# NMMB331 - HW4

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## 1

1. $\sim_{EA}$ is reflexive since we can take $A = id = B$ (identities) and $C = 0$ (null map).

2. If $A, B$ are affine permutations and $C$ is an affine map s.t. $G(x) = A \circ F \circ B(x) + C(x) \iff F \sim_{EA} G$ then we can write $A^{-1} \circ G \circ B^{-1}(x) + A^{-1} \circ C \circ B^{-1}(x) = F(x) \iff G \sim_{EA} F$. Since $A, B$ are affine permutations then their inverses are also affine permutations and also $A^{-1} \circ C \circ B^{-1}$ is affine since $C$ is affine and $A^{-1}, B^{-1}$ are affine permutations. This proves symmetry.

3. Assume $F \sim_{EA} G$, $G \sim_{EA} H$ and we want to show $F \sim_{EA} H$. By definition we have $G(x) = A_1 \circ F \circ B_1(x) + C_1(x)$ and $H(x) = A_2 \circ G \circ B_2(x) + C_2(x)$. $A_2(x) = M(x) + y$ for some $M$ linear permutation and $y$ vector.

$$\begin{gathered}
H(x) = A_2 \circ G \circ B_2(x) + C_2(x) = A_2(A_1 \circ F \circ B_1(B_2(x)) + C_1(B_2(x))) + C_2(x) = \\
M(A_1 \circ F \circ B_1(B_2(x)) + C_1(B_2(x))) + y + C_2(x) = \\
M(A_1 \circ F \circ B_1(B_2(x))) + M(C_1(B_2(x))) + y + C_2(x) = \\
(M \circ A_1) \circ F \circ (B_1 \circ B_2)(x) + (M \circ C_1 \circ B_2(x) + y + C_2(x))
\end{gathered}$$

$M \circ A_1$ is an affine permutation since $M$ is linear permutation and $A_1$ is affine permutation. $B_1 \circ B_2$ is affine permutation since both are affine permutations. $M \circ C_1 \circ B_2$ is affine map since $M, B_2$ are affine permutations and $C_1$ affine map, $C_2(x) + y$ is an affine map. Sum of affine maps is affine map. This proves transitivity.