1) $\vec{K} = (3x^2y\,;\, yz^2\,;\, -xz)$ Je $\vec{K}$ potenciálové pole?

$$\vec{\nabla} \times \vec{K} \overset{?}{=} \vec{0}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ K_x & K_y & K_z \end{vmatrix} = (0 - 2yz\,;\, -z - 0\,,\, 0 - 3x^2) \neq \vec{0}$$

$\vec{K}$ není potenciálové.

2) $\vec{K} = (2xy + z^3,\, x^2 + 2y,\, 3xz^2 - 2)$

$$\vec{\nabla} \times \vec{K} \overset{?}{=} \vec{0}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ K_x & K_y & K_z \end{vmatrix} = (0 - 0,\, 3z^2 - 3z^2,\, 2x - 2x) = \vec{0}$$

$$\vec{K} = -\nabla\varphi \Rightarrow K_x = -\frac{\partial\varphi}{\partial x},\quad K_y = -\frac{\partial\varphi}{\partial y},\quad K_z = -\frac{\partial\varphi}{\partial z}$$

$$\varphi = -\int K_x\,dx = -x^2y - z^3x + C(y,z)$$

$$\varphi = -\int K_y\,dy = -x^2y - y^2 + C(x,z)$$

$$\varphi = -\int K_z\,dz = -xz^3 + 2z + C(x,y)$$

$$C(y,z) = -y^2 + 2z + C$$

$$C(x,z) = -z^3x + 2z + C$$

$$C(x,y) = -x^2y - y^2 + C$$

$$\underline{\underline{\varphi = -x^2y - xz^3 - y^2 + 2z + C}}$$

Pole je potenciálové

③ $\vec{K} = (1+2xy,\ x^2+3y^2,\ 0)$

$\vec{\nabla} \times \vec{K} = \vec{0}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ K_x & K_y & K_z \end{vmatrix} = (0-0,\ 0-0,\ 2x-2x) = \vec{0}$$

$\vec{K} = \nabla \cdot \varphi$

$\varphi = -\int K_x dx = -x - x^2 y + C(y,z)$

$\varphi = -\int K_y dy = -x^2 y - y^3 + C(x,z)$

$\varphi = -\int K_z dz = 0 + C(x,y)$

$C(y,z) = -y^3 + C$

$C(x,z) = -x + C$

$C(x,y) = -x - x^2 y - y^3 + C$

$\underline{\underline{\varphi = -x - x^2 y - y^3 + C}}$

(4)

$$c_n m_n (T_n - T) = c_v m_v (T - T_0) + C(T - T_0)$$

$$c_n m_n T_n - c_n m_n T = c_v m_v T - c_v m_v T_0 + CT - CT_0$$

$$c_n m_n T_n + c_v m_v T_0 + CT_0 = CT + c_v m_v T + c_n m_n T$$

$$T = \frac{c_n m_n T_n + c_v m_v T_0 + CT_0}{C + c_v m_v + c_n m_n}$$

$$t = T - 273{,}15 \doteq 32{,}8\,°C$$

(5) $M = 2000\,kg$; $v = 25\,m \cdot s^{-1}$; $n = 84$ (počet brzd) ; $c_ž = 45\,J \cdot K^{-1} \cdot kg^{-1}$

$m = 9\,kg$

$$\frac{1}{2} M v^2 = n \cdot m \cdot c_ž \Delta t$$

$$\Delta t = \frac{M v^2}{n \cdot m \cdot 2 c_ž} \doteq 38{,}6\,°C$$

(6) Ponorný vařič

$P = 620\,W$; $V = 1\,l$; $t_1 = 20\,°C$; $t_2 = 100\,°C$; $\rho = 1000\,kg\,m^{-3}$

$c_v = 4190\,J \cdot K^{-1} \cdot kg^{-1}$

$$P \Delta \tau = c_v \rho V (t_2 - t_1)$$

$$\Delta \tau = \frac{c_v \rho V (t_2 - t_1)}{P} \doteq 541\,°C$$

7 Vodík H_2 za $p = 10^5\ Pa$ $t = 0\,°C$

$$\rho = \frac{m}{V}$$

$$V_m = \frac{V}{n} \Rightarrow V = V_m n$$

$$M = \frac{m}{n} \Rightarrow m = M \cdot n$$

$$\rho = \frac{M}{V_m}$$

$$V_m = 22{,}414\ l \cdot mol^{-1} = 0{,}022414\ m^3 \cdot mol^{-1}$$

$$\rho = 8{,}9 \cdot 10^{-2}\ kg \cdot m^{-3}$$

$$M = 2g \cdot mol^{-1} = 2 \cdot 10^{-3}\ kg \cdot mol^{-1}$$

8 Kosmická loď: $V = 20\,m^3$, $t = -100\,°C$, vakuum

Jaký bude tlak vodních par, když do lodi unikne kapka vody (H_2O) $m = 1\,g$

$$pV = n R_m T$$

$$T = t + 273{,}15\ K$$

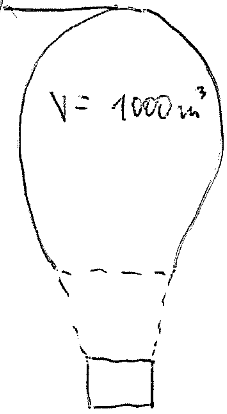
$$p = \frac{R_m (t + 273{,}15)}{MV} \doteq \underline{\underline{3{,}99\ Pa}}$$

$$M = \frac{m}{n} \Rightarrow n = \frac{m}{M}$$

$$M_{H_2O} = 18\ g \cdot mol^{-1}$$

$$R_m = 8{,}314\ J \cdot K^{-1} \cdot mol^{-1}$$

10 BALON



$m = 200\ kg$

$t = ?$ (teplota)

$p_{atm} = 10^5\ Pa$

$\rho_{or} = 1{,}2\ kg \cdot m^{-3}$

$t_{or} = 20°C$

$$\vec{F}_g = m\vec{g} + \rho_{vB} V \vec{g}$$

$$\vec{F}_v = -\rho_{or} V \vec{g}$$

$$\vec{F}_g + \vec{F}_v = \vec{0}$$

$$(m + \rho_{vB} V - \rho_{or} V)\,\vec{g} = \vec{0}$$

$$m + \rho_{vB} V - \rho_{or} V = 0$$

STAVOVÁ RCE:

$$\frac{pV}{T} = konst.$$

$$\frac{pV}{T} = \frac{\overset{101325\,Pa}{p_0}\ V_0}{\underset{273{,}15\,K}{T_0}}$$

$$\underbrace{\frac{p_0 V_m}{T_0}}_{R_m = 8{,}314\ J \cdot K^{-1} mol^{-1}} n = R_m \cdot n$$

n - počet molů

$N = n \cdot N_A$ ← Avogadrova konst. $= 6{,}023 \cdot 10$

$V_0 = V_m \cdot n \qquad V_m = 22{,}4 \cdot 10^{-3}\ m^3$

$$pV = n R_m T$$

$$\frac{pm}{\rho} = n R_m T \Rightarrow \frac{p}{\rho} = \frac{n}{m} \cdot R_m T \qquad M = \frac{m}{n} \text{ molární hm.}$$

$$\boxed{\frac{p}{\rho} = \frac{R_m T}{M}}$$

BALON – POKR.

$$\boxed{\frac{p}{\varrho} = \frac{R_m T}{M}}$$

$$\varrho_{OV} T_{OV} = \text{konst} = \varrho_{VB} T_{VB}$$

$$m + \varrho_{VB} V - \varrho_{OV} V = 0$$

$$\varrho_{OV} \cdot \frac{T_{OV}}{T_{VB}} V - \varrho_{OV} V = -m$$

$$\varrho_{OV} \left(\frac{T_{OV}}{T_{VB}} - 1 \right) V = -m \Rightarrow \frac{T_{OV}}{T_{VB}} = 1 - \frac{m}{\varrho_{OV} V} \Rightarrow T_{VB} = \frac{T_{OV}}{1 - \frac{m}{\varrho_{OV} V}} = 352 \text{ K}$$

$(79\,°C)$

11

$$\delta W = p\,dV$$

$$W = \int_{V_1}^{V_2} p\,dV = p\int_{V_1}^{V_2} dV = p(V_2 - V_1) = p\,\frac{n R_m}{p}(T_2 - T_1) =$$

$$\boxed{W = n R_m (T_2 - T_1)}$$

$$V_2 = \frac{n R_m}{p} T_2$$

$$V_1 = \frac{n R_m}{p} T_1$$

$$dU = n C_v\,dT$$

$$\Delta U = n C_v \int_{T_1}^{T_2} dT = n C_v (T_2 - T_1)$$

$$\boxed{\Delta U = n C_v (T_2 - T_1)}$$

$$\delta Q = dU + \delta W$$

$$Q = \Delta U + W = n C_v (T_2 - T_1) + n R_m (T_2 - T_1) = n(C_v + R_m)(T_2 - T_1)$$

$$C_p = C_v + R_m$$

$$\boxed{Q = n C_p (T_2 - T_1)}$$

12

$T = \text{konst.} \rightarrow dT = 0$

$$Q = \Delta U + W$$

$$\delta Q = C_v dT + p dV$$

$$\delta Q = \delta W = p dV$$

$$dU = C_v dT = 0 \quad \Rightarrow \quad \underline{\underline{\Delta U = 0}}$$

$$\delta Q = \delta W = p dV$$

$$pV = n R_m T$$

$$p = \frac{n R_m T}{V}$$

$$Q = W = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} \frac{n R_m T}{V} dV =$$

$$= n R_m T \int_{V_1}^{V_2} \frac{dV}{V} = \underline{\underline{n R_m T \ln \frac{V_2}{V_1}}}$$

13

$V = \text{konst.} \Rightarrow dV = 0$

$$Q = \Delta U + W$$

$$\delta Q = C_v dT + p dV \quad \Rightarrow \quad \underline{\underline{W = 0}}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V}{T_2}$$

$$p_1 T_2 = p_2 T_1$$

$$T_2 = \frac{p_2}{p_1} T_1$$

$$\delta Q = dU = C_v dT$$

$$Q = \Delta U = \int_{T_1}^{T_2} C_v dT = C_v \int_{T_1}^{T_2} dT = C_v (T_2 - T_1) = C_v \left(\frac{p_2}{p_1} T_1 - T_1 \right) =$$

$$= \underline{\underline{C_v T_1 \left(\frac{p_2}{p_1} - 1 \right)}}$$

14

1. $p = \text{konst.} = p_1 \qquad \cancel{V_1} \to V_2 \;;\; V_1 < V_2$

2. $V = \text{konst.} = V_2 \qquad p_1 \to p_2 \;;\; p_1 > p_2 \qquad \boxed{W = 0}$

3. $p = \text{konst.} = p_2 \qquad V_2 \to V_1 \;;\; V_2 > V_1$

4. $V = \text{konst.} = V_1 \qquad p_2 \to p_1 \;;\; p_2 < p_1 \qquad \boxed{W = 0}$

$W = ?$

$$\delta W = p\,dV$$

1-2 $$W_{1,2} = \int_{V_1}^{V_2} p_1\,dV = p_1 (V_2 - V_1)$$

3-4 $$W_{3,4} = \int_{V_2}^{V_1} p_2\,dV = -\int_{V_1}^{V_2} p_2\,dV = -p_2 (V_2 - V_1)$$

$$W = W_{1,2} + W_{3,4} = p_1 (V_2 - V_1) - p_2 (V_2 - V_1) = \underline{\underline{(p_1 - p_2)(V_2 - V_1)}}$$

15

pracovní médium: n molů id. plynu

1. $T = konst. = T_1$ $\quad V_1 \to V_2 \;;\; V_1 < V_2$

2. $V = konst. = V_2$ $\quad T_1 \to T_2 \;;\; T_1 > T_2$ $\quad \boxed{W = 0}$

3. $T = konst. = T_2$ $\quad V_2 \to V_1 \;;\; V_2 > V_1$

4. $V = konst. = V_1$ $\quad T_2 \to T_1 \;;\; T_2 < T_1$ $\quad \boxed{W = 0}$

$W = ?$

$\delta W = p\,dV$ $\qquad pV = n R_m T$

$\delta W = \frac{n R_m T}{V} dV$ $\qquad p = \frac{n R_m T}{V}$

1-2
$$W_{12} = \int_{V_1}^{V_2} \frac{n R_m T_1}{V} dV = n R_m T_1 \int_{V_1}^{V_2} \frac{dV}{V} = n R_m T_1 \ln \frac{V_2}{V_1}$$

3-4
$$W_{34} = \int_{V_2}^{V_1} \frac{n R_m T_2}{V} dV = - n R_m T_2 \int_{V_1}^{V_2} \frac{dV}{V} = - n R_m T_2 \ln \frac{V_2}{V_1}$$

$$W = W_{12} + W_{34} = n R_m T_1 \ln \frac{V_2}{V_1} - n R_m T_2 \ln \frac{V_2}{V_1} =$$

$$= \underline{\underline{n R_m (T_1 - T_2) \ln \frac{V_2}{V_1}}}$$

16

$t_1 = 45°C$ $\qquad T_1 = (273{,}15 + 45)\ K$

$V_1 = 630\ cm^3 = 630 \cdot 10^{-6}\ m^3$

$V_2 = 30\ cm^3 = 30 \cdot 10^{-6}\ m^3$

$\varkappa = 1{,}37$

adiabatické stlačení $V_1 \to V_2$

$t_2 = ?$

$p V^{\varkappa} = konst.$

předpokládáme id. plyn:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$p_1 = \frac{T_1 p_2 V_2}{T_2 V_1}$$

$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}$

$$\frac{T_1 \cancel{p_2} V_2}{T_2 V_1} V_1^{\varkappa} = \cancel{p_2} V_2^{\varkappa}$$

$$T_2 = \frac{T_1 V_2 V_1^{\varkappa}}{V_1 V_2^{\varkappa}} = T_1 \cdot V_2^{1-\varkappa} \cdot V_1^{\varkappa-1}$$

$t_2 = T_2 - 273{,}15 = \underline{\underline{708\ °C}}$

(17) adiabatické stlačení

$$V_0,\ p_0,\ \varkappa \longrightarrow V = \frac{V_0}{n},\ p,\ \varkappa \qquad W = ?$$

$$p_0 V_0^{\varkappa} = p V^{\varkappa}$$

$$p = \frac{p_0 V_0^{\varkappa}}{V^{\varkappa}}$$

$$\delta W = p\,dV$$

$$W = \int_{V_0}^{\frac{V_0}{n}} \frac{p_0 V_0^{\varkappa}}{V^{\varkappa}}\,dV = -p_0 V_0^{\varkappa}\left[\frac{V^{-\varkappa+1}}{-\varkappa+1}\right]_{\frac{V_0}{n}}^{V_0} = -p_0 V_0^{\varkappa}\left(\frac{V_0^{-\varkappa+1}}{-\varkappa+1} - \frac{V_0^{-\varkappa+1}\frac{1}{n^{-\varkappa+1}}}{-\varkappa+1}\right) =$$

$$= \frac{-p_0 V_0^{\varkappa} V_0^{-\varkappa+1}\left(1 - n^{\varkappa-1}\right)}{-\varkappa+1} = - \frac{p_0 V_0 \left(n^{\varkappa-1} - 1\right)}{\varkappa - 1}$$

že znaménkové dohody: platí $W < 0 \Rightarrow$ na soustavě je konána práce

Dodaná práce $W_{dod} = -W = \dfrac{p_0 V_0 \left(n^{\varkappa-1} - 1\right)}{\varkappa - 1}$.

18

Carnotův cyklus

$\eta = 0{,}4$

$\eta' = 0{,}5$

$t_{ch} = 27°C$

$\Delta T = -T_0 + T_0' = ?$

$$\eta = \frac{|Q_{DOD}| - |Q_{ODEVZ}|}{|Q_{DOD}|}$$

$T_{CH} = t_{CH} + 273{,}15$

$$\eta = \frac{T_0 - T_{CH}}{T_0}$$

$$T_0 = \frac{T_{CH}}{1-\eta}$$

$$T_0' = \frac{T_{CH}}{1-\eta'}$$

$$\Delta t = \Delta T = T_0' - T_0 = \frac{T_{CH}}{1-\eta'} - \frac{T_{CH}}{1-\eta}$$

$\Delta t = 100°C$

19

Ottův cyklus

1. $Q = \cancel{\text{konst.}}\ 0$ $\quad p_1 \to p_2$ $\quad V_1 \to V_2\ ;\ V_1 < V_2$ $\quad T_1 \to T_2$

2. $V = \text{konst.} = V_2$ $\quad T_2 \to T_3$ $\quad T_2 > T_3$ $\quad p_2 \to p_3$

3. $Q = \cancel{\text{konst.}}\ 0$ $\quad p_3 \to p_4$ $\quad V_2 \to V_1\ ;\ V_2 > V_1$ $\quad T_3 \to T_4$

4. $V = \text{konst.} = V_1$ $\quad T_4 \to T_1$ $\quad p_4 \to p_1$

$$\eta = \frac{|Q_D| - |Q_0|}{Q_D} = \frac{\overset{T_1 > T_4}{|T_1 - T_4|} - |T_3 - T_2|^{\,T_3 < T_2}}{|T_1 - T_4|} = 1 - \frac{T_2 - T_3}{T_1 - T_4}$$

(4-1) $\delta Q_D = c_v\, dT + \underbrace{p\, dV}_{0} \Rightarrow Q_D = c_v (T_1 - T_4)$

(2-3) $Q_0 = c_v (T_3 - T_2)$

$p V^{\varkappa} = K$ (konstanta) $\qquad \frac{pV}{T} = C$ (konstanta) $\Rightarrow \frac{T}{V} V^{\varkappa} = \frac{C}{K} \Rightarrow T V^{\varkappa - 1} = \text{konst.}$

(1-2) $T_1 V_1^{\varkappa - 1} = T_2 V_2^{\varkappa - 1}$

(3-4) $T_3 V_2^{\varkappa - 1} = T_4 V_1^{\varkappa - 1}$ $\quad \Big\}\ \ominus$

$$(T_1 - T_4) V_1^{\varkappa - 1} = (T_2 - T_3) V_2^{\varkappa - 1}$$

$$\frac{T_2 - T_3}{T_1 - T_4} = \left(\frac{V_1}{V_2}\right)^{\varkappa - 1}$$

$$\eta = 1 - \frac{T_2 - T_3}{T_1 - T_4} = 1 - \left(\frac{V_1}{V_2}\right)^{\varkappa - 1}$$

21

$m = 10g = 0{,}01\ kg$

$t_1 = 50°C$

$t_2 = -10°C$

$c_v = 651\ J \cdot kg^{-1} \cdot K^{-1}$

$M = 32 g \cdot mol^{-1}$

$n = \frac{m}{M}$

a) vratný děj $t_1 \to t_2$; $V = konst.$

$$dS = \frac{\delta Q}{T} = \frac{c_v dT + p\overset{0}{\widetilde{dV}}}{T} = \frac{m c_v dT}{T}$$

$$\Delta S = m c_v \int_{T_1}^{T_2} \frac{dT}{T} = m c_v \ln \frac{T_2}{T_1} = \underline{\underline{-1{,}34\ J \cdot K^{-1}}}$$

b) vratný děj $t_1 \to t_2$; $p = konst.$

$$dS = \frac{m c_v dT + p dV}{T}$$

$$\Delta S = m c_v \int_{T_1}^{T_2} \frac{dT}{T} + \int_{V_1}^{V_2} \frac{n R_m}{V} dV =$$

$$= m c_v \ln \frac{T_2}{T_1} + n R_m \ln \frac{V_2}{V_1} =$$

$$pV = n R_m T$$

$$p = \frac{n R_m T}{V}$$

$$\frac{p V_1}{T_1} = \frac{p V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

$$n = \frac{m}{M}$$

$$= m c_v \ln \frac{T_2}{T_1} + \frac{m}{M} R_m \ln \frac{T_2}{T_1} =$$

$$= \left(c_v + \frac{R_m}{M}\right) m \ln \frac{T_2}{T_1} = \underline{\underline{-1{,}87\ J \cdot K^{-1}}}$$

22

$m_1 = 80 g = 0{,}08 kg$ $\qquad R_m = 8{,}314 \ J \cdot K^{-1} \cdot mol^{-1}$

$m_2 = 20 g = 0{,}02 kg$ $\qquad n = \frac{m}{M}$

$t_1 = 90°C \quad T_1 = 363{,}15 K$

$t_2 = 10°C \quad T_2 = 283{,}15 K$ $\qquad M = 18 g \cdot mol^{-1}$

$c = 4190 \ J \cdot kg^{-1}$

$$m_1 c (T_1 - T) = m_2 c (T - T_2)$$

$$m_1 T_1 - m_1 T = m_2 T - m_2 T_2$$

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = 347{,}15 K$$

$$\Delta S_1 = m_1 c \ln \frac{T}{T_1} + n R_m \ln \frac{T}{T_1} = \left(c + \frac{R_m}{M}\right) m_1 \ln \frac{T}{T_1}$$

$$\Delta S_2 = m_2 c \ln \frac{T}{T_2} + n R_m \ln \frac{T}{T_2} = \left(c + \frac{R_m}{M}\right) m_2 \ln \frac{T}{T_2}$$

entropie je extenzivní veličina:

$$\Delta S = \Delta S_1 + \Delta S_2 = \left(c + \frac{R_m}{M}\right)\left(m_1 \ln \frac{T}{T_1} + m_2 \ln \frac{T}{T_2}\right) =$$

$$= 2{,}19 \ J \cdot K^{-1}$$

?? CHYBA ??

oficiální výsledek:

$$\boxed{\Delta S = 1{,}97 \ J \cdot K^{-1}}$$

23

$t_1 = 10°C$ $\qquad$ $\alpha_m = 1{,}9 \cdot 10^{-5}\ K^{-1}$

$t_2 = 100°C$ $\qquad$ $\alpha_h = 2{,}4 \cdot 10^{-5}\ K^{-1}$

$l_{1_h} = l_{1_m} = l_1 = 2m$

$$\Delta X = X_0 \cdot \alpha \cdot \Delta T$$

$$\Delta l_h = \alpha_h \cdot l_1 \cdot (t_2 - t_1)$$

$$\Delta l_m = \alpha_m \cdot l_1 \cdot (t_2 - t_1)$$

$$\Delta l = \Delta l_h - \Delta l_m = (\alpha_h - \alpha_m) \cdot l \cdot (t_2 - t_1) = \underline{\underline{8 \cdot 10^{-4}\ m}}$$

24

$t_1 = 10°C$

$t_2 = 25°C$

$\alpha = 1{,}7 \cdot 10^{-5}\ K^{-1}$

$\tau_1 = 1s$

$$\tau_1 = \pi \sqrt{\frac{2l_0}{3g}} \Rightarrow \frac{\tau_1^2 \cdot 3g}{2\pi^2} = l_0$$

$$\Delta l = l_0 \cdot \alpha \cdot \Delta t$$

$$\tau_2 = \pi \sqrt{\frac{2(l_0 + \Delta l)}{3g}} = \sqrt{\frac{2\pi^2 (l_0 + l_0 \alpha \Delta t)}{3g}} = \sqrt{\frac{\cancel{2\pi^2} \cdot \frac{\tau_1^2 \cdot \cancel{3g}}{\cancel{2\pi^2}} (1 + \alpha \Delta t)}{\cancel{3g}}} =$$

$$= \sqrt{\tau_1^2 (1 + \alpha \Delta t)} = \underline{\underline{1{,}000\,13\ s}}$$

25

$V = 200\ cm^3$

$\Delta t = 30°C$

$\alpha = 9{,}0 \cdot 10^{-6}\ K^{-1}$

$\beta = 0{,}182 \cdot 10^{-3}\ K^{-1}$

$V = a \cdot b \cdot c$

$V_2 = (a + \Delta a)(b + \Delta b)(c + \Delta c) = a \cdot b \cdot c\,(1 + \alpha \Delta t) = V(1 + \alpha \Delta t)^3$

$\Delta x = x \cdot \alpha \cdot \Delta t$

$\Delta V_s = V_2 - V_1 = V\left((1 + \alpha \Delta t)^3 - 1\right)$

$\Delta V_{Hg} = V \cdot \beta \cdot \Delta t$

$V_v = \Delta V_{Hg} - \Delta V_s = V\left(\beta \Delta t + 1 - (1 + \alpha \Delta t)^3\right) = \underline{\underline{0{,}93\ cm^3}}$

26

$V = 2\ l$

$t = 10°C$

$t_2 = 100°C$

$\tau_1 = 5\ min = 300\ s$

$W = konst.$

$c = 4190\ J \cdot kg^{-1} \cdot K^{-1}$

$l_v = 2{,}256 \cdot 10^6\ J \cdot kg^{-1}$

$m = 2\ kg$ $\qquad \Delta t = t_2 - t$

$Q_1 = c m \Delta t$ $\qquad Q_2 = m\, l_v$

$W = \frac{Q_1}{\tau_1} = \frac{c m \Delta t}{\tau_1}$ $\qquad W = \frac{Q_2}{\tau_2} = \frac{m\, l_v}{\tau_2}$

$\frac{c m \Delta t}{\tau_1} = \frac{m\, l_v}{\tau_2} \Rightarrow \tau_2 = \frac{\cancel{m}\, l_v\, \tau_1}{c \cancel{m} \Delta t} = \underline{\underline{1795\ s = 29\ min\ 55\ s}}$

27

$t_1 = 27\,°C$ $\qquad$ $c = 129\ J \cdot kg^{-1} \cdot K^{-1}$

$t_t = 328\,°C$ $\qquad$ $l_t = 23{,}2 \cdot 10^3\ J \cdot kg^{-1}$

$$E_k = \frac{1}{2} m v^2 \qquad \Delta t = t_t - t_1$$

$$Q = mc\Delta t + m l_t$$

$$E_k = Q$$

$$\frac{1}{2}\cancel{m} v^2 = \cancel{m} c \Delta t + \cancel{m} l_t$$

$$v = \sqrt{2c(t_t - t_1) + 2 l_t} = 352\ m \cdot s^{-1}$$

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$V = 0{,}5\ l \Rightarrow m_i = 0{,}5\ kg$ $\qquad$ $c_v = 4190\ J \cdot kg^{-1} \cdot K^{-1}$

$t_i = 80\,°C$ $\qquad$ $c_e = 2220\ J \cdot kg^{-1} \cdot K^{-1}$

$m_e = 200 g = 0{,}2\ kg$ $\qquad$ $l_t = 333\ 000\ J \cdot kg^{-1}$

$t_e = -10$

$t_0 = 0\,°C$

$$Q_i = m_i c_v (t_i - t_x)$$

$$Q_e = m_e c_e (t_0 - t_e) + m_e l_t + m_e c_v (t_x - t_0)$$

$$Q_i = Q_e$$

$$m_i c_v t_i - \underline{m_i c_v t_x} = \underbrace{m_e c_e t_0}_{t_0} - m_e c_e t_e + m_e l_t + \underline{m_e c_v t_x} - \underbrace{m_e c_v t_0}_{0}$$

$$t_x (m_e c_v + m_i c_v) = m_i c_v t_i + m_e c_e t_e - m_e l_t$$

$$t_x = \frac{m_i c_v t_i + m_e c_e t_e - m_e l_t}{m_e c_v + m_i c_v} = \underline{\underline{32{,}9\,°C}}$$

(29) $u = u(x,t)$

$c = \text{konst.}$

$\alpha = \text{konst.}$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - \alpha^2 c^2 \frac{\partial^4 u}{\partial x^4}$$

předpokládáme řešení ve tvaru:

$$u(x,t) = u_0 e^{j(\omega t - kx)}$$

$$\frac{\partial u}{\partial x} = u_0 e^{j(\omega t - kx)} (-jk)$$

$$\frac{\partial^2 u}{\partial x^2} = u_0 e^{j(\omega t - kx)} (-k^2)$$

$$\frac{\partial^3 u}{\partial x^3} = u_0 e^{j(\omega t - kx)} (jk^3)$$

$$\frac{\partial^4 u}{\partial x^4} = u_0 e^{j(\omega t - kx)} (k^4)$$

$$\frac{\partial u}{\partial t} = u_0 e^{j(\omega t - kx)} (j\omega)$$

$$\frac{\partial^2 u}{\partial t^2} = u_0 e^{j(\omega t - kx)} (-\omega^2)$$

$$-\omega^2 = c^2(-k^2) - a^2 c^2 k^4$$

$$\boxed{\omega^2 = c^2 k^2 + a^2 c^2 k^4}$$ DISPERZNÍ RELACE

$$\boxed{v_F = \frac{\omega}{k} = c\sqrt{1 + a^2 k^2}}$$ FÁZOVÁ RYCHLOST

$$\frac{\omega^2}{k^2} = c^2 + a^2 c^2 k^2$$

(30) $\omega = \sqrt{gk} \Rightarrow \omega = k\sqrt{\frac{g}{k}}$

$$v_f = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$$

$$v_g = \frac{d\omega}{dk} = \sqrt{g}\,\frac{d\sqrt{k}}{dk} = \sqrt{g} \cdot \frac{1}{2\sqrt{k}} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{v_f}{2}$$

(31) $v_f = \frac{a}{\lambda} = \frac{k \cdot a}{2\pi}$; $a = \text{konst.}$; $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k}$

$$v_g = \frac{d\omega}{dk} = \frac{d(v_f \cdot k)}{dk} = v_f + \frac{dv_f}{dk}k = v_f + \frac{a}{2\pi} \cdot k = v_f + v_f = 2v_f$$

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$\omega^2 = \omega_p^2 + c^2k^2$; c = konst. ; ω_p = konst.

$$v_f = \sqrt{\frac{\omega_p^2}{k^2} + c^2} = c\sqrt{\left(\frac{\omega_p}{ck}\right)^2 + 1}$$

$$v_g = \frac{d\omega}{dk} = \frac{d\left(\sqrt{\omega_p^2 + c^2k^2}\right)}{dk} = \frac{1 \cdot c^2 \cdot 2k}{2\sqrt{\omega_p^2 + c^2k^2}} = \frac{c^2k}{ck\sqrt{\left(\frac{\omega_p}{ck}\right)^2 + 1}} =$$

$$= \frac{c}{\sqrt{\left(\frac{\omega_p}{ck}\right)^2 + 1}}$$

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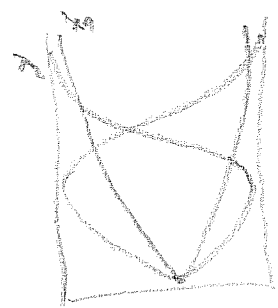
ČVRTVLNNÝ REZONÁTOR

$l = 17\ \text{cm} = 0{,}17\ \text{m}$

$c = 340\ \text{m}\cdot\text{s}^{-1}$



$$\lambda = \frac{c}{f}$$

$$l = \frac{2m+1}{4}\lambda_{m+1} \quad ; \quad m = 0, 1, 2$$

$$l = \frac{2m+1}{4} \cdot \frac{c}{f_{m+1}} \Rightarrow f_{m+1} = \frac{2m+1}{4} \cdot \frac{c}{l}$$

$$\boxed{f_1 = \frac{1}{4}\frac{c}{l} = 500\ \text{Hz}}$$

$$\boxed{f_2 = \frac{3}{4}\frac{c}{l} = 1500\ \text{Hz}}$$

$$\boxed{f_3 = \frac{5}{4}\frac{c}{l} = 2.500\ \text{Hz}}$$

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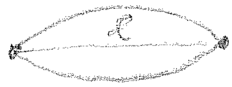
$f_e = 660\ \text{Hz}$

$f_a = 880\ \text{Hz}$



$\lambda = \frac{c}{f} \qquad l = \frac{\lambda}{2} = \frac{c}{2f}$

$l_e = \frac{c}{2 f_e}$

$l_a = \cancel{x}(1-x)\, l_e = \frac{c}{2 f_a} \Rightarrow c = (1-x)\, l_e \cdot 2 f_a$

$l_e = \frac{(1-x)\, l_e \cdot 2 f_a}{2 f_e} \Rightarrow \frac{f_e}{f_a} = 1 - x \Rightarrow x = 1 - \frac{f_e}{f_a} = \frac{1}{4}$

Strunu musíme zkrátit o 25 %.