# 25.1.2024

čtvrtek 25. ledna 2024 20:28

(A)

| | |
|---|---|
| 1 | C |
| 2 | B |
| 3 | A |
| 4 | D |

1) $x_1 \in M \Rightarrow f(x_1) = b$
$x_2 \in M \Rightarrow f(x_2) = b$

$f(3x_1 - 2x_2) = 3f(x_1) - 2f(x_2) = 3b - 2b = b$

2) $P \cdot P = P$ je vlastnost projekcí
viz MFF skripta 42

navíc je lze snížit i geometricky

3) $B = (a(x), b(x), c(x))$

a) $[\pi\, b(x)]_B = \begin{pmatrix} 0 \\ \pi \\ 0 \end{pmatrix}$

$[(17+\sqrt{3})\, c(x)]_B = \begin{pmatrix} 0 \\ 0 \\ 17+\sqrt{3} \end{pmatrix}$

$[6\, a(x)]_B = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$

a matice $\begin{pmatrix} 0 & 0 & 6 \\ \pi & 0 & 0 \\ 0 & 17+\sqrt{3} & 0 \end{pmatrix}$ má rank 3

4) příklad:
$2p$ řeší $(A|2b) \Leftrightarrow A2p = 2b \quad / \cdot \frac{1}{2}$

$\Downarrow$

$Ap = b$

(B)

dobírano v
sbírce Lucy sbírky 2019/ dubnový 2019 II
12. příklad

(C)

G-S:

$$q_1 = b_2 - \sum_{i=1}^{2-1} \frac{\langle q_i | b_2 \rangle}{\langle q_i | q_i \rangle} q_i$$

$q_1 = b_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \quad \|q_1\| = \sqrt{5} \rightsquigarrow q_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$q_2 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} - \frac{(2\,1\,0)\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}}{5} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \quad \|q_2\| = \sqrt{5} \rightsquigarrow q_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

$q_3 = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} - \frac{(2\,1\,0)\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}}{5} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{(-1\,2\,0)\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}}{5} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} =$

$= \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} - \frac{11}{5} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \frac{3}{5} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} =$

$= \frac{1}{5} \left( \begin{pmatrix} 25 \\ 5 \\ 15 \end{pmatrix} - \begin{pmatrix} 22 \\ 11 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix} \right) = \frac{1}{5} \begin{pmatrix} 0 \\ 0 \\ 15 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \quad \|q_3\| = 3 \rightsquigarrow q_3 = \frac{1}{3} \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$

$\Rightarrow \quad G = \left( \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}; \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \right)$

$Q = \left( \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}; \frac{1}{\sqrt{5}} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}; \frac{1}{3} \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \right)$