a priori* information used in the construction of the vertical basis functions. For the simulations, representative vertical profiles can be obtained from the original data, and then these vertical profiles can be used to create a realistic set of vertical profiles to be used for *a priori* information. For practical ionospheric tomography, vertical profiles can be obtained from any one of several ionospheric models.

Figure 2 shows a reconstruction using Legendre polynomials as basis functions in both the horizontal and vertical directions. The choice of Legendre polynomials as basis functions means that there is no *a priori* information used in the reconstruction. The reconstruction of Figure 2 shows two peaks at close to the correct latitude and altitude; however, the reconstruction is smeared in the vertical direction due to poor vertical resolution, and the magnitude of the peaks is seriously underestimated.

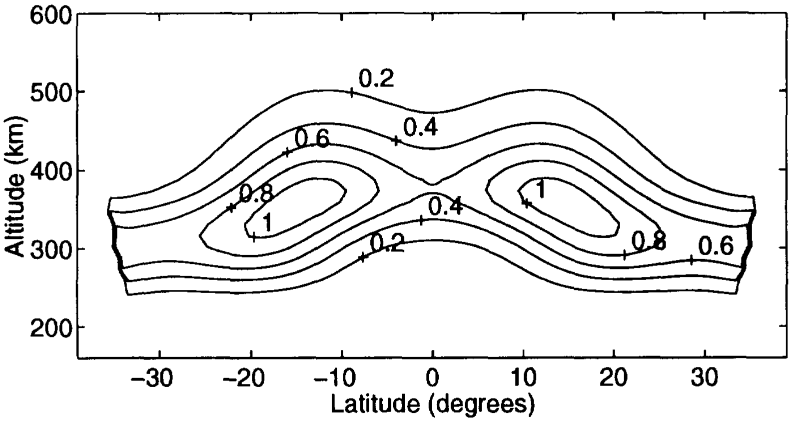

Figure 3: Reconstruction using *a priori* information.

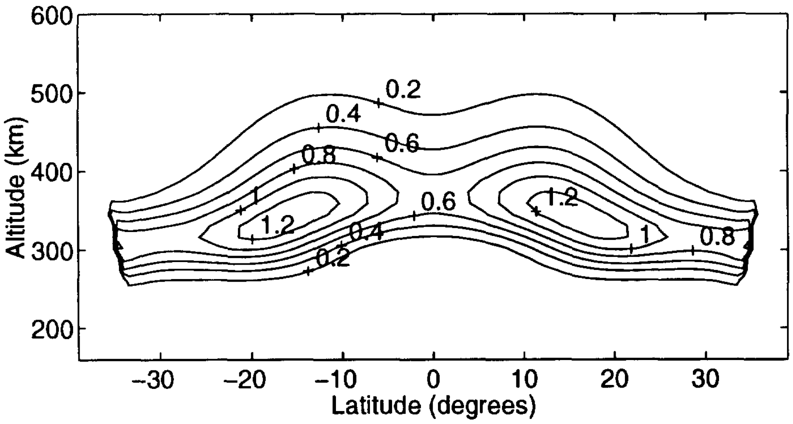

Figure 4: Reconstruction using more precise *a priori* information.

The reconstruction of Figure 2 can be improved by using *a priori* information in the reconstruction algorithm. In order to simulate the approximate nature of *a priori* information, a large set of vertical profiles was constructed so that the altitude of the maximum electron density covered a wider range than the altitude of the maximum electron density in the original image. Basis functions derived from this large set of vertical profiles were used for the reconstruction shown in Figure 3. The reconstruction of Figure 3 is much closer to the original image; however, there is still some smearing in the vertical direction, and the magnitude of the peaks is still underestimated. On the other hand, note that the variation in altitude of the maximum electron density found in the original image has been reproduced in the reconstruction, even though this information was not part of the *a priori* information used in the construction of the vertical basis functions.

Figure 4 shows a reconstruction using basis functions derived from the original data. This provides an idealized set of vertical basis functions for this particular reconstruction, and results in further improvement over the reconstruction shown in Figure 3. Notice that the peaks are narrower in the vertical direction and greater in magnitude then in the reconstruction of Figure 3. Thus, an improvement in the quality of *a priori* information results in an improvement in the vertical resolution of the reconstruction.

4. CONCLUSION

Due to the limitations of the imaging system, images of ionospheric electron density reconstructed without *a priori* information exhibit very poor vertical resolution. The vertical resolution can be dramatically improved by incorporating *a priori* information into the reconstruction algorithm.

In the orthogonal decomposition algorithm, the image is expressed as the sum of a set of orthonormal basis functions. The basis functions are constructed based upon *a priori* information. The reconstruction is then constrained to lie in a space spanned by a set of reasonable solutions.

For ionospheric tomography, the fundamental resolution of the imaging system in the vertical direction is very poor. When the image space is constrained in the vertical direction through the use of *a priori* information, the vertical resolution is improved. The amount of improvement in the vertical resolution depends on the quality of the *a priori* information. However, the increased resolution is obtained without weighting the solution toward a particular *a priori* model.

5. REFERENCES

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