NMAG436 - HW3

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Let $L \coloneqq \mathbb{F}_p(V_{w_a})$. w_a is a short WEP by definition for every p and a.

First let $p = 2$. $w_0 = y^2 + x^3, w_1 = y^2 + x^3 + 1$ with partial derivatives $\frac{\partial w_0}{\partial x}(x, y) = x^2$, $\frac{\partial w_0}{\partial y}(x, y) = 0$, $\frac{\partial w_1}{\partial x}(x, y) = x^2$, $\frac{\partial w_1}{\partial y}(x, y) = 0$. We see that $V_{w_0}(\mathbb{F}_2) = \{(0, 0), (1, 1)\}, V_{w_1}(\mathbb{F}_2) = \{(1, 0), (0, 1)\}$. We see that w_0 is not smooth at $(0, 0)$ and w_1 at $(0, 1)$ therefore they are not smooth.

Theorem 8.4 tells us that L is an EFF iff w is smooth therefore L is not an EFF. Proposition 8.3 5) tell us that the only other option is that the genus of L is 0.

Now let $p > 2$. Let $f \coloneqq x^3 + a$. We want to know for which a is f separable. By definition we want to know when $GCD_{\mathbb{F}_p}(f, f') = 1$. Since $f' = 3x^2$ for every a, we can see that f, f' are not coprime iff $a = 0$. From that we see:

If $a = 0$ then w_a is not smooth by 3.12. 3) which implies that the genus of L is 0 (same reasoning as in the case $p = 2$). If $a \neq 0$ then w_a is smooth and by 8.4 the genus of L is 1.