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Ray Based Diffraction Tomography of the Ionosphere and Laboratory Inhomogeneous Plasma[1, 2]

Yu. A. Kravtsov[1, 2] and M. V. Tinin[3]

[1] *Space Research Institute, Russian Academy of Sciences, Moscow, Russia*
[2] *Space Research Center, Polish Academy of Sciences, Warsaw, Poland*
[3] *Irkutsk State University, Irkutsk, Russia*
e-mail: kravtsov@asp.iki.rssi.ru, vmt@api.isu.runnet.ru

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Abstract—A new procedure for restoration of the plasma inhomogeneities with improved resolution is suggested. The procedure deals with the double weighted Fourier transform (DWFT) of the observed wavefield in coordinates of both receivers $\boldsymbol{\rho} = (x, y)$ and sources $\boldsymbol{\rho}_0 = (x_0, y_0)$ [1]. Phase increments between the sources and receivers, being found from DWFT representation, can be used for extracting information on small perturbations of the dielectric constant $\tilde{\varepsilon}(\boldsymbol{\rho}, z)$ in a way similar to traditional radio tomography. The resulting resolution of the method is close to the diffraction limit $\Delta\boldsymbol{\rho} = \lambda h/D$ in the horizontal direction and $\Delta z = \lambda(h/D)^2$ in the vertical direction, where h is the height of inhomogeneities and D is the length of the ground-based receiving system.

1. INTRODUCTION

Traditional radio tomography (TRT) of the ionosphere plasma (see, for instance, papers [2, 3]) is based on geometrical optics representation of the wave field emitted by a satellite and observed by the ground-based receivers. The radius of the Fresnel zone a_F, estimated as

$$a_F \approx \sqrt{\lambda}\left(\frac{1}{h} + \frac{1}{H-h}\right)^{-1/2}, \qquad (1)$$

(here λ is a wavelength; H is a satellite altitude, say, $H = 1000$ km; and h is a typical inhomogeneities altitude, say, $h \approx 200$–400 km), serves as diffraction limit of resolution for ray-based representation of the wave field. At the frequency $f = 200$ MHz (corresponding wavelength is 1.5 m) and at the typical inhomogeneities height $h = 300$ km, the Fresnel radius is about 0.7 km, so that the TRT technique can restore inhomogeneities no less than 0.7 km in size.

In this paper we describe an improved measurement scheme, based on a double weighted Fourier transform (DWFT) of the wave field [1]. This approach accounts for diffraction effects and provides better resolution than the Fresnel radius (1).

2. MEASUREMENT SCHEME

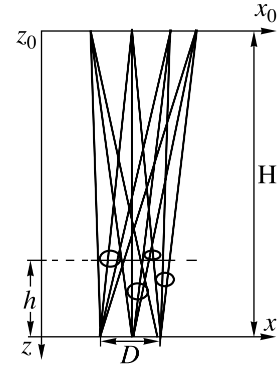
The inhomogeneities which are less in size than $a_F \sim (\lambda h)^{1/2}$ can noticeably focus and defocus a wave field from satellite sources and thereby cause strong amplitude fluctuations. In these conditions a multireceiver measurement scheme can be applied which implies coherent processing of data at many receiving points (figure). Such a scheme provides a diffraction resolution limit about

$$l_{\min} \sim \lambda\frac{h}{D}, \qquad (2)$$

where D is the horizontal length of a receiving system. In fact, formula (2) is similar to the expression for the transversal size of a focal spot of a lens of focal length h and diameter D [4]. According to Eq. (2), for frequency $f = 200$ MHz, $h \approx 300$ km, and $D = 30$ km

$$l_{\min} = 15 \text{ m}, \qquad (3)$$

what is about 50 times less than the Fresnel radius (1) at the same frequency. The minimal distance between



Multi-element ground-based receiving system for the ionosphere tomography with improved resolution.

[1] This paper was submitted by the authors in English.
[2] This paper was reported on the 5th International Symposium on Research and Application of Plasmas, PLASMA-2001 held in Warsaw, Poland on September 19–22, 2001.

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receivers ΔD is determined by a characteristic length l_a of the amplitude fluctuations: $\Delta D \le l_a$. If $\Delta D \sim l_a \sim 1 \sim 1$ km, then for providing resolution (3) the total number of receivers should be about $N = D/\Delta D \sim 30$.

Vertical resolution $\Delta z_{\min}$ is estimated by formula $\Delta z_{\min} \sim \lambda(h/D)^2$, which is analogous to the longitudinal size of the focal spot in the optical system. For the values used above, $\Delta z_{\min} \sim 0.15$ km.

3. PROCESSING PROCEDURE

Let us denote the wave field emitted by a satellite in the point ($\boldsymbol{\rho}_0$, z_0) and fixed by a receiver in a point ($\boldsymbol{\rho}_0$, z), as $U(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z, z_0$, where $\boldsymbol{\rho}_0$ is a 2D vector in the "source plane" (z_0) and $\boldsymbol{\rho}$ is the corresponding vector in the "receiving plane" z. Generalizing the DWFT method, initially developed for a 1D system of sources and receivers [1], for a 2D system, one can express the wave field $U(\boldsymbol{\rho}_0, z_0; \boldsymbol{\rho}, z)$ in the form of a double Fourier transform

$$U(\boldsymbol{\rho}_0, \boldsymbol{\rho}_0, z, z_0) = \frac{C_1}{(z-z_0)^3}\exp\{ik[z-z_0 + (\boldsymbol{\rho}+\boldsymbol{\rho}_0)^2/2(z-z_0)]\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{ik\left[\frac{2}{z-z_0} \times (\boldsymbol{\rho}_0'\boldsymbol{\rho}' - \boldsymbol{\rho}_0'\boldsymbol{\rho} - \boldsymbol{\rho}'\boldsymbol{\rho}_0) + \tilde{\Phi}(\boldsymbol{\rho}', \boldsymbol{\rho}_0', z, z_0)\right]\right\}d^2\boldsymbol{\rho}_0' d^2\boldsymbol{\rho}', \quad (4)$$

where $C_1 = 2k^2\exp(i3\pi/4)/(2\pi)^3$ and

$$\tilde{\Phi}(\boldsymbol{\rho}', \boldsymbol{\rho}_0', z, z_0) = 0.5\int_{z_0}^{z}\tilde{\varepsilon}\left(\boldsymbol{\rho}_0'\frac{z-z'}{z-z_0} + \boldsymbol{\rho}'\frac{z'-z_0}{z-z_0}, z'\right)dz'. \quad (5)$$

Such a representation of the wave field is convenient not only for solution of direct propagation problems, but also for determination of plasma inhomogeneities $\tilde{\varepsilon}(r)$, which are considered to be small as compared to the background value ε_0: $\varepsilon = \varepsilon_0 + \tilde{\varepsilon}(r)$. In spite of the smallness of fluctuations $\tilde{\varepsilon}$, integral representation (4) can describe strong amplitude fluctuations due to focusing of the partial waves composing the integral.

Inversing integral representation (4), one can obtain

$$\hat{U}(\boldsymbol{\rho}^*, \boldsymbol{\rho}_0^*, z, z_0) = \left(\frac{k}{\pi}\right)^4\frac{C_1^{-1}}{z-z_0}\times \iint d^2\boldsymbol{\rho}\, d^2\boldsymbol{\rho}_0 U(\boldsymbol{\rho}, \boldsymbol{\rho}_0, z, z_0) \exp\left\{ik\left(-z+z_0-\frac{(\boldsymbol{\rho}+\boldsymbol{\rho}_0)^2}{2(z-z_0)}+\frac{2(\boldsymbol{\rho}_0^*\boldsymbol{\rho}+\boldsymbol{\rho}^*\boldsymbol{\rho}_0)}{z-z_0}\right)\right\} = \exp\left\{-ik\left[2\frac{\boldsymbol{\rho}^*\boldsymbol{\rho}_0^*}{z-z_0}+\tilde{\Phi}(\boldsymbol{\rho}^*, \boldsymbol{\rho}_0^*, z, z_0)\right]\right\}, \quad (6)$$

where $\boldsymbol{\rho}*$ and $\boldsymbol{\rho}_0^*$ – are fixed points in the receiver and source planes correspondingly.

Thus, processing measurement data $U(\boldsymbol{\rho}_0, z_0; \boldsymbol{\rho}, z)$ according to (6), that is, performing the double weighted Fourier transform, one can obtain the phase variations (5). For ionospheric satellite-based measurements integration over source coordinate $\boldsymbol{\rho}_0 = \mathbf{v}_0 t$ in fact is performed by integration over time t; however, integration over $\boldsymbol{\rho}$ demands a sufficiently large amount of receivers.

Applying the tomographic procedure to phase variations (5), one can extract the value $\varepsilon(\boldsymbol{\rho}, z)$. Unlike ray radio tomography of the ionosphere [3] inversion of (4) takes into account diffraction effects connected with strong fluctuations due to wave focusing by large-scale random inhomogeneities.

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