Homogenní soustavy DR

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$$\begin{cases} y_1' = -2y_1 + 4y_2 \\ y_2' = y_1 + 2y_2 \end{cases} \quad \begin{cases} y_1(0) = 4 \\ y_2(0) = -1 \end{cases}$$

$$A = \begin{pmatrix} -2 & 4 \\ 1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -2-\lambda & 4 \\ 1 & 1-\lambda \end{pmatrix}$$

$$\begin{vmatrix} -2-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = (-2-\lambda)(1-\lambda) - (4) =$$

$$-6 + \lambda + \lambda^2 = \lambda^2 + \lambda - 6$$

$$D = 1 + 24$$

$$\sqrt{D} = 5$$

$$\lambda_{1,2} = \frac{-1 \pm 5}{2} \begin{cases} 2 \\ -3 \end{cases}$$

$\lambda = 2 \Rightarrow$

$$\begin{pmatrix} -4 & 4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

volím $v_1 = 1$

$-4v_1 + 4v_2 = 0 \Rightarrow -4 + 4v_2 = 0 \Rightarrow 4v_2 = 4 \Rightarrow v_2 = 1$

$v_1 - v_2 = 0 \Rightarrow v_1 = v_2 \Rightarrow v_1 = 1$

$\Rightarrow \vec{y}_a(x) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2x}$

$\lambda = -3 \Rightarrow$

$$\begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

volím $v_2 = 1$

$v_1 + 4v_2 = 0 \Rightarrow v_1 = -4v_2 \Rightarrow v_1 = -4$

$v_1 + 4v_2 = 0 \Rightarrow v_2 = 1$

$\Rightarrow \vec{y}_b(x) = \begin{pmatrix} -4 \\ 1 \end{pmatrix} e^{-3x}$

obecné řeš.

$$\vec{y} = a \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2x} + b \cdot \begin{pmatrix} -4 \\ 1 \end{pmatrix} e^{-3x} = \begin{pmatrix} a e^{2x} & -4b e^{-3x} \\ a e^{2x} & b e^{-3x} \end{pmatrix}$$

$y_1(x) = a e^{2x} - 4b e^{-3x}; x \in \mathbb{R}$

$y_2(x) = a e^{2x} + b e^{-3x}; x \in \mathbb{R}$

stac. řeš. $\begin{cases} y_1(x) = y_{01} \\ y_2(x) = y_{02} \end{cases} \Rightarrow y_1' = y_2' = 0 \Rightarrow$ podmínka stac. řeš.

$$\text{chci} \begin{pmatrix} -2y_1 + 4y_2 = 0 \\ 1y_1 + 1y_2 = 0 \end{pmatrix} \Rightarrow \text{triviální řeš.} \quad \begin{matrix} y_1 = 0 \\ y_2 = 0 \end{matrix}$$

→ stac. řeš.

$y_1(x) = 0; y_2(x) = 0; x \in \mathbb{R}$

partikulární řeš.

$\lambda = 2; -3$

$0 \to \lambda = 0 \Rightarrow A e^{0x} \Rightarrow A$

$0 \to \lambda = 0 \Rightarrow B$ → hodnoce

$y_{1p}(x) = A$

$y_{2p}(x) = Bx + C$

derivaci

$$(y_{1p}(x))' = -2y_{1p}(x) + 4y_{2p}(x)$$

$$(y_2(x))' = y_{1h}(x) + y_{2h}(x)$$

$$0 = -2A + 4Bx + 4C$$
$$B = A + Bx + C$$

$$0 = (-2A + 4C) + x(4B)$$
$$0 = (A - B + C) + x(B)$$

$$-2A + 4C = 0 \qquad 4B = 0$$
$$A - B + C = 0 \qquad B = 0$$

$$A = 2C \rightarrow A = 0$$
$$3C = B \rightarrow C = 0$$

$y_{1h}(x) = 0$
$y_{2h}(x) = 0$

obecné řeš.

$$y_1(x) = ae^{2x} - 4be^{-3x} ; x \in \mathbb{R}$$
$$y_2(x) = ae^{2x} + be^{-3x} ; x \in \mathbb{R}$$

poč. podmínky

$y_1(0) = 4 \Rightarrow 4 = ae^0 - 4be^0 \Rightarrow 4 = a - 4b \Rightarrow 4 = -b - 1 - 4b \Rightarrow 5b = -5 \Rightarrow b = -1$

$y_2(0) = -1 \Rightarrow -1 = ae^0 + be^0 \Rightarrow -1 = a + b \Rightarrow a = -b - 1 \Rightarrow a = 0$

$$y_1(x) = 4be^{-3x} ; x \in \mathbb{R}$$
$$y_2(x) = -e^{-3x} ; x \in \mathbb{R}$$