Electromagnetic Field Theory (BAB17EMP)
Useful Mathematical Identities

September 24, 2024

Contents

1 Operations on Radius Vectors

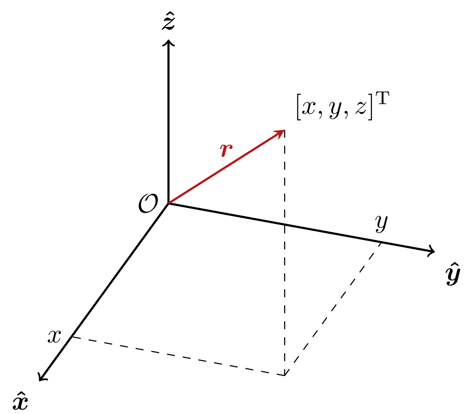

Figure 1: Radius vector $\boldsymbol{r}$.

Observation vector:

$$\boldsymbol{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}} \tag{1}$$

Vector magnitude:

$$r = |\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2} \tag{2}$$

Unit vector:

$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{r} \tag{3}$$

Gradient applied to r:

$$\nabla r = \frac{x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}}{\sqrt{x^2 + y^2 + z^2}} = \hat{\boldsymbol{r}} \tag{4}$$

Gradient applied to $1/r$:

$$\nabla \frac{1}{r} = -\frac{x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3} = -\frac{\boldsymbol{r}}{r^3} \tag{5}$$

Divergence applied to $\boldsymbol{r}$:

$$\nabla \cdot \boldsymbol{r} = 3 \tag{6}$$

Curl applied to $\boldsymbol{r}$:

$$\nabla \times \boldsymbol{r} = 0 \tag{7}$$

1.1 Identities Involving Separation Vector

Source vector:

$$\boldsymbol{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = x'\hat{\boldsymbol{x}} + y'\hat{\boldsymbol{y}} + z'\hat{\boldsymbol{z}} \tag{8}$$

Separation vector $\boldsymbol{R}$

$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' = \begin{bmatrix} x - x' \\ y - y' \\ z - z' \end{bmatrix} = (x - x')\,\hat{\boldsymbol{x}} + (y - y')\,\hat{\boldsymbol{y}} + (z - z')\,\hat{\boldsymbol{z}} \tag{9}$$

Magnitude of Separation vector $\boldsymbol{R}$

$$R = |\boldsymbol{r} - \boldsymbol{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \tag{10}$$

$$\nabla R = \frac{(x - x')\,\hat{\boldsymbol{x}} + (y - y')\,\hat{\boldsymbol{y}} + (z - z')\,\hat{\boldsymbol{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|} \tag{11}$$

$$\nabla\left(\frac{1}{R}\right) = -\frac{(x - x')\,\hat{\boldsymbol{x}} + (y - y')\,\hat{\boldsymbol{y}} + (z - z')\,\hat{\boldsymbol{z}}}{\left(\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}\right)^3} = -\frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \tag{12}$$

$$\nabla \cdot \boldsymbol{R} = 3 \tag{13}$$

$$\nabla \times \boldsymbol{R} = 0 \tag{14}$$

2 Trigonometric Identities

$$\sin^2 \xi + \cos^2 \xi = 1 \tag{15}$$

$$\sin^2 \xi = \frac{1 - \cos 2\xi}{2} \tag{16}$$

$$\cos^2 \xi = \frac{1 + \cos 2\xi}{2} \tag{17}$$

$$\mathrm{e}^{\mathrm{j}\xi} = \cos \xi + \mathrm{j} \sin \xi \tag{18}$$

$$\cos 2\xi = \cos^2 \xi - \sin^2 \xi \tag{19}$$

$$\sin 2\xi = 2 \sin \xi \cos \xi \tag{20}$$

$$\sin(\xi \pm \zeta) = \sin \xi \cos \zeta \pm \cos \xi \sin \zeta \tag{21}$$

$$\cos(\xi \pm \zeta) = \cos \xi \cos \zeta \mp \sin \xi \sin \zeta \tag{22}$$

3 Coordinate System Transformations

3.1 Point Transformations

Cylindrical to Cartesian:

$$x = \rho \cos \phi \tag{23}$$
$$y = \rho \sin \phi \tag{24}$$
$$z = z \tag{25}$$

Cartesian to cylindrical:

$$\rho = \sqrt{x^2 + y^2} \tag{26}$$
$$\phi = \arctan \frac{y}{x} \tag{27}$$
$$z = z \tag{28}$$

Spherical to Cartesian:

$$x = r \cos \phi \sin \theta \tag{29}$$
$$y = r \sin \phi \sin \theta \tag{30}$$
$$z = r \cos \theta \tag{31}$$

Cartesian to Spherical:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{32}$$
$$\theta = \arctan \frac{\sqrt{x^2 + y^2}}{z} \tag{33}$$
$$\phi = \arctan \frac{y}{x} \tag{34}$$

3.2 Vector Transformations

Cylindrical to Cartesian:

$$\hat{\boldsymbol{x}} = \hat{\boldsymbol{\rho}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi \tag{35}$$
$$\hat{\boldsymbol{y}} = \hat{\boldsymbol{\rho}} \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi \tag{36}$$
$$\hat{\boldsymbol{z}} = \hat{\boldsymbol{z}} \tag{37}$$

Spherical to Cartesian:

$$\hat{\boldsymbol{x}} = \hat{\boldsymbol{r}} \sin \theta \cos \phi + \hat{\boldsymbol{\theta}} \cos \theta \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi \tag{38}$$
$$\hat{\boldsymbol{y}} = \hat{\boldsymbol{r}} \sin \theta \sin \phi + \hat{\boldsymbol{\theta}} \cos \theta \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi \tag{39}$$
$$\hat{\boldsymbol{z}} = \hat{\boldsymbol{r}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta \tag{40}$$

4 Differential Operators

4.1 Rectangular Coordinate System

$$\boldsymbol{F} = F_x \hat{\boldsymbol{x}} + F_y \hat{\boldsymbol{y}} + F_z \hat{\boldsymbol{z}} \tag{41}$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}} \tag{42}$$

$$\nabla \cdot \boldsymbol{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{43}$$

$$\nabla \times \boldsymbol{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{z}} \tag{44}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{45}$$

$$\nabla^2 \boldsymbol{F} = \nabla^2 F_x\, \hat{\boldsymbol{x}} + \nabla^2 F_y\, \hat{\boldsymbol{y}} + \nabla^2 F_z\, \hat{\boldsymbol{z}} \tag{46}$$

4.2 Polar Coordinate System

$$\boldsymbol{F} = F_\rho \hat{\boldsymbol{\rho}} + F_\phi \hat{\boldsymbol{\phi}} \tag{47}$$

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} \tag{48}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} \tag{49}$$

4.3 Cylindrical Coordinate System

$$\boldsymbol{F} = F_\rho \hat{\boldsymbol{\rho}} + F_\phi \hat{\boldsymbol{\phi}} + F_z \hat{\boldsymbol{z}} \tag{50}$$

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}} \tag{51}$$

$$\nabla \cdot \boldsymbol{F} = \frac{1}{\rho}\frac{\partial\left(\rho F_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \tag{52}$$

$$\nabla \times \boldsymbol{F} = \left(\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \hat{\boldsymbol{\rho}} + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \right) \hat{\boldsymbol{\phi}} + \left(\frac{1}{\rho} \frac{\partial \left(\rho F_\phi \right)}{\partial \rho} - \frac{1}{\rho} \frac{\partial F_\rho}{\partial \phi} \right) \hat{\boldsymbol{z}} \tag{53}$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 z}{\partial z^2} \; f? \tag{54}$$

4.4 Spherical Coordinate System

$$\boldsymbol{F} = F_r \hat{\boldsymbol{r}} + F_\theta \hat{\boldsymbol{\theta}} + F_\phi \hat{\boldsymbol{\phi}} \tag{55}$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{\boldsymbol{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} \tag{56}$$

$$\nabla \cdot \boldsymbol{F} = \frac{1}{r^2} \frac{\partial \left(r^2 F_r \right)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial \left(F_\theta \sin \theta \right)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi} \tag{57}$$

$$\nabla \times \boldsymbol{F} = \frac{1}{r \sin \theta} \left(\frac{\partial \left(F_\phi \sin \theta \right)}{\partial \theta} - \frac{\partial F_\theta}{\partial \phi} \right) \hat{\boldsymbol{r}} + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial \left(r F_\phi \right)}{\partial r} \right) \hat{\boldsymbol{\theta}} + \frac{1}{r} \left(\frac{\partial \left(r F_\theta \right)}{\partial r} - \frac{\partial F_r}{\partial \theta} \right) \hat{\boldsymbol{\phi}} \tag{58}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} \tag{59}$$

5 Vector Identities

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \tag{60}$$

$$|\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}} \tag{61}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} u_1 & v_1 & \hat{\boldsymbol{e}}_1 \\ u_2 & v_2 & \hat{\boldsymbol{e}}_2 \\ u_3 & v_3 & \hat{\boldsymbol{e}}_3 \end{vmatrix} = \left(u_2 v_3 - u_3 v_2 \right) \hat{\boldsymbol{e}}_1 + \left(u_3 v_1 - u_1 v_3 \right) \hat{\boldsymbol{e}}_2 + \left(u_1 v_2 - u_2 v_1 \right) \hat{\boldsymbol{e}}_3 \tag{62}$$

$$\frac{\partial \boldsymbol{F}}{\partial q} = \frac{\partial F_x}{\partial q} \hat{\boldsymbol{x}} + \frac{\partial F_y}{\partial q} \hat{\boldsymbol{y}} + \frac{\partial F_z}{\partial q} \hat{\boldsymbol{z}} \tag{63}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}, \tag{64}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\,\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\,\mathbf{w} \tag{65}$$

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1} \tag{66}$$

$$(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} \tag{67}$$

$$\frac{\partial \mathbf{A}}{\partial x} = \begin{bmatrix} \dfrac{\partial a_{11}}{\partial x} & \cdots & \dfrac{\partial a_{1n}}{\partial x} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial a_{n1}}{\partial x} & \cdots & \dfrac{\partial a_{nn}}{\partial x} \end{bmatrix} \tag{68}$$

6 Differential Identities

$$\nabla \times \nabla f = 0 \tag{69}$$

$$\nabla \cdot \nabla \times \boldsymbol{F} = 0 \tag{70}$$

$$\nabla \times \nabla \times \boldsymbol{F} = \nabla\nabla \cdot \boldsymbol{F} - \nabla^2 \boldsymbol{F} \tag{71}$$

$$\nabla \cdot (f\boldsymbol{F}) = f\,(\boldsymbol{r})\,\nabla \cdot \boldsymbol{F} + \boldsymbol{F}\,(\boldsymbol{r}) \cdot \nabla f \tag{72}$$

$$\nabla \times (f\boldsymbol{F}) = f\nabla \times \boldsymbol{F} + \nabla f \times \boldsymbol{F} \tag{73}$$

$$\nabla \cdot (\boldsymbol{F} \times \boldsymbol{G}) = (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{G} - (\nabla \times \boldsymbol{G}) \cdot \boldsymbol{F} \tag{74}$$

7 Integration Identities

Substitution of cylindrical coordinates:

$$\iiint\limits_V f(x, y, z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint\limits_V f(\rho, \phi, z)\,\rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z \tag{75}$$

Substitution of spherical coordinates:

$$\iiint\limits_V f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint\limits_V f(r,\theta,\phi)\,r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{76}$$

The line integral of the scalar field:

$$\int\limits_\ell f(\boldsymbol{r})\ \mathrm{d}l = \int\limits_a^b f(\boldsymbol{r}(u))\cdot\left|\frac{\mathrm{d}\boldsymbol{r}(u)}{\mathrm{d}u}\right|\mathrm{d}u \tag{77}$$

The line integral of the vector field:

$$\int\limits_\ell \boldsymbol{F}(\boldsymbol{r})\cdot\,\mathrm{d}\boldsymbol{l} = \int\limits_\ell \boldsymbol{F}(\boldsymbol{r})\cdot\boldsymbol{\tau}\,\mathrm{d}l = \int\limits_a^b \boldsymbol{F}(\boldsymbol{r}(u))\cdot\frac{\mathrm{d}\boldsymbol{r}(u)}{\mathrm{d}u}\,\mathrm{d}u \tag{78}$$

example

$$\int\limits_{\ell:u\hat{\boldsymbol{x}}+u\hat{\boldsymbol{y}},u\in[0,1]} \left(u^2\hat{\boldsymbol{x}}+u^2\hat{\boldsymbol{y}}\right)\cdot\,\mathrm{d}\boldsymbol{l} = \int\limits_0^1\left(u^2\hat{\boldsymbol{x}}+u^2\hat{\boldsymbol{y}}\right)\cdot\frac{\mathrm{d}\left(u\hat{\boldsymbol{x}}+u\hat{\boldsymbol{y}}\right)}{\mathrm{d}u}\,\mathrm{d}u = \int\limits_0^1 2u^2\,\mathrm{d}u = \frac{2}{3} \tag{79}$$

The surface integral of the scalar field:

$$\iint\limits_S f(\boldsymbol{r})\ \mathrm{d}S = \iint\limits_S f(\boldsymbol{r}(u,s))\left|\frac{\partial\boldsymbol{r}(u,s)}{\partial u}\times\frac{\partial\boldsymbol{r}(u,s)}{\partial s}\right|\mathrm{d}u\,\mathrm{d}s \tag{80}$$

The surface integral of the vector field:

$$\iint\limits_S \boldsymbol{F}(\boldsymbol{r})\cdot\,\mathrm{d}\boldsymbol{S} = \iint\limits_S \boldsymbol{F}(\boldsymbol{r})\cdot\boldsymbol{n}\,\mathrm{d}S = \iint\limits_S \boldsymbol{F}(\boldsymbol{r}(u,s))\cdot\left(\frac{\partial\boldsymbol{r}(u,s)}{\partial u}\times\frac{\partial\boldsymbol{r}(u,s)}{\partial s}\right)\mathrm{d}u\,\mathrm{d}s \tag{81}$$

example $S: r(\rho,\phi) = \hat{\boldsymbol{x}}\rho\cos\phi + \hat{\boldsymbol{y}}\rho\cos\phi, 0\le\rho\le 1, 0\le\phi\le 2\pi$

$$\iint\limits_S \hat{\boldsymbol{z}}\cdot\,\mathrm{d}\boldsymbol{S} = \int\limits_0^1\int\limits_0^{2\pi}\hat{\boldsymbol{z}}\cdot\left[\left(\hat{\boldsymbol{x}}\cos\phi+\hat{\boldsymbol{y}}\sin\phi\right)\times\left(-\hat{\boldsymbol{x}}\rho\sin\phi+\hat{\boldsymbol{y}}\rho\cos\phi\right)\right]\,\mathrm{d}\phi\,\mathrm{d}\rho = \int\limits_0^1\int\limits_0^{2\pi}\rho\,\mathrm{d}\phi\,\mathrm{d}\rho = \pi \tag{82}$$

Curl theorem:

$$\oint\limits_{\partial S}\boldsymbol{F}(\boldsymbol{r})\cdot\,\mathrm{d}\boldsymbol{l} = \iint\limits_S \nabla\times\boldsymbol{F}\cdot\,\mathrm{d}\boldsymbol{S} \tag{83}$$

Divergence theorem:

$$\oiint\limits_{\partial V}\boldsymbol{F}(\boldsymbol{r})\cdot\,\mathrm{d}\boldsymbol{S} = \iiint\limits_V \nabla\cdot\boldsymbol{F}\,\mathrm{d}V \tag{84}$$

8 Fourier Transform

(maybe not needed for this course)

$$\mathscr{F}\left\{\boldsymbol{F}\left(\boldsymbol{r},t\right)\right\} = \hat{\boldsymbol{F}}\left(\boldsymbol{r},\omega\right) \tag{85}$$

$$\mathscr{F}\left\{\frac{\partial \boldsymbol{F}\left(\boldsymbol{r},t\right)}{\partial t}\right\} = \mathrm{j}\omega\hat{\boldsymbol{F}}\left(\boldsymbol{r},\omega\right) \tag{86}$$

$$\mathscr{F}\left\{f\left(\omega\right) * \boldsymbol{F}\left(\boldsymbol{r},t\right)\right\} = \hat{f}\left(\omega\right)\hat{\boldsymbol{F}}\left(\boldsymbol{r},\omega\right) \tag{87}$$

9 Useful Functions

9.1 Sinc Function

$$\mathrm{Sinc}\left(x\right) = \frac{\sin x}{x} = \int\limits_{-1}^{1} \mathrm{e}^{-\mathrm{j}kx}\,\mathrm{d}k \tag{88}$$

9.2 Dirac Delta Function

$$\int\limits_{-\infty}^{\infty} \delta\left(x\right)\,\mathrm{d}x = 1 \tag{89}$$

$$\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \delta\left(y\right)\delta\left(x\right)\delta\left(z\right)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 1 \tag{90}$$

$$\int\limits_{-\infty}^{\infty}\int\limits_{0}^{2\pi}\int\limits_{0}^{\infty} \frac{1}{\rho}\delta\left(\rho\right)\delta\left(\phi\right)\delta\left(z\right)\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z = 1 \tag{91}$$

$$\int\limits_{0}^{\pi}\int\limits_{0}^{2\pi}\int\limits_{0}^{\infty} \frac{1}{r^2\sin\theta}\delta\left(\rho\right)\delta\left(\phi\right)\delta\left(z\right)\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}\theta = 1 \tag{92}$$

$$\int\limits_{V'} f\left(\boldsymbol{r}\right)\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right)\,\mathrm{d}V' = \begin{cases} f\left(\boldsymbol{r}'\right), & \boldsymbol{r}' \in V' \\ 0, & \text{otherwise} \end{cases} \tag{93}$$

9.3 Heaviside Step Function

$$H(x) = \int_{-\infty}^{x} \delta(x')\,\mathrm{d}x' = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{94}$$

$$\delta(x) = \frac{\partial H(x)}{\partial x} \tag{95}$$

9.4 Rectangular Function

$$\mathrm{rect}\left(\frac{t-X}{Y}\right) = H\left(t - X + \frac{Y}{2}\right) - H\left(t - X - \frac{Y}{2}\right) \tag{96}$$

10 Series Expansions

10.1 Taylor Series

$$f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots \tag{97}$$

10.2 The Generalized Binomial Theorem

$$(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^{r-k} y^k \tag{98}$$

for $|x| > |y|$ real numbers and any complex number r.

10.3 Fourier Series

$$f\left(x\right) = \sum_{n=-\infty}^{\infty} c_n \mathrm{e}^{\mathrm{j}k_0 x} \tag{99}$$