Electromagnetic Field Theory (BAB17EMP)
Formulas

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Contents

1 Physical Constants

Constant	**Symbol**	**Value**	**Units**
Speed of light in vacuum	c	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Planck's constant	h	6.626×10^{-34}	$\mathrm{J\,s}$
Gravitational constant	G	6.674×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$
electronvolt	eV	1.602×10^{-19}	J
Elementary charge	e	1.602×10^{-19}	C
Electron mass	m_e	9.109×10^{-31}	kg
Proton mass	m_p	1.673×10^{-27}	kg
Permittivity of free space	ϵ_0	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	1.257×10^{-6}	$\mathrm{H\,m^{-1}}$
Avogadro's number	N_{A}	6.022×10^{23}	$\mathrm{mol^{-1}}$
Boltzmann constant	k_{B}	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$

Table 1: Physical constants.

Quantity	**Symbol**	**Unit**	**SI Units**
Charge	q	Coulomb (C)	$\mathrm{A\,s}$
Potential	φ, V	Volt (V)	$\mathrm{kg\,m^2\,A^{-1}\,s^{-3}}$
Electric field intensity	$\boldsymbol{E}$	Volt per meter (V/m)	$\mathrm{kg\,m\,A^{-1}\,s^{-3}}$
Electric displacement field	$\boldsymbol{D}$	Coulomb per meter (C/m)	$\mathrm{m^{-1}\,A\,s}$
Capacitance	C	Farad (F)	$\mathrm{A^2\,s^4\,kg^{-1}\,m^{-2}}$
Electric flux	Φ_E	Volt meter (V/m)	$\mathrm{kg\,m\,A^{-1}\,s^{-3}}$
Polarization	$\boldsymbol{P}$	Coulomb per square meter (C/m^2)	$\mathrm{m^{-2}\,A\,s}$
Current	I	Ampere (A)	A
Resistance	R	Ohm (Ω)	$\mathrm{kg\,m^2\,A^{-2}\,s^{-3}}$
Magnetic field intensity	$\boldsymbol{H}$	Ampere per meter (A/m)	$\mathrm{A\,m^{-1}}$
Magnetic field	$\boldsymbol{B}$	Tesla (T)	$\mathrm{kg\,A^{-1}\,s^{-2}}$
Inductance	L	Henry (H)	$\mathrm{kg\,m^2\,A^{-2}\,s^{-2}}$
Magnetic flux	Φ_B	Weber (Wb)	$\mathrm{kg\,m^2\,A^{-1}\,s^{-2}}$
Magnetization	$\boldsymbol{M}$	Ampere per meter (A/m)	$\mathrm{A\,m^{-1}}$
Electromagnetic force	$\boldsymbol{F}$	Netwon (N)	$\mathrm{kg\,m\,s^{-2}}$
Energy	U	Joule (J)	$\mathrm{kg\,m^2\,s^{-2}}$
Energy density	u	Joule per cubic meter (J/m^3)	$\mathrm{kg\,m^{-1}\,s^{-2}}$
Power	P	Watt (W)	$\mathrm{kg\,m^2\,s^{-3}}$

Table 2: Several fundamental electromagnetic quantities and their units.

2 Electromagnetic Quantities and Units

3 Fundamental Relations

3.1 General Electromagnetism

Linear, surface, volumetric charge density

$$Q = \int_{\ell'} \tau(\boldsymbol{r}')\,\mathrm{d}\ell' \tag{1}$$

$$Q = \iint_{S'} \sigma(\boldsymbol{r}')\,\mathrm{d}S' \tag{2}$$

$$Q = \iiint_{V'} \rho(\boldsymbol{r}')\,\mathrm{d}V' \tag{3}$$

Electric current

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{4}$$

$$I = \iint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} \tag{5}$$

Maxwell's equations

differential form

$$\nabla \cdot \boldsymbol{D} = \rho_{\text{free}} \tag{6}$$
$$\nabla \cdot \boldsymbol{B} = 0 \tag{7}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{8}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{9}$$

integral form

$$\oiint_{\partial V} \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = Q_{\text{free}} \tag{10}$$
$$\oiint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0 \tag{11}$$
$$\oint_{\partial S} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I + \iint_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{S} \tag{12}$$
$$\oint_{\partial S} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S} \tag{13}$$

Material relations

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{14}$$
$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right) \tag{15}$$

linear media

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \epsilon_0 \chi_e \boldsymbol{E} = \varepsilon \boldsymbol{E} \tag{16}$$
$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \chi_m \boldsymbol{M}\right) = \mu \boldsymbol{H} \tag{17}$$

free space

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{18}$$
$$\nabla \cdot \boldsymbol{B} = 0 \tag{19}$$
$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c_0^2} \frac{\partial \boldsymbol{E}}{\partial t} \tag{20}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{21}$$

3.2 Electrostatics

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{22}$$
$$\nabla \times \boldsymbol{E} = 0 \tag{23}$$

$$\boldsymbol{E} = -\nabla \varphi \tag{24}$$
$$\varphi(\boldsymbol{r}) = \int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} + K \tag{25}$$

$$\nabla \cdot \nabla \varphi = \nabla^2 \varphi = -\frac{\rho}{\epsilon_0} \tag{26}$$
$$\nabla \times \nabla \varphi = 0 \tag{27}$$

$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 (\boldsymbol{r}_1 - \boldsymbol{r}_2)}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3} \tag{28}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \sum_n \frac{Q_n (\boldsymbol{r} - \boldsymbol{r}'_n)}{|\boldsymbol{r} - \boldsymbol{r}'_n|^3} \tag{29}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{V'} \frac{\rho(\boldsymbol{r}')(\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \mathrm{d}V' \tag{30}$$

3.3 Magnetostatics

$$\nabla \cdot \boldsymbol{B} = 0 \tag{31}$$
$$\nabla \times \boldsymbol{B} = \mu \boldsymbol{J} \tag{32}$$

Biot-Savart law

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \mathrm{d}^3(\boldsymbol{r}) \tag{33}$$

magnetic vector potential

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{34}$$
$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}^3(\boldsymbol{r}) \tag{35}$$

3.4 Electrodynamics

Lorentz Force Law

$$\boldsymbol{F} = Q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right) \tag{36}$$

Continuity equation

$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0 \tag{37}$$

Poynting vector

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H} \tag{38}$$

Poynting theorem

$$\frac{\partial u}{\partial t} + \nabla \cdot \boldsymbol{S} = -\boldsymbol{J} \cdot \boldsymbol{E} \tag{39}$$

Wave equation

$$\nabla^2 \boldsymbol{E} - \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \tag{40}$$

$$\nabla^2 \boldsymbol{B} - \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0 \tag{41}$$

Boundary conditions

$$\hat{\boldsymbol{n}} \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0 \tag{42}$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_2 - \boldsymbol{D}_1) = \sigma \tag{43}$$

$$\hat{\boldsymbol{n}} \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{K} \tag{44}$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{B}_2 - \boldsymbol{B}_1) = 0 \tag{45}$$