4.1.

$\vec{x} = (1,2,3)$

$\vec{y} = (-1,0,1)$

$\|\vec{x}\| = \sqrt{1^2 + 2^2 + 3^2} = \underline{\underline{\sqrt{14}}}$

$\|\vec{x} - \vec{y}\| = \|(1,2,3) - (-1,0,1)\| = \|(2,2,2)\| = \sqrt{2^2+2^2+2^2} = \sqrt{12} = \underline{\underline{2\sqrt{3}}}$

$$\cos\varphi = \frac{(1,2,3)^T \cdot (-1,0,1)}{\|(1,2,3)\| \|(-1,0,1)\|} = \frac{1\cdot(-1) + 2\cdot 0 + 3\cdot 1}{\sqrt{14}\cdot\sqrt{(-1)^2+0^2+1^2}} = \frac{2}{\sqrt{14}\cdot\sqrt{2}} = \frac{2}{\sqrt{28}} = \frac{2}{2\sqrt{7}} = \frac{1}{\sqrt{7}}$$

$\varphi = \arccos\left(\frac{1}{\sqrt{8}}\right) = \underline{\underline{1{,}1832 \text{ rad}}}$

4.3.

$\operatorname{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

řešení: $\underline{\underline{\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}}$

4.5.

(a) $\|x\| = \|y\| \Rightarrow (x+y) \perp (x-y)$

$(x+y)^T(x-y) = 0$

$x^Tx + \underbrace{y^Tx - x^Ty}_{0} - y^Ty = 0$

$x^Tx - y^Ty = 0$

$x^Tx = y^Ty$

$\sqrt{x^Tx} = \sqrt{y^Ty}$

$\|x\| = \|y\|$ ✓

(b) $x \perp y \Rightarrow \|x\|^2 + \|y\|^2 = \|x-y\|^2$

$\|x^2\| + \|y\|^2 = \|x-y\|^2$?

$x^Tx + y^Ty = (x-y)^T(x-y)$

$x \perp y \Rightarrow x^Ty = 0$

$y^Tx = 0$

$$x^Tx + y^Ty = x^Tx - \underbrace{x^Ty}_{0} - \underbrace{y^Tx}_{0} + y^Ty$$

protože $x \perp y$

$$x^Tx + y^Ty = x^Tx + y^Ty \quad ✓$$

4.10. $f: \mathbb{R}^3 \to \mathbb{R}^4$

$f(1,-1,2) = (1,2,-1,1)$

$f(1,1,0) = (0,1,-1,0)$

isometrie = zachovává vzdálenosti

$$\left.\begin{array}{l} \|(1,-1,2)\| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6} \\ \|(1,2,-1,1)\| = \sqrt{1^2+2^2+(-1)^2+1^2} = \sqrt{7} \end{array}\right\} \text{nerovná se} \Rightarrow \underline{\underline{\text{není izometrie!}}}$$

$$\left.\begin{array}{l} \|(1,1,0)\| = \sqrt{1^2+1^2+0^2} = \sqrt{2} \\ \|(0,1,-1,0)\| = \sqrt{0^2+1^2+(-1)^2+0^2} = \sqrt{2} \end{array}\right\} \text{rovná se}$$

4.13. $\operatorname{span}\{x,y\} = \operatorname{span}\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}\right\}$

$$x = \begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$y = \begin{pmatrix}1\\2\\3\end{pmatrix} - \alpha\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$x^Ty = 0 \Rightarrow \begin{pmatrix}0\\1\\1\end{pmatrix}^T \cdot \left[\begin{pmatrix}1\\2\\3\end{pmatrix} - \alpha\begin{pmatrix}0\\1\\1\end{pmatrix}\right] = 0$$

$$\begin{pmatrix}0\\1\\1\end{pmatrix}^T \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} - \alpha\begin{pmatrix}0\\1\\1\end{pmatrix}^T \cdot \begin{pmatrix}0\\1\\1\end{pmatrix} = 0$$

$$0+2+3-\alpha(0+1+1) = 0$$

$$5 - 2\alpha = 0$$

$$\alpha = \frac{5}{2}$$

$$\underline{\underline{x = \begin{pmatrix}0\\1\\1\end{pmatrix} \quad y = \begin{pmatrix}1\\-1/2\\1/2\end{pmatrix}}}$$