NMMB331 - HW3

Jan Oupický

1

Let $x \in \mathbb{F}_2^n$. Then

$$\frac{1}{2^n}\sum_{u\in\mathbb{F}_2^n} F_g(u)(-1)^{\langle u,x\rangle} = \frac{1}{2^n}\sum_{u\in\mathbb{F}_2^n}\sum_{y\in\mathbb{F}_2^n} g(y)(-1)^{\langle u,y\rangle}(-1)^{\langle u,x\rangle} =$$

$$\frac{1}{2^n}\sum_{y\in\mathbb{F}_2^n} g(y)\sum_{u\in\mathbb{F}_2^n}(-1)^{\langle u,x+y\rangle}$$

There is exactly one y s.t. $x + y = 0 \iff x = y$. Using annihilator lemma we get:

$$\frac{1}{2^n}g(x)2^n + 0 = g(x)$$

$\square$

2

First assume $u = 0$.

$$F_f(0) = \sum_{x\in\mathbb{F}_2^n} f(x)(-1)^{\langle 0,x\rangle} = \sum_{x\in\mathbb{F}_2^n} f(x) = \sum_{x\in\mathbb{F}_2^n, f(x)=1} 1$$

$$2^{n-1} - \frac{1}{2}W_f(0) = \frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n}(1-(-1)^{f(x)})\right) = \frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n, f(x)=1}(1-(-1))\right) =$$

$$\frac{1}{2}\sum_{x\in\mathbb{F}_2^n, f(x)=1} 2 = F_f(0)$$

Now $u \neq 0$:

$$F_f(u) = \sum_{x\in\mathbb{F}_2^n} f(x)(-1)^{\langle u,x\rangle} = \sum_{x\in\mathbb{F}_2^n, f(x)=0} 0(-1)^{\langle u,x\rangle} + \sum_{x\in\mathbb{F}_2^n, f(x)=1} 1(-1)^{\langle u,x\rangle} =$$

$$\sum_{x\in\mathbb{F}_2^n, f(x)=1}(-1)^{\langle u,x\rangle}$$

$$-\frac{1}{2}W_f(u) = \frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n}(-1)^{\langle u,x\rangle} - W_f(u)\right) = \frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n}(-1)^{\langle u,x\rangle} - (-1)^{f(x)+\langle u,x\rangle}\right) =$$

$$\frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n}(-1)^{\langle u,x\rangle}(1-(-1)^{f(x)}\right) = \frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n, f(x)=0}(-1)^{\langle u,x\rangle}(1-1)\right) +$$

$$\frac{1}{2}\left(\sum_{x\in\mathbb{F}_2^n, f(x)=1}(-1)^{\langle u,x\rangle}(1-(-1))\right) = 0 + F_f(u) = F_f(u)$$

$\square$

3

$$F_{f\oplus g}(u) = \sum_{x\in\mathbb{F}_2^n} (f\oplus g)(x)(-1)^{\langle u,x\rangle} = \sum_{x\in\mathbb{F}_2^n} (f(x)\oplus g(x))(-1)^{\langle u,x\rangle} =$$
$$\sum_{x\in\mathbb{F}_2^n} (f(x)+g(x)-2f(x)g(x))(-1)^{\langle u,x\rangle} =$$
$$\sum_{x\in\mathbb{F}_2^n} f(x)(-1)^{\langle u,x\rangle} + \sum_{x\in\mathbb{F}_2^n} g(x)(-1)^{\langle u,x\rangle} + \sum_{x\in\mathbb{F}_2^n} 2f(x)g(x)(-1)^{\langle u,x\rangle} =$$
$$F_f(u) + F_g(u) + \sum_{x\in\mathbb{F}_2^n} 2(fg)(x)(-1)^{\langle u,x\rangle} = F_f(u) + F_g(u) + F_{2fg}(u)$$

$\square$

4