①

	Nar	Svat.	Témat.	
Svat	12	6	12	30
Tm	28	4	18	50
	40	10	30	80

$\boxed{\text{2obj./den}}$

a) $P(\text{Svat}) = \underline{\frac{30}{80}}$ b) $P(\text{Tém} \mid \text{Svat.}) = \underline{\underline{\frac{12}{30}}}$

c) X … počet narozeninových dortů za 3 dny
$X \sim Po(3)$, tj. $P(X=k) = \frac{3^k}{k!} e^{-3}$ pro $k = 0, 1, \ldots$
$P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X=0) + P(X=1)) = \underline{\underline{1 - \left(\frac{3^0}{0!} e^{-3} + \frac{3^1}{1!} e^{-3}\right)}}$

d) X … doba do objednání svatebního dortu [dny]
$X \sim Exp\left(\frac{1}{4}\right)$, tj. $f(x) = \frac{1}{4} e^{-\frac{x}{4}}$ a $F(x) = 1 - e^{-\frac{x}{4}}$ pro $x > 0$
$P(X \leq 2) = \int_{-\infty}^{2} f(x)\,dx = \int_{0}^{2} \frac{1}{4} e^{-\frac{x}{4}}\,dx = \frac{1}{4} \cdot \left[\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}\right]_0^2 = \cancel{1} - e^{-\frac{2}{4}} - (-e^{-0})$
$= \underline{\underline{1 - e^{-\frac{1}{2}}}}$

nebo

$= F(2) = 1 - e^{-\frac{2}{4}} = \underline{\underline{1 - e^{-\frac{1}{2}}}}$

NEBO
Y … počet svat dortů za dva dny $\sim Po\left(\frac{1}{2}\right)$, tj. $P(Y=k) = \frac{\left(\frac{1}{2}\right)^k}{k!} e^{-\frac{1}{2}}$ pro $k = 0, 1, \ldots$
$P(Y \geq 1) = 1 - P(Y=0) = 1 - \frac{\left(\frac{1}{2}\right)^0}{0!} e^{-\frac{1}{2}} = \underline{\underline{1 - e^{-\frac{1}{2}}}}$

e) X … počet nar nebo témat dortů před prvním svatebním
$X \sim Geom\left(\frac{1}{8}\right)$, tj. $P(X=k) = \left(\frac{7}{8}\right)^k \frac{1}{8}$ pro $k = 0, 1, \ldots$
$P(X \leq 3) = \underline{\underline{\sum_{k=0}^{3} \left(\frac{7}{8}\right)^k \cdot \frac{1}{8} = 1 - \left(\frac{7}{8}\right)^4}}$

NEBO Y počet svat dortů mezi dalšími čtyřmi

$Y \sim \text{Binom}(4; \frac{1}{8})$, tj. $P(Y=k) = \binom{4}{k}(\frac{1}{8})^k(\frac{7}{8})^{4-k}$ pro $k = 0, 1, \dots, 4$

$$P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{4}{0}\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^4 = \underline{1 - \left(\frac{7}{8}\right)^4}$$

f) X počet tématických dortů mezi 6 dorty $\sim \text{Binom}(6, \frac{3}{8})$

$$\underline{P(X \leq 2) = \binom{6}{0}\left(\frac{3}{8}\right)^0\left(\frac{5}{8}\right)^6 + \binom{6}{1}\left(\frac{3}{8}\right)^1\left(\frac{5}{8}\right)^5 + \binom{6}{2}\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^4}$$

g) $$\left.\begin{array}{l} X_i = 1, \text{ pokud } i\text{-tý objednaný dort je tématický} \\ X_i = 0, \quad \text{—} \quad \text{''} \quad \text{—} \quad \text{není} \quad \text{-''-} \end{array}\right\} X_i \sim \text{Alt}\left(\frac{3}{8}\right) \Rightarrow \begin{array}{l} EX_i = \frac{3}{8} \\ \text{var}\, X_i = \frac{5}{8} \end{array}$$

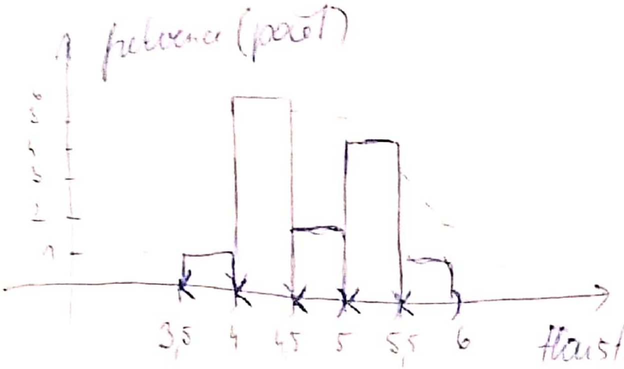
$\forall i = 1, \dots, 60$

$$P\left(\sum_{i=1}^{60} X_i \leq 20\right) = P\Bigg(\underbrace{\frac{\sum_{i=1}^{60} X_i - 60 \cdot \frac{3}{8}}{\sqrt{60 \cdot \frac{3}{8} \cdot \frac{5}{8}}}}_{Z} \leq \underbrace{\frac{20 - 60 \cdot \frac{3}{8}}{\sqrt{60 \cdot \frac{3}{8} \cdot \frac{5}{8}}}}\Bigg) = (*)$$

$$\frac{\frac{160-180}{8}}{\sqrt{4 \cdot 3 \cdot 5 \cdot \frac{3}{8} \cdot \frac{5}{8}}} = \frac{-\frac{20}{8}}{\frac{30}{8}} = -\frac{2}{3}$$

$$(*) = P\left(Z \leq -\frac{2}{3}\right) = \Phi\left(-\frac{2}{3}\right) = 1 - \Phi\left(\frac{2}{3}\right) \doteq 1 - 0{,}747 = \underline{0{,}253}$$

② a)



X tloušťka ledovce v náhodně vybraném místě [m]

$X \sim N(\mu, \sigma^2)$

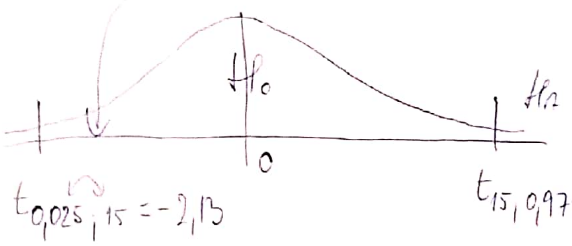
b) $\hat{\mu} = \frac{1}{16} \sum x_i = \frac{1}{16} \cdot 75 = \underline{\underline{4{,}7}}$ "$\bar{X}_{16}$"

$\hat{\sigma}^2 = S_{16}^2 = \frac{1}{15} \sum (x_i - \bar{x}) = \frac{1}{15} \cdot 5{,}7175 = \underline{\underline{0{,}36}}$

c) $H_0 : EX = 5$ m

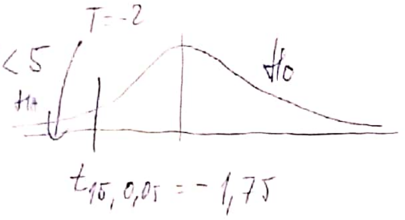
$H_A \; EX \neq 5$

$T = \frac{4{,}7 - 5}{\sqrt{0{,}36}} \cdot \sqrt{16} = \frac{-0{,}3}{0{,}6} \, 4 = \boxed{-2}$



⇒ H_0 nezamítáme ve prospěch $H_A \cdot EX \neq 5$ na hladině $\alpha = 5\%$

(nebo: $H_A \cdot EX < 5$



⇒ H_0 zamítáme ve prospěch $H_A : EX < 5$ na hladině $\alpha = 5\%$)

③

příjem \ typ	zděný	stand. dřev.	pasivní	
vysoký	60	40	100	200
st. vysoký	150	200	150	500
prům / nižší	90	160	50	300
	300	400	300	1000

a) $P(X=1) = \frac{300}{1000} = \underline{\underline{0{,}3}}$ $P(Y=1) = \frac{200}{1000} = \underline{\underline{0{,}2}}$

$P(X=2) = \frac{400}{1000} = \underline{\underline{0{,}4}}$ $P(Y=2) = \frac{500}{1000} = \underline{\underline{0{,}5}}$

$P(X=3) = \frac{300}{1000} = \underline{\underline{0{,}3}}$ $P(Y=3) = \frac{300}{1000} = \underline{\underline{0{,}3}}$

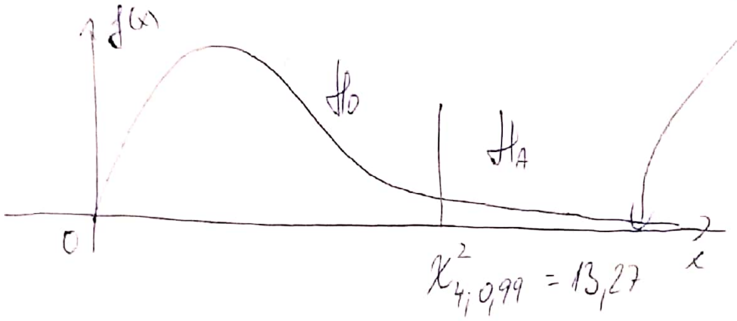
b) H_0: příjem a typ domu jsou nezávislé $(\alpha = 1\%)$

H_A: — " — nejsou — " —

$$\chi^2 = \underbrace{\frac{\left(60-\frac{200\cdot300}{1000}\right)^2}{\frac{20\ 30}{1000}}}_{0} + \frac{\left(40-\frac{200\cdot400}{1000}\right)^2}{\frac{30\ 40}{1000}} + \frac{\left(100-\frac{200\cdot300}{1000}\right)^2}{\frac{20\ 30}{1000}} + \underbrace{\frac{\left(150-\frac{500\cdot300}{1000}\right)^2}{\frac{50\ 30}{1000}}}_{0} + \underbrace{\frac{\left(200-\frac{500\cdot400}{1000}\right)^2}{\frac{50\cdot40}{1000}}}_{0} +$$

$$+ \underbrace{\frac{\left(150-\frac{500\cdot300}{1000}\right)^2}{\frac{50\cdot30}{1000}}}_{0} + \underbrace{\frac{\left(90-\frac{300\cdot300}{1000}\right)^2}{\frac{30\ 30}{1000}}}_{0} + \frac{\left(160-\frac{300\cdot400}{1000}\right)^2}{\frac{30\ 40}{1000}} + \frac{\left(50-\frac{300\cdot200}{1000}\right)^2}{\frac{30\cdot30}{1000}} =$$

$$= \frac{40^2}{80} + \frac{40^2}{60} + \frac{40^2}{120} + \frac{40^2}{90} = 20 + 26,\bar{6} + 13,\bar{3} + 17,\bar{7} = \boxed{77,\bar{7}}$$



$\Rightarrow$ H_0 zamítám ve prospěch H_A na hladině 1%

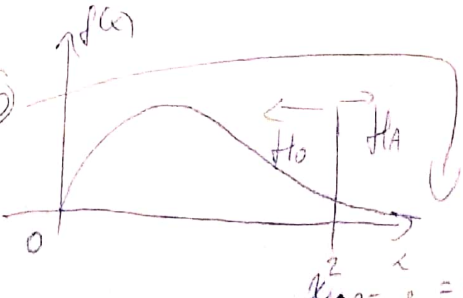
c) H_0: $p_Z = p_D = p_P = \frac{1}{3}$, kde p_i = pst, že zákazník projeví zájem o dům typu $i \in \{Z, D, P\}$

H_A: aspoň jedno p_i je jiné

$\alpha = 5\%$

$$\chi^2 = \frac{\left(300-\frac{1000}{3}\right)^2}{\frac{1000}{3}} + \frac{\left(400-\frac{1000}{3}\right)^2}{\frac{1000}{3}} + \frac{\left(300-\frac{1000}{3}\right)^2}{\frac{1000}{3}} = \frac{\left(\frac{-100}{3}\right)^2 + \left(\frac{200}{3}\right)^2 + \left(\frac{-100}{3}\right)^2}{\frac{1000}{3}}$$

$$= \frac{\frac{60000}{9}}{\frac{1000}{3}} = \boxed{20}$$



$\chi^2_{0,95;2} = 5,99$

$\Rightarrow$ H_0 zamítám ve prospěch H_A na hladině $\alpha = 5\%$

Ústní část:

- X a Y (diskrétní) jsou nezávislé, jestliže $P(X=i, Y=j) = P(X=i) \cdot P(Y=j)$ (nebo třeba i $P(X \leq i, Y \leq j) = P(X \leq i) \cdot P(Y \leq j)$ $\forall i,j \in \mathbb{R}$) $\forall i,j \in \mathbb{R}$

- (i) Víme, že: X,Y nezávislé $\Rightarrow \operatorname{cov}(X,Y) = 0 \Rightarrow$ parametry mohou být.
  ~~$a = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, $b=$~~ & součet marg. pstí $= 1 \Rightarrow l = 1$
  $j = k = \frac{1}{4}$, $a = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, $b = c = d = g = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$, $e = f = h = i = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

  (ii) Víme, že $X = a \cdot Y$ pro $a > 0 \Rightarrow \operatorname{corr}(X,Y) = 1 \Rightarrow$ parametry mohou být: $k = \frac{1}{4} = j$, $l = 1$, $a = \frac{1}{2}$, $e = \frac{1}{4}$, $i = \frac{1}{4}$, $b = c = d = f = g = h = 0$